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# A Model for Crank-Angle-Resolved Engine Cylinder Pressure Estimation

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## Outline

- **Introduction**
- **Model for Engine Cylinder Pressure Estimation**
  - Description
  - Tests: Engine 1 Sets (3), Engine 2 Sets (10)
- **Hardware Implementation**
  - Self-Reconfigurable embedded systems
  - Hardware architecture
  - Embedded implementation

## Introduction

- **Cylinder Pressure Estimation:** This useful information for engine operation can be used to: balance the power given to different cylinders to the engine crankshaft, detect disturbance in the engine operation, compute the optimal spark timing, etc.
- *Traditional methodology:* Most of today's vehicle engines rely on pre-computed values of cylinder pressure for different operating conditions. This look-up table approach does not scale well with changes in operating conditions or parameters, as the amount of required memory can grow very quickly.
- *Our method:* We develop a model to compute an estimation of the engine pressure based on specific conditions (e.g.: speed, amount of fuel being used, engine parameters, etc.). This model can then be used to generate meaningful parameters such as instant torque, optimal spark timing, etc.
- For real-time operation, this model is best suited for dedicated hardware implementation for real-time cylinder pressure estimation.

## Model for Engine Cylinder Pressure Estimation

- Relationship among heat release rate, pressure, volume, and heat lost (heat release) for a *closed cylinder engine* (using 1<sup>st</sup> Law of Thermodynamics):

$$\frac{dQ_{HR}}{d\theta} = \frac{\gamma}{\gamma - 1} P \frac{dV}{d\theta} + \frac{1}{\gamma - 1} V \frac{dP}{d\theta} + \frac{dQ_{HT}}{d\theta}$$

- Discrete Model:

$$\frac{dQ_{HR}}{d\theta} = \frac{\gamma}{\gamma - 1} P(n) \frac{dV}{d\theta} + \frac{1}{\gamma - 1} V(n) \left( \frac{P(n+1) - P(n)}{\theta(n+1) - \theta(n)} \right) + \frac{dQ_{HT}}{d\theta}$$

- Then:

$$P(n+1) = P(n) + \Delta\theta \left[ \frac{(\gamma - 1)}{V(n)} \frac{dQ_{HR}}{d\theta} \Big|_n - \gamma \frac{P(n)}{V(n)} \frac{dV}{d\theta} \Big|_n - \frac{(\gamma - 1)}{V(n)} \frac{dQ_{HT}}{d\theta} \Big|_n \right]$$

## Model for Engine Cylinder Pressure Estimation

- We have an *empirical model* for the Heat Transfer Rate:

$$\left. \frac{dQ_{HT}}{d\theta} \right|_n = \frac{dQ_{HT}}{dt} \frac{dt}{d\theta} = h_{corr}(n) A_{ch}(n) (T_g(n) - T_w) \frac{30}{N\pi}, N = rpm$$

$$h_{corr}(n) = c \times 0.013 \times V(n)^{-0.06} \times P(n)^{0.8} \times T_g(n)^{-0.4} \times (v_p + 1.4)^{0.8}$$

- Heat Release Rate: We approximate the function with:

$$\left. \frac{dQ_{HR}}{d\theta} \right|_n = \begin{cases} 0, \text{ for } \theta_n \leq \theta_0. & \theta_0: \text{spark time} \\ \frac{\eta_c m_f LHV \alpha (\beta + 1)}{\Delta\theta_B (1 - e^{-\alpha})} \left( \frac{\theta_n - \theta_0}{\Delta\theta_B} \right)^\beta \times e^{\left[ -\alpha \left( \frac{\theta_n - \theta_0}{\Delta\theta_B} \right)^{\beta+1} \right]}, & \text{for } \theta_n > \theta_0 \end{cases}$$

## Model for Engine Cylinder Pressure Estimation - Calibration

- Most heat transfer model parameters are directly obtained by the engine data and the operating conditions (rpm, pressure of chamber, etc.).
- Given actual pressure data, we initially assume some constants in the Heat Transfer Rate, and then we get the Heat Release Rate.
- We then plot the cumulative Heat Release. The maximum value divided by the actual fuel energy released ( $\eta_c m_f LHV$ ), also called mass fraction, should be 1. We adjust the constants in the Heat Transfer Rate until the mass fraction reaches 1.
- With the mass fraction, we can get the burn duration ( $\Delta\theta_B$ : 0 to 97% of the mass fraction). Non-linear curve-fitting is applied to fit the Wiebe function to the cumulative heat release curve, resulting in  $\alpha$  and  $\beta$  (heat release model parameters).
- With the complete heat transfer and heat release models, we calculate the estimated pressure trace. This process can be performed for pressure traces covering the entire operating range. To cover the full operating space, the parameter space can be interpolated. We only need to store the heat transfer and heat release rate parameters in order to generate the estimated pressure trace.

## Experimental Validation

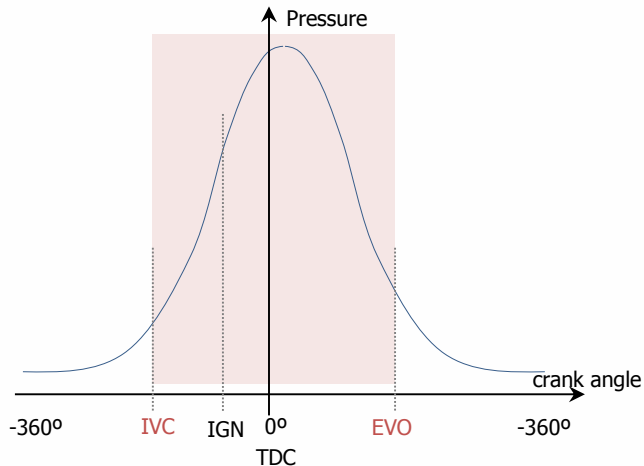
- Actual pressure traces were acquired from two production engines from two OEMs” Engine 1 (3 sets) and Engine 2 (10 sets).
- We compare the Heat Release Model with the actual Heat Release. We also compare the estimated pressure against the real one. The metric used is Relative Error:

$$\text{Relative Error} = \frac{\text{Estimated value} - \text{Actual Value}}{\text{Actual Value}}$$

- With the engine and operating information, the calibration of the model can be completed for each engine. The final results are a modeled heat release rate and an estimated pressure trace. The results will be plotted and compared (actual vs. estimated) for all the 13 sets, using relative error (max., avg., and at peak pressure).
- **Reference for crank angles:** The engine cycle goes from  $-360^\circ$  to  $360^\circ$  (1 cycle or 2 revolutions). The pressure trace will be estimated from IVC to EVO. Also, the heat release rate will be plotted from IGN (ignition time= $\theta_0$ ).

