Proceedings of the Eighth IASTED International Conference
SIGNAL AND IMAGE PROCESSING
August 14–16, 2006, Honolulu, Hawaii, USA
ISBN Hardcopy: 0-88986-583-3 / CD: 0-88986-585-X

A LOWER BOUND ON THE VARIANCE OF ALGEBRAIC ELLIPSOID-FITTING CENTER ESTIMATOR

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ABSTRACT
Ellipsoid fitting is a widely used technique in 3D shape modeling, which simultaneously estimate the center and orientation of 3D object. This paper explores the limits of performance for the ellipsoid-fitting center estimator. It is shown that the noise in the surface sample data can be approximated by a Gaussian distribution when the signal to noise ratio is high. The Cramér-Rao lower bound is applied to yield a bound on the variance of unbiased ellipsoid-fitting center estimator. The simulation results show that the bound is approachable by the center estimator developed from Bookstein's ellipsoid fitting method when the noise level is low.

KEY WORDS
Ellipsoid-fitting, center estimation, Cramér-Rao bound

1 Introduction

Fitting ellipse or ellipsoid to scattered data is widely employed in many applications of pattern recognition and computer vision due to its ability to simultaneously estimate the center and the orientation of an object [1]. For example, a common shape modeling technique uses basis functions, such as spherical harmonics, B-splines, and Fourier series to approximate the boundary of an object [2, 3]. To use such basis functions efficiently and to compute the shape parameters accurately, it is important to properly choose the object center, such that the origin of the coordinate system can be aligned with it. In some applications, we have prior information of the object geometry, or the measurement system may define a natural geometry of the object, and the optimum center of the object is well defined. For example, when an object has symmetry relative to a center, aligning the origin with the object symmetric center is an optimum choice because many basis functions are defined to be symmetric to the origin and some high spatial frequencies can thus be avoided in shape modeling. In the applications that no prior knowledge of the object geometry is available, the optimum center of the object may vary with different assumptions and objectives. The object gravity center is often a natural and reliable choice as the center for shape modeling [3, 4]. However, if the surface is not evenly sampled, it is hard to compute the gravity center of an object. And the noise in the surface sample data also increases the difficulty in center estimation.

In many cases of pattern recognition and image registration, it is also necessary to extract information of an object orientation based on principal axes. Given a set of surface sample data, a commonly used method that can jointly estimate the object center and the principal axes is ellipsoid fitting [3]. The method fits an ellipsoid to the surface sample data. The symmetry axes of the ellipsoid are then used as principal axes. For unevenly distributed sample data, it is known that ellipsoid fitting method is more robust in estimating object center than computing the gravity center of the sample data, and also has better performance than the second moments method in estimating principal axes [3].

The existing methods for ellipsoid fitting are usually based on the least-squares fitting of the scattered data, and can be classified into two categories according to their error definitions, i.e. algebraic fitting and geometric fitting, respectively. It is well known that the quadric curves and surfaces can be represented by an implicit equation $F(\mathbf{v}, \mathbf{x})$, where $\mathbf{v}$ is the parameter vector and $\mathbf{x}$ is the surface point coordinates. For a given point $\mathbf{x}_i$, if it is not on the curve or surface, $F(\mathbf{v}, \mathbf{x}_i) \neq 0$. In algebraic fitting, the functional value of $F(\mathbf{v}, \mathbf{x}_i)$ is regarded as the error distance or algebraic distance, and the ellipsoid fitting is done through minimizing $\sum_i^k [F(\mathbf{v}, \mathbf{x}_i)]^2$, the square sum of the errors [5, 6, 7]. In geometric fitting, the error is defined as the shortest distance from the scattered data to the best fitting curve or surface, so it is also known as orthogonal distance fitting [8, 9]. The advantages of algebraic fitting lies on its elegant analytical solution and low computing cost, while geometric fitting claims advantages in accuracy with much higher computing cost associated with its iterative solution. In our study of articular cartilage deformations, the center and orientation information of large amount of cells are obtained by fitting ellipsoid to microscopic image segmentation results. The computational cost consideration motivates us to choose algebraic fitting method to solve the ellipsoid fitting problem. However, it is desirable to know how well the curve and surface parameters can be estimated for a given signal to noise ratio of the scattered data so that the performance of algebraic fitting algorithms can be compared to the limit and evaluated.

