Solutions - Homework 1

(Due date: January 20th @ 5:30 pm)

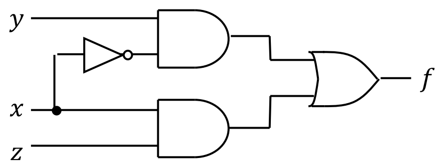
Presentation and clarity are very important! Show your procedure!

PROBLEM 1 (25 PTS)

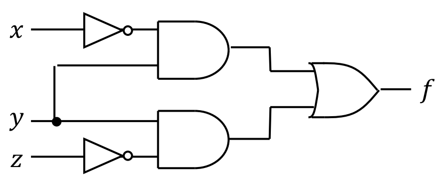
a) Simplify the following functions using ONLY Boolean Algebra Theorems. For each resulting simplified function, sketch the logic circuit using AND, OR, XOR, and NOT gates. (15 pts)

✓ $F(x,y,z) = \prod(M_0, M_1, M_4, M_6)$ ✓ $F = \overline{x(y\oplus\bar{z}) + \bar{y}}$ ✓ $F = (\bar{A}C + \bar{D})(A + \bar{C} + D)$

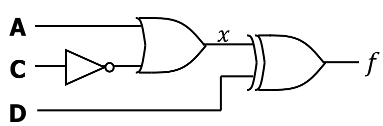
✓ $F(x,y,z) = \prod(M_0, M_1, M_4, M_6) = \sum(m_2, m_3, m_5, m_7) = \bar{X}Y\bar{Z} + \bar{X}YZ + X\bar{Y}Z + XYZ = \bar{X}Y(\bar{Z}+Z) + XZ(\bar{Y}+Y)$
$= \bar{X}Y + XZ$



✓ $F = \overline{x(y\oplus\bar{z}) + \bar{y}} = \overline{x(y\oplus\bar{z})}.y = (\bar{x} + y\oplus z)y = (\bar{x} + y\bar{z} + \bar{y}z)y = \bar{x}y + y\bar{z}$



✓ $F = (A + \bar{C} + D)(\bar{A}C + \bar{D}) = (X + D)(\bar{X} + \bar{D}) = X\bar{D} + \bar{X}D, \quad X = A + \bar{C}$
$= (A + \bar{C})\bar{D} + \bar{A}CD = A\bar{D} + \bar{C}\bar{D} + \bar{A}CD$



b) Determine whether or not the following expression is valid, i.e., whether the left- and right-hand sides represent the same function. Suggestion: complete the truth tables for both sides: (5 pts)

$$\overline{x_1}x_2 + x_1x_3 + \overline{x_2}\,\overline{x_3} = \overline{x_1}\,\overline{x_3} + x_2x_3 + x_1\overline{x_2}$$

Left-hand side:

$\overline{x_1}x_2(x_3+\overline{x_3}) + x_1(x_2+\overline{x_2})x_3 + (x_1+\overline{x_1})\overline{x_2}\,\overline{x_3} = \overline{x_1}x_2x_3 + \overline{x_1}x_2\overline{x_3} + x_1x_2x_3 + x_1\overline{x_2}x_3 + x_1\overline{x_2}\,\overline{x_3} + \overline{x_1}\,\overline{x_2}\,\overline{x_3}$
$= \sum m(0,2,3,4,5,7)$

Right-hand side:

$\overline{x_1}(x_2+\overline{x_2})\overline{x_3} + (x_1+\overline{x_1})x_2x_3 + x_1\overline{x_2}(x_3+\overline{x_3}) = \overline{x_1}x_2\overline{x_3} + \overline{x_1}\,\overline{x_2}\,\overline{x_3} + x_1x_2x_3 + \overline{x_1}x_2x_3 + x_1\overline{x_2}x_3 + x_1\overline{x_2}\,\overline{x_3}$
$= \sum m(0,2,3,4,5,7)$

Both left-hand and right-hand equations represent the same Boolean function.