## Experimental Validation

- For each set, engine data and operating conditions (e.g.: fuel, IVC, EVO, load, rpm) are provided in the table.
- Reference for crank angles:**

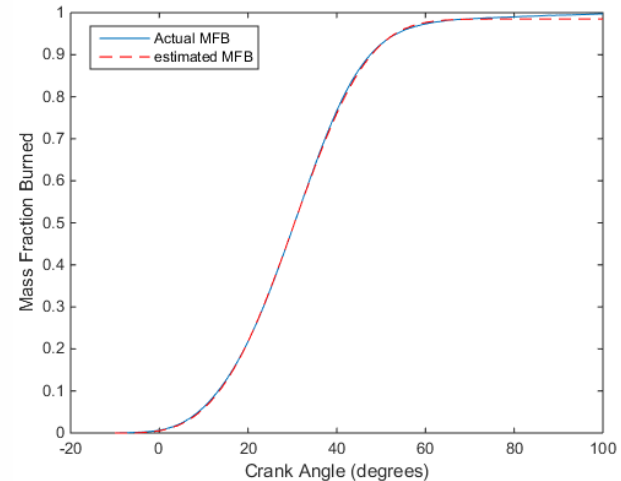
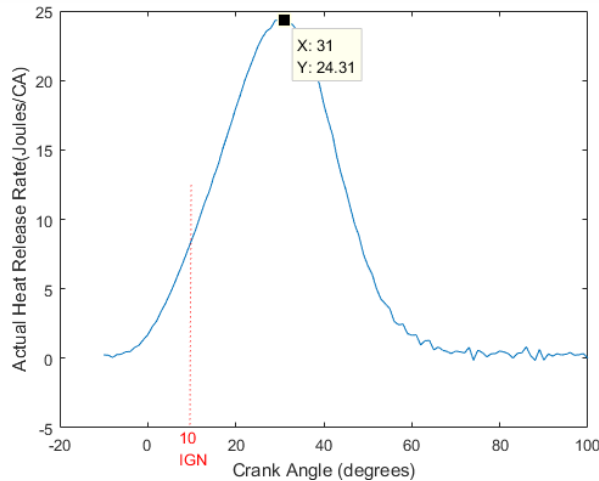


	Set	$\theta_0$	rpm	BMEP (bar)	Resid. fract.	Fuel: mg/cycle	A/F ratio	IVC/EVO
Engine 1	1	-10	1300	-	0.108	18.68	13.90	-95/100
	2	-26		-	0.129	16.70		
	3	-40		-	0.125	17.08		
Engine 2	1	-23	1250	2	0.15	13.66	13.81	-88/123
	2	-19	1250	4	0.125	21.88	13.82	-87/124
	3	-13	1250	6	0.125	29.70	13.84	-87/124
	4	-29	2000	2	0.125	14.01	13.85	-87/124
	5	-22	2000	6	0.125	29.81	13.90	-87/124
	6	-16	2000	8	0.110	38.12	13.91	-95/116
	7	-24	3200	4	0.125	22.65	13.94	-87/124
	8	-19	3200	6	0.100	30.67	13.88	-91/120
	9	-21	4000	4.5	0.125	25.75	13.92	-91/120
	10	-11	4000	wot	0.125	62.58	11.09	-109/102



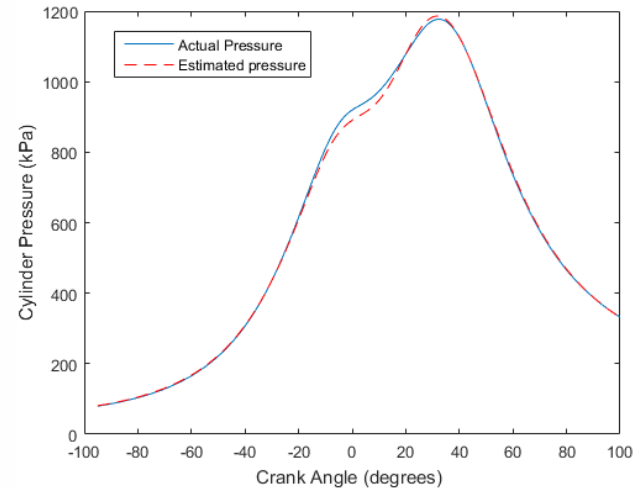
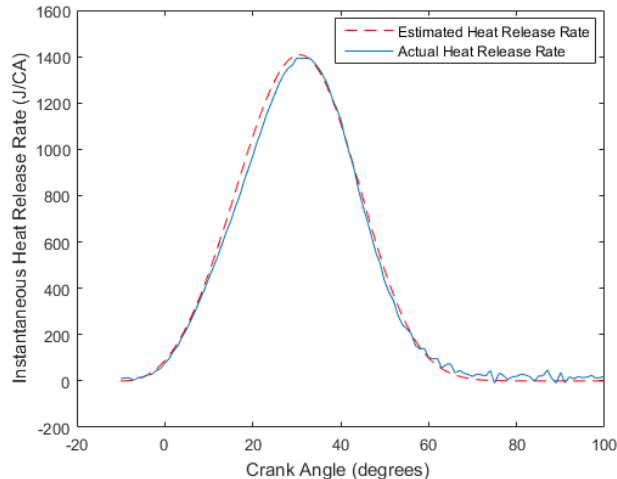
## Results – Engine 1, Set 1- Mass fraction

- We show the actual heat release rate for IGN ( $-10^\circ$ ) to EVO ( $100^\circ$ ).
- The mass fraction vs. crank angle (standardized curve). Note that  $\Delta\theta_B=64^\circ$  (0 to 97% of the mass fraction). We also show the approximation of the mass fraction (dotted red) with the Wiebe function.



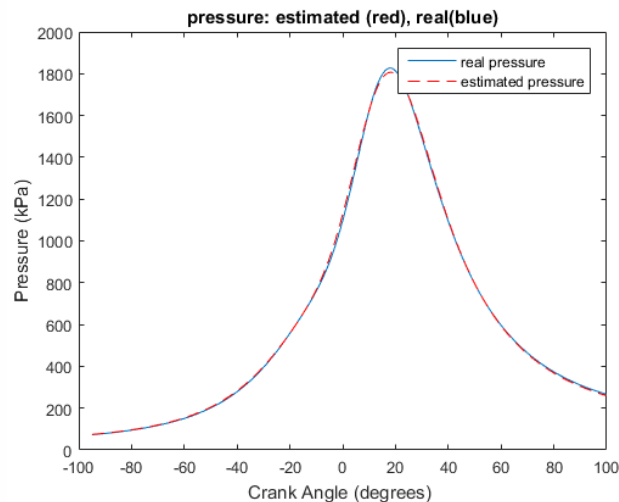
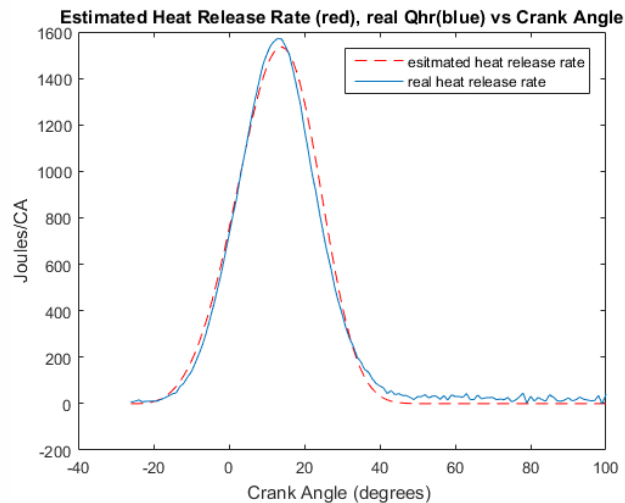
## Results – Engine 1, Set 1 (1300 rpm)

- Heat Release Rate. From IGN (-10°) to EVO (100°): The **red**-dotted line is the estimated HR rate. Curve fitting parameters:  $\alpha = 3.45$ ,  $\beta = 2.48$ .
- Pressure. IVC (-95°) to EVO (100°)
  - Estimated pressure trace (**red**-dotted line).