This paper develops a lower bound on the variance of unbiased ellipsoid-fitting center estimator based on a Gaussian distribution model of the surface sample data. The rest of this paper is organized as following: In section 2, the ellipsoid fitting method is briefly described. In section 3, a Gaussian noise model is set up for the surface sample data, and then a lower bound on the variance of unbiased center estimator is derived by applying the Cramér-Rao lower bound. In section 4, the simulation results are presented and compared with the developed lower bound.

2 Ellipsoid Fitting

The surface of an ellipsoid is a special case of quadric surface which has a general expression $F(\mathbf{v}, \mathbf{x})$ as:

$$Ax^2+By^2+Cz^2+Dxy+Exz+Fyz+Gx+Hy+Iz+J=0 \tag{1}$$

where $\mathbf{x} = (x, y, z)$ is the cartesian coordinates of the point on the ellipsoid surface, $\mathbf{v} = (A, B, C, ..., J)$ is the parameter vector which describes the location, size and orientation of the ellipsoid. There are nine degrees of freedom in the description of an ellipsoid, which include three coordinates of the centroid, the lengths of three symmetry axes and three orientation angles. Since the above expression has ten parameters, they must be constrained by some relations.

The 3D ellipsoid fitting method described here is generalized from the 2D ellipse fitting method proposed by Bookstein [5]. And a constraint

$$A^2 + B^2 + C^2 + \frac{1}{2}(D^2 + E^2 + F^2) = 1, \tag{2}$$

which is in a similar format as proposed in [5], is adopted so that the fitting result is invariant under equiform transformations.

Let $\{\mathbf{x}_i = (x_i, y_i, z_i), i = 1 \cdots k\}$ be the set of scattered surface samples to be fitted, where k is the total number of sample points in the data set. The algebraic error is defined as $e_i = F(\mathbf{v}, \mathbf{x}_i)$, and fitting is done by searching for $\mathbf{v}$ that can minimize $\sum_i^k [F(\mathbf{v}, \mathbf{x}_i)]^2$ with the constraint over $\mathbf{v}$. Define the vector

$$\mathbf{m}_i = (x_i^2, y_i^2, z_i^2, x_i y_i, x_i z_i, y_i z_i, x_i, y_i, z_i, 1) \tag{3}$$

which is determined by the sample point $\mathbf{x}_i$, and the matrix

$$\mathbf{S} = \sum_i^k \mathbf{m}_i^T \mathbf{m}_i, \tag{4}$$

the objective function $\sum_i^k [F(\mathbf{v}, \mathbf{x}_i)]^2$ can be expressed as $\mathbf{vSv}^T$.

To add the constraint (2) as Lagrangian, the vectors $\mathbf{v}$ and $\mathbf{m}_i$ can be decomposed into two parts so that $\mathbf{v} = [\mathbf{v}_1, \mathbf{v}_2]$ and $\mathbf{m}_i = [\mathbf{m}_{i1}, \mathbf{m}_{i2}]$, where $\mathbf{v}_1 = (A, B, C, D, E, F)$, $\mathbf{v}_2 = (G, H, I, J)$, $\mathbf{m}_{i1} = (x_i^2, y_i^2, z_i^2, x_i y_i, x_i z_i, y_i z_i)$, and $\mathbf{m}_{i2} = (x_i, y_i, z_i, 1)$.

The matrix $\mathbf{S}$ can be written as

$$\mathbf{S} = \begin{bmatrix} \mathbf{S}_{11} & \mathbf{S}_{12} \\ \mathbf{S}_{21} & \mathbf{S}_{22} \end{bmatrix} \tag{5}$$

where $\mathbf{S}_{11} = \sum_i \mathbf{m}_{i1}^T \mathbf{m}_{i1}$, $\mathbf{S}_{12} = \sum_i \mathbf{m}_{i1}^T \mathbf{m}_{i2}$, $\mathbf{S}_{21} = \sum_i \mathbf{m}_{i2}^T \mathbf{m}_{i1}$, and $\mathbf{S}_{22} = \sum_i \mathbf{m}_{i2}^T \mathbf{m}_{i2}$. The objective function now becomes

