A New Method of Spectrum Sensing in Cognitive Radio Based on Statistical Covariance Matrix

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Abstract. Spectrum sensing is a significant part of technique in a cognitive radio that detecting the presence of primary users in an authorized spectrum. the method that based on the statistical covariance matrix is one of main spectrum sensing techniques, using the difference of statistical covariance between the received signal and noise. In this paper, the new sensing method we proposed is also based on the statistical covariance. The new method compare to some traditional covariance algorithms has decrease the complexity of algorithm, at the same time, ensured the accuracy of detection. We give the statistics of detection, and we also find the threshold of the method when the probability of false alarm is given. The analysis and derivation process of threshold are provided in behind. Using Matlab for simulation to validate the correctness of the method and making the comparison with some typical detection method.

Keywords: Cognitive radio · Statistical covariance matrix · Energy detection Threshold · Eigenvalue

1 Introduction

With the rapid development of wireless communication technologies and low spectrum utilization rate in many frequency bands, which have increased the demand of usage of spectrum resource. To solve this problem, Cognitive Radio (CR) [1, 2] has is a promising technology that exploit the underutilized spectrum in an dynamic access manner and allow secondary users (SUs) to share the licensed spectrum of PU provided that PU is not harmfully interfered [3].

For spectrum sensing, it is the based technique and the key for cognitive radio. However, there are several challenges that we should to overcome. First, the signal-to-noise radio (SNR) may be in a low sensing environment. Second, the sensing problem caused by multipath fading and time dispersion of the wireless channels. Third, the noise uncertainty.

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Common spectrum sensing strategies include the energy detection [4, 5], and the cyclostationary detection [5], the matched filtering detection [6], spectrum sensing based on the statistical covariance matrix. the detection methods above have their advance and shortage. For instance, the matched filtering detection need the prior knowledge [7] of channel and accurate synchronization. Energy detection does not need any prior knowledge of the signal, but the false estimation of noise power could cause the high probability of false alarm.

In this paper, the method of detection that we propose have the better accuracy than energy detection. Comparing with classical algorithm of statistical covariance matrix like the Eigenvalue-Based spectrum sensing algorithms, the complexity of our algorithm is lower than it. The theoretical basis of the method is to assume that the signal samples are correlated. Making full use difference of correlation of signal between PU and noise. The oversampled signal, the time dispersed propagation channel and the correlated original signal are the key to make signal samples being correlated.

The organization of this paper is as follows. Section 2 presents the system model. In Sect. 3 gives the performance analysis and finds thresholds for the algorithms. Simulation results will be given in Sect. 4. Conclusion are drawn in Sect. 5.

2 System Model

$x(t) = s(t) + \eta(t)$ is the continuous-time received signal, where $s(t)$ is the primary user's signal and $\eta(t)$ is the noise. $\eta(t)$ is assumed to be a stationary process satisfying $E(\eta(t)) = 0$, $E(\eta^2(t)) = \sigma^2$, and $E(\eta(t)\eta(t+\tau)) = 0$ for any $\tau \neq 0$. f_s is our sample rate, where $f_s \geq W$. $T_s = 1/f_s$ is the sampling period. To discretize the continuous signal, we define $x(n) = x(nT_s)$, $s(n) = s(nT_s)$, and $\eta(n) = \eta(nT_s)$. There are two hypotheses: 1, H_0 signal does not exit 2, H_1, the signal exits.

$$x_i(n) = \begin{cases} \eta_i(n) & H_0 \\ s(n) + \eta_i(n) & H_1 \end{cases} \tag{1}$$

In this formula $x_i(n)$ denote the nth signal sample taken by ith sensing user(second user), $s(n)$ is the receive signal which is effect by path loss and multipath fading. η_i is the receive white Gaussian noise by which the ith SU is interfered and assumed to be i.i.d (independent and identically distributed), and with mean zero and variance σ_η^2.

Let consider M is the number of SUs, and different SU sample the receive signal from the same primary signal. Then we obtain a matrix.

$$X = [x_1, x_2, x_3, \ldots, x_M]^T,$$

As the same:

$$S = [\mathrm{s}, \mathrm{s}, \ldots, \mathrm{s}]^T,$$

$$\eta = [\eta_1, \eta_2, \ldots, \eta_M]^T,$$

So we can express the matrix as follows.

$$X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_M \end{bmatrix} = \begin{bmatrix} x_1(1) & x_1(2) & \cdots & x_1(N) \\ x_2(1) & x_2(2) & \cdots & x_2(N) \\ \cdots & \cdots & \cdots & \cdots \\ x_M(1) & x_M(2) & \cdots & x_M(N) \end{bmatrix}, \tag{2}$$

Assume s and η are independence. when in situation H_1, consider the Covariance Matrix in M SUs:

$$R_x = E[XX^H]. \tag{3}$$

The Covariance Matrix in of primary signal after channel is

$$R_s = E[SS^H], \tag{4}$$

then

$$R_x = R_s + \sigma^2 I_M, \tag{5}$$

I_M is the identity matrix, σ^2 is the variance of noise.