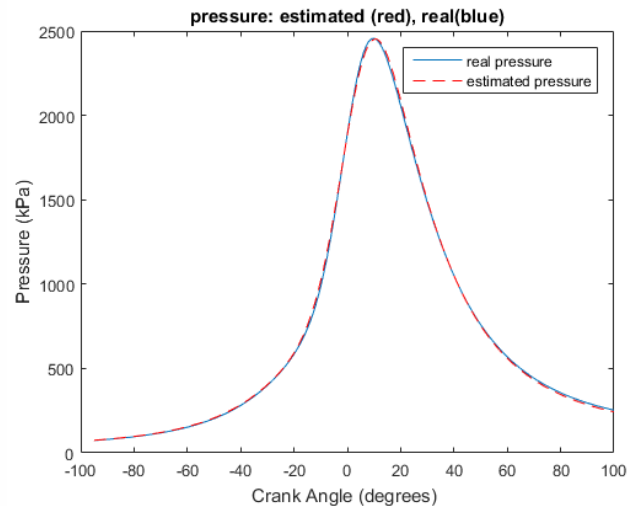
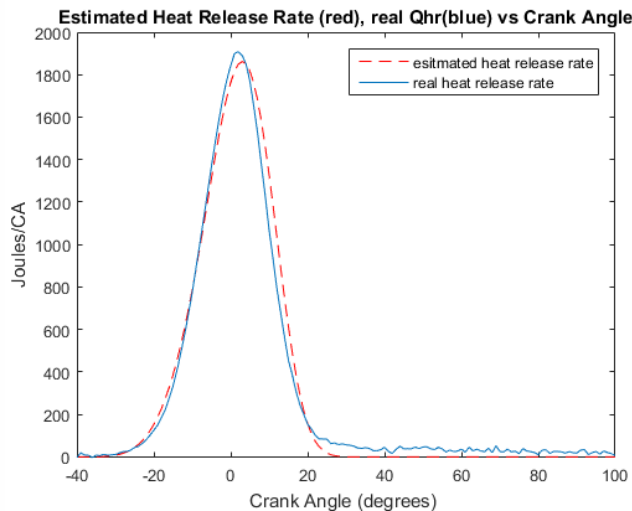
## Results – Engine 1, Set 2 (1300 rpm)

- Heat Release Rate. From IGN (-26°) to EVO (100°): The **red**-dotted line is the estimated HR rate. Curve fitting parameters:  $\alpha = 4.45$ ,  $\beta = 3.154$ .
- Pressure. IVC (-95°) to EVO (100°)
  - Estimated pressure trace (**red**-dotted line).



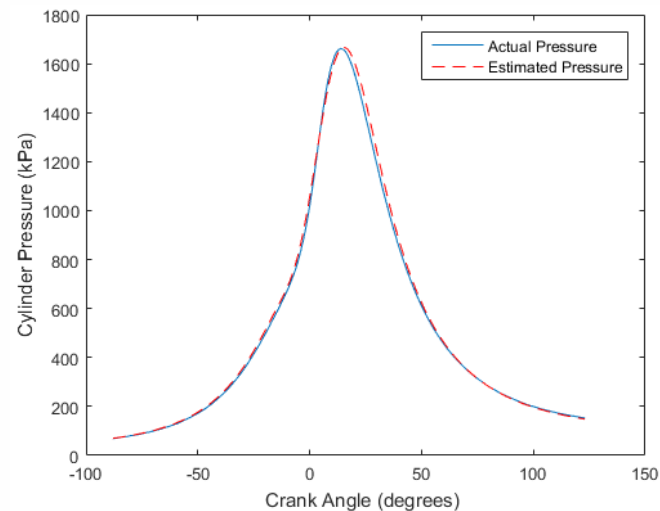
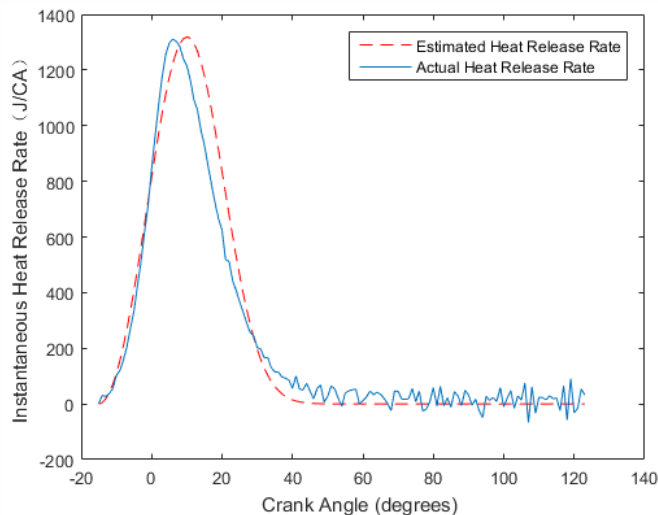
## Results – Engine 1, Set 3 (1300 rpm)

- Heat Release Rate. From IGN (-40°) to EVO (100°): The **red**-dotted line is the estimated HR rate. Curve fitting parameters:  $\alpha = 10.68$ ,  $\beta = 4.3$ .
- Pressure. IVC (-95°) to EVO (100°)
  - Estimated pressure trace (**red**-dotted line).



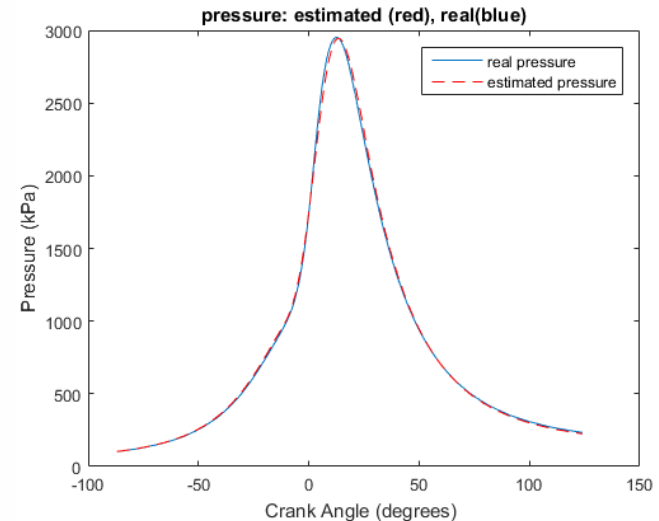
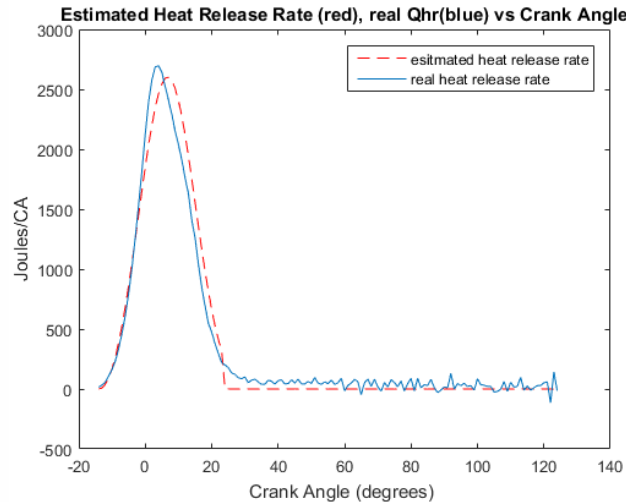
## Results – Engine 2, Set 1 (1250 rpm)

- Heat Release Rate. From IGN ( $-14^{\circ}$ ) to EVO ( $123^{\circ}$ ): The **red**-dotted line is the estimated HR rate. Curve fitting parameters:  $\alpha = 3.243$ ,  $\beta = 1.947$ . IGN delayed. Original IGN:  $-23^{\circ}$ .
- Pressure. IVC ( $-88^{\circ}$ ) to EVO ( $123^{\circ}$ ): Estimated pressure trace (**red**-dotted line).



