Transpired Turbulent Boundary Layers Subject to Forced Convection and External Pressure Gradients

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The problem of forced convection transpired turbulent boundary layers with external pressure gradient has been studied by using different scalings proposed by various researchers. Three major results were obtained: First, for adverse pressure gradient boundary layers with suction, the mean deficit profiles collapse with the free stream velocity, U_{∞} , but into different curves depending on the strength of the blowing parameter and the upstream conditions. Second, the dependencies on the blowing parameter, the Reynolds number, and the strength of pressure gradient are removed from the outer flow when the mean deficit profiles are normalized by the Zagarola/Smits [Zagarola, M. V., and Smits, A. J., 1998, "Mean-Flow Scaling of Turbulent Pipe Flow," J. Fluid Mech., 373, 33–79] scaling, $U_{\infty}\delta_*/\delta$. Third, the temperature profiles collapse into a single curve using the new inner and outer scalings proposed by Wang and Castillo [Wang, X., and Castillo, L., 2003, "Asymptotic Solutions in Forced Convection Turbulent Boundary Layers," J. Turbulence, 4(006)], which produce the true asymptotic profiles even at finite Péclet number. [DOI: 10.1115/1.1842790]

1 Introduction

In recent years, the field of flow control in turbulent boundary layers has become urgently important due to the wide variety of fundamental and industrial applications, particularly in controlling heat transfer, separation, and drag reduction. In this study, various scalings for both the velocity and temperature profiles will be investigated for the boundary layer subjected to the effects of transpiration, heat transfer and pressure gradient. The similarity analysis of the RANS equations for zero pressure gradient (ZPG) and adverse pressure gradient (APG) developed by George and Castillo [1] and Castillo and George [2], respectively, has been

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applied to obtain the velocity scales in turbulent boundary layers. The same theory has been developed for thermal turbulent boundary layers by Wang and Castillo [3]. In this investigation, the scales obtained from the similarity analysis of turbulent boundary layers will be applied to analyze the data obtained by Orlando, Kays and Moffat [4], which is subject to transpiration and heat transfer. These similarity scalings will be compared with the classical scalings and scalings obtained by the other investigators, and the advantage and disadvantage of each scaling are clearly observed.

2 Various Velocity and Temperature Scalings

Tables 1 and 2 summarize the available scalings for both the velocity and the temperature profiles, respectively. Table 1 shows the scalings in inner variables from various investigators for both the velocity and temperature profiles. The inner velocity scaling, as shown in row two, is the same for all investigators, which is described by the friction velocity, u_* . In the classical view, this scaling was supposed to collapse both inner and outer parts of the boundary layer as described by Millikan [5], but many measurements predicted the opposite. The inner temperature scaling obtained by the Reynolds analogy shown in the fourth row is different from the scaling by George, Wosnik, and Castillo [6] (known here as GWC) and Wang/Castillo [3] (known here as WC) shown in the last row. The similarity length scale in inner variables is different for all cases in the temperature field. Similarly, Table 2 displayed the scalings in outer variables. The outer velocity scaling obtained from the classical approach (row 2) is different from the scaling obtained by Castillo/George [2] (row 3) (known here as CG), and the Zagarola/Smits [7] (row 4) (known here as ZS). Notice that the outer length scale is the same for all investigators. As for the temperature field, row six shows the scaling used in the Reynolds analogy [8], which again assumed that existence of a single scaling for inner and outer flow. The scaling by GWC using similarity analysis is shown in the seventh row and finally, the results obtained by Wang and Castillo [3] labeled as WC are shown in the eighth row.

3 Velocity and Temperature Profiles

Figure 1 shows the velocity profile normalized by the scalings listed in Tables 1 and 2. Figure 1(*a*) shows the velocity profiles in a semi-log scale normalized with the friction velocity, u_* , and the length scale, ν/u_* . Using the inner scaling, u_* , the profiles collapse in the inner region, but fail to collapse in the outer region for this particular low Reynolds number data ($834 \le \text{Re}_{\theta} \le 3144$). Although the profiles have a similar range of Reynolds number, the profiles move towards the wall as the magnitude of the blowing parameter is increased. Also, notice the overlap region

1	Investigator	Scaling: U _{si}	Length Scale: y ⁺
2	Classical and other investigators $\frac{U}{U_{si}}$	$U_{si} = u_*$	$y^+ = \frac{yu_*}{\nu}$
3	Investigator	Scaling: T _{si}	Length Scale: y_T^+
4	Reynolds Analogy $\frac{T-T_{\infty}}{T_{si}}$	$T_{si} = T_{\tau} = \frac{q_w}{\rho C_p u_*}$	$y_T^+ = y^+ = \frac{yu_*}{\nu}$
5	George/Wosnik/Castillo $\frac{T-T_{\infty}}{T_{si}}$	$T_{si} = T_w - T_\infty$	$y_T^+ = \frac{yq_w/\rho C_p}{\alpha(T_w - T_\infty)}$
6	Wang/Castillo $\frac{T_W - T}{T_{si}}$	$T_{si} = Pr(T_w - T_\infty) \sqrt{St}$	$y_T^+ = \frac{yU_\infty}{\nu} \sqrt{St}$