$$\begin{aligned} \mathbf{vSv}^T &= (\mathbf{v}_1, \mathbf{v}_2) \begin{bmatrix} \mathbf{S}_{11} & \mathbf{S}_{12} \\ \mathbf{S}_{21} & \mathbf{S}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{v}_1^T \\ \mathbf{v}_2^T \end{bmatrix} \\ &= \mathbf{v}_1 \mathbf{S}_{11} \mathbf{v}_1^T + 2\mathbf{v}_1 \mathbf{S}_{12} \mathbf{v}_2^T + \mathbf{v}_2 \mathbf{S}_{22} \mathbf{v}_2^T. \end{aligned} \tag{6}$$

Setting the derivative $\frac{d(\mathbf{vSv}^T)}{d(\mathbf{v}_2)}$ to zero leads to $\mathbf{v}_2 = -\mathbf{v}_1 \mathbf{S}_{12} \mathbf{S}_{22}^{-1}$. Substituting it back into (6), we have

$$\mathbf{vSv}^T = \mathbf{v}_1 (\widetilde{\mathbf{S}}) \mathbf{v}_1^T \tag{7}$$

where $\widetilde{\mathbf{S}} = \mathbf{S}_{11} - \mathbf{S}_{12} \mathbf{S}_{22}^{-1} \mathbf{S}_{21}$.

Let us define $\mathbf{D} = \text{diag}(1, 1, 1, \frac{1}{2}, \frac{1}{2}, \frac{1}{2})$, the constraint (2) can be described in the form $\mathbf{v}_1 \mathbf{D} \mathbf{v}_1^T = 1$. Therefore, the constrained minimization of $\mathbf{vSv}^T$ has been converted to the minimization of $\mathbf{v}_1 \widetilde{\mathbf{S}} \mathbf{v}_1^T - \lambda \mathbf{v}_1 \mathbf{D} \mathbf{v}_1^T$. Taking the derivative over $\mathbf{v}_1$ and setting it to zero, we obtain $2\widetilde{\mathbf{S}} \mathbf{v}_1^T - 2\lambda \mathbf{D} \mathbf{v}_1^T = 0$. This indicates that the vector $\mathbf{v}_1^T$ should be the eigenvector of $\widetilde{\mathbf{S}}$. In the implementation, the eigenvector of best geometric fit is chosen to be the final solution of parameter estimation. The center and principal axes of an ellipsoid are completely determined by the parameter vector $\mathbf{v}$. So the algebraic fitting method simplifies the estimation problem to the problem of solving eigenvectors of a generalized symmetric matrix.

3 Lower Bound on the variance of Ellipsoid-fitting Center Estimator

To evaluate the performance of the ellipsoid fitting method described in section 2, we develop a lower bound on the variance of unbiased ellipsoid-fitting center estimators under a Gaussian noise model. Our derivation of the lower bound follows similar steps as the derivation of a Cramér-Rao bound. The effectiveness of an unbiased estimator can be characterized by its variance. Cramér-Rao bound, the inverse of Fisher information matrix, describes the minimum obtainable mean square error associated with a given estimate of a set of parameters. The derivation of such a bound for the ellipsoid parameter vector $\mathbf{v}$ is described as follows.

First, we set up the noise model of the surface sample data. Write equation (1) in the spherical coordinate system and arrange it into a quadratic form of r:

$$A_1 r^2 + B_1 r + C_1 = 0 \tag{8}$$

where

$$A_1 = A(\sin\theta\cos\phi)^2 + B(\sin\theta\sin\phi)^2 + C(\cos\theta)^2 +$$
$$\frac{1}{2}[D(\sin^2\theta\sin 2\phi) + E(\sin 2\theta\cos\phi) + F(\sin 2\theta\sin\phi)]$$
$$B_1 = G\sin\theta\cos\phi + H\sin\theta\sin\phi + I\cos\theta$$
$$C_1 = J. \tag{9}$$