c) For the following Truth table with two outputs: (8 pts)
- Provide the Boolean functions using the Canonical Sum of Products (SOP), and Product of Sums (POS). (4 pts)
- Express the Boolean functions using the minterms and maxterms representations.
- Sketch the logic circuits as Canonical Sum of Products and Product of Sums. (3 pts)

x	y	z	f_1	f_2
0	0	0	0	0
0	0	1	1	1
0	1	0	1	0
0	1	1	0	0
1	0	0	1	0
1	0	1	0	0
1	1	0	0	1
1	1	1	1	1

Sum of Products

$f_1 = \bar{x}\bar{y}z + \bar{x}y\bar{z} + x\bar{y}\bar{z} + xyz$

$f_2 = \bar{x}\bar{y}z + xy\bar{z} + xyz$

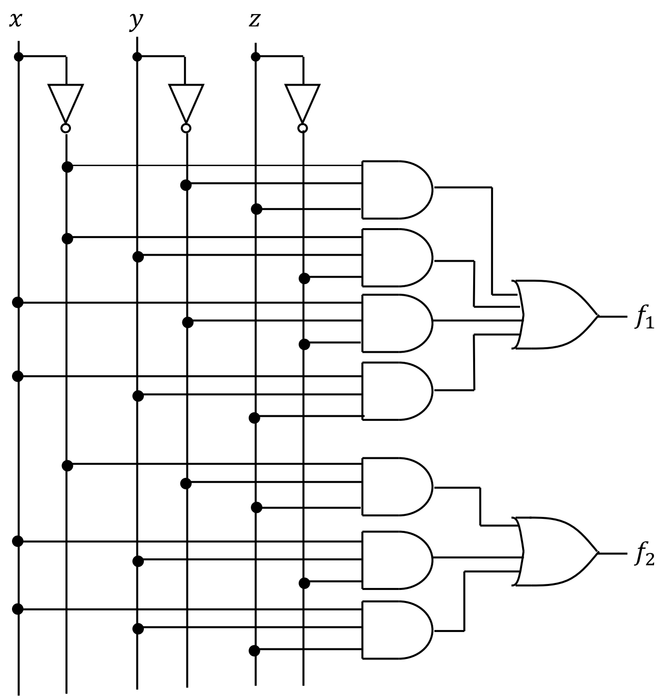
Product of Sums

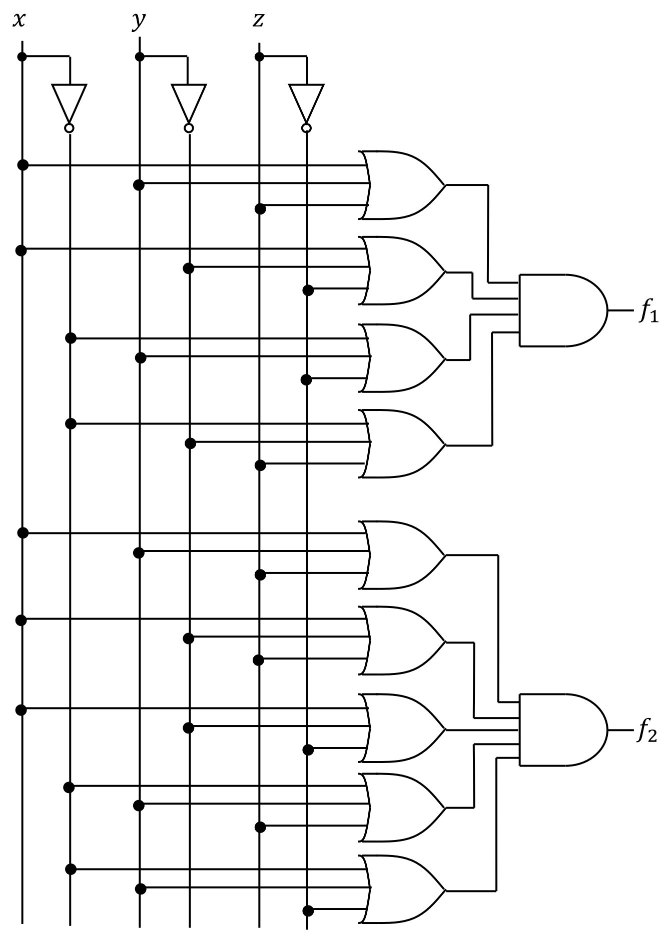
$f_1 = (x + y + z)(x + \bar{y} + \bar{z})(\bar{x} + y + \bar{z})(\bar{x} + \bar{y} + z)$

$f_2 = (x + y + z)(x + \bar{y} + z)(x + \bar{y} + \bar{z})(\bar{x} + y + z)(\bar{x} + y + \bar{z})$

Minterms and maxterms: $f_1 = \sum(m_1, m_2, m_4, m_7) = \prod(M_0, M_3, M_5, M_6)$.

$f_2 = \sum(m_1, m_6, m_7) = \prod(M_0, M_2, M_3, M_4, M_5)$.





PROBLEM 2 (10 PTS)

- The following is a truth table for logic functions f_1 and f_2. Note that an 'X' on the input means that the logical value can be either '0' of '1'. So, if the input $xyzw$ is 01XX, it means that for the output f_1 to be 1, we only need $x = 0$ and $y = 1$.

  ✓ Provide the simplified Boolean expressions for f_1 and f_2.

x	y	z	w	f_1	f_2
1	X	X	X	1	1
0	1	X	X	1	0
0	0	1	X	0	1
all others				0	0

$f_1 = x.f(y,z,w) + \bar{x}y.g(z,w)$

$f_2 = x.f(y,z,w) + \bar{x}\bar{y}z.h(w)$

$f(y,z,w) = \sum m(0,1,2,3,4,5,6,7) = 1,\ g(z,w) = \sum m(0,1,2,3) = 1,\ h(w) = w + \bar{w} = 1$

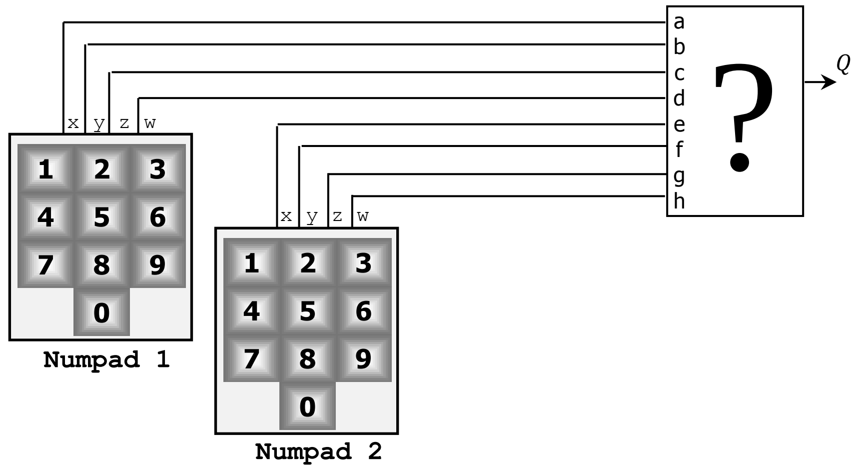
Then: $f_1 = x + \bar{x}y = x + y$

$f_2 = x + \bar{x}\bar{y}z = x + \bar{y}z$

PROBLEM 3 (11 PTS)

- We want to design a logic circuit that opens a lock ($Q = 1$) whenever the user presses the correct number on each numpad (numpad 1: **8**, numpad2: **3**). The numpad encodes each decimal number using BCD encoding (see figure). We expect that the 4-bit groups generated by each numpad be in the range from `0000` to `1001`. Note that the values from `1010` to `1111` are assumed not to occur.
- Provide the simplified expression for $Q(a,b,c,d,e,f,g,h)$ and sketch the logic circuit.
  Suggestion: Create two circuits: one that verifies the first number (**8**), and another that verifies the second number (**3**). Then perform the AND operation on the two outputs. This avoids creating a truth table with 8 inputs.

Number pressed	BCD code x	y	z	w
0	0	0	0	0
1	0	0	0	1
2	0	0	1	0
3	0	0	1	1
4	0	1	0	0
5	0	1	0	1
6	0	1	1	0
7	0	1	1	1
8	1	0	0	0
9	1	0	0	1



---

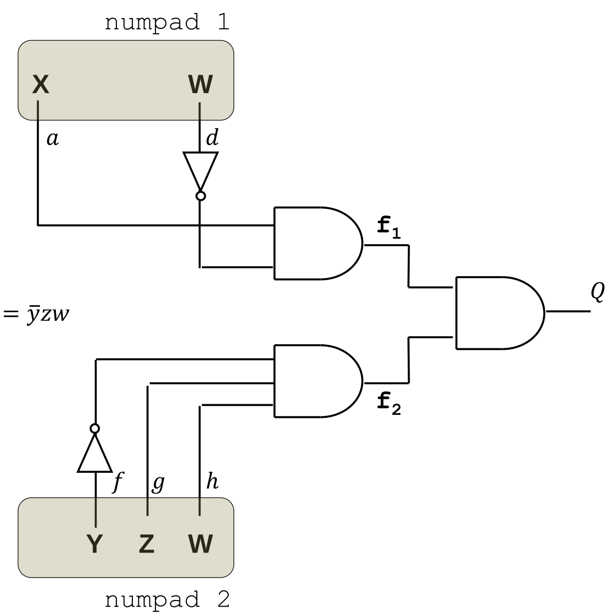
x	y	z	w	f_1	f_2
0	0	0	0	0	0
0	0	0	1	0	0
0	0	1	0	0	0
0	0	1	1	0	1
0	1	0	0	0	0
0	1	0	1	0	0
0	1	1	0	0	0
0	1	1	1	0	0
1	0	0	0	1	0
1	0	0	1	0	0
1	0	1	0	X	X
1	0	1	1	X	X
1	1	0	0	X	X
1	1	0	1	X	X
1	1	1	0	X	X
1	1	1	1	X	X

zw \ xy	00	01	11	10
00	0	0	X	1
01	0	0	X	0
11	0	0	X	X
10	0	0	X	X

$f_1 = x\overline{w}$

zw \ xy	00	01	11	10
00	0	0	X	0
01	0	0	X	0
11	1	0	X	X
10	0	0	X	X

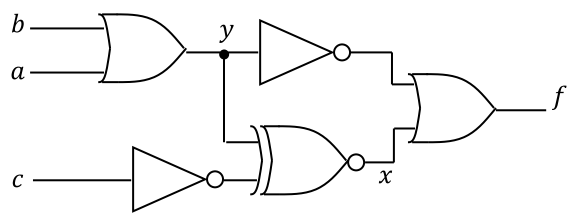
$f_2 = \overline{y}zw$

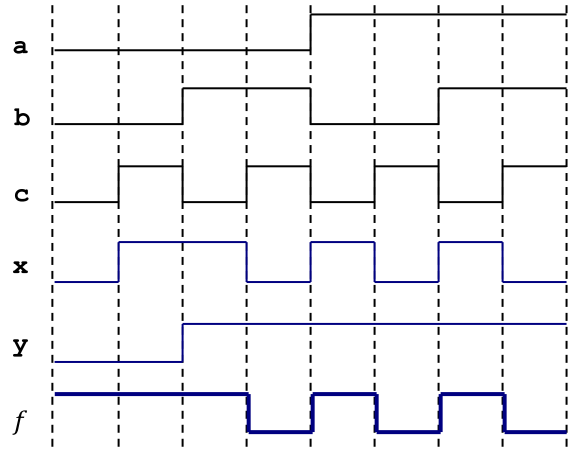


$Q = a\overline{d}\overline{f}gh = (a\overline{d})(\overline{f}g)h$

PROBLEM 4 (26 PTS)