Table 1 The inner velocity and temperature scalings

Table 2 The outer velocity and temperature scalings

1	Investigator	Scaling: U _{so}	Length Scale: \overline{y}
2	Classical Scaling $\frac{U_{\infty} - U}{U_{so}}$	$U_{so} = u_*$	$\overline{y} = \frac{y}{\delta_{99}}$
3	Castillo/George $\frac{U_{\infty} - U}{U_{so}}$	$U_{so} = U_{\infty}$	$\overline{y} = \frac{y}{\delta_{99}}$
4	Zaragola/Smits $\frac{U_{\infty} - U}{U_{so}}$	$U_{so} = U_{\infty}(\delta_* / \delta)$	$\overline{y} = \frac{y}{\delta_{99}}$
5	Investigator	Scaling: T _{so}	Length Scale: \bar{y}_T
6	Reynolds Analogy $\frac{T-T_{\infty}}{T_{so}}$	$T_{so} = T_{\tau} = \frac{q_w}{\rho C_p u_*}$	$\overline{y}_T = \frac{y}{\delta_T}$
7	George/Wosnik/Castillo $\frac{T-T_{\infty}}{T_{so}}$	$T_{so} = (T_w - T_\infty) \frac{St}{C_f/2}$	$\overline{y}_T = \frac{y}{\delta_T}$
8	Wang/Castillo $\frac{T-T_{\infty}}{T_{so}}$	$T_{so} = (T_w - T_\infty) \frac{\delta_T^*}{\delta_T}$	$\bar{y}_T = \frac{y}{\delta_T}$

increases with increasing value of V_0^+ while the wake region decreases. The blowing paramter V_0^+ was determined from the similarity analysis and was given as $V_0^+ = V_0 / U_{\infty} d \, \delta / dx$ according to Cal [9].

Figures 1(*b*), 1(*c*), and 1(*d*) show the normalized velocity deficit profiles. Figure 1(*b*) shows the profiles normalized using the classical scaling, u_* , and δ_{99} . Notice that in the classical view, the profiles should collapse as a single curve. Clearly, this figure



Fig. 1 Velocity profiles in APG normalized by different outer scalings and δ_{99} subject to suction and forced convection using the data of Orlando et al. [4]



Fig. 2 Comparison of different inner temperature scalings in APG subject to suction and forced convection using the data of Orlando et al. [4]

shows the opposite effect. In Fig. 1(c), the profiles are now normalized by the CG scaling, U_∞ . It can be seen that the profiles collapse, but to different curves showing the effect of the blowing parameter. Notice that as the magnitude of V_0^+ increases, the profiles move closer to the wall; thus increasing the skin friction coefficient. This scaling is obtained by means of the similarity analysis proposed by Castillo and George [2], which is contrary to the classical view that the scaling is chosen before any type of analysis. Finally when the ZS scaling is used as shown in Fig. 1(d), the profiles collapse to one single curve. More importantly, this scaling successfully removes all effects including the pressure gradient, the upstream conditions and the blowing parameter from the outer flow. Also, notice that the slope in the overlap region is close to 1/2, which is consistent with the 1/2 power theory of Perry [10]. In addition, the blowing parameter influences the outer flow and the inner flow, but mostly in the inner region.

Figure 2 displays the temperature profiles scaled in inner variables in a semi-log scale. Figure 2(*a*) shows the APG temperature profiles with suction using the Reynolds analogy scaling, T_{τ} . Notice that the classical scaling fails to collapse the profiles in the overlap region and near the wall. However, the GWC profiles shown in Fig. 2(*b*) show a better collapse than the classical scaling. However, the profiles with strong suction (i.e., $V_0^+ = -0.2757 \rightarrow -0.2045$) are far away from other profiles. The more negative the parameter (increase in suction) the closer the profiles move toward to the wall. The WC inner temperature scaling collapses the profiles very well as shown in Fig. 2(*c*). Interestingly,

the effects of the blowing parameter are nearly removed from the inner flow, but the blowing parameter has an effect on the overlap region. However, Fig. 2(d) shows various APG and ZPG flows without suction normalized with the new scaling proposed by Wang and Castillo [3]. Notice the remarkable collapse of these profiles even with APG over the entire boundary layer. Therefore, Fig. 2(c) shows that suction effects do have a significant influence in the inner flow. Therefore, this inner scaling related to the heat transfer coefficient only is not enough to remove the effect of suction on the inner flow, and a scaling which includes the mass transfer information may do a better job.

