Dynamic Dual Fixed-Point CORDIC Implementation

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Main Contributions

- ***Extra layer of flexibility added to Dual Fixed Point (DFX) CORDIC:*** *Run-time reconfiguration of DFX CORDIC allow for Dynamic DFX CORDIC implementation.*
- ***Methodology for self-reconfiguration based on user input and output data****: The run-time reconfigurable embedded system swaps hardware configurations based on external requirements and arithmetic overflow (issued via an interrupt).*
- ***Comparison with CORDIC architectures on DFX, FX, and FP arithmetic****: This is to assess whether the hardware and software overhead of DDFX CORDIC is justified by the dynamic range and accuracy improvements.*

Dynamic DFX CORDIC

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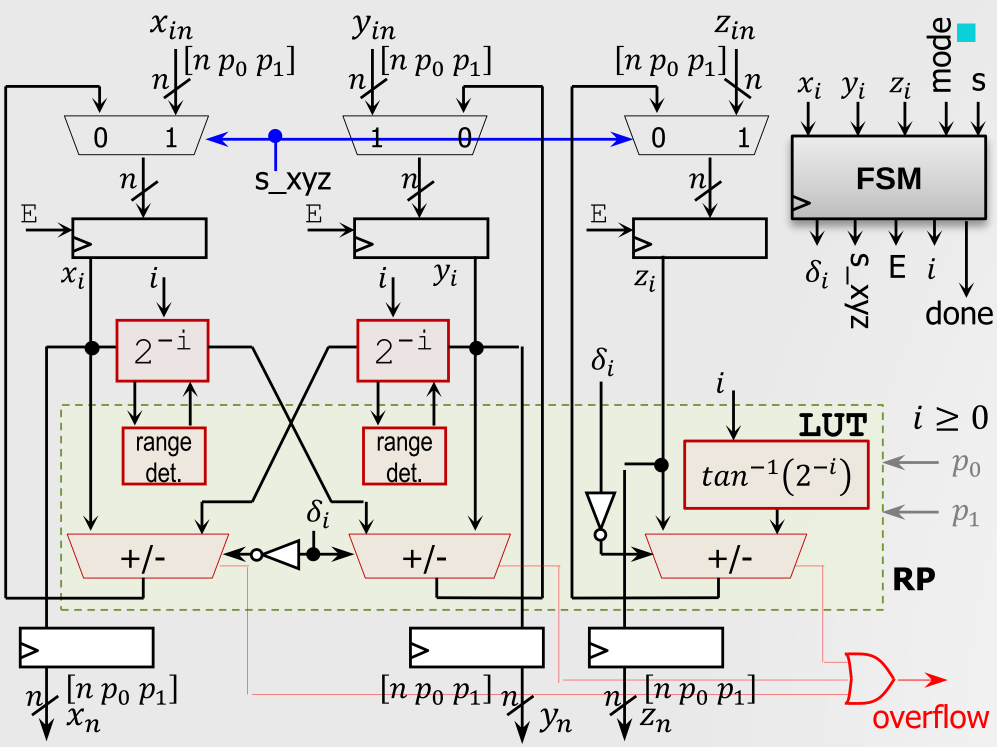
- ***DDFX***: *Set of DFX formats sharing the same word-length*
  - *Example: [32 26 4 6]: It includes DFX formats that guarantee increasing range of values between DFX formats:*
    [32 26 20], [32 18 12], [32 10 4].
  - *DFX format:* $[n\ p_0\ p_1]$*: It can thought as two FX formats* $[n-1\ p_0]$ *and* $[n-1\ p_1]$*, where* $p_0 > p_1$*.*
  - *DDFX: Implemented by modifying the DFX format at run-time.*



- ***DDFX CORDIC***: *It uses the DFX CORDIC hardware.*
  - *DFX format: Adjusted (vary* p_0 *and* p_1*, n fixed) by altering the combinational logic that depends on* p_0 *and* p_1*: this is the run-time alterable area (green-shaded).*
  - *Overflow: Issued to request a numerical format with higher dynamic range.*

