Quantization of UWB TR Receiver with Slightly Frequency Shifted Reference

Weihong Niu, Jia Li, and Timothy Talty

Oakland Unversity, Rochester, MI 48309, USA and GM Research, Warren, MI 48902, USA

***Abstract*— To eliminate the requirement for a delay unit capable of processing wideband signals in a conventional UWB transmitted reference (TR) receiver, the slightly frequency shifted (SFS) reference UWB signaling scheme has been proposed in the literature. Motivated by the flexibility of the digital systems and the availability of sophisticated digital signal processing circuits, this paper proposes a digital implementation of the SFS TR UWB receiver. Performance of such a digital receiver is derived and evaluated based on the channel measurements from the intra-vehicle environments.**

***Index Terms*— UWB, Quantization, Digital, SFS TR Receiver.**

I. Introduction

A main challenge in the design of UWB systems is the implementation of low-cost, low-complexity and high performance receivers. In the UWB literature, the conventional rake receiver employed in spread spectrum systems has been intensively analyzed [1] [2]. However, there are two major disadvantages with the UWB rake receiver. Firstly, the energy in a UWB signal spreads over tens of resolved multi-path components and tens of fingers are required to capture as much energy as possible. This makes it too complicated and costly to implement the rake receiver. Secondly, rake receiver also requires a robust estimation of the delays and strengths of all multi-path components. This is very difficult when the noise level is high due to the ultra-low energy in each UWB component [1].

To overcome the drawbacks of the conventional rake receiver, the TR signaling scheme has been applied to UWB systems [3]. A TR system transmits the UWB pulses in pair with some delay between them. The first pulse is a reference signal and the second is a data-bearing one. At the TR receiver, the received reference signal is correlated with the data-bearing one for the detection of the transmitted data. Although the TR receiver suffers from the problem of a noise-corrupted reference signal which is used as the correlator template, it still attracts the attention of many UWB researchers because the structure is obviously very simple. A lot of work has been done to overcome the noisy template such as using the average of several reference signals to correlate with the data-bearing signal, and a lot of effort to derive the performance of TR receivers has been reported [4] [5]. Although the architecture of the UWB TR receiver is simple in theory, the practical implementation is difficult because it requires an analog delay unit capable of processing an ultra-wideband signal.

To eliminate the need for a time delay unit in the above time-shifted UWB TR receiver, a slightly frequency shifted TR receiver is proposed in [6]. SFS TR signaling scheme uses an unmodulated UWB signal shifted in frequency domain as the reference and transmits it together with the data-bearing signal. Performance analysis of such a receiver shows that it works fine for low-data-rate applications, such as an intra-vehicle UWB sensor network [6] [7]. Although this SFS TR receiver removes the requirement for an analog delay unit and simplifies the implementation, it is still less flexible than a digital version of itself. Due to the availability of integrated and sophisticated digital signaling circuits, a digital method provides convenient access to powerful digital signal processing (DSP) algorithms, reduces the complexity and increases the flexibility in the implementation of a receiver. This paper proposes a digital version of the SFS TR receiver. Because the analog-to-digital (ADC) converter in such a receiver has to support a high sampling rate in the order of gigahertz for UWB signals, the resolution of the ADC will be proposed to be less than three bits to make its implementation feasible and less costly.

II. System Model

The digital version of the SFS TR receiver is illustrated in Fig. 1. The received signal $\tilde{r}(t)$ is filtered by an ideal low-pass filter (LPF) to remove the noise outside the signal bandwidth B. The signal $r(t)$ output from the filter is first sampled with rate f_s larger than Nyquist rate and then quantized by an ADC. The digitized signal $r_q(n)$ is fed into a digital signal processor together with the digitized version of a cosine signal. In the DSP unit, $r_q(n)$ is first multiplied by the digital cosine signal and then correlated with itself to decide the received data.

In a single user analog SFS TR signaling scheme, the transmitted signal in one symbol interval consisting of N_f frames of length T_f is given by $s(t) = \sqrt{\frac{E_s}{2}}[u(t) + b\sqrt{2}\cos(2\pi f_0 t)u(t)](0 \leq t < T_s)$, where $u(t) = \sum_{k=0}^{N_f-1} p(t-kT_f)$ is a sequence of N_f unmodulated UWB pulses whose energy is normalized to be $1/N_f$, E_s is the