Figure 3(a) shows the outer temperature profiles normalized using the Reynolds analogy. Notice that the profiles do not collapse as expected from the classical theory; consequently they show a Péclet number and blowing parameter dependence. The same experimental data scaled using GWC scaling is shown in Fig. 3(b). Clearly, the outer temperature profiles with suction collapse into a single curve, but different from the profile without suction, which means that GWC scaling cannot remove the effects of suction. In Fig. 3(c), the profiles are now normalized using the new proposed outer temperature scaling found by Wang and Castillo [3]. Evidently, the profiles collapse into one curve regardless different Péclet number and effects of suction, specially for the outer region as shown in the semi-log scale figure. Thus, the asymptotic profiles are found at finite Péclet number. Notice that these profiles collapse better than the profiles normalized using the GWC scaling even near the wall region.



(c) Wang/Castillo Scaling: Outer

Fig. 3 Comparison of different outer temperature scalings in APG subject to suction and forced convection using the data of Orlando et al. [4]

Summary and Conclusions 4

Both velocity scalings and temperature scalings have been tested for forced convected turbulent boundary layer subject to suction and pressure gradients. The mean deficit profiles collapse with the free-stream velocity, but to different curves depending on the blowing parameter. This is true as long as the upstream conditions are kept fixed, such as the wind-tunnel speed. The dependencies on the upstream conditions, strength of pressure gradient, the Reynolds number and the blowing parameter are removed from the mean deficit profiles when normalized by the Zagarola/ Smits scaling, $U_{\infty}\delta_*/\delta$. It was also shown that the velocity profiles normalized in inner variables are affected by the blowing parameter in the overlap region and in the wake region. Increasing the blowing parameter, increases the overlap region, and decreases the wake region.

The effects of the blowing parameter are completely removed from the outer temperature profiles when normalized with the outer scaling $T_{so} = (T_w - T_\infty) \delta_T^* / \delta_T$; thus the profiles collapse into a single curve. However, the profiles in inner variables using the scaling, $T_{si} = Pr(T_w - T_\infty)\sqrt{St}$, exhibit a dependence on the blowing parameter V_0^+ .

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Nomenclature

- δ = boundary layer thickness, δ_{99}
- St = Stanton number, St = $q_w / \rho C_p U_\infty (T_W T_\infty)$
- T_{∞} = free stream temperature
- δ_T^* = thermal displacement thickness
- $\delta_* = \text{displacement thickness}, \ \delta_* = \int_0^\infty (1 U/U_\infty) dy$
- δ^+ = local Reynolds number dependence, $\delta^+ = \delta u_* / v$
- * = dependence on upstream conditions
- V_0^+ = blowing parameter, $V_0^+ = V_0 / U_\infty d \delta / dx$
- = thermal boundary layer thickness
- δ_T
- $T_{\tau} = \text{friction temperature, } T_{\tau} = q_w / \rho C_p u_*$
- T_w = wall temperature
- Pr = Prandtl number
- U_{∞} = free-stream velocity

$$U_{\infty} - U =$$
 velocity deficit

$$U_{\infty}\delta_*/\delta =$$
 Zagarola/Smits scaling

 $u_* = \text{friction velocity}, u_*^2 = \tau_w / \rho$

References

- [1] George, W. K., and Castillo, L., 1997, "Zero-Pressure Gradient Turbulent Boundary Layer," Appl. Mech. Rev., 50(11), pp. 689–729. [2] Castillo, L., and George, W. K., 2001, "Similarity Analysis for Turbulent
- Boundary Layer With Pressure Gradient: Outer Flow," AIAA J., 39(1), pp. 41 - 47.
- [3] Wang, X., and Castillo, L., 2003, "Asymptotic Solutions in Forced Convection Turbulent Boundary Layers," J. Turbulence, 4(006).
- [4] Orlando, A. F., Kays, W. M., and Moffat, R. J., 1974, "Turbulent Transport of

Heat and Momentum in a Boundary Layer Subject to Deceleration, Suction, and Variable Wall Temperature," Report No. HMT-17, Dept. of Mech. Eng., Stanford University, Stanford, CA.

- [5] Millikan, C. M., 1938, "Critical Discussion of Turbulent Flows in Channels and Circular Tubes," Proc. 5th Int. Congr. Applied Mech., Wiley, New York, pp. 386-392.
- [6] George, W. K., Wosnik, M., and Castillo, L., 1997, "Similarity Analysis for Forced Convection Thermal Boundary Layer," Transp. Phenomena Thermal Sci. Process Eng., 1, pp. 239-244.
- [7] Zagarola, M. V., and Smits, A. J., 1998, "Mean-Flow Scaling of Turbulent Pipe Flow," J. Fluid Mech., 373, pp. 33–79.
 [8] Monin, A. S., and Yaglom, A. M., 1971, *Statistic Fluid Mechanics*, MIT Press,
- Cambridge, MA.
- [9] Cal, R. B., 2003, "Similarity Analysis of Transpired Turbulent Boundary Layers," M.S. thesis, Rensselaer Polytechnic Institute.
 [10] Perry, A. E., Bell, J. B., and Joubert, P. N., 1966, "Velocity and Temperature
- Profiles in Adverse Pressure Gradient Turbulent Boundary Layers," J. Fluid Mech., 25, pp. 299-320.