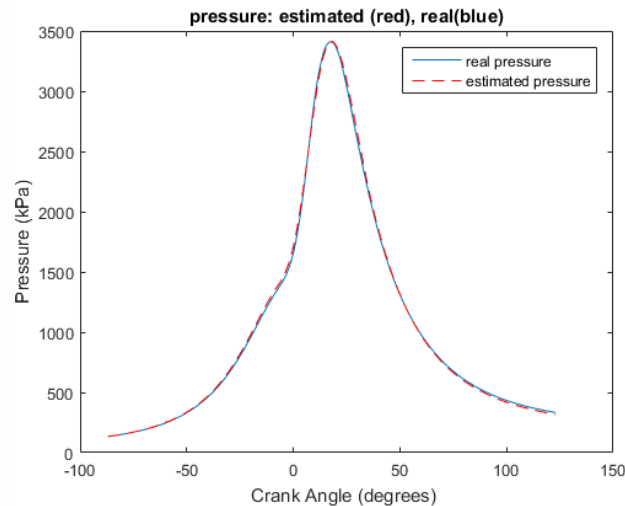
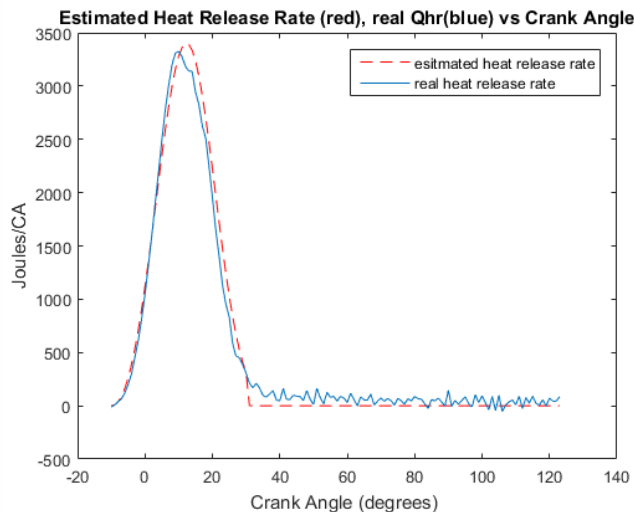
## Results – Engine 2, Set 2 (1250 rpm)

- Heat Release Rate. From IGN ( $-14^{\circ}$ ) to EVO ( $124^{\circ}$ ): The **red**-dotted line is the estimated HR rate. Curve fitting parameters:  $\alpha = 4.719$ ,  $\beta = 2.066$ . IGN delayed. Original IGN:  $-19^{\circ}$ .
- Pressure. IVC ( $-87^{\circ}$ ) to EVO ( $124^{\circ}$ ): Estimated pressure trace (**red**-dotted line).



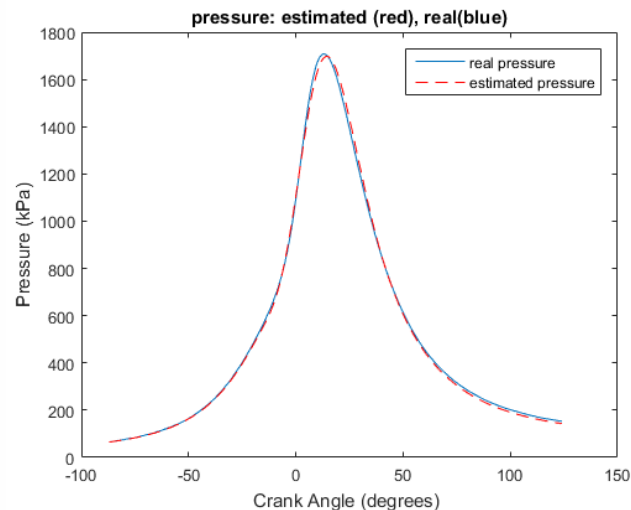
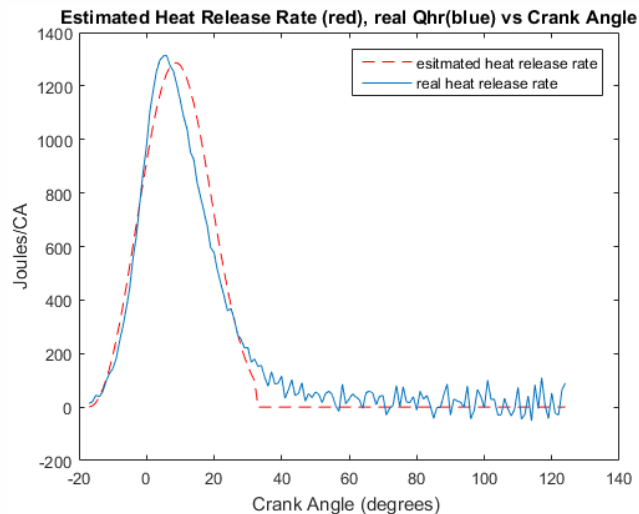
## Results – Engine 2, Set 3 (1250 rpm)

- Heat Release Rate. From IGN ( $-10^\circ$ ) to EVO ( $124^\circ$ ): The **red**-dotted line is the estimated HR rate. Curve fitting parameters:  $\alpha = 4.719$ ,  $\beta = 2.173$ . IGN delayed. Original IGN:  $-13^\circ$ .
- Pressure. IVC ( $-87^\circ$ ) to EVO ( $124^\circ$ ): Estimated pressure trace (**red**-dotted line).



## Results – Engine 2, Set 4 (2000 rpm)

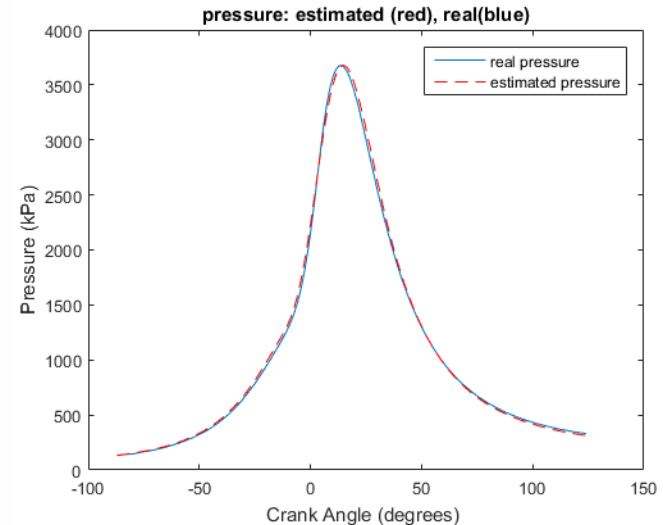
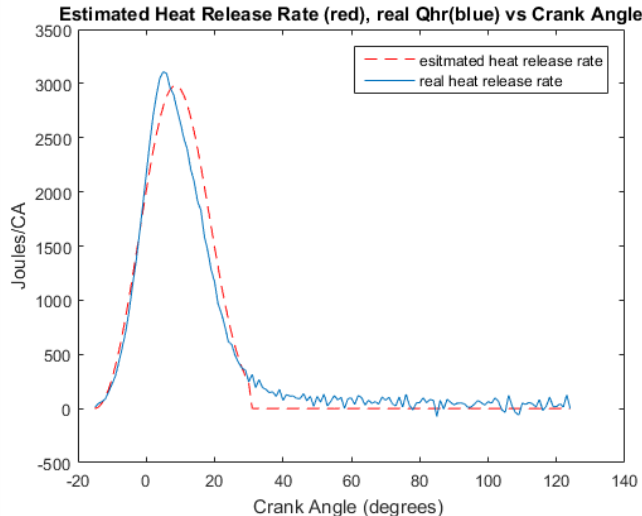
- Heat Release Rate. From IGN ( $-17^\circ$ ) to EVO ( $124^\circ$ ): The **red**-dotted line is the estimated HR rate. Curve fitting parameters:  $\alpha = 4.782$ ,  $\beta = 1.961$ . IGN delayed. Original IGN:  $-29^\circ$ .
- Pressure. IVC ( $-87^\circ$ ) to EVO ( $124^\circ$ ): Estimated pressure trace (**red**-dotted line).





