Genus-Zero Shape Classification Using Spherical Normal Image

Shaojun Liu
Oakland University
Rochester, MI 48309, USA
sliu2@oakland.edu

Jia Li
Oakland University
Rochester, MI 48309, USA
li4@oakland.edu

Abstract

A new method for three dimensional (3D) genus-zero shape classification is proposed. It conformally maps a 3D mesh onto a unit sphere and uses normal vectors to generate a spherical normal image (SNI). Unlike extended Gaussian images which have an ambiguity problem, the SNI is unique for each shape. Spherical harmonics coefficients of SNIs are used as feature vectors and a self-organizing map is adopted to explore the structure of a shape model database. Since the method compares only the SNIs of different objects, it is computationally more efficient than the methods which compare multiple 2D views of 3D objects. The experimental results show that the proposed method can discriminate collected 3D shapes very well, and is robust to mesh resolution and pose difference.

1 Introduction

Available 3D models on the Internet increase dramatically with advancement of modeling and digitizing techniques. Efficiently searching relevant shape models is desired in many fields like entertainment, engineering and science. Shape-based retrieval of 3D data has been an active research area in disciplines such as computer vision, mechanical engineering and chemistry. The performance of 3D shape search engine, however, is far behind as compared with that of image and text, such as Google search engine.

Comparison of shape similarity is a basis for shape recognition, matching, and classification. Methods based on 2D visual similarity require multiple views of a 3D object [1]. Gu et al. proposed a 2D geometry image to represent an original 3D mesh [2]. It cuts the 3D mesh open and maps it onto a unit square. Based on geometry images, Laga et al. proposed a shape matching method to save comparison of multiple 2D views [6]. However, similar 3D shape models are not guaranteed to have the same cut since there are multiple choices of cutting paths. As a result their geometry images may be quite different due to different cutting, adding variance to the similarity comparison based on geometry images. Extended gaussian images (EGI) use normal vectors as geometric features to compare shape similarity. However, EGI is not unique to non-convex objects, referred as an ambiguity problem, and EGI does not incorporate local spatial maps either.

We propose a new shape similarity comparison method based on spherical normal images (SNI). The normal vectors are stored in the conformal map of a 3D mesh over a unit sphere, which is one to one mapping without cutting the 3D mesh open. The overall approach follows the sequence of pose alignment, conformal mapping, feature extraction, and similarity search as shown in Fig. 1. We use a self-organizing map to classify 3D models collected from the Internet.

The paper is organized as following. In Section 2, we briefly describe the background of pose alignment and conformal mapping of 3D meshes to a unit sphere. The feature extraction step is introduced in Section 3. In Section 4, we present experimental results and analysis. Section 5 concludes the paper and discusses future directions.

2 Background

Traditional shape recognition methods that are based on geometric features follow the strategy of registration and recognition. The registration eliminates the variance of feature vectors caused by different poses. Factors to be considered usually include scaling, translation, and rotation. Translation is usually removed by shifting the center of a mesh to the origin, and scaling is removed through normalization. As for rotation, Vranić compared the shape comparison methods using principle component analysis (PCA) in pose alignment and the methods without rotation processing, and concluded that the PCA based methods have better performance [10]. We adopt a Continuous PCA (CPCA) method in our pose alignment process, which computes sums of integrals over triangles instead of those over vertices to reduce dependency on surface tessellation and mesh resolution [10]. It works well with most of the shapes collected from the Internet.

To analyze geometric features of a 3D object, it is con-

0-7695-2521-0/06/$20.00 (c) 2006 IEEE

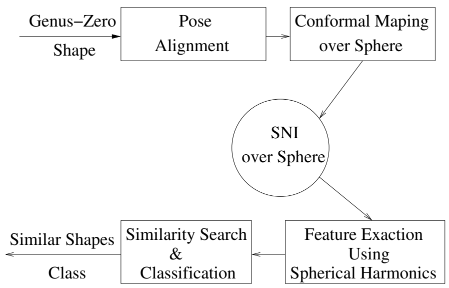
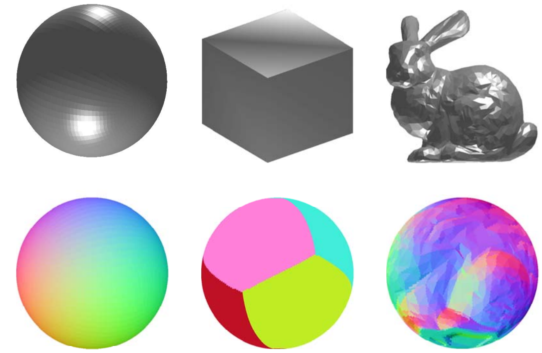


Figure 1. The procedure of genus-zero shape classification.



(a) Ball (b) Cube (c) Bunny

Figure 2. Original Models and their SNI.

venient to map the object surface onto the region of a plane or sphere first. The surface analysis is then carried over the plane or sphere domain. For a closed surface, mapping it onto a sphere, if possible, yields less distortion than onto a plane. Conformal mapping preserves angles and the mapped mesh and the original differs only in a scaling factor in terms of the first fundamental form. In other words, shape is preserved locally in the sense that distances and areas are only changed by a scaling factor. We adopt Gu and Yau's algorithm to compute a conformal map over a sphere, which is a six dimensional Mobius transformation group [3]. It translates the mass center of a mesh to the origin to guarantee a unique mapping. For a non-convex shape in which a vector starting from the origin may intersect the surface more than once, the conformal mapping can converge to a valid mesh on the sphere without overlapping. Thus we avoid the ambiguity problem associated with EGI approaches. Nevertheless, a conformal map over a sphere is limited to genus-zero shapes only. As for non-zero genus shapes, they have to be cut open and mapped onto a plane.

After mapped onto a sphere, geometric features are further indexed to facilitate the shape classification process. Schudy and Ballard used spherical harmonics (SH) to fit a surface as a function over a sphere [8]. In our approach, the geometric feature stored in a conformal map over a sphere, is regarded as a radial function $f : S^2 \rightarrow \Re$. It can be expanded as a linear combination of SH: $f(\theta,\phi) = \sum_{l=0}^{\infty}\sum_{m=-l}^{l} C_l^m Y_l^m(\theta,\phi)$, where $Y_l^m(\theta,\phi)$ is the SH and the coefficients C_l^m are uniquely determined by $C_l^m = \int_0^{\pi}\int_0^{2\pi} Y_l^{m*}(\theta,\phi) f(\theta,\phi)\sin\theta d\phi d\theta$. A feature vector composed by the coefficient C_l^m, is used to represent the original shape in further classification.

3 Shape Classification

In our approach, we take normal vectors $\vec{N} = \{N_x, N_y, N_z\}$ as the geometric feature and store them in a conformal map over a unit sphere. A SNI is generated by interpolating grids of longitude and latitude. We use Gnomonic mapping to interpolate a grid point P inside a spherical triangle ABC: $P = \frac{uA+vB+wC}{||uA+vB+wC||}$, where $0 \leq u, v, w \leq 1$ and $u+v+w = 1$. And P is given the same normal vector as that of the original triangle ABC. To illustrate, we show $\{N_x, N_y, N_z\}$ as $\{R, G, B\}$ color for shape models in Fig. 2. Compared with geometry images, the SNI incurs less distortion by mapping a closed surