GFSK Demodulation Using Sequential Monte Carlo Technique

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***Abstract*—This letter applies the Sequential Monte Carlo (SMC) approach along with a low complex detection algorithm to non-coherently demodulate a Gaussian frequency shift keying (GFSK) signal in Bluetooth receivers. Sequential importance resampling (SIR) is used to estimate the state of the GFSK signal, while a low complex detection algorithm is used to decide on the received bit. SIR is the original and most used particle filtering algorithm, which approximates the filtering distribution by a weighted set of particles. The bit error rate (BER) analysis is used as the performance metrics of the proposed receiver at the physical layer. Applying SMC technique to GFSK demodulator shows significant performance improvement compared to several other existing techniques that are presented in the literature. In addition, it does not require a pre-detection filter, which is usually used to reject out-of-band interference. Simulation results show approximately 6 dB of BER improvement over the commonly used limiter discriminator with an integrator (LDI) receiver.**

***Index Terms*—Bluetooth, Gaussian frequency shift keying, sequential Monte Carlo, sequential importance resampling.**

I. Introduction

In signal processing, receivers designs have been profoundly focused on by many researchers because of the importance nature of interpreting the correct data at the receiving end. Designing non-coherent and high power efficient receivers have been the ultimate goal of researchers.

Bluetooth [1] devices often employ a simple limiter discriminator with an integrator (LDI) receiver for detection of the GFSK-modulated data [2]. Other approaches that are proposed in the literature such as optimized differential receiver [3], and Bluetooth zero-crossing matched filter (BT-ZXMF) receiver [4] are simple to implement but suffer from low power efficiency. The power efficiency is defined as the achievable bit error rate (BER) for a certain signal-to-noise power ratio (SNR). The zero-crossing detector detects the time instants of the received signal at which the received signal equals to zero and has a positive slope. The optimized differential detector focuses on taking the difference between the averages of the first and the last 35% of the received phase curves after performing a phase unwrapping algorithm on them. Both techniques show approximately 1–2 dB performance improvement over the LDI.

Other Bluetooth receivers consider Viterbi algorithm to search for the minimum Euclidean distance path of the state trellis while assuming a nominal value for modulation index h [5], [6]. For Bluetooth, the modulation index has a relatively wide range of variation which ranges between 0.28 and 0.35; such variations results in varying trellis structures for sequence detection. These receivers achieve significant performance improvements over the LDI, but a small mismatch between the actual and the assumed h results in severe performance degradation. Furthermore, Scholand *et al.* proposed a new receiver design that searches through four-state trellis by applying the max-log-maximum-likelihood (MLM) symbol estimation after processing the received signal through an LDI receiver [7]. The MLM-LDI receiver improves the LDI performance by 4 dB. In [8], Lampe *et al.* proposed a new receiver that consisted of a single filter and a subsequent non-coherent sequence detector (NSD) that operates on a two-state trellis, achieving an improvement of more than 4 dB compared to the LDI. In [9], Ibrahim *et al.* presented a receiver design based on Laurents decomposition. Although the receivers performance has significant improvement compared to the LDI, a mismatch in the modulation modulation index causes the receiver to search for an adaptation period, which causes significant performance degradation.

In this article, a new GFSK demodulation technique is developed by applying Sequential Monte Carlo (SMC) to estimate the state of the GFSK signal, and a low complex detection algorithm is used to decide on the received bit. The proposed receiver does not require prior knowledge of the modulation index or a pre-detection filter to reject out-of-band interference. The proposed receiver significantly outperforms some other techniques that were presented in the literature, and it was compared to the GFSK theoretical performance limit known as the maximum likelihood sequence detection (MLSD) [10].

The remainder of the letter is organized as follows. In Section II, introduction of the system definition will be presented. Section III will briefly introduce Bluetooth transmission channel. In Section IV, a receiver based on SIR filtering will be proposed. Simulation results and discussion will be presented in Section V, followed by the conclusions in Section VI.

II. System Definition

A passband transmitted GFSK signal can be expressed as follows [10], [11]

$$s(t, \boldsymbol{\alpha}, h) = \sqrt{\frac{2E_s}{T}} \cos\left\{2\pi f_0 t + \varphi(t, \boldsymbol{\alpha}, h) + \varphi_0\right\}, \qquad (1)$$

where E_s denotes the signal energy per modulation interval T, f_0 is the carrier frequency, φ_0 is an arbitrary constant phase shift,

Manuscript received March 25, 2015; revised August 10, 2015; accepted August 28, 2015. Date of publication September 3, 2015; date of current version December 15, 2015. The associate editor coordinating the review of this paper and approving it for publication was G. M. Vitetta.