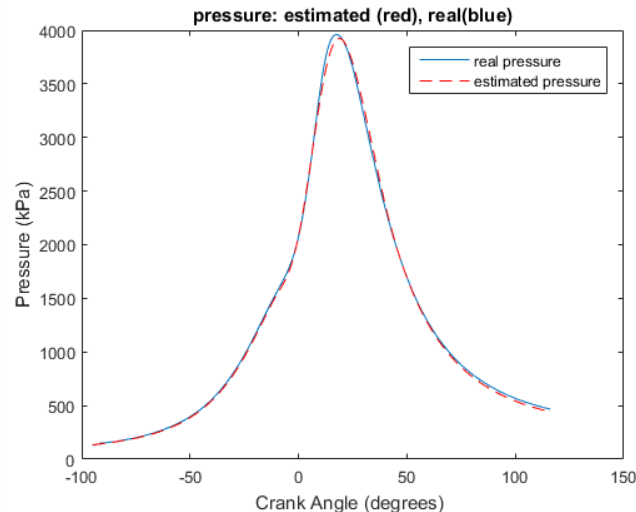
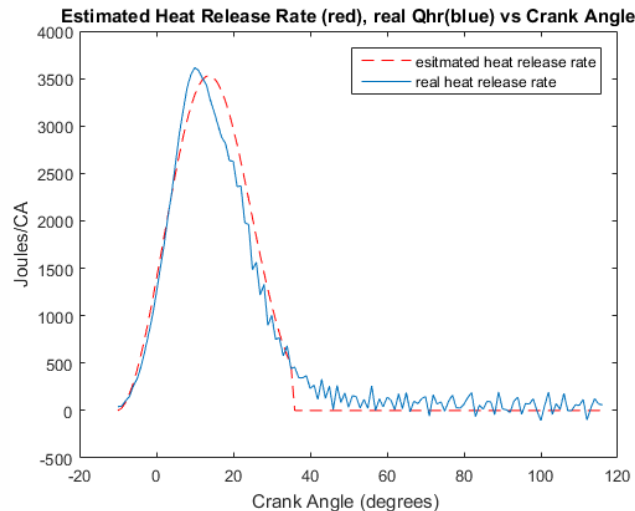
## Results – Engine 2, Set 5 (2000 rpm)

- Heat Release Rate. From IGN ( $-15^{\circ}$ ) to EVO ( $124^{\circ}$ ): The **red**-dotted line is the estimated HR rate. Curve fitting parameters:  $\alpha = 4.47$ ,  $\beta = 1.952$ . IGN delayed. Original IGN:  $-22^{\circ}$ .
- Pressure. IVC ( $-87^{\circ}$ ) to EVO ( $124^{\circ}$ ): Estimated pressure trace (**red**-dotted line).



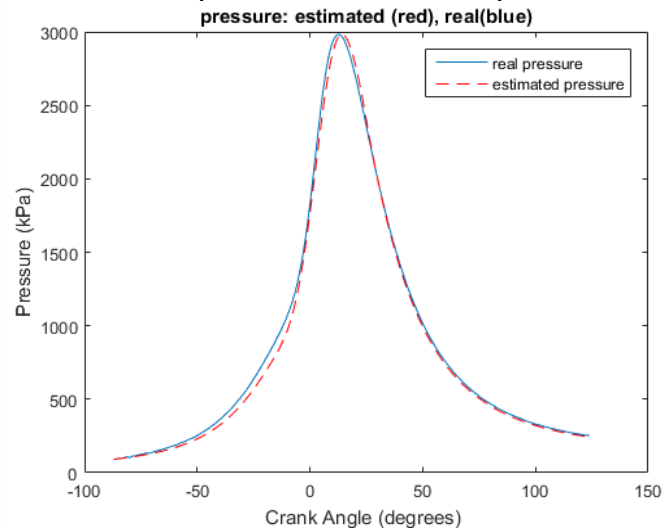
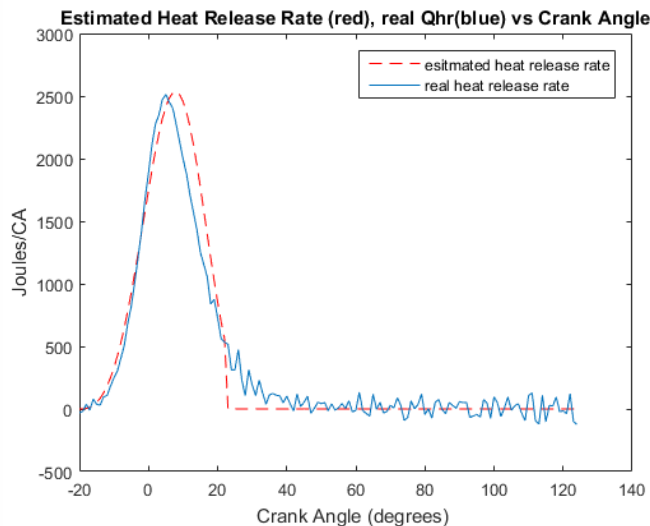
## Results – Engine 2, Set 6 (2000 rpm)

- Heat Release Rate. From IGN ( $-10^\circ$ ) to EVO ( $116^\circ$ ): The **red**-dotted line is the estimated HR rate. Curve fitting parameters:  $\alpha = 3.243$ ,  $\beta = 1.947$ . IGN delayed. Original IGN:  $-16^\circ$ .
- Pressure. IVC ( $-95^\circ$ ) to EVO ( $116^\circ$ ): Estimated pressure trace (**red**-dotted line).



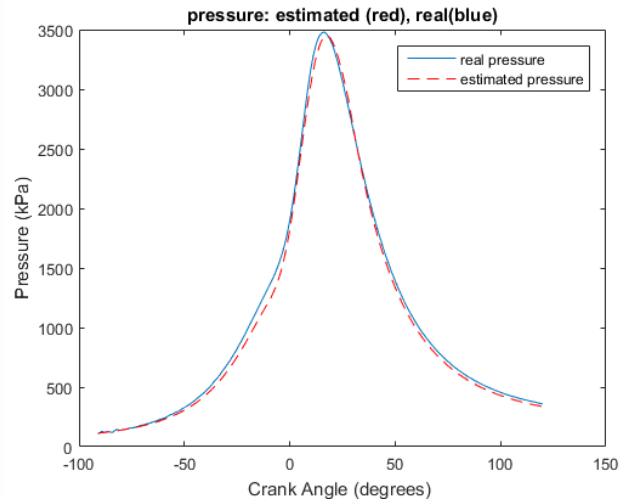
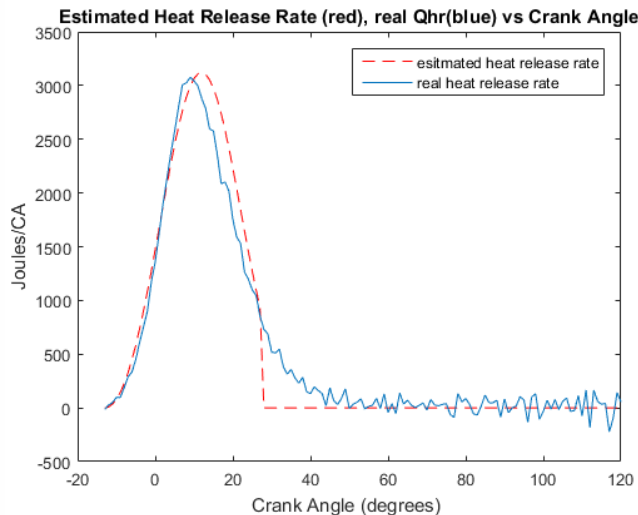
## Results – Engine 2, Set 7 (3200 rpm)

- Heat Release Rate. From IGN (-20°) to EVO (124°): The **red**-dotted line is the estimated HR rate. Curve fitting parameters:  $\alpha = 3.347$ ,  $\beta = 2.69$ . IGN delayed. Original IGN: -24°.
- Pressure. IVC (-87°) to EVO (124°): Estimated pressure trace (**red**-dotted line).