Because of the statistical covariance matrix can only be acquired by using a limited number of signal samples, we couldn't get R_x accurately. Since R_x can be approximated by the sample covariance matrix defined as

$$R_x(N) = \frac{1}{N}\sum_{n=0}^{N-1} XX^H. \tag{6}$$

In hypothesis H_0, $s(n)$ is not exist, ideally as the result $R_s = 0$, $R_x = \sigma^2 I_M$.

$$R_X = \begin{bmatrix} \sigma^2 & 0 & \ldots & 0 \\ & \sigma^2 & \ddots & \vdots \\ & & \ddots & 0 \\ & & & \sigma^2 \end{bmatrix} \tag{7}$$

In other hand, in hypothesis H_1, $R_s \neq 0$. The elements in Upper and down triangle is not zero. So we can use difference of Covariance Matrix when PU is existed or not to make statistics:

$$T_1 = \frac{1}{M}\sum_{i=1}^{M}\sum_{j=1}^{M} |C_{ij}|, \tag{8}$$

the sum of elements in the matrix divided M

$$T_2 = \frac{1}{M}\sum_{i=1}^{M} |C_{ii}|, \tag{9}$$

the sum of diagonal divided M

$$T_{CDV} = T_1 - T_2, \tag{10}$$

$T_{CDV} = 0$ when primary signal is not existed. (the noise is white Gaussian noise $T_{CDV} \neq 0$ when primary signal is existed. However, it works in an ideal situation, because of limited number of signal samples. So we need setting the threshold.

3 Threshold Determination

As a good detection method, should have the high P_d and low P_{fa}. P_d and P_{fa} largely determine the choice of threshold γ (P_d is the probability of detection, P_{fa} is the probability of false alarm.). How to set the threshold? We don't have any information of the signal (in real sensing scene we do not even know whether the signal is exiting or not), it is difficult to set the threshold based on P_d. As the result, we choose the threshold based on P_{fa}. First we set an ideal P_{fa}. Then, there are two ways to find the threshold based on P_{fa}, theoretical derivation and computer simulation. In this paper we choose the way of theoretical derivation to find the threshold. It is necessary for us to find the statistical distribution of $T_{CDV} = T_1 - T_2$, which is a hard working. However, we get some references to support statistical distribution.

We can get a conclusion from the reference, when the PU signal does not exist, the expectations of statistics of T_1 and T_2 is:

$$E(T_1) = [1 + (M-1)\sqrt{\frac{2}{\pi N}}]\sigma^2, \tag{11}$$

$$E(T_2) = \sigma^2 \; D(T_2) = \frac{2\sigma^4}{N}, \tag{12}$$

To find the threshold based on P_{fa}, we can get probability distribution function:

$$P_{fa} = p_r\{T_{CDV} > \gamma\}$$

$$P_{fa} = p_r\{T_1 - T_2 > \gamma\}$$

When the number of sample is large, we can know from the central limit theorem that T_1 and T_2 follow Gaussian distribution.

$$P_{fa} = p_r\{T_1 - T_2 > \gamma\} \approx p_r\{[1 + (M-1)\sqrt{\frac{2}{\pi N}}]\sigma^2 - T_2 > \gamma\} \tag{13}$$

$$= p_r\{T_2 < [1 + (M-1)\sqrt{\frac{2}{\pi N}}]\sigma^2 - \gamma\}$$

$$= p_r\left\{\frac{T_2 - \sigma^2}{\sqrt{\frac{2}{N}}\sigma^2} < \frac{[1 + (M-1)\sqrt{\frac{2}{\pi N}}]\sigma^2 - \gamma - \sigma^2}{\sqrt{\frac{2}{N}}\sigma^2}\right\}$$

$$= 1 - Q\left[\frac{[1 + (M-1)\sqrt{\frac{2}{\pi N}}]\sigma^2 - \gamma - \sigma^2}{\sqrt{\frac{2}{N}}\sigma^2}\right],$$

So we figure out the γ

$$\gamma = \sigma^2\left[(M-1)\sqrt{\frac{2}{\pi N}} - \sqrt{\frac{2}{N}}Q^{-1}(1 - P_{fa})\right]. \tag{14}$$

The Q function is: $Q(t) = \frac{1}{\sqrt{2\pi}} \int\limits_{t}^{+\infty} e^{\frac{-u^2}{2}} du.$

From the threshold expression we can get the conclusion that if we set the value of the P_{fa}, and know the variance of noise, we can get the value of threshold.

