An Improved Blind Spectrum Sensing Algorithm Based on QR Decomposition and SVM

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Abstract. Spectrum sensing, a basic functionality in cognitive radio, aims at detecting the presence or absence of primary user (PU). As one of the most popular spectrum sensing methods, Covariance-based sensing works based on the correlation between signal samples. However, its performance sharply declines in low Signal Noise Ratio (SNR) environment. To improve detection performance of covariance-based sensing as far as possible, an improved blind spectrum sensing scheme is proposed in this paper on the basis of QR matrix decomposition and support vector machine (SVM). In the proposed scheme, QR matrix decomposition is applied to the co-variance matrix of received signal firstly, and then the main features are constituted by extracting and arranging orderly the upper triangular elements of R matrix. After that, SVM is used to conduct the obtained features and determine whether PU exists. The proposed algorithm does not need the prior information of PU and noise. Simulation results demonstrate that the proposed method has a better performance than conventional covariance-based methods, especially in low SNR scenarios.

Keywords: Spectrum sensing · Covariance-based sensing · QR decomposition SVM

1 Introduction

In recent years, spectrum scarcity has caused widespread concern. However, it is reported by FCC that the usage rate of fixed spectrum varies from 15% to 85% [1], which means that the traditional spectrum allocation method leads to the incomplete utilization of spectrum. To solve the problem, cognitive radio (CR) was proposed.

Spectrum sensing (SS), a key technique in CR, which aims at detecting the existence of PU. Conventional sensing methods include energy detection, cyclostationary feature detection, likelihood ratio test and so on [2].

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All of the above algorithms need the prior information about signal or noise, which may not be realistic in practice. Therefore, some blind spectrum sensing algorithms emerged. Among these blind sensing algorithms, eigenvalue-based [3, 4], and covariance-based sensing methods [5] are widely accepted. Because they do not need any prior information and take the correlation between signal samples into account. It was voted that [6] proposed a blind scheme, features from the Cholesky decomposition of covariance matrix as the criterion to determine whether the radio frequency band is vacant, which was proved to have good detection performance. In addition, to overcome channel fading and hidden terminal, [7] put forward a cooperative sensing algorithm.

As we all know, SS is actually a binary classification problem. Therefore, more and more machine learning classification algorithms are applied in SS in recent years. In [8], Awe OP extracted eigenvalues as the features of signal then used SVM for classification, it is proved by experiment that the algorithm has better performance. Then, a paper proves the superiority of SVM [9]. Considering the mobility of secondary users, [10] used the random forest to achieve a better network throughput.

In this paper, motivated by the work in [5–10], we proposed a blind spectrum sensing algorithm based on QR decomposition and SVM. Firstly, the covariance matrix of cooperative secondary users (SUs) is estimated. Secondly, we employ QR decomposition of the covariance matrix to extract the features of signals when PU exit or not. Finally, SVM is applied to classify whether PU presents.

The rest of this paper is organized as follows. In Sect. 2, the system model is given. In Sect. 3, the process of feature extraction and the application of SVM is described. In Sect. 4, simulation results are discussed. Finally, we draw our conclusion in Sect. 5.

2 System Model

2.1 Spectrum Sensing

The main task of spectrum sensing is to detect the presence of PU. Thus, spectrum sensing can be represented as a binary hypothesis testing problem, which may be written as

$$\begin{cases} H_1: & y(k) = s(k) + n(k) \\ H_0: & y(k) = n(k) \end{cases} \quad k = 1, 2, \ldots K \tag{1}$$

where K denotes the number of samples and $y(k)$ indicates the signal of SU received from the PU transmitter. Additionally, $s(k)$ and $n(k)$ respectively denote the signal of PU and noise. When PU exists, SU receives the signal of PU and noise, represented as H_1. Otherwise, SU only get noise signal, represented as H_0.