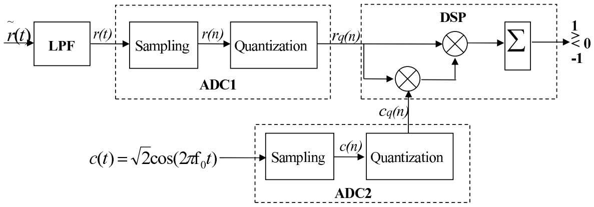

Fig. 1. Structure of the Digital SFS TR Receiver.

total signal energy within the symbol period, b is the data to be transmitted which equals "-1" or "1", and f_0 is the frequency used to shift $u(t)$ [6]. Assuming the length of a symbol is $T_s = N_f T_f$, the value of f_0 is set to be $\frac{1}{T_s}$ so that it is much less than the channel coherence frequency [6]. It can be seen that the transmitted signal is a sum of the reference signal $u(t)$ and its frequency shifted version modulated by the transmission information. Assuming $h(t)$ is the combined impulse response of the channel and the LPF, the received signal can be expressed as

$$r(t) = x(t) + v(t) \tag{1}$$

where $x(t) = s(t) * h(t)$ is the received noise-free signal after the ideal LPF, and noise $v(t)$ is a zero mean, AWGN random process with two-sided power spectral density $N_0/2$. Without ADC, the output from the correlator of an analog SFS TR receiver used to decide the transmitted data is given by $Z = \int_0^{T_s} r(t)r(t)\sqrt{2}c(t)dt$, where $c(t) = \cos(2\pi f_0 t)$ [6].

Accordingly, if the received signal $r(t)$ after the LPF is sampled with Nyquist rate f_s (hence the sample interval $t_s = 1/f_s$) to get the discrete time full-resolution signal $r(n)$ and then quantized with stepsize Δ by the ADC to get the quantized digital signal $r_q(n)$, we have

$$r_q(n) = r(n) + e_q(n) \tag{2}$$

$$r(n) = x(n) + v(n) = s(n) * h(n) + v(n) \tag{3}$$

$$s(n) = \sqrt{\frac{E_s}{2}}[u(n) + b\sqrt{2}c(n)u(n)] \tag{4}$$

in which $e_q(n)$ is the quantization error, $s(n)$, $h(n)$, $u(n)$, $c(n)$ are the discrete time signals corresponding to $s(t)$, $h(t)$, $u(t)$, $c(t)$ respectively, and $v(n)$ is the i.i.d. zero mean white Gaussian noise sequence with variance $N_0 f_s/2$.

III. Performance of The Full-resolution Digital Receiver

Considering an ADC of infinite resolution in the digital SFS TR receiver, when the data bit $b = 1$ is transmitted and N samples are taken in a symbol period, the received signal without noise is given by $x(n) = \sqrt{\frac{E_s}{2}}u(n)*h(n)+\sqrt{E_s}[u(n)c(n)] * h(n)$ and the correlator output is

$$Z = \sum_{n=0}^{N-1}\sqrt{2}r^2(n)c(n) = \sqrt{2}\sum_{n=0}^{N-1}x^2(n)c(n) + 2\sqrt{2}\sum_{n=0}^{N-1}x(n)v(n)c(n) + \sqrt{2}\sum_{n=0}^{N-1}v^2(n)c(n) \tag{5}$$

Let Z_1, Z_2 and Z_3 respectively denote the three terms in the above equation of Z. The expectation of the correlator output is $\mu_z = \sqrt{2}\sum_{n=0}^{N-1}x^2(n)c(n)$. To get the performance expression of the receiver, the variance of the correlator output is $\sigma_z^2 = 4N_0 f_s \sum_{n=0}^{N-1}c^2(n)x^2(n) + \frac{NN_0^2 f_s^2}{2}$.

Based on the above derivations and the assumption that the transmitted data equals -1 or 1 with the same probability, the bit error probability (BEP) of the discrete time full-resolution SFS TR receiver can be expressed as

$$P_e = Q\left(\sqrt{\frac{2\left[\sum_{n=0}^{N-1}x^2(n)c(n)\right]^2}{4N_0 f_s\sum_{n=0}^{N-1}[x^2(n)c^2(n)] + \frac{NN_0^2 f_s^2}{2}}}\right) \tag{6}$$