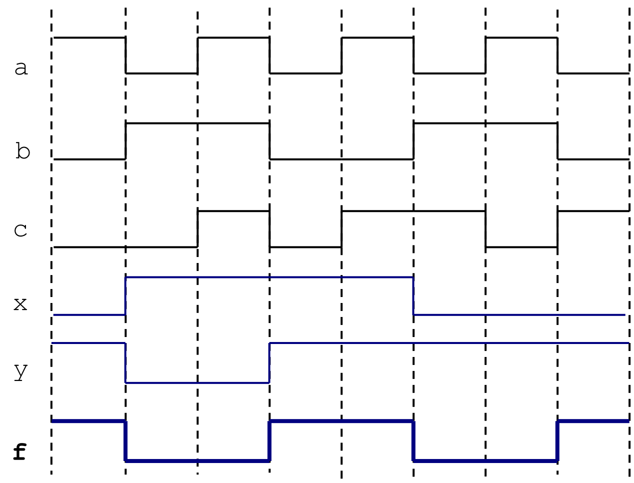
a) Complete the timing diagram of the following circuit: (5 pts)





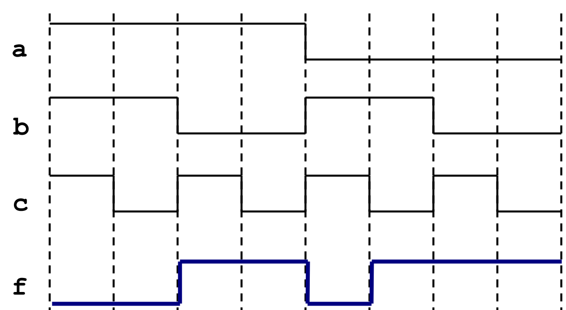
b) Complete the timing diagram of the logic circuit whose VHDL description is shown below: (7 pts)

```
library ieee;
use ieee.std_logic_1164.all;

entity circ is
  port ( a, b, c: in std_logic;
         f: out std_logic);
end circ;

architecture struct of circ is
  signal x, y: std_logic;

begin
  f <= y and (not b);
  x <= not(a) xor c;
  y <= x nand b;
end struct;
```



c) The following is the timing diagram of a logic circuit with 3 inputs. Sketch the simplified logic circuit that generates this waveform. Then, complete the VHDL code. (8 pts)

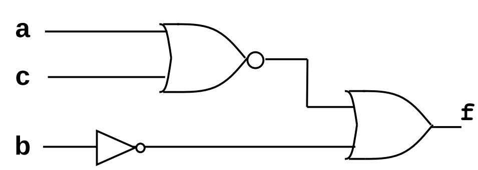
```
library ieee;
use ieee.std_logic_1164.all;

entity wav is
  port ( a, b, c: in std_logic;
            f: out std_logic);
end wav;

architecture struct of wav is

begin
   f <= not(b) or (a nor c)
end struct;
```



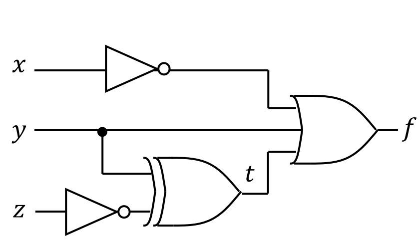
a	b	c	f
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	0

c \ ab	00	01	11	10
0	1	1	0	1
1	1	0	0	1

$f = \bar{b} + \bar{a}\bar{c} = \bar{b} + \overline{a + c}$



d) Construct the truth table describing the output of the following circuit and write the simplified Boolean equation (6 pts).