If we assume the origin of the spherical coordinate system is inside the ellipsoid, the true radial value of the ellipsoid in each direction (θ, ϕ) is determined by:

$$R(\theta, \phi) = \frac{\sqrt{B_1^2 - 4A_1C_1} - B_1}{2A_1}. \tag{10}$$

It has been proved in [5] that $F(\mathbf{v}, \mathbf{x}_i) = F(\mathbf{v}, r_i, \theta_i, \phi_i) \propto (\frac{r_i}{R(\theta_i, \phi_i)})^2 - 1$, where $R(\theta_i, \phi_i)$ is the true radius in the sample direction (θ_i, ϕ_i) determined by (10). Therefore, the ellipsoid fitting method implements a maximum likelihood estimation of the parameter vector if the noise in each direction (θ_i, ϕ_i) is uncorrelated and the segmentation data follows the probability density function

$$f(r_i|\mathbf{v}) = \alpha_i \cdot \exp[-\frac{(\frac{r_i^2}{R^2(\theta_i,\phi_i)} - 1)^2}{\sigma^2}], \qquad r_i > 0 \tag{11}$$

where α_i is the normalization factor. If $r_i - R(\theta_i, \phi_i) \ll R(\theta_i, \phi_i)$, we have $(\frac{r_i^2}{R^2(\theta_i,\phi_i)} - 1)^2 \approx 4(\frac{r_i - R(\theta_i,\phi_i)}{R(\theta_i,\phi_i)})^2$, and $f(r_i|\mathbf{v})$ can be approximated by

$$\tilde{f}(r_i|\mathbf{v}) = \alpha_i \cdot \exp[-4(\frac{r_i - R(\theta_i, \phi_i)}{\sigma R(\theta_i, \phi_i)})^2], \qquad r_i > 0 \tag{12}$$

Notice that $\tilde{f}(r_i|\mathbf{v})$ is not a probability density function with respect to $r_i \in (-\infty, \infty)$ unless we modify the normalization factor α_i as

$$\tilde{\alpha}_i = 1/\int_{-\infty}^{\infty} \exp[-4(\frac{x}{\sigma R(\theta_i, \phi_i)})^2]dx = \frac{2}{R(\theta_i, \phi_i)\sigma\sqrt{\pi}}.$$

This approximation is illustrated in Figure 1.

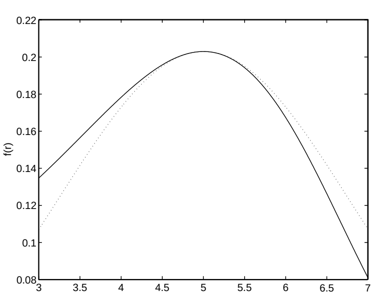


Figure 1. Approximation of $f(r)$ by $\tilde{f}(r)$ under the condition $r - R \ll R$. Here $R = 5$ and $\sigma = 1$. The solid line represents $f(r)$ and the dotted line represents $\tilde{f}(r)$

During the derivation of lower bound and the simulation of the ellipsoid fitting performance, we adopt $\tilde{f}(r|\mathbf{v})$ as the probability density function of the surface sample data. This is equivalent to a Gaussian noise model. In the following, we show an example of deriving the Fisher information for parameter A with such a noise model.

Taking the logarithm in both side of (12), we have

$$\ln \tilde{f}(r|\mathbf{v}) = -4(\frac{r - R}{\sigma R})^2 - \ln R - \ln(\frac{\sigma\sqrt{\pi}}{2}). \tag{13}$$

The expectation of $\frac{\partial^2 \ln \tilde{f}(r|\mathbf{v})}{\partial A^2}$ can be obtained as:

$$E[\frac{\partial^2 \ln \tilde{f}(r|\mathbf{v})}{\partial A^2}] = -(\frac{8}{\sigma^2 R^2} + \frac{2}{R^2})(\frac{\partial R}{\partial A_1})^2 \cdot (\frac{\partial A_1}{\partial A})^2, \tag{14}$$

where $\frac{\partial R}{\partial A_1} = -(\frac{C_1}{A_1\sqrt{B_1^2 - 4A_1C_1}} + \frac{\sqrt{B_1^2 - 4A_1C_1} - B_1}{2A_1^2})$ and $\frac{\partial A_1}{\partial A} = (\sin\theta\cos\phi)^2$.