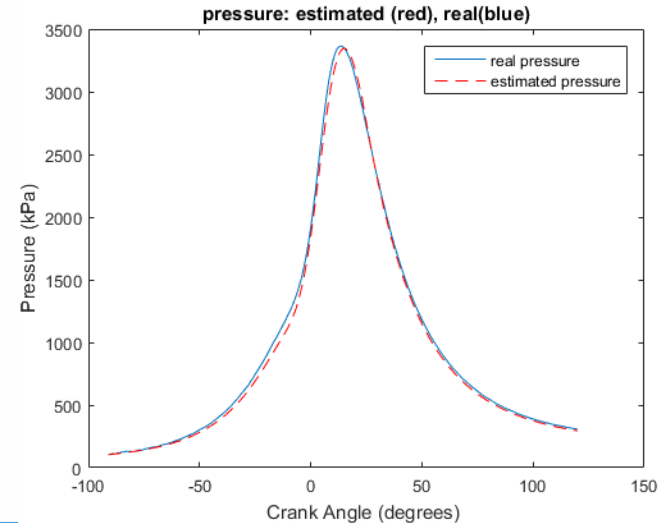
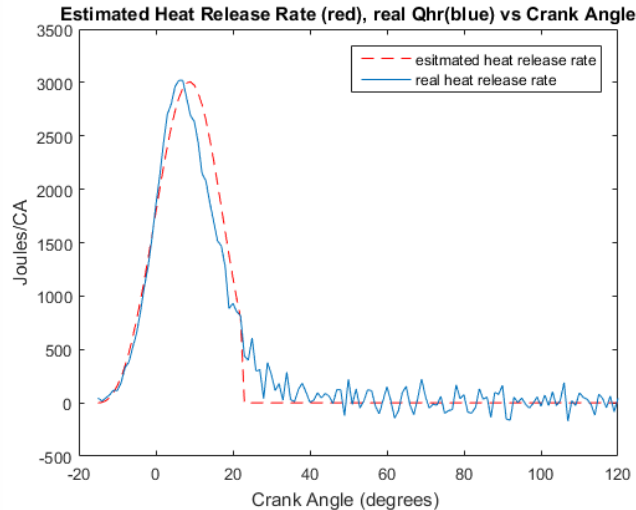
## Results – Engine 2, Set 8 (3200 rpm)

- Heat Release Rate. From IGN ( $-13^\circ$ ) to EVO ( $120^\circ$ ): The **red**-dotted line is the estimated HR rate. Curve fitting parameters:  $\alpha = 3.068$ ,  $\beta = 2.047$ . IGN delayed. Original IGN:  $-19^\circ$ .
- Pressure. IVC ( $-91^\circ$ ) to EVO ( $120^\circ$ ): Estimated pressure trace (**red**-dotted line).



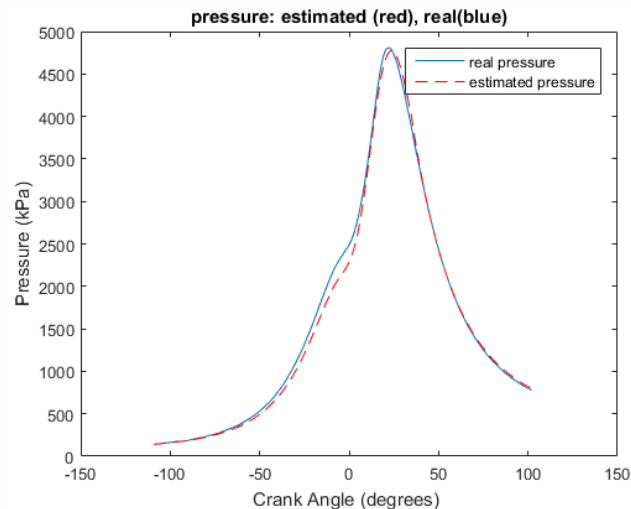
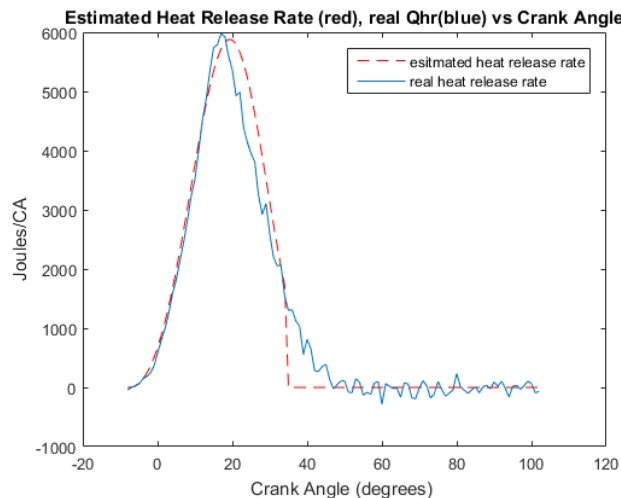
## Results – Engine 2, Set 9 (4000 rpm)

- Heat Release Rate. From IGN ( $-15^{\circ}$ ) to EVO ( $120^{\circ}$ ): The **red**-dotted line is the estimated HR rate. Curve fitting parameters:  $\alpha = 3.346$ ,  $\beta = 2.33$ . IGN delayed. Original IGN:  $-21^{\circ}$ .
- Pressure. IVC ( $-91^{\circ}$ ) to EVO ( $120^{\circ}$ ): Estimated pressure trace (**red**-dotted line).



## Results – Engine 2, Set 10 (4000 rpm, WOT)

- Heat Release Rate. From IGN (-8°) to EVO (102°): The **red**-dotted line is the estimated HR rate. Curve fitting parameters:  $\alpha = 3.125$ ,  $\beta = 2.327$ . IGN delayed. Original IGN: -11°.
- Pressure. IVC (-109°) to EVO (102°): Estimated pressure trace (**red**-dotted line).



## Results

- The table shows the final results for all data sets that include the parameters of the heat release rate model ( $\alpha, \beta$ ), the constant in the heat transfer model ( $c$ ), the burn duration ( $\Delta\theta_B$ ), and the relative error (maximum, average, and at peak pressure).
- For the case of Engine 2, the heat transfer model should be further tuned.

	Set	$\Delta\theta_B$	$\alpha$	$\beta$	$c$	Relative error (%)		
						Max.	Avg.	At peak pressure
Engine 1	1	64	3.45	2.48	1.45	3.19	1.01	0.75
	2	61	4.45	3.15	1.43	2.68	1.19	1.10
	3	70	10.68	4.30	1.52	4.22	1.62	0.19
Engine 2	1	43	3.24	1.95	1.35	4.63	2.11	0.06
	2	38	4.72	2.06	1.2	4.05	2.00	0.25
	3	41	4.72	2.17	1.3	5.19	1.81	0.15
	4	103	4.78	1.96	1.45	6.41	2.49	1.97
	5	45	4.47	1.95	1.45	5.44	3.07	0.05
	6	46	4.00	1.76	1.5	5.47	2.22	1.49
	7	42	3.34	2.69	1.45	11.2	5.44	0.33
	8	41	3.07	2.05	1.55	9.29	4.51	1.38
	9	38	3.34	2.33	1.2	8.24	3.60	0.80
	10	43	3.12	2.32	1.45	9.11	4.34	1.92

## Hardware Implementation

- **Hardware:** The nature of the calculations in the model suggests that a dedicated hardware implementation inside an embedded system can deliver real-time performance for computing the estimated pressure at every crank angle.
- **Closed Cylinder Engine:** Based on input parameters, the hardware should compute estimated pressure at every crank angle. Moreover, the hardware can adapt to different operating conditions.
- **Static Approach:** the parameters are inputs to our circuit. If we need to update the parameters, we just upload them into input registers in order to update the estimated pressure trace.

