

# A Spectrum Sensing Scheme with Multiple Users

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**Abstract.** Spectrum sensing has attracted much concern of researchers due to its significant contribution to the spectral efficiency. However, the corresponding work mainly focuses on the sensing event of single primary users within a certain band and the investigation of the effect of PU traffic on the sensing performance is considered rarely. In this paper, a spectrum sensing scheme is presented to explore the co-existence of multiple users in the same frequency band based on subspace filtering. To remove uncertain noise as much as possible, subspace filtering is applied to the received signal of a cognitive radio, where the received signal is decomposed into two parts: noise subspace and signal-plus-noise subspace. Then the closed-form solution of the detection and false alarm probabilities with multiple users is given on the basis of the signal-plus-noise subspace in Rayleigh fading channel. Eventually, simulations are made to validate the proposed scheme.

**Keywords:** Spectrum sensing · Subspace filtering · Rayleigh fading channel

## 1 Introduction

With the development of wireless communication technology, scarce spectrum resource catches our attention. However, according to the investigation of Federal Communications Commission (FCC) [1], most of the registered spectrum is unoccupied in time or space. To take full advantage of the vacant spectrum, Cognitive Radio (CR) technology is presented to improve the level of spectrum utilization by allowing some unlicensed (secondary) users to have access in an opportunistic and non-interfering manner some licensed bands temporarily unoccupied by the licensed (primary) users [2].

In a CR network, one aim is to protect the primary user (PU) from the case that the registered user is present within a certain frequency range while the secondary user (SU) also uses the same frequency range for opportunistic wireless transmissions.

To avoid this situation, SU is required to sense and monitor periodically the radio frequency environment for opportunistic occupation within a certain amount of time to decide whether PU exists. Another purpose considered in CR is the achievable throughput for the secondary network. For a higher throughput (spectrum efficiency) in a secondary network, the case should be strictly restricted that SU detects the presence of PU while the frequency band is idle factually. The core technology behind the reuse of spectrum in CR network is Spectrum Sensing (SS). However, the dynamic and unpredictable sensing environment greatly lowers the detection efficiency of SU, resulting in the poor protection of PU and a lower achievable throughput for the secondary network.

In the literature of SS, a number of algorithms have been proposed to identify the presence of the primary signal as well as to improve the detection efficiency by inhibiting the disgusting interference in the sensing environment. Some examples of the existing proposals include Energy Detector (ED) [2], Matched Filtering (MF) [3], Covariance-Based Detection (CBD) [4] and Cyclostationary Feature Detection (CFD) [5]. The proposed solutions devote to optimizing sensing time, complexity or detection capabilities (the necessary prior knowledge of the primary signal or environment) for a possible tradeoff. However, the obtained performance is always at the expense of one or more of the other factors. For example, ED based methods are widely accepted for its low complexity while its detection performance works badly, especially under a low SNR. Although the CBD based method outperforms others on detection performance due to its nearly independence of noise uncertainty, the required sampling frequency is much higher than the normal conditions, leading to a higher complexity of implementation.

Interference from sensing environment almost determines the accuracy of judging whether the given frequency band is used, for most of the SS schemes. Consequently, that how to remove interference of the frequency bands has been significant for the improvement of detection accuracy and applicability to sensing scenarios. In the context of CR, close attention is paid to this issue. Andrea Mariani et al. [6] explored the SNR wall phenomenon caused by uncertain noise for ED based method, addressing the threshold design and giving the conditions for the existence of the SNR wall. Valentin Rakovic et al. [7] proposed an optimization approach for cooperative spectrum sensing utilizing ED with estimated noise power.

As a result, a subspace filtering is firstly applied to the receiver in this paper, where the observed signal of SU is divided into two subspaces: noise subspace and signal-plus-noise subspace. The noise subspace only contains background noise while the signal-plus-noise subspace contains the whole signal and some residual noise [8]. By clearing the noise subspace and noise contribution in the signal-plus-noise subspace, the remainder components mainly consist of the primary signal. Due to the possible removal of environmental interference, the existing interference is dramatically reduced and the noise uncertainty is restrained commendably. Furthermore, detection capability will be greatly heightened for the decrease of background noise.

On the other hand, most of the related works always neglect the influence from the multiple primary users on the spectrum sensing performance of the CR's energy detector. However, in several widely used wireless communication standards, such as long term evolution advanced (LTE-A), WiFi and WiMAX, where code-division

multiple-access (CDMA) is used, users simultaneously operate in the same frequency band [9]. This motivates a consideration of multiple users in the same frequency band. Factually, few researches are made to consider the effect of PU traffic on the sensing performance. In addition, these considerations of PU traffic are based on the uncertain environmental noise, which lowers the sensing efficiency and performance. In this paper, a spectrum sensing scheme with multiple users is proposed based on subspace filtering, where the closed-form solution of the detection and false alarm probabilities is given on the basis of subspace filtering. Some simulations based on MATLAB platform are made to validate the proposed method.