With the assumption that noises over different direction (θ_i, ϕ_i) are independent, the joint probability density function for the set of sample data is $\prod_{i=1}^{K} \tilde{f}(r_i|\mathbf{v})$, where $\tilde{f}(r_i|\mathbf{v})$ is the probability density function of the segmentation data in direction (θ_i, ϕ_i). Therefore, the Fisher information for the parameter A is:

$$I_A = \sum_{i=1}^{K} -E[\frac{\partial^2 \ln \tilde{f}(r_i|\mathbf{v})}{\partial A^2}]. \tag{15}$$

Similarly, we can compute the other entries in the Fisher information matrix of the ellipsoid parameter vector $\mathbf{v}$.

Once we have estimated the parameter vector $\mathbf{v}$ through ellipsoid fitting, the coordinates of the center (x, y, z) can be determined by:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = -\begin{pmatrix} 2A & D & E \\ D & 2B & F \\ E & F & 2C \end{pmatrix}^{-1} \begin{pmatrix} G \\ H \\ I \end{pmatrix}. \tag{16}$$

Based on the inverse of the Fisher information matrix of $\mathbf{v}$, we can obtain a lower bound on the covariance of the center estimator. Let $\hat{\mathbf{x}} = (\hat{x}, \hat{y}, \hat{z})^T$ represent the center estimator, and define $\hat{\mathbf{K}} = \begin{pmatrix} 2\hat{A} & \hat{D} & \hat{E} \\ \hat{D} & 2\hat{B} & \hat{F} \\ \hat{E} & \hat{F} & 2\hat{C} \end{pmatrix}^{-1}$ and $\hat{\mathbf{b}} = (\hat{G}, \hat{H}, \hat{I})^T$. We rewrite equation (16) in the form of $\hat{\mathbf{x}} = -\hat{\mathbf{K}}\hat{\mathbf{b}} = -(\bar{\mathbf{K}} + \mathbf{K}_e)(\bar{\mathbf{b}} + \mathbf{b}_e)$, where $\bar{\mathbf{K}}$ and $\bar{\mathbf{b}}$ are the mean values of $\hat{\mathbf{K}}$ and $\hat{\mathbf{b}}$, and $\mathbf{K}_e$ and $\mathbf{b}_e$ represent errors in $\hat{\mathbf{K}}$ and $\hat{\mathbf{b}}$. The lower bound for the covariance of the center estimator can be obtained from following computation:

$$\begin{aligned} \text{cov}(\hat{\mathbf{x}}) &= E\left[(\hat{\mathbf{K}}\hat{\mathbf{b}} - \bar{\mathbf{K}}\bar{\mathbf{b}})(\hat{\mathbf{K}}\hat{\mathbf{b}} - \bar{\mathbf{K}}\bar{\mathbf{b}})^T\right] \\ &= \text{cov}(\bar{\mathbf{K}}\mathbf{b}_e) + \text{cov}(\mathbf{K}_e\bar{\mathbf{b}}) + \text{cov}(\mathbf{K}_e\mathbf{b}_e) \\ &\geq \bar{\mathbf{K}}\mathbf{F}^{-1}(\mathbf{b})\bar{\mathbf{K}}^T + \text{cov}(\mathbf{K}_e\bar{\mathbf{b}}) \end{aligned} \tag{17}$$

where $\mathbf{F}^{-1}(\mathbf{b})$ denotes the inverse of Fisher information matrix of the parameter vector (G, H, I). In the above derivation, we have assumed that $\text{cov}(\mathbf{K}_e\mathbf{b}_e)$ is much smaller than $\text{cov}(\bar{\mathbf{K}}\mathbf{b}_e)$ and $\text{cov}(\mathbf{K}_e\bar{\mathbf{b}})$. $\bar{\mathbf{K}}\mathbf{F}^{-1}(\mathbf{b})\bar{\mathbf{K}}^T$ is a Cramér-Rao lower bound of $\text{cov}(\bar{\mathbf{K}}\mathbf{b}_e)$. To further simplify the computation of the lower bound, we let $\bar{\mathbf{b}}$ equal zero in our experiment so that the term $\text{cov}(\mathbf{K}_e\bar{\mathbf{b}})$ in (17) can be ignored.