2.2 Cooperative Covariance-Based Detection

In covariance-based detection [5], $s(k)$ and $n(k)$ meet the basic assumptions: (1) $n(k)$ is an independent, identically distributed Gaussian signal, satisfying $E(n(k)) = 0$, $E(n^2(k)) = \sigma_n^2$. (2) The samples of $s(k)$ are correlated.

Supposing that the number of cooperative SUs is L. Then we get the following vectors:

$$Y = \begin{bmatrix} y_1(k) & y_2(k) & \ldots & y_L(k) \end{bmatrix}^T \tag{2}$$

$$S = \begin{bmatrix} s_1(k) & s_2(k) & \ldots & s_L(k) \end{bmatrix}^T \tag{3}$$

$$N = \begin{bmatrix} n_1(k) & n_2(k) & \ldots & n_L(k) \end{bmatrix}^T \tag{4}$$

where

$$Y = \begin{bmatrix} y_1(1) & y_1(2) & \ldots & y_1(K) \\ y_2(1) & y_2(2) & \ldots & y_2(K) \\ \vdots & \vdots & & \vdots \\ y_L(1) & y_L(2) & \ldots & y_L(K) \end{bmatrix}$$

Then the $L \times K$ (0 < L/K < 1) statistical covariance matrix of Y, S, N can be written as

$$R_Y = E\left[Y \cdot Y^H\right] \tag{5}$$

$$R_s = E\left[S \cdot S^H\right] \tag{6}$$

$$R_N = E\left[N \cdot N^H\right] = \sigma_n^2 I_L \tag{7}$$

$$\begin{cases} H_1: & R_Y = R_s + \sigma_n^2 I_L \\ H_0: & R_Y = \sigma_n^2 I_L \end{cases} \tag{8}$$

From the above formulae, we can notice that when PU is absent, $R_s = 0$. Because the samples of noise are independent, R_Y is a diagonal matrix. When PU is present, due to the correlation of PU signal, the off-diagonal entries of R_s are non-zero. In this case, R_Y is not a diagonal matrix.

3 The Improved Algorithm

3.1 The QR Decomposition of Signal

In pattern recognition, QR decomposition has been widely used. Because Q matrix in QR decomposition is a group of orthogonal eigenvectors, R matrix covers all information of matrix and reflects the main features of signal. And the main advantage of QR decomposition is the numerical stability, which is suitable for spectrum sensing with Gaussian signal and noise. In [11], QR decomposition is successfully applied in channel identification.

Based on the above theories, we decompose R_Y by using QR decomposition.

$$R_Y = QR \tag{9}$$

where

$$Q = [\, q_1 \quad q_2 \quad \ldots \quad q_L \,]$$

$$q_i \cdot q_j = 0 \quad 1 \le i \le L$$

In the hypothesis of H_0, $R_Y = R_N = \sigma_n^2 I_L$. According to (9), we get

$$R = \sigma_n^2 I_L = \begin{bmatrix} \sigma_n^2 & & & \\ & \sigma_n^2 & & \\ & & \ddots & \\ & & & \sigma_n^2 \end{bmatrix}$$

In the hypothesis of H_1, because of the correlation between SUs, R matrix of R_Y is not a diagonal matrix.

From the above analysis, we get

$$\begin{cases} H_1: \quad R = \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1L} \\ & r_{22} & \cdots & r_{2L} \\ & & \ddots & \vdots \\ & & & r_{LL} \end{bmatrix} \\ H_0: \quad R = \begin{bmatrix} \sigma_n^2 & & & \\ & \sigma_n^2 & & \\ & & \ddots & \\ & & & \sigma_n^2 \end{bmatrix} \end{cases} \tag{10}$$

Therefore, the upper triangular matrix R can be used to differentiate PU from noise. After arranging the elements of R in rows, the features when PU present or not are extracted and represented as a vector.

3.2 SVM Based Sensing Algorithm

In the classification algorithms, SVM is regarded as one of the best classifiers. In this paper, we use nonlinear SVM to achieve the detection of PU.