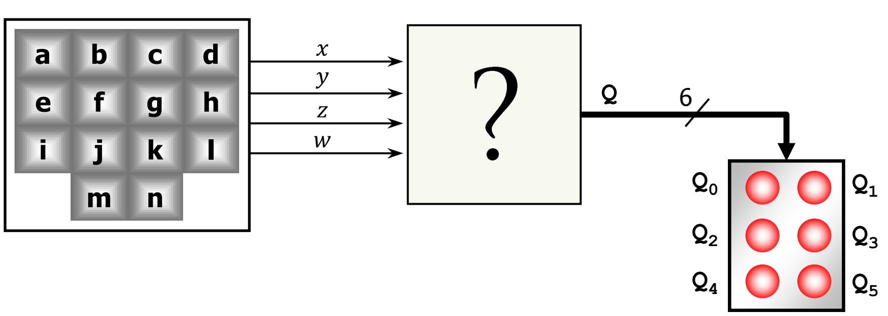
x	y	z	t	f
0	0	0	1	1
0	0	1	0	1
0	1	0	0	1
0	1	1	1	1
1	0	0	1	1
1	0	1	0	0
1	1	0	0	1
1	1	1	1	1

$f = \bar{x} + y + \bar{z}$

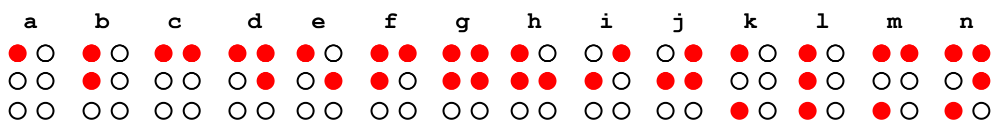
PROBLEM 5 (25 PTS)

- A 14-letter keypad produces a 4-bit code as shown in the table. We want to design a logic circuit that converts those 4-bit codes to Braille code, where the 6 dots are represented by LEDs. A raised (or embossed) dot is represented by an LED ON (logic value of '1'). A missing dot is represented by a LED off (logic value of '0').

✓ Complete the truth table for each output (Q_0-Q_5). (4 pts)
✓ Provide the simplified expression for each output (Q_0-Q_5). Use Karnaugh maps for Q_5,Q_4,Q_1,Q_0 and the Quine-McCluskey algorithm for Q_3-Q_2. Note it is safe to assume that the codes `1110` and `1111` will not be produced by the keypad.



x	y	z	w	Letter
0	0	0	0	a
0	0	0	1	b
0	0	1	0	c
0	0	1	1	d
0	1	0	0	e
0	1	0	1	f
0	1	1	0	g
0	1	1	1	h
1	0	0	0	i
1	0	0	1	j
1	0	1	0	k
1	0	1	1	l
1	1	0	0	m
1	1	0	1	n
1	1	1	0	
1	1	1	1	



x	y	z	w	Q_5	Q_4	Q_3	Q_2	Q_1	Q_0	Letter
0	0	0	0	0	0	0	0	0	1	a
0	0	0	1	0	0	0	1	0	1	b
0	0	1	0	0	0	0	0	1	1	c
0	0	1	1	0	0	1	0	1	1	d
0	1	0	0	0	0	1	0	0	1	e
0	1	0	1	0	0	0	1	1	1	f
0	1	1	0	0	0	1	1	1	1	g
0	1	1	1	0	0	1	1	0	1	h
1	0	0	0	0	0	0	1	1	0	i
1	0	0	1	0	0	1	1	1	0	j
1	0	1	0	0	1	0	0	0	1	k
1	0	1	1	0	1	0	1	0	1	l
1	1	0	0	0	1	0	0	1	1	m
1	1	0	1	0	1	1	0	1	1	n
1	1	1	0	X	X	X	X	X	X	
1	1	1	1	X	X	X	X	X	X	