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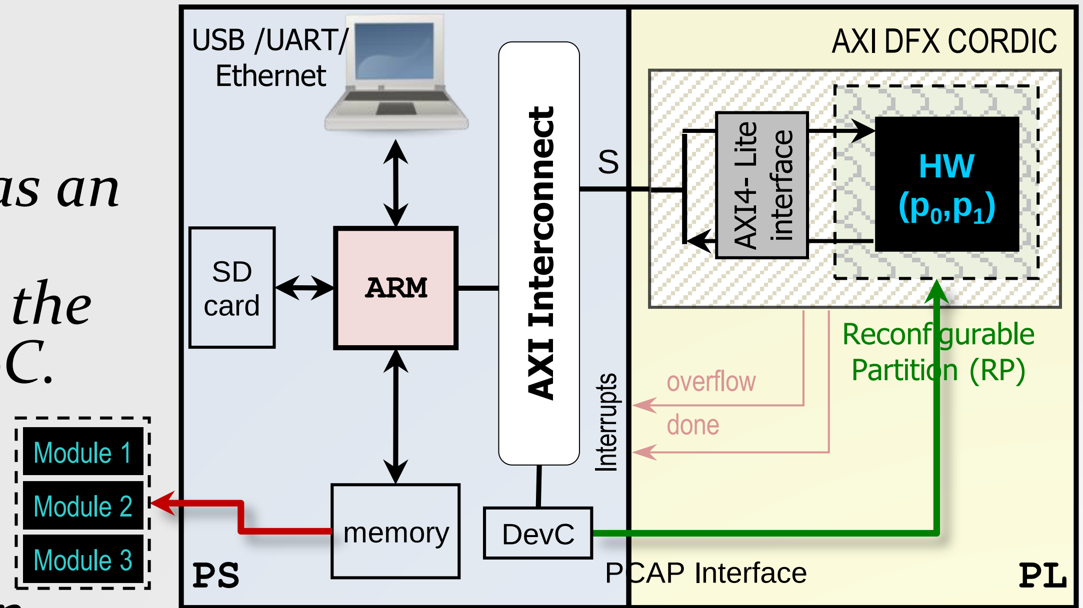
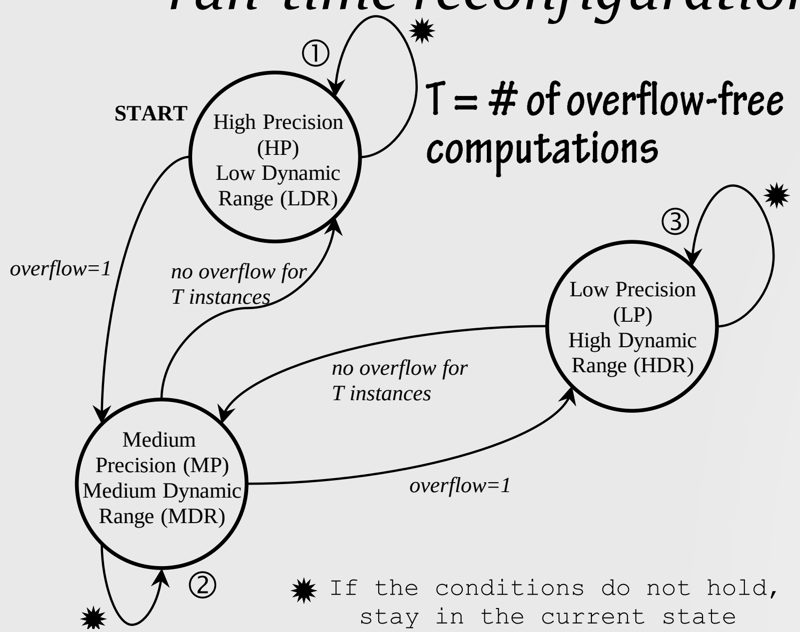
DDFX CORDIC