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Digital Object Identifier 10.1109/LWC.2015.2476460

and $\varphi(t, \boldsymbol{\alpha}, h)$ is the continuous phase of the signal that can be expressed as [10]

$$\varphi(t, \boldsymbol{\alpha}, h) = 2\pi h \sum_{k=n-L+1}^{n} q(t - kT\alpha_k) + \pi h \sum_{k=-\infty}^{n-L} \alpha_k, \quad (2)$$

where the total number of transmitted bits are $nT \le t \le (n+1)T$, h is the modulation index, $\boldsymbol{\alpha} = \ldots, \alpha_{-2}, \alpha_{-1}, \alpha_0, \alpha_1, \alpha_2, \ldots$ is an infinitely long sequence of independent and identically distributed data symbols, belong to the alphabet $\alpha_k \in \{\pm 1\}$. L is the length of the frequency pulse $g(t)$ (e.g. $L = 2$ for Bluetooth). The normalized phase pulse $q(t) = \int_{-\infty}^{t} g(\tau)d\tau$ is obtained from integrating the frequency impulse $g(t)$ [8]

$$g(t) = \frac{1}{2T}\left(Q\left(c.B\left(t - \frac{T}{2}\right)\right) - Q\left(c.B\left(t + \frac{T}{2}\right)\right)\right),$$

with constant $c = 2\pi/\sqrt{\log(2)}$, BT is the time bandwidth product of the pre-modulation filter that corresponds to a minimum carrier separation to ensure orthogonality between signals in adjacent channels, and Q() is the Gaussian Q-function. In Bluetooth standard, the 3 dB bandwidth-time product is specified as $BT = 0.5$, with $T = 10^{-6}$.

III. Bluetooth Transmission Channel

An AWGN channel is a transmission channel that adds white Gaussian noise to a signal. The ratio between the signal's power i.e. P_{signal} and the noise's power i.e. P_{noise} is expressed by

$$SNR_{dB} = 10 \log\left(\frac{P_{signal}}{P_{noise}}\right),$$

This type of channel has been widely considered for Bluetooth performance for several reasons that were described in [12], and they are listed again for completeness. (1) Because Bluetooth communication is deployed mainly in an indoor environment, fast fading has only a slight impact on Bluetooth performance and can be neglected. (2) Line of sight communication from the receiver to the transmitter exists due to Bluetooth transmission power restrictions. (3) Interfering signals that might affect the performance of Bluetooth, such as IEEE 802.11b operating in the same ISM band, can be approximated by noise with a constant spectral density.

IV. Proposed Receiver

A. Received Signal

The complex envelope of the received signal may be expressed as [8], [13]

$$y(t) = s(t, \boldsymbol{\alpha}, h)e^{j\psi(t)} + \gamma(t), \quad (3)$$

where $\psi(t)$ is the phase rotation introduced by the channel, modeled as as continuous-time Wiener process with incremental variance over a signaling interval equal to $\sigma_{\triangle}^2$. $\psi(t)$ is assumed to be slowly varying such that it can be considered constant over the symbol period. $\gamma(t)$ is a complex-valued Gaussian white noise process with independent components, each with two sided power density A_0 [8], [11].

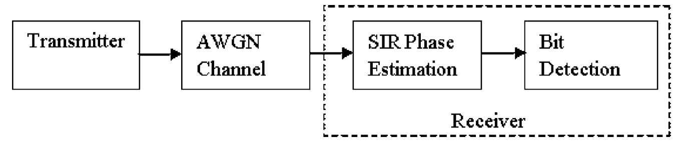

Fig. 1. Receiver architecture design.

B. GFSK State and Measurement Equations

Sampling the received signal (Eq. (3)) at time kT, y_k can be expressed as [14]

$$y_k = s(k, \boldsymbol{\alpha_k}, h_k)e^{j\psi_k} + \gamma_k. \quad (4)$$

Using the sampled received signal from (Eq. (4)), a state space transition model is derived to non-coherently estimate the phase of the GFSK modulated signal. Let $\theta_k \in \Re^{n_\theta}$ and $y_k \in \Re^{n_y}$ denote the state and measurement representation of the state space model respectively. n_θ and n_y are the dimensions of the state and measurement vectors [15]. Assuming that the state is equal to the sampled received phase θ_k, and there is no external effect to the system i.e. $u() = 0$, the state space transition and observation model may be expressed as [15], [16]

$$\theta_k = \arg\left(s(k, \boldsymbol{\alpha_k}, h_k)\right) + \psi_k \quad (5)$$

$$y_k = z_k \exp(j\theta_k) + \gamma_k, \quad (6)$$

where the sampled amplitude of the received sampled signal given by $z_k = \sqrt{\frac{2E_s}{T}}$.

C. Particle Filter Algorithm

The proposed receiver architecture is shown in Fig. 1. The essence of particle filtering is to estimate the posterior density function as a weighted set of samples [15], [16],

$$\hat{p}(\theta_k|y_{1:k}) = \sum_{i=1}^{N} w_k^i \delta\left(\theta_k - \theta_k^i\right), \quad (7)$$

where N is the number of resampling performed on each particle, δ is the Dirac delta distribution, and w_k^i is the normalized importance weight associated with particle θ_k^i. To evaluate the above expression of the weight, an alternative representation of the state space model that describes the dynamics and measurement relationship using probability density functions is expressed as [16]–[18]

$$\theta_{k+1} \sim p(\theta_{k+1}|\theta_k), \quad (8)$$

$$y_k \sim p(y_k|\theta_k), \quad (9)$$

where $p(\theta_{k+1}|\theta_k)$ and $p(y_k|\theta_k)$ denote the probability of the state and measurement distribution. Following the derivations from [16], [17], the normalized importance weight can be calculated by

$$w_k^i = \frac{p\left(y_k|\theta_k^i\right)p\left(\theta_k^i|\theta_{k-1}\right)}{q\left(\theta_k^i|\theta_{k-1}^i, y_t\right)} w_{k-1}^i, \quad (10)$$

TABLE I
SIR ALGORITHM

1. Initialization: Initialize the particles, $\theta_0^i \sim p(\theta_0^i), i = 1,, N$ and the weights, $w_0^i = \frac{1}{N}, i = 1, ..., N$ and let $k := 1$.
2. Measurement update: For $i = 1, ..., N$, evaluate the importance weights $w_k^i = p(y_k\|\theta_k^i)w_{k-1}^i$ according to the prior.
3. Estimation: The filtering density is approximated $\hat{p}(\theta_{1:k}\|y_{1:k}) = \sum_{i=1}^N w_{k\|k}^i \delta(\theta_{1:k} - \theta_{1:k}^i)$. For each $i = 1,, N$ draw a new particle θ_k^i according to, $P\left(\theta_k^i = \theta_k^j\right) = w_k^j, j = 1, ..., N$.
4. Resampling: If $\hat{w}_k^i =< Threshold$, resample θ_k^i with replacement according to $Pr(\theta_{k\|k}^i = \theta_{k\|k-1}^j) = \hat{w}_k^j$. 5. Time update: Generate preiction according to the proposal distribution $\theta_{k+1}^i \sim q(\theta_{k+1}\|\theta_k^i, y_{k+1})$, where $q(\theta_k\|\theta_{k-1}^i, y_k) = p(\theta_k\|\theta_{k-1}^i)$.
6. Set $k = k + 1$ and repeat from step 2.

where $p(\theta_k^i|\theta_{k-1})$ is the prior of θ_k. $q(\theta_k|\theta_{k-1}^i, y_t)$ is a proposal distribution. For SIR, choosing the right proposal distribution is very crucial step since there are infinite number of choices. The optimal proposal distribution is the one that minimizes the variance of the importance weights conditional on $\theta_{0:k-1}$ and $y_{1:k}$. In practice, however, finding the optimal proposal is very difficult if not impossible. Instead, the SIR filter have chosen to trade the optimality with easy implementation by using the conditional prior of the state vector as proposal distribution i.e. $q(\theta_k^i|\theta_{k-1}^i, y_t) = p(\theta_k^i|\theta_{k-1})$. Equation (10) reduces to [16], [17]

$$w_k^i = p\left(y_k|\theta_k^i\right) w_{k-1}^i. \tag{11}$$

Assuming the initial density function $p(\theta_0^i)$ is known and θ_0^i is the initial state, then initialization of particles and their weights can be performed by

$$\theta_0^i \sim p\left(\theta_0^i\right), \qquad i = 1, \ldots, N. \tag{12}$$

$$w_0^i = \frac{1}{N}, \qquad i = 1, \ldots, N. \tag{13}$$

Substituting equations (14) and (15) into equation (11) produce,

$$\hat{p}(\theta_0) = \sum_{i=1}^{N} \frac{1}{N} \delta\left(\theta_k - \theta_0^i\right). \tag{14}$$

For all $k \geq 1$, we assume that the following estimation

$$\hat{p}(\theta_{k-1}|y_{1:k-1}) = \sum_{i=1}^{N} w_{k-1}^i \delta\left(\theta_{k-1} - \theta_{k-1}^i\right). \tag{15}$$

is available. Table I summarizes the sequence of the SIR algorithm [16].

D. Bit Decision Algorithm

Similar to the conventional differential demodulator technique [10, Ch. 7], the decision algorithm depends on obtaining the instantaneous change $\triangle\hat{\theta}_k$ from the estimated states by calculating the difference between the current $\hat{\theta}_k$ and the neighboring state $\hat{\theta}_{k-1}$ [3]

$$\triangle\hat{\theta}_k = \hat{\theta}_k - \hat{\theta}_{k-1}. \tag{16}$$

The decision then can be made on the sign of $\triangle\hat{\theta}_k$ i.e. "1" if the sign is positive, "0" otherwise [3], [10]. However, due to the phase wrapping issue, the estimated states $\hat{\theta}_k$ are unwrapped by simply adding $\pm 2\pi$ when the absolute change between the consecutive phase samples are greater than the jump tolerance π [3].

TABLE II
COMPLEXITY ANALYSIS OF SIR ALGORITHM

Big O notation of SIR Algorithm		
Step Name	SIR Algorithm	Computational Ste
SIR Initialize	$\theta_0^i \sim p(\theta_0^i)\ w_0^i = \frac{1}{N}$	$O(1)$
Measurement update	$\hat{\theta}_k^i \sim q(\theta_k\|\theta_{k-1}^i, y_k)$ $w_k^i = p(y_k\|\theta_k^i)w_{k-1}^i$	$O(N)$
Estimation	$w_k^i = \frac{p\left(y_k\|\theta_k^i\right)\hat{w}_t^i}{\sum_{j=1}^N p\left(y_k\|\theta_k^j\right)\hat{w}_k^j}$	$O(N^2)$
Resampling	$\hat{\theta}_k^i \sim \sum_{i=1}^N w_{k-1}^i \delta(\theta_{k-1} - \theta_{k-1}^i)$	$O(Nlog_2(N))$
Time update	$\theta_{k+1}^i \sim q(\theta_{k+1}\|\theta_k^i, y_{k+1})$	$O(N)$
Total		$O(N^2)$

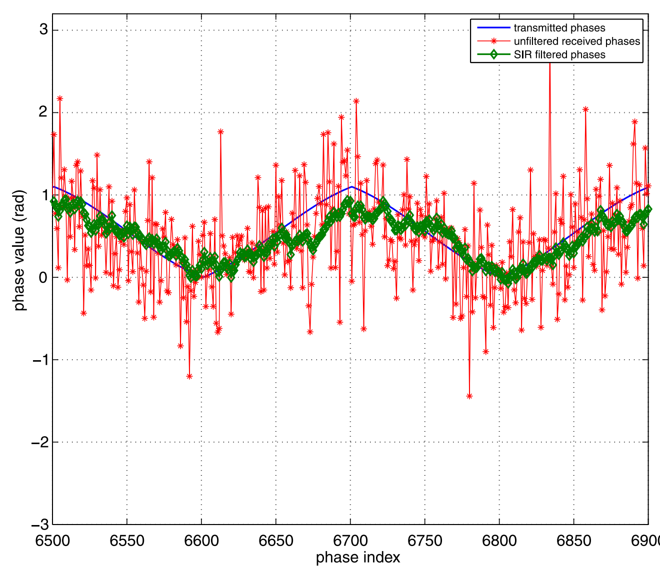

Fig. 2. Transmitted phases vs. Phase estimation using SIR with $N = 10$, $h = 1/3$, and $SNR = 17$.

E. Complexity Analysis of SIR Algorithm

The complexity analysis of the SIR filter is described in Table II. The table shows the Big O notation of each step of the SIR filter. The overall complexity the proposed receiver is $O(N^2)$ [16].

V. PERFORMANCE RESULTS AND DISCUSSION

In this section, the performance of the proposed receiver is evaluated with extensive MATLAB simulations. Fig. 2 shows the comparison between the transmitted phases, the unfiltered received phases, and the estimated phases using SIR filter while resampling the data 10 times, i.e. $N = 10$, $h = 1/3$, $SNR = 17$. The results show the the SIR filter tracking the transmitted phases very well. In addition, the performance of the proposed receiver is studied and analyzed at the physical level layer in term of bit error rate (BER). Fig. 3 shows results for the SIR particle filter based receiver in term of $10\log_{10}\left(\frac{E_s}{N_0}\right)$ [dB] that is required to achieve $BER = 10^{-3}$, which is the required BER

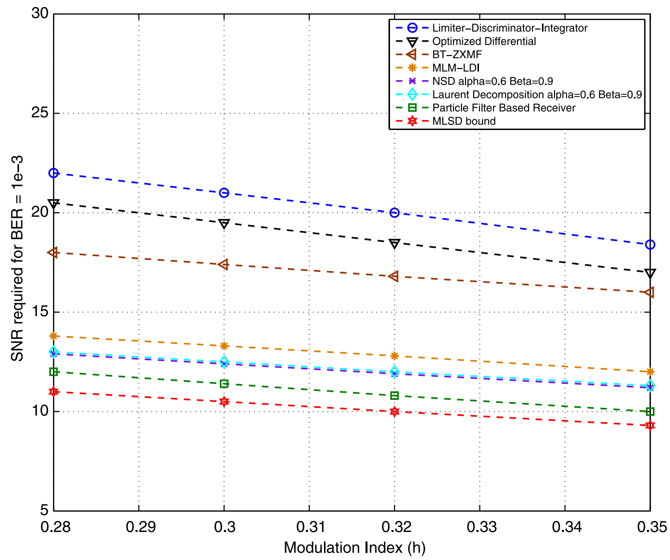

Fig. 3. BER performance of the proposed SIR detector as a function of (h).

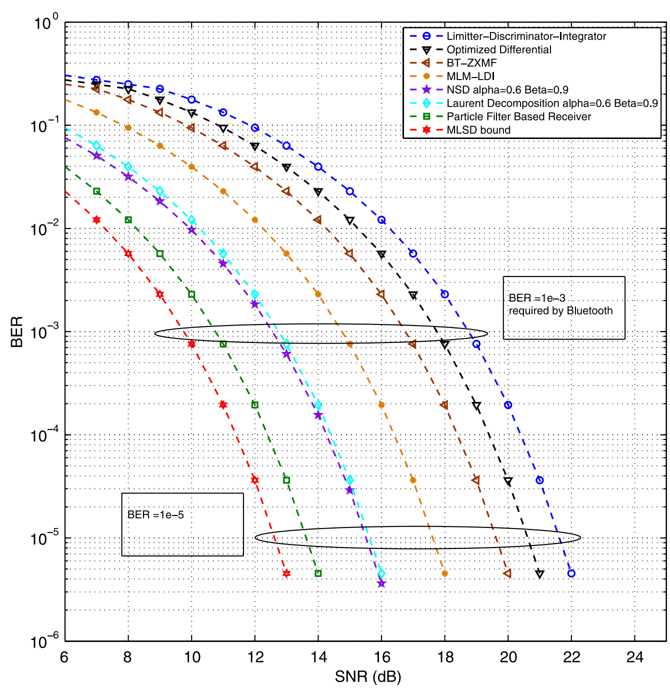

Fig. 4. Performance comparison of several receivers designs in terms of BER for $h = 1/3$.

value in Bluetooth standard [1], as a function of modulation index h. It is also worth noting that for the cases of NSD and Laurent decomposition based receivers, the proposed receivers were only compared to the more realistic and favorable choices and that is when ($\alpha = 0.6$) and ($\beta = 0.9$), suggested by both references [8], [9]. Fig. 3 shows significant improvement in the BER using the new proposed receiver. It outperforms some of the compared receivers. Fig. 4 shows the raw BER vs SNR for several receivers designs presented in the literature compared to the new proposed SIR based receiver for modulation index 1/3. The figure shows the performance improvement using the proposed receiver.

VI. Conclusion

In conclusion, a new GFSK receivers design using particle filtering theory i.e., SIR is proposed in this article. This new proposed receiver can be used as a Bluetooth receiver to better detect and demodulate the received signal. From the results and the analysis of its performance, it can be seen that it outperforms several other receivers that were presented in the literature. The proposed receiver does not require knowledge of the modulation index or a pre-detection filter for detecting GFSK modulated signals. The performance of this demodulator has been studied by extensive MATLAB simulations at the physical levels. The proposed receiver shows approximately 6 dB improvements over the LDI receiver for an AWGN channel noise for different modulation indices. And, it is as close as $\approx$1 dB to the MLSD bound. Also, the complexity of the proposed receiver has been studied and found to be $O(N^2)$.

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