4 Simulation

In this section, we will give the simulation of new method, and we use the receiver operating characteristic (ROC) curve to show the performance of the new method.

1. The simulation of new method in different SNR situation.

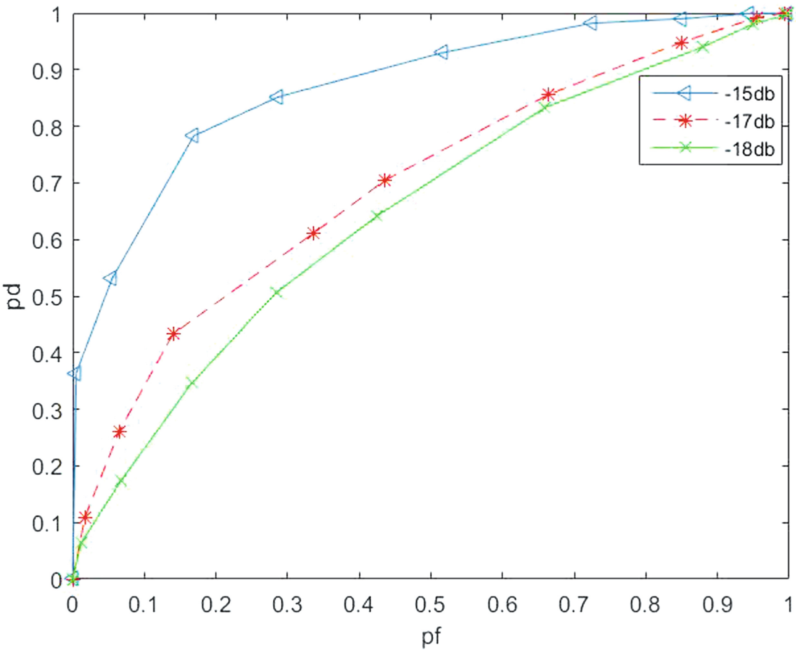

Fig. 1. The simulation of new method in different SNR situation.

From the result of Fig. 1, we can see clearly that the abscissa is the given p_f, the ordinate is the corresponding p_d under the given p_f. The lines respectively represent the simulation is under the SNR of −15 dB, −17 dB, −18 dB. The new method also has the good performance in big SNR situation.

2. The simulation of new method compare to traditional cooperative Energy Detection and Maximum-minimum Eigenvalue Detection.

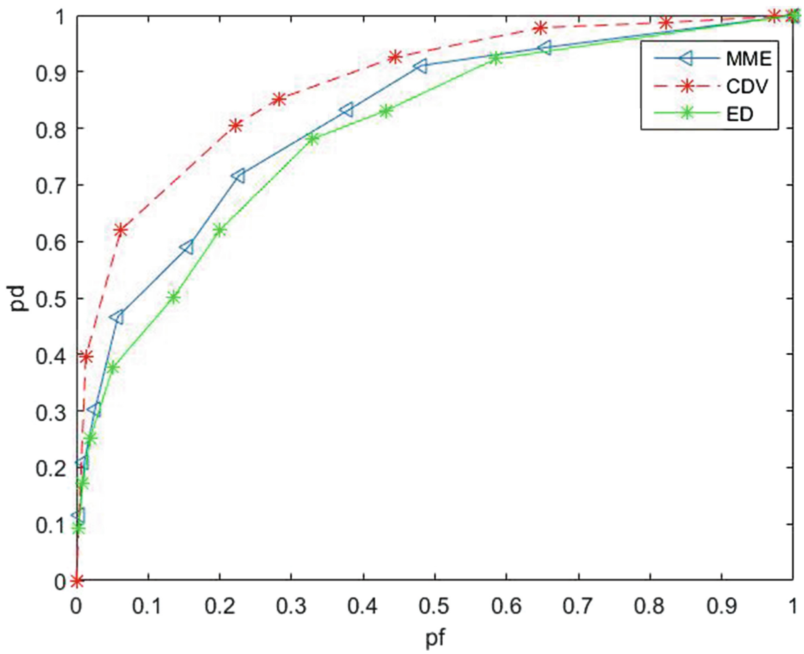

Fig. 2. Compare to the traditional methods

In this simulation Fig. 2, showing the different detected methods. So we can get the result very clearly that the new method is always better than the traditional Energy Detection and Maximum-minimum Eigenvalue Detection. The simulation is under the SNR of −15 dB. The abscissa is the given p_f, the ordinate is the corresponding pd under the given p_f.

5 Conclusion

In this paper, we proposed a new method of spectrum sensing in cognitive radio based on Statistical Covariance. We give a new Statistical algorithm, based on coherence of signal and noise. Then we analysis distribution of this statistic to get the expression of threshold. Last the simulation under the different SNR situation and compare to the other spectrum sensing methods show our new method performance is well.

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