- ***Self-reconfigurable Embedded System***
  - *DFX CORDIC is placed as an AXI4 peripheral in an embedded system inside the XC7Z010 Zynq-7000 PSoC.*
  - *Overflow, done: Connected to interrupt sources to trigger run-time reconfiguration*





- ***Run-time management***
  - *DDFX CORDIC: Implemented by run-time swapping DFX CORDIC cores based on a State Diagram.*
  - *State Diagram: Implemented as software routine. It specifies transitions to different DFX formats based on overflow and user input.*

Experimental Setup

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- Functions tested: atan(x), $A_n\sqrt{x^2+y^2}$. T=50.
- Input domains: They were selected so that the State Diagram can transition forward and backward across all states.
- DDFX formats tested (32 and 24 bits), along with the DFX, FX, FP CORDIC realization we perform comparisons against:

DDFX Format	Set of DFX formats	Range of Values	Max/Min Resolution	Dynamic Range	DFX	FX	FP
[32 26 4 6]	[32 26 20] [32 18 12] [32 10 4]	$[-2^{26}, 2^{26}-2^{-4}]$	1.49×10^{-8} 6.25×10^{-2}	313.07dB	[32 10 4]	[32 4]	32-bit
[32 29 1 8]	[32 29 21] [32 19 11] [32 9 1]	$[-2^{29}, 2^{29}-2^{-1}]$	1.86×10^{-9} 5×10^{-1}	349.12dB	[32 9 1]	[32 1]	32-bit
[24 20 4 4]	[24 20 16] [24 14 10] [24 8 4]	$[-2^{18}, 2^{18}-2^{-4}]$	9.54×10^{-7} 6.25×10^{-2}	228.78dB	[24 8 4]	[24 4]	24-bit EW:7, FW:16
[24 23 1 6]	[24 23 17] [24 15 9] [24 7 1]	$[-2^{21}, 2^{21}-2^{-1}]$	1.19×10^{-7} 5×10^{-1}	264.91dB	[24 7 1]	[24 1]	24-bit EW:7, FW:16