Assuming that the UWB channel impulse response is $h(t) = \sum_{l=0}^{L-1}\alpha_l\delta(t-\tau_l)$, in which L, α_l and τ_l are the number of paths, the amplitude and the delay of the paths respectively. In such a channel, the received noise-free signal can be written as $x(t) = \sum_{l=0}^{L-1}\alpha_l s(t-\tau_l)$ whose digital expression after being sampled with rate f_s is $x(n) = \sum_{l=0}^{L-1}\alpha_l s(n-D_l)$. Consequently, if the number of samples in a frame time is F, the sum in the denominator of equation (6) is calculated as

$$\begin{aligned}\sum_{n=0}^{N-1}x^2(n)c^2(n) &= \sum_{n=0}^{N-1}[\sum_{l=0}^{L-1}\alpha_l s(n-D_l)]^2c^2(n)\\ &= E_s\sum_{l=0}^{L-1}\sum_{m=0}^{L-1}\alpha_l\alpha_m\sum_{n=0}^{N-1}[\sum_{k=0}^{N_f-1}p(n-kF-D_l)]\\ &\quad[\sum_{i=0}^{N_f-1}p(n-iF-D_m)][\sqrt{\frac{1}{2}}+c(n-D_l)]\\ &\quad[\sqrt{\frac{1}{2}}+c(n-D_m)]c^2(n)\end{aligned} \tag{7}$$

It is assumed that the paths in the channel impulse response do not overlay with each other, i.e. the time delay between any two paths is larger than the width of the UWB pulse. For the lth path and the mth path in equation (7), if $l \neq m$, we have $[\sum_{k=0}^{N_f-1}p(n-kF-D_l)][\sum_{i=0}^{N_f-1}p(n-iF-D_m)] = 0$. As a result, $\sum_{n=0}^{N-1}x^2(n)c^2(n) = 0$. If $l = m$, we have $\sum_{n=0}^{N-1}x^2(n)c^2(n) = \frac{E_s}{t_s}\sum_{l=0}^{L-1}\alpha_l^2[\frac{3}{8}+\frac{1}{4}c^2(D_l)]$. Similarly, $\sum_{n=0}^{N-1}x^2(n)c(n) = \frac{1}{\sqrt{2}}\frac{E_s}{t_s}\sum_{l=0}^{L-1}\alpha_l^2 c(D_l)$. By substituting these two equations into equation (6), we get the performance of the discrete time full-resolution SFS TR receiver as

$$P_e = Q\left(\frac{\frac{E_s}{N_0}\sum_{l=0}^{L-1}\alpha_l^2 c(D_l)}{\sqrt{\frac{E_s}{N_0}\sum_{l=0}^{L-1}\alpha_l^2\left[\frac{3}{2}+c^2(D_l)\right]+\frac{1}{2}N}}\right) \quad (8)$$

IV. Performance of The Quantized Low Resolution Digital Receiver

In the structure of digital UWB receiver shown in Fig. 1, the implementation of a high sampling rate high resolution low power ADC is difficult and too expensive to achieve with current CMOS technology [8] [9]. Thus this paper will work on low resolution quantizers which quantize the signals by no more than 3 bits per sample. Assuming a roundoff quantizer with a uniform stepsize Δ is used in the ADC and the quantizer is not overloaded, the input to the quantizer is the samples of the received signal $r(n)$. It relates to the quantizer output $r_q(n)$ by equation (2). Now the decision variable of the quantized digital receiver is

$$Z_q = \sum_{n=0}^{N-1}\sqrt{2}r_q^2(n)c_q(n) = \sqrt{2}\sum_{n=0}^{N-1}[r(n)+e_q(n)]^2 c_q(n)$$

in which $c_q(n)$ is the quantized samples of the cosine function $c(n)$ in equation (4). If the variance and the expectation of Z_q are represented as σ_{zq}^2 and μ_{zq}, we have

$$\mu_{zq} = \sqrt{2}\sum_{n=0}^{N-1}E[r^2(n)+e_q^2(n)+2r(n)e_q(n)]c_q(n) \quad (9)$$

$$\begin{aligned}\sigma_{zq}^2 &= E[Z_q^2]-\mu_{zq}^2 = 2N\sigma^4+8\sigma^2\sum_{n=0}^{N-1}x^2(n)c_q^2(n)\\ &-2\sum_{n=0}^{N-1}E[e_q^2(n)]^2c_q^2(n)-4\sum_{n=0}^{N-1}E[r^2(n)]E[e_q^2(n)]c_q^2(n)\\ &+8\sum_{n=0}^{N-1}E[r^3(n)e_q(n)]c_q^2(n)+8\sum_{n=0}^{N-1}E[e_q^3(n)r(n)]c_q^2(n)\\ &-8\sum_{n=0}^{N-1}E[r(n)e_q(n)]^2c_q^2(n)+12\sum_{n=0}^{N-1}E[r^2(n)e_q^2(n)]c_q^2(n)\\ &-8\sum_{n=0}^{N-1}E[r^2(n)]E[r(n)e_q(n)]c_q^2(n)+2\sum_{n=0}^{N-1}E[e_q^4(n)]c_q^2(n)\\ &-8\sum_{n=0}^{N-1}E[e_q^2(n)]E[r(n)e_q(n)]c_q^2(n) \quad (10)\end{aligned}$$