Combing the features and corresponding labels, the training set and the testing set can be obtained. Assume the testing set as $T = \{(x_1, y_1), (x_2, y_2), \cdots, (x_M, y_M)\}$, M is the number of training samples. x_i, a $1 \times (L \times (L+1)/2)$ vector, is the feature, consisting of the upper triangular elements of matrix. y_i, the corresponding label, equals 1 when PU present, otherwise -1.

The nonlinear SVM achieves classification based on the optimal separation hyper-plane. The optimal hyperplane and classification decision function can be represented respectively as [10].

$$\begin{aligned} & w^* \cdot \varphi(x) + b^* = 0 \\ & f(x) = sign(w^* \cdot \varphi(x) + b^*) \end{aligned} \tag{11}$$

where $\varphi(x)$ is a mapping function, which maps x_i into a high dimensional space.

According to the conditions of the optimal hyperplane, Lagrange function can be constructed. Then the question of the optimal hyperplane can be converted to

$$\begin{aligned} & \min_{\alpha} \tfrac{1}{2} \sum_{i=1}^{M} \sum_{j=1}^{M} \alpha_i \alpha_j y_i y_j K(x_i, x_j) - \sum_{i=1}^{M} \alpha_i \\ & s.t. \quad \sum_{i=1}^{M} \alpha_i y_i = 0 \\ & \qquad 0 \le \alpha_i \le C, \quad i = 1, 2, \cdots, M \end{aligned} \tag{12}$$

where α represent Lagrange multiplier. $K(x_i, x_j) = \varphi(x_i) \cdot \varphi(x_i)$. C is a positive constant penalty parameter.

Based on (12), we can get the optimal solution of α as $\alpha^* = (\alpha_1^*,\ \alpha_2^*,\ \cdots \alpha_M^*)^T$. Then the optimal solution of w is calculated by

$$w^* = \sum_{i=1}^{M} \alpha_i^* y_i x_i \tag{13}$$

Selecting α_i^*, which satisfies $0 \le \alpha_i^* \le C$, the optimal solution of b can be obtained

$$b^* = y_j - \sum_{i=1}^{M} \alpha_i^* y_i K(x_i, x_j) \tag{14}$$

Finally, the classification decision function can be written as

$$f(x) = sign(\sum_{i=1}^{M} \alpha_i^* y_i K(x \cdot x_i) + b^*) \tag{15}$$

4 The Simulation Result and Discussion

In simulation, the modulation of PU signal and the noise are respectively assumed to be OFDM and AWGN. The carrier frequency and the sampling frequency are respectively 100 MHz and 400 MHz. The value of L is 5, and the value of K is 1000. The simulation mainly includes three steps as follows.

1. We generate the training set and testing set according to the third section of this paper. The size of them is respectively 8000 and 2000.
2. The training set then is sent into SVM to get the classification decision function.
3. Classification decision function obtained from step 2 is applied to feature vectors of the testing set. Then the output of SVM are compared with corresponding labels in testing set to compute the probability of detection P_d and false alarm P_{fa}.

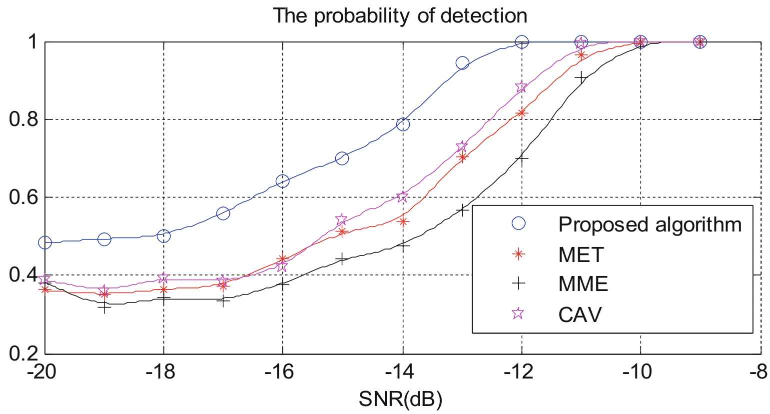

Fig. 1. The comparisons of the detection probability

Figure 1 compares the proposed algorithm with maximum-minimum eigenvalue (MME) [3], maximum-trace ratio (MET) [4], and covariance absolute value (CAV) [5]. In this figure, their P_{fa} at the certain SNR are same. It is obvious that the P_d of proposed method reaches 49% at the SNR of −20 dB, while the P_d of MME, MET, and CAV are below 40%. When SNR between −20 dB and −12 dB, the P_d of proposed method outperforms other algorithms about 15%.