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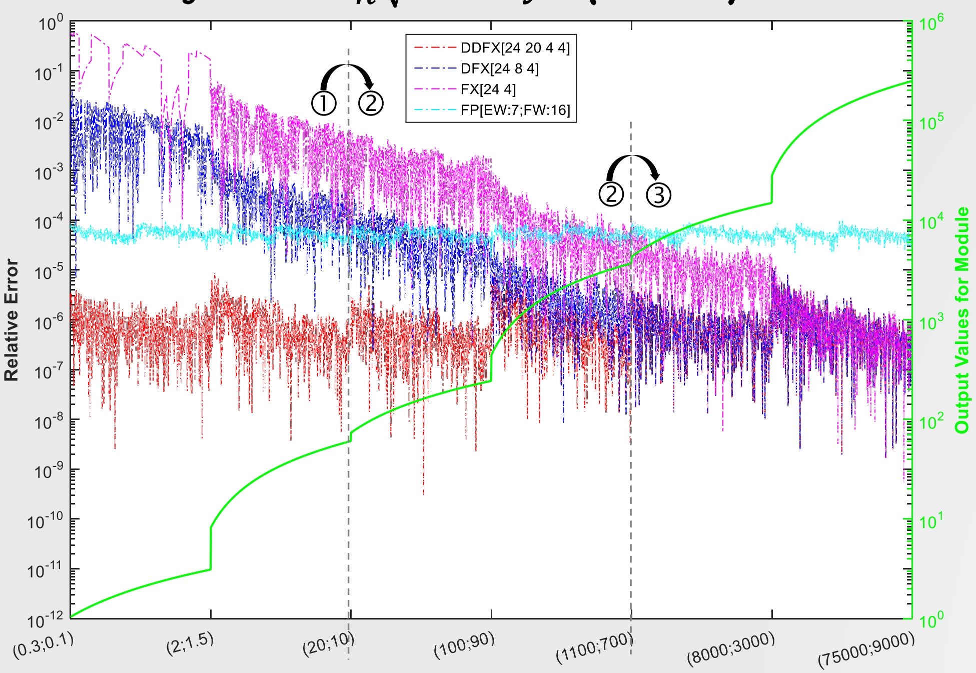
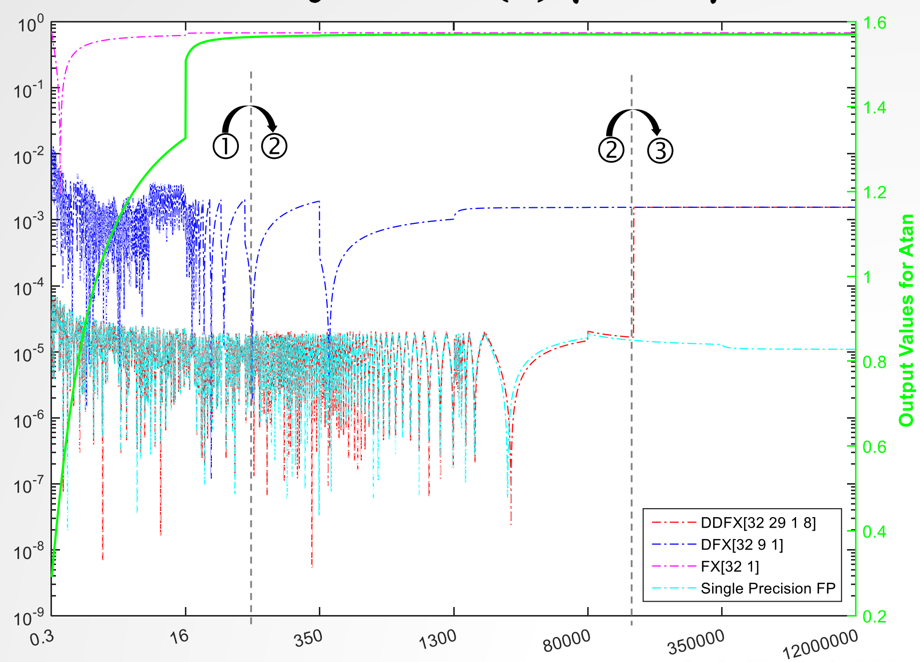
Results

- ***Accuracy (Relative Error)***
  - *Increasing monotonic domains: Note how states transition from states 1 to 3. State transitions are caused by output overflow.*
  - *DDFX looks more accurate most of the time (even when using 24 bits) than single-precision FP. However, DDFX is less accurate than FP when output values are very large.*

Magnitude: $A_n\sqrt{x^2+y^2}$ (24 bits)



Arc Tangent: $\mathrm{atan}(x)$ (32 bits)



Relative Error: all values are compared against MATLAB® built-in functions in 64-bit FP.