Q_4

zw \ xy	00	01	11	10
00	0	0	1	0
01	0	0	1	0
11	0	0	X	1
10	0	0	X	1

Q_1

zw \ xy	00	01	11	10
00	0	0	1	1
01	0	1	1	1
11	1	0	X	0
10	1	1	X	0

Q_0

zw \ xy	00	01	11	10
00	1	1	1	0
01	1	1	1	0
11	0	1	X	1
10	1	1	X	1

$Q_5 = 0$
$Q_4 = xz + xy$
$Q_3 = \bar{x}y\bar{w} + \bar{x}zw + x\bar{z}w$

$Q_2 = yz + \bar{x}\bar{z}w + x\bar{y}\bar{z} + x\bar{y}w$
$Q_1 = \bar{x}\bar{y}z + y\bar{z}w + \bar{x}z\bar{w} + x\bar{z}$
$Q_0 = \bar{x} + y + z$

- $Q_3 = \sum m(3,4,6,7,9,13) + \sum d(14,15)$.

Number of ones	4-literal implicants	3-literal implicants	2-literal implicants	1-literal implicants
1	m_4 = 0100 ✓	**$m_{4,6}$ = 01-0**	**$m_{6,14,7,15}$ = -11-** ~~$m_{6,7,14,15}$ = -11-~~	We can't combine any further, so we stop here
2	m_3 = 0011 ✓ m_6 = 0110 ✓ m_9 = 1001 ✓	**$m_{3,7}$ = 0-11** $m_{6,7}$ = 011- ✓ $m_{6,14}$ = -110 ✓ **$m_{9,13}$ = 1-01**		
3	m_7 = 0111 ✓ m_{13}= 1101 ✓ m_{14}= 1110 ✓	$m_{7,15}$ = -111 ✓ **$m_{13,15}$ = 11-1** $m_{14,15}$ = 111- ✓		
4	m_{15}= 1111 ✓			

Prime Implicants		Minterms					
		3	4	6	7	9	13
$m_{4,6}$	$\bar{x}y\bar{w}$		**X**	X			
$m_{3,7}$	$\bar{x}zw$	**X**			X		
$m_{9,13}$	$x\bar{z}w$					**X**	X
$m_{13,15}$	xyw						X
$m_{6,14,7,15}$	yz			X	X		

$\rightarrow Q_3 = \bar{x}y\bar{w} + \bar{x}zw + x\bar{z}w$

- $Q_2 = \sum m(1,5,6,7,8,9,11) + \sum d(14,15)$.

Number of ones	4-literal implicants	3-literal implicants	2-literal implicants	1-literal implicants
1	m_1 = 0001 ✓ m_8 = 1000 ✓	**$m_{1,5}$ = 0-01** **$m_{1,9}$ = -001** **$m_{8,9}$ = 100-**	**$m_{6,7,14,15}$ = -11-** ~~$m_{6,14,7,15}$ = -11-~~	We can't combine any further, so we stop here
2	m_5 = 0101 ✓ m_6 = 0110 ✓ m_9 = 1001 ✓	**$m_{5,7}$ = 01-1** $m_{6,7}$ = 011- ✓ $m_{6,14}$ = -110 ✓ **$m_{9,11}$ = 10-1**		
3	m_7 = 0111 ✓ m_{11}= 1011 ✓ m_{14}= 1110 ✓	$m_{7,15}$ = -111 ✓ **$m_{11,15}$ = 1-11** $m_{14,15}$ = 111- ✓		
4	m_{15}= 1111 ✓			

Prime Implicants		Minterms						
		1	5	6	7	8	9	11
$m_{1,5}$	$\bar{x}\bar{z}w$	**X**	X					
$m_{1,9}$	$\bar{y}\bar{z}w$	X					X	
$m_{8,9}$	$x\bar{y}\bar{z}$					**X**	X	
$m_{5,7}$	$\bar{x}yw$		X		X			
$m_{9,11}$	$x\bar{y}w$						X	**X**
$m_{11,15}$	xzw							X
$m_{6,7,14,15}$	yz			**X**	X			

$\rightarrow Q_2 = yz + \bar{x}\bar{z}w + x\bar{y}\bar{z} + x\bar{y}w$