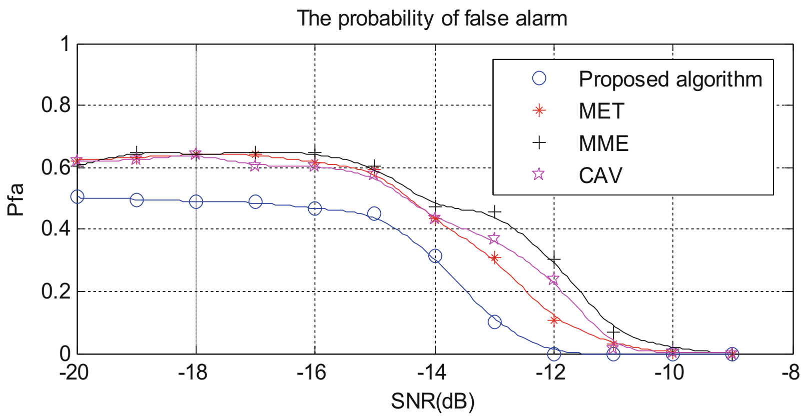

Fig. 2. The comparisons of the false alarm probability

Figure 2 compares their P_{fa} for given P_d at various SNRs. It is worth noting that the P_{fa} of proposed method is lower than other methods. At the SNR of −12 dB, the P_{fa} of the proposed method converges to 0, while others are above 0.1.

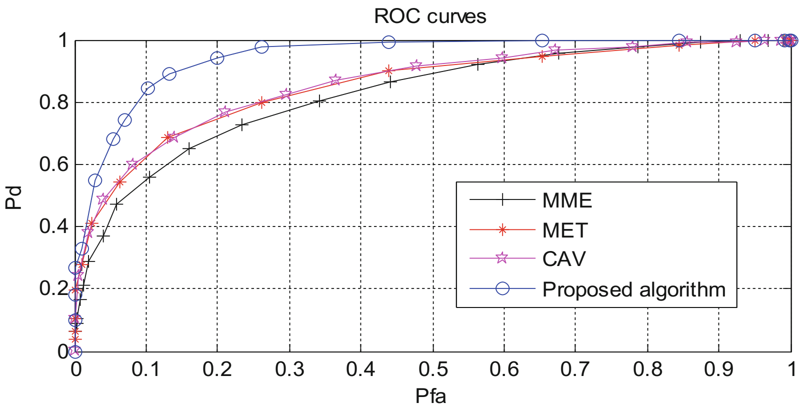

Fig. 3. The ROC curves at the SNR of −15 dB

Figure 3 shows their ROC curves at the SNR of −15 dB. When P_{fa} equals 0.1, the P_d of proposed method reaches 0.84, which is greater than other algorithms. It is clear that the proposed method has better detection performance than MME, MET, and CAV.

In summary, the detection performance of the proposed method is better than conventional covariance-based methods. The first two figures show that the proposed method is more stable than MME, MET, and CAV when SNR is low. The reason is that the traditional methods cannot distinguish between signal and noise at low SNRs. While features extracted from QR decomposition cover the all key information, which can distinguish signal and noise effectively. The last figure shows the superiority of the combination of QR decomposition and SVM.

5 Conclusions

In this paper, a blind cooperative spectrum sensing algorithm is put forward combing QR decomposition and SVM. This method does not demand the information of signal or noise and has good performance than conventional covariance-based methods. Simulation results verified the superiority of the proposed scheme, and our future work will focus on the influence of different modulation types.

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