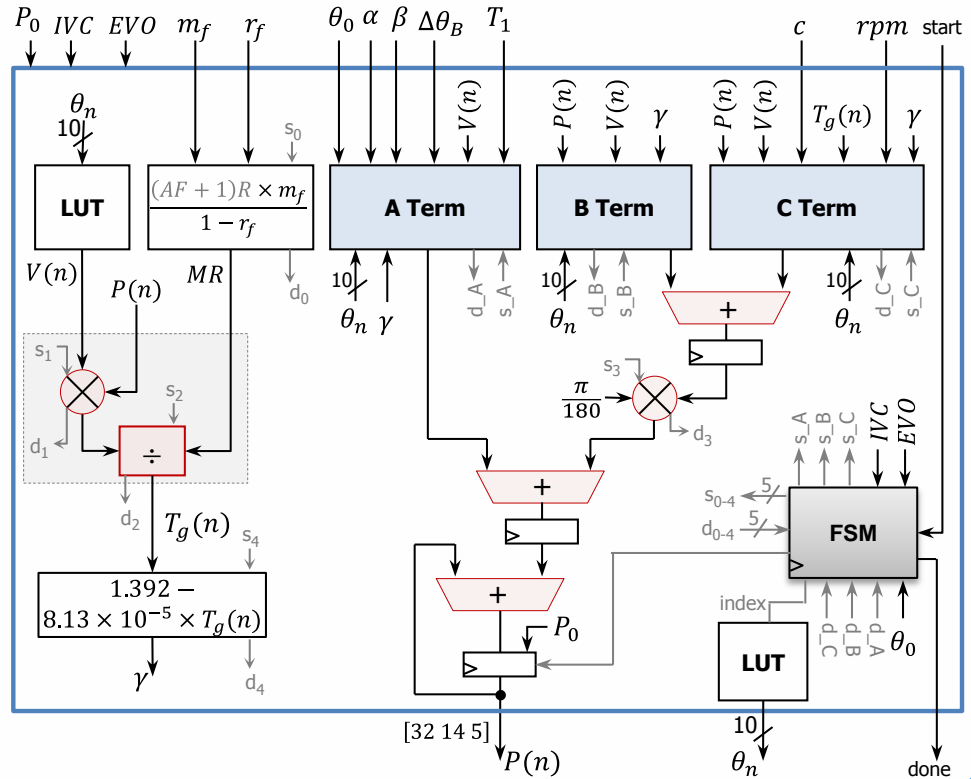


## Hardware Implementation

- Dual Fixed-Point Arithmetic: A compromise between Floating Point Arithmetic (high hardware usage) and Fixed Point (reduced range of numerical values).
- Inputs: remaining parameters are hardwired in the architecture.
  - engine parameters:  $rpm, IVC, EVO, m_f, r_f, P_0, \theta_0, \Delta\theta_B, T_1 = \frac{\eta_c m_f LHV \alpha (\beta + 1)}{\Delta\theta_B (1 - e^{-\alpha})}$
  - model parameters:  $\alpha, \beta, c$
- Output:  $P(n)$  (estimated pressure trace for every crank angle).
- It was determined the 32-bit DFX format [32 14 5] to be the optimal one, based on the range of values of the datasets.
- Hardware architecture: The discrete model equations were slightly modified to comply with the data units. The design includes custom DFX units for arithmetic operations (addition/subtraction, multiplication, division), LUTs as well as exponential and powering functions (based on the CORDIC algorithm).

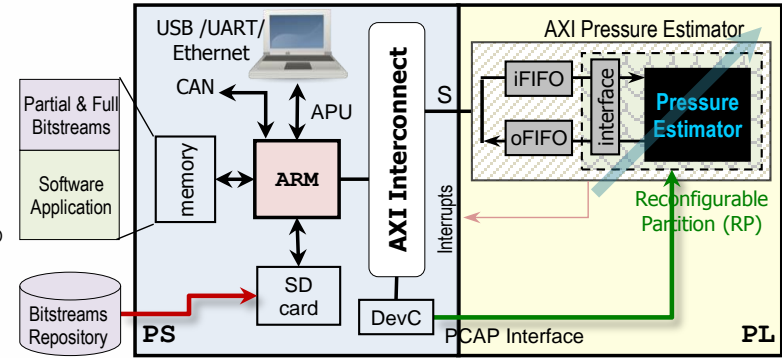
## Hardware Implementation

- The *start* signal begins the process (data inputs are captured at this time). The signal *done* is asserted each time a new  $P(n)$  value is computed.
- Any change in the engine operating conditions is addressed by loading new model and engine parameters.
- The A, B, C terms implement portions of the pressure eq.
- Data transfers are orchestrated by a Finite State Machine (FSM) that controls the *start* and *done* signals of every unit.



## Embedded System Design

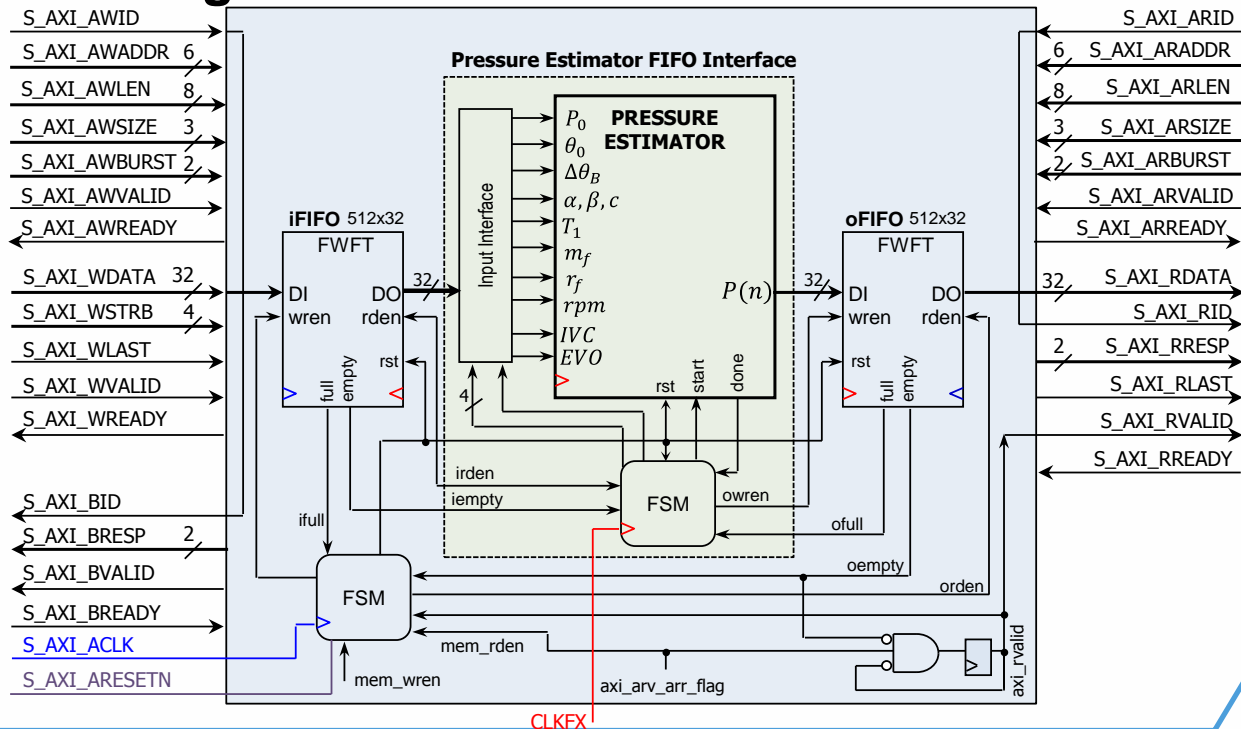
- For real-time hardware validation, the pressure estimator was placed in a reconfigurable embedded system.
- It was implemented on a Programmable System-on-Chip (SoC) that integrates:
  - Processing System (PS)*: dual-core ARM® processor and common peripheral
  - Programmable Logic (PL)*: run-time reconfigurable fabric (FPGA).
- The PS feeds data to and extracts data from the Pressure Estimator via an AXI4-Full Interface.
- AXI Pressure Estimator Peripheral: It includes our design. It is located in the PL, and it runs at 50 MHz.