Results

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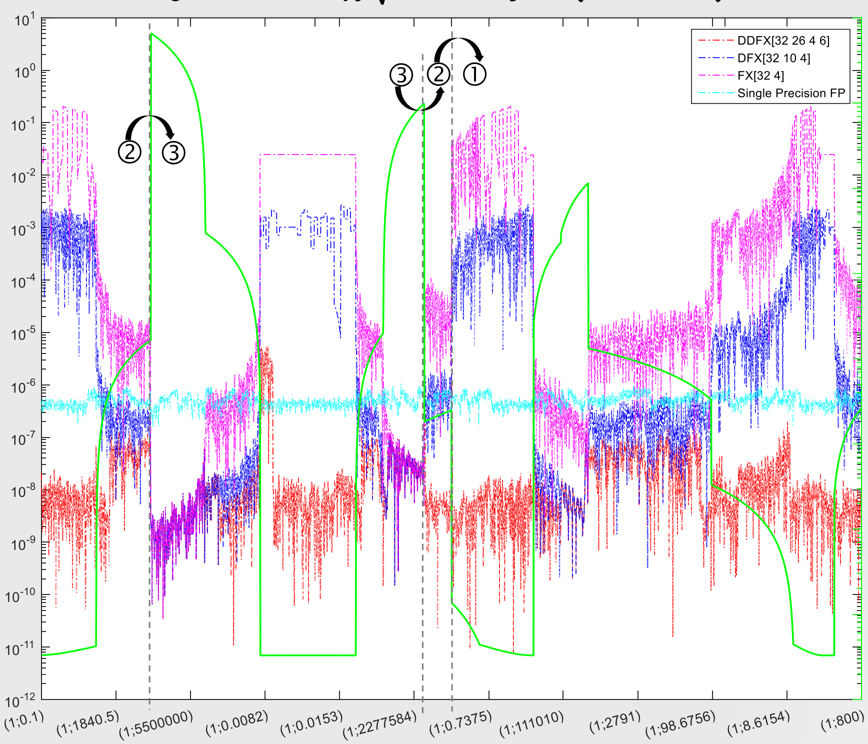
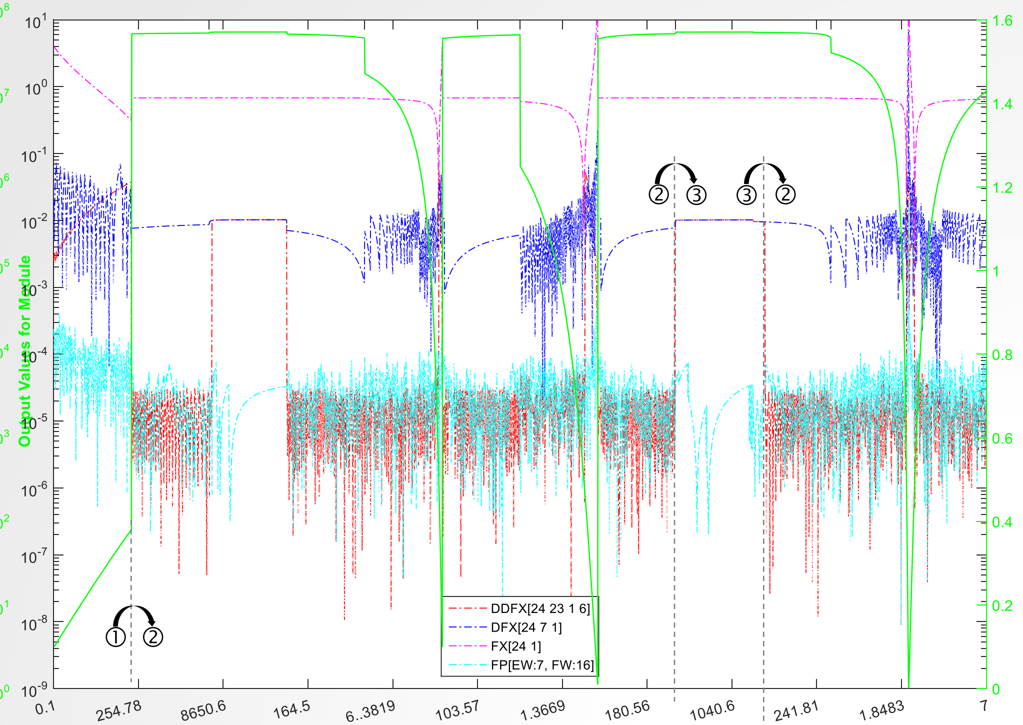
- ***Accuracy (Relative Error)***
  - *Non-monotonic domains: disjoint piece-wise vectors. This forces the States to transition back and forth between 1 and 3.*
  - *For FP, relative error is steady. DDFX is more accurate than FP except in some cases with State 3 (with large output values).*

Magnitude: $A_n\sqrt{x^2+y^2}$ (32 bits)



Arc Tangent: $\mathrm{atan}(x)$ (24 bits)



Relative Error: all values are compared against MATLAB® built-in functions in 64-bit FP.

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