4 Simulation Results

To evaluate the performance of the ellipsoid-fitting center estimator generalized from Bookstein's method, we have simulated noisy surface sample data and applied the ellipsoid fitting method to estimate the object center. In the simulation, segmentation data in each sample direction (θ_i, ϕ_i) has been generated independently with Gaussian distribution $\tilde{f}(r_i|\mathbf{v}) = \tilde{\alpha}_i \cdot \exp[-4(\frac{r_i - R(\theta_i,\phi_i)}{\sigma R(\theta_i,\phi_i)})^2]$, where $R(\theta_i, \phi_i)$ is the true radial value. Sampling direction (θ, ϕ) is evenly distributed over a grid on $[0, \pi] \times [0, 2\pi)$. The true surface used in the simulation is an ellipsoid $\frac{x^2}{7^2} + \frac{y^2}{6^2} + \frac{z^2}{5^2} = 1$. One such simulated segmentation in a 2D cross section of the ellipsoid is shown in Figure 2. We think that these simulated noises are representative of errors incurred by automatic segmentation of a noisy boundary. It is known that the segmentation error can be modeled by a Gaussian random variable in 1D edge detection. If we regard the 3D surface segmentation as implemented through 1D edge detection along each sampling direction and assume that surface curvature has no significant influence over the detection, our noise model will simulate the segmentation error very well. However, if the sampling density is relatively high as compared to the object size, the segmentation noise in neighborhood will be correlated.

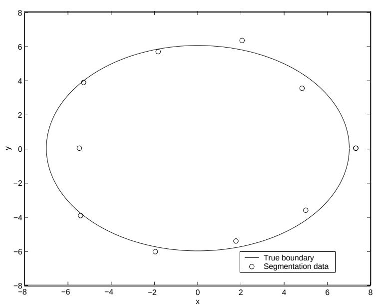
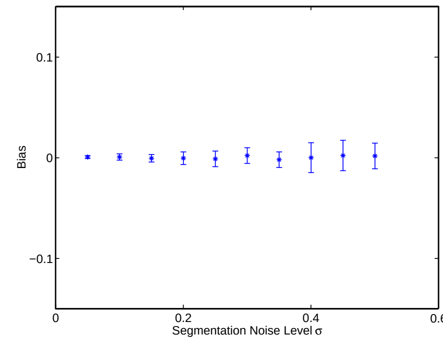
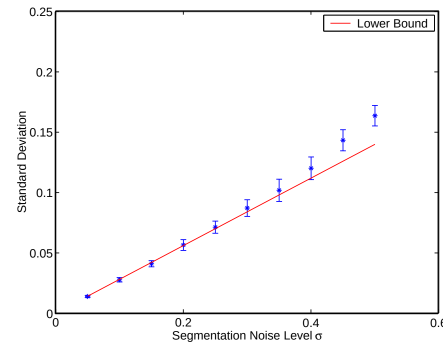


Figure 2. Segmentation data on a cross section of the ellipsoid, $\sigma = 0.2$.

For each noise level, 200 sets of simulated surface sample data have been generated. Figure 3(a) shows the bias of the center coordinate estimator $\hat{\mathbf{x}}$. Figure 3(b) compares the variance of $\hat{\mathbf{x}}$ with the developed lower bound. When the noise level is low, the variance of $\hat{\mathbf{x}}$ is very close to the lower bound. This proves that this center estimator is an efficient maximum likelihood estimator when the noise level is low. We have also simulated the segmentation data with σ larger than 0.5 with the same ellipsoid used above. The results show that the performances of the ellipsoid fitting center estimator is not stable because the method may generate other types of quadric surfaces due to the outliers in the segmentation data.



(a) Bias of $\hat{x}$



(b) Standard deviation of $\hat{x}$

Figure 3. Performance of the ellipsoid fitting center estimator

5 Conclusion

We established that the estimator of the ellipsoid parameter vector developed from Bookstein's 2D ellipse fitting method is a maximum likelihood estimator when the segmentation noise level is low. A lower bound has been derived for the variance of unbiased ellipsoid-fitting center estimator with Gaussian noise model of the segmentation data. The simulated results show that the described ellipsoid-fitting center estimator is efficient when the noise level is low.

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