The rest of this paper is organized as follows. Section 2 presents the primary principle of subspace filtering. The main contribution of this paper is shown in Sect. 3, which consists in the derivation of the closed-form solution of detection and false alarm probabilities. Simulation experiments and result analysis are accomplished in Sect. 4. Finally, Sect. 5 concludes this paper.

## 2 Subspace Filtering

### 2.1 System Model

Generally, the background noise of a certain radio band we are interested in is modeled as Gaussian, independent and identically distributed random process with symbol  $u(t)$ . The clean primary signal is denoted as  $s(t)$  and  $s(t)$  is usually independent of  $u(t)$  for convenience to talk about. The received signal is with central frequency  $f_c$  and bandwidth  $W$ . Then a system model at the receiver for a CR could be formulated as

$$r(t) = u(t) + s(t). \quad (1)$$

To sample the received signal at the frequency of  $f_s$ , the signal at the receiver could be described as

$$r(n) = u(n) + s(n). \quad (2)$$

Factually, several primary users may operate in the same frequency band in some CDMA applications. Therefore,  $s(n)$  generally exists with another form

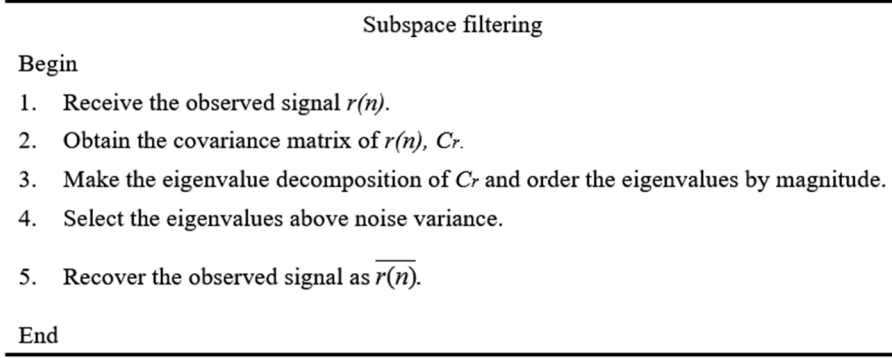
$$s(n) = \sum_{i=1}^M s_i(n), \quad (3)$$

where  $M$  represents the number of primary users in some a frequency band.

### 2.2 Subspace Filtering

In this subsection, the primary principle of subspace based filtering is discussed. Subspace filtering is a widely accepted approach in signal processing, where the received signal is decomposed into two orthogonal subspaces: noise subspace and

signal-plus-noise subspace. The main purpose of subspace filtering consists in signal enhancement by the removal of noise subspace and noise contribution in signal-plus-noise subspace. As it has been analyzed on several open literatures, only a flowchart is provided here as follows (Fig. 1).



**Fig. 1.** The general procedure of subspace filtering.

Note that the recovered observation signal after subspace filtering is usually written as

$$\overline{r(n)} = V\Lambda_x(\Lambda_x + \mu\Lambda_u)^{-1}V^{-1}r(n), \quad (4)$$

where  $V$  is the eigenvector of  $C_r$ ,  $\Lambda_x$  is the eigenvalue matrix after eigenvalue selection,  $\Lambda_u$  is the unit matrix with the same dimension as  $\Lambda_x$  and  $\mu$  is a given Lagrange constant.

### 3 Multiple Users Based Spectrum Sensing

The energy of the remainders after the removal of noise can be approximatively shown as

$$E_r \approx \sum_{i=1}^M |h_i(n)s_i(n)|^2 = \sum_{i=1}^M a_i(n)|h_i(n)|^2, \quad (5)$$

Assume that  $|h_i(n)|$  follows Rayleigh distribution with Probability Distribution Function (PDF)

$$f(x; k) = \frac{x^{k/2-1}}{2^{k/2}\Gamma(k/2)} e^{-x/2} U(x), \quad (6)$$

and Cumulative Distribution Function (CDF) shown as

$$F(x; k) = \frac{\gamma(k/2, x/2)}{\Gamma(k/2)} U(x). \quad (7)$$

Assume that  $a_i(n) = (1/M) \sum_{i=1}^M a_i(n) = a$  is a constant value,  $|h_i(n)|^2$  follows gamma distribution with the same parameters  $N$  and  $2\sigma^2/a$ , the PDF of  $E_r$  in (18) can be rewritten as

$$f_r(z; M, 2\sigma^2/a) = \frac{(2\sigma^2/a)^M z^{M-1} \exp(-2\sigma^2 z/a)}{(M-1)!}, \quad (8)$$

with its CDF denoted as

$$\begin{aligned} F_r(z; M, 2\sigma^2/a) &= \frac{\gamma(M, 2\sigma^2 z/a)}{(M-1)!} \\ &= 1 - \sum_{n=0}^{M-1} \frac{1}{n!} \exp(-2\sigma^2 z/a) (2\sigma^2 z/a)^n. \end{aligned} \quad (9)$$