According to the modified Quantization Theorem as proposed in [10], the probability density of the quantization error $e_q(n)$ is

$$f_{eq}(\epsilon) = \begin{cases}\frac{1}{\Delta}+\frac{1}{\Delta}\sum_{k\neq 0}\Phi_r(\frac{2\pi k}{\Delta})e^{\frac{-j2\pi k\epsilon}{\Delta}}, & \frac{-\Delta}{2}\leq\epsilon\leq\frac{\Delta}{2}\\ 0, & otherwise\end{cases} \quad (11)$$

We also know from equation (3) that the input $r(n)$ to the quantizer is a Gaussian random sequence with mean $x(n)$ and variance $\sigma^2 = \frac{N_0 f_s}{2}$, thus the characteristic function of $r(n)$ is $\Phi_r(\omega) = e^{jx(n)\omega-\sigma^2\omega^2/2}$. Moreover, in [10] the characteristic function of the quantizer output $r_q(n)$ is given as $\Phi_{rq}(\omega) = \sum_{k=-\infty}^{\infty}\Phi_r(\omega-\frac{2k\pi}{\Delta})\frac{\sin(\Delta\omega/2-k\pi)}{\Delta\omega/2-k\pi}$. At the same time, the correlation between the quantizer input $r(n)$ and the quantization error $e_q(n)$ is derived as

$$\begin{aligned}E[r(n)e_q(n)] &= \sum_{k=1}^{\infty}\sin\frac{2k\pi x(n)}{\Delta}e^{\frac{-2k^2\pi^2\sigma^2}{\Delta^2}}\frac{(-1)^k\Delta x(n)}{k\pi}\\ &+2\sum_{k=1}^{\infty}\cos\frac{2k\pi x(n)}{\Delta}e^{\frac{-2k^2\pi^2\sigma^2}{\Delta^2}}(-1)^k\sigma^2 \quad (12)\end{aligned}$$

In addition, from equation (11), we can calculate the following expectations of the quantization error $e_q(n)$ as

$$E[e_q^2(n)] = \frac{\Delta^2}{12}+\sum_{k=1}^{\infty}\cos\frac{2k\pi x(n)}{\Delta}e^{\frac{-2k^2\pi^2\sigma^2}{\Delta^2}}\frac{(-1)^k\Delta^2}{k^2\pi^2} \quad (13)$$

$$\begin{aligned}E[e_q^4(n)] &= \frac{\Delta^4}{80}+\\ &\sum_{k=1}^{\infty}\cos\frac{2k\pi x(n)}{\Delta}e^{\frac{-2k^2\pi^2\sigma^2}{\Delta^2}}[\Delta^4(\frac{(-1)^k}{2k^2\pi^2}-\frac{3(-1)^k}{k^4\pi^4})] \quad (14)\end{aligned}$$

Finally, because we have

$$\begin{aligned}E[r_q^4(n)] =&E[r^4(n)]+E[e_q^4(n)]+4E[r^3(n)e_q(n)]\\ &+4E[r(n)e_q^3(n)]+6E[r^2(n)e_q^2(n)] \quad (15)\end{aligned}$$

and at the same time it is also established that

$$E[r_q^4(n)] = \Phi_{rq}^{(4)}(\omega)|_{\omega=0} \quad (16)$$

Following similar procedure in the derivation of $E[r(n)e_q(n)]$ in [10], by calculating equation (16) and comparing its result with equation (15), we can get

$$\begin{aligned}E[r^2(n)e_q^2(n)] &= \frac{\Delta^2(x^2(n)+\sigma^2)}{12}\\ &-\sum_{k=1}^{\infty}\Phi_r^{(2)}(\frac{-2k\pi}{\Delta})\frac{(-1)^k\Delta^2}{2k^2\pi^2} \quad (17)\end{aligned}$$

$$E[r^3(n)e_q(n)] = -\sum_{k=1}^{\infty}\Phi_r^{(3)}(\frac{-2k\pi}{\Delta})\frac{(-1)^k\Delta}{k\pi} \quad (18)$$

$$E[r(n)e_q^3(n)] = \sum_{k=1}^{\infty}\Phi_r'(\frac{-2k\pi}{\Delta})(-1)^k(\frac{\Delta^3}{8k\pi}-\frac{3\delta^3}{4k^3\pi^3}) \quad (19)$$

Substitute equations (12) - (14) and equations (17) - (19) into equations (9) and (10), we can get the mean μ_{zq} and the variance σ_{zq}^2 of Z_q. The performance of the quantized digital receiver is just

$$P_{eq} = Q\left(\frac{\mu_{zq}}{\sqrt{\sigma_{zq}^2}}\right), \quad (20)$$

V. Simulations and Numerical Results

Simulations have been run to evaluate the accuracy of the BEP expression for the discrete time full-resolution SFS TR receiver in equation (8) and the BEP for the quantized digital SFS TR receiver in equations (20). The second derivative of Gaussian

$$p(t) = \left\{1 - 4\pi \left[\frac{t - T_p/2}{0.39T_p}\right]^2\right\} exp\left(-2\pi \left[\frac{t - T_p/2}{0.39T_p}\right]^2\right)$$

is used as the shape of the transmitted UWB pulse and the width of the pulse T_p is set to be 1 ns [1]. The impulse responses randomly selected from the intra-vehicle UWB channel measurement beneath chassis is used in the simulations to characterize the multi-path channels and the sampling rate is set to be 4GHz [7]. In the simulations the data transmitted are set to be 1s and -1s with equal probabilities. For the same set of channel impulse responses, the BEPs are also calculated via equations (8) and (20).

In the case of the discrete time full-resolution SFS TR receiver, results from both the simulations and the theoretical calculations are shown in Fig. 2, where the dotted lines represent BEPs from the theoretical calculations. The frame length is set to be 4.5 ns and the numbers of frames in a symbol period are 20, 15, 10, 5 and 2 corresponding to the data rates from 11.114 Mbps to 111.420 Mbps in Fig. 2. It can be seen that the calculated BEPs match the simulation results very well when the data rate is less than 10.259Mbps. This validates the accuracy of equation (8).

Fig. 3 shows the BEP from the quantization simulation in solid lines together with the calculated BEP in dashed lines for the quantized digital receiver. The quantizer scale is set to be equal to the maximum amplitude in the received signal $r(n)$ and the quantization levels is 8, corresponding to the quantization resolution of 3-bit. In addition, the value of k increases from 1 to 10. Again, the simulated digital receiver performance is very close to the theoretically calculated result, with an EpN0 difference of about 0.1dB when the error rate is 10^{-4}. This can be caused by the use of a finite maximum value for k.

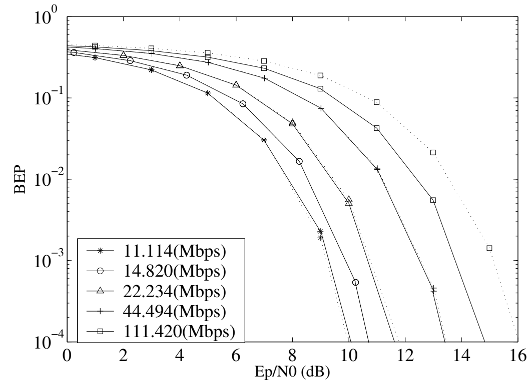

Fig. 2. Simulated and calculated BEP versus UWB pulse energy for the discrete time full-resolution SFS TR receiver.

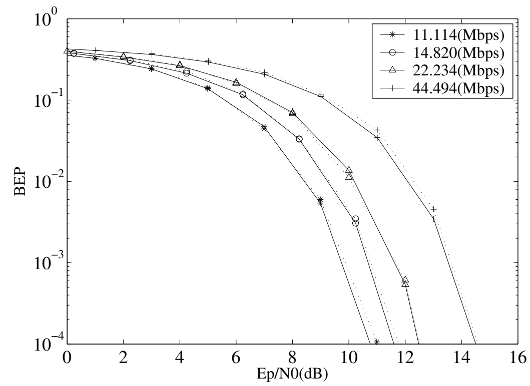

Fig. 3. Simulated and calculated BEP versus UWB pulse energy for the quantized digital SFS TR receiver with 3-bit resolution.

VI. Summary

This paper works on quantization of the UWB SFS TR receiver. Simulated performance based on measured channel data from the intra-vehicle environment highly agrees with the calculated performance in the case of 3-bit quantization resolution. This indicates the possibility of implementing a low cost low complexity digital UWB SFS TR receiver.

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