- Target device: Xilinx® XC7020 Zynq-7000 All Programmable SoC.
- It was tested on a ZED Development Board.

## Embedded System Design

- Pressure Estimator Peripheral: our design + a 32-bit AXI4-Full Slave Interface (2 FIFOs, control, and extra logic).
- With this configuration, we feed the 13 sets of data and then retrieve the resulting pressure traces via the AXI4-Full Interface.

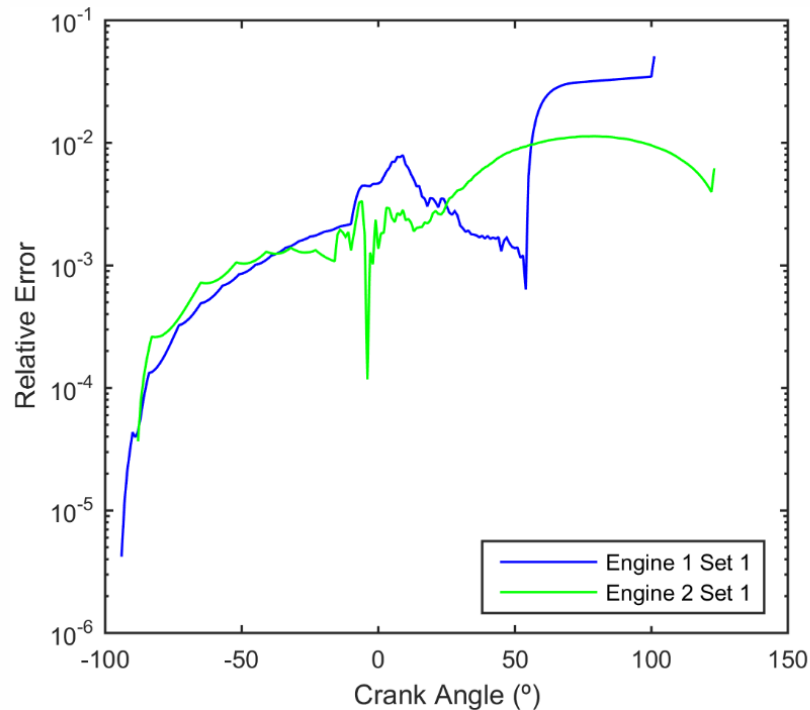
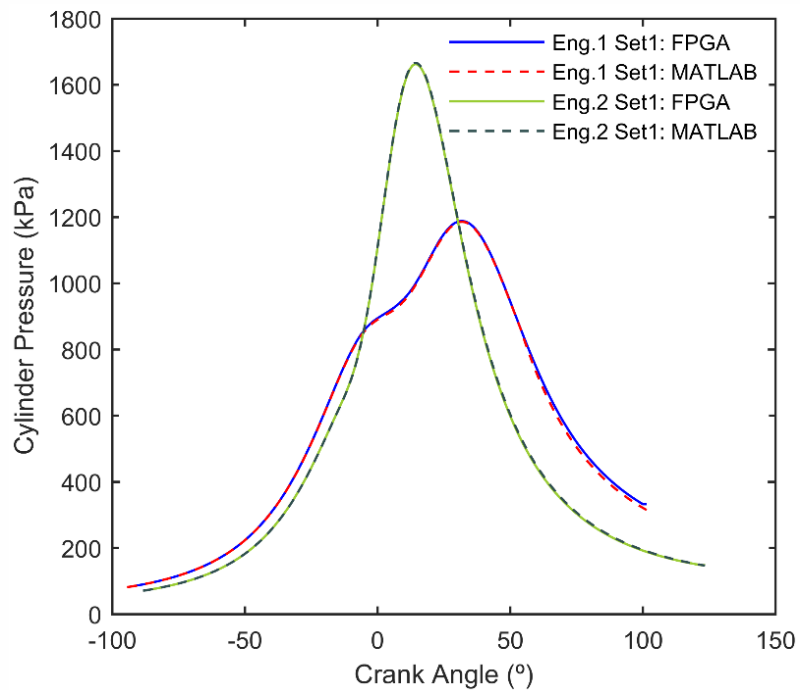


## Hardware Implementation - Results

- Engine 1, Set 1; Engine 2, Set 1:
  - We show estimated pressure traces (from IVC to EVO) for both the 64-bit floating-point MATLAB® model and the 32-bit DFX hardware. Note that the curves are pretty close. This occurs for all the 13 sets.
- The relative error between the MATLAB® results and the hardware results are shown. For Engine 1, Set 1, the relative error is at most 5%. On average, the relative error is 0.86%. The table depicts these relative error results (maximum, average) for all the 13 sets.
- This error is the quantization error between 64-bit floating-point and 32-bit DFX arithmetic. These results suggest that the use of dual fixed-point arithmetic provides results close to those of floating-point without the large hardware overhead of floating-point arithmetic.

SET		RELATIVE ERROR (%)	
		AVG.	MAX.
ENGINE 1	1	0.86%	4.99%
	2	0.70%	4.02%
	3	1.61%	9.26%
ENGINE 2	1	0.47%	1.13%
	2	1.15%	6.29%
	3	1.12%	4.97%
	4	0.83%	3.75%
	5	1.53%	6.02%
	6	1.08%	4.31%
	7	1.29%	6.83%
	8	2.41%	7.95%
	9	2.16%	8.45%
	10	1.47%	6.75%

## Hardware Implementation - Results



## Hardware Implementation - Results

- **Execution Time:**
- A pressure data point  $P(n)$  is computed in a maximum of  $T_p = 753$  clock cycles for [32 14 5].
- Hardware operating frequency: 50 MHz.
- Pressure Trace Computation:  
 $(IVC - EVO)^\circ \times T_p$  cycles
- Processing time: It depends on IVC-EVO, crank angle resolution, and operating frequency. For a crank angle resolution of  $1^\circ$ , and 50 MHz, the pressure trace computation can take up to:
  - 2.951 ms** for the Engine 1 sets (IVC-EVO=196°)
  - 3.192 ms** for the Engine 2 sets (IVC-EVO=212°).
- The hardware can support real-time pressure computation at  $1^\circ$  crank angle resolution and keep up with speeds up to **10000 rpm** (at 10000 rpm, the IVC-EVO time duration is 3.26 ms for Engine 1 Sets and 3.53 ms for Engine 2 sets). The table shows the actual processing times for the 13 sets, which are shorter than the reported maximum times

SET		RPM	IVC-EVO (ms)	PROC. TIME (ms)
ENGINE 1	1	1300	46.15	2.6671
	2			2.6613
	3			2.6609
ENGINE 2	1	1250	48.00	2.8800
	2	1250	48.00	2.8959
	3	1250	48.00	2.8988
	4	2000	30.00	2.8833
	5	2000	30.00	2.8979
	6	2000	30.00	2.9083
	7	3200	18.75	2.8967
	8	3200	18.75	2.9033
	9	4000	15.00	2.9068
	10	4000	15.00	2.9260

## Conclusions

- A model for Pressure Estimation was completed.
  - Better tweaking of the parameters of the Heat Transfer Rate might be required. Engine 1 gives better results than Engine 2 data, as it seems that the parameters have been calibrated for the GM engine.
- Hardware implementation completed using a non-standard numerical representation (DFX) for resource optimization.
- DFX Hardware successfully validated. It can handle large numerical ranges with reasonable resource requirements. Numerical results show that the accuracy of the DFX architecture is close to that of a double-precision software realization.
- Hardware design tested in real-time using a reconfigurable embedded system.
- Results are very encouraging:
  - Pressure model estimated results matches very closely real pressure traces.
  - Hardware results match very closely those of the model in MATLAB®.
  - The hardware design can keep up with engine speeds of up to 10000 rpm with 1° crank angle resolution.





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## Thank you

Daniel Llamocca