The corresponding detection and false alarm probabilities are formulated as follows

$$\begin{aligned} P_d &= 1 - F_r(\lambda; M, 2\sigma^2/a|H_1) \\ &= \sum_{n=0}^{M-1} \frac{1}{n!} \exp(-2\lambda\sigma^2/a) (2\lambda\sigma^2/a)^n, \end{aligned} \quad (10)$$

$$\begin{aligned} P_f &= 1 - F_r(\lambda; M, 2\sigma^2/a|H_0) \\ &= 1 - \frac{\gamma(M, 2\sigma^2 \lambda/a)}{(M-1)!}. \end{aligned} \quad (11)$$

## 4 Simulation and Analysis

Simulations are conducted in this section to verify the performance of the proposed method.

Figure 2 gives ROC (Receiver Operating Characteristic) of different scenarios for the case that  $a_i(n) = (1/M) \sum_{i=1}^M a_i(n) = a$  is a constant value according to the formula (10) and (11). In Fig. 2,  $M$  represents the number of PU in the same frequency band,  $\text{var}$  denotes the variance  $\sigma^2$  in (8) and  $a = (i, j)$  indicates the amplitude of  $H_1$  and  $H_0$  under the identical scenarios, where  $a = i$  denotes the squared amplitude of residual noise in  $H_0$  case after subspace based filtering and  $a = j$  denotes squared amplitude of  $H_1$  case after subspace based filtering.

Figure 2-a mainly discusses how the variance of  $|h_i(n)|$  influences the detection performance of a CR for multiple PUs at a certain frequency band. From the simulation results, different variances of  $|h_i(n)|$  have few effects on the detection performance, which indicates the detection performance is independent of the variance of  $|h_i(n)|$ . Figure 2-b exhibits the influence from the signal squared magnitude on the detection performance, where the detection performance improves as the rise of the squared signal magnitude when the noise squared magnitude is invariant. This declares that the signal magnitude is one of the factors to determine the detection accuracy for a set of CR equipment. Similarly, Fig. 2-c shows the influence from the residual noise squared magnitude on the detection performance. Obviously, the ROC increases as the decline of residual noise squared magnitude. Figure 2-b and c indicate that the detection performance of a CR in the case of multiple PUs also suffers from the influence of SNR (signal-to-noise-ratio), where the performance is in positive relation to the variation of SNR. How the number of PU alters the detection performance is investigated in Fig. 2-d. It is emphasized that the detection performance arises with the increase of PU number in the same frequency, which results from the removal of background noise. The removal of background noise is equivalent to a rise of SNR, which will make the sensing performance turn better in return.

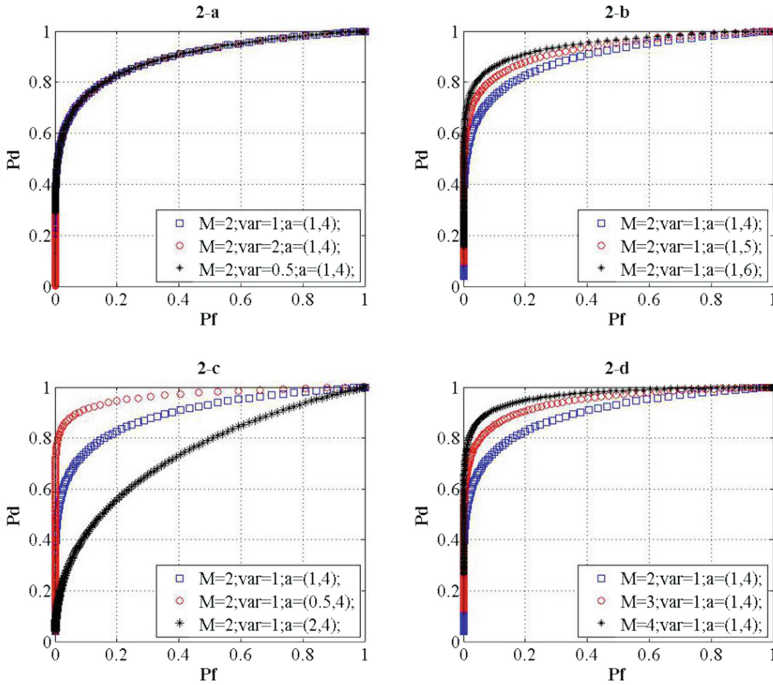


Fig. 2. ROC curve of different situations

## 5 Conclusions

A multiple users based spectrum sensing scheme is explored on the basis of subspace filtering in this paper. The corresponding closed-form solutions on the detection and false alarm probabilities are given on the assumption that the signal magnitude is identical. The case with different signal magnitude will be discussed for the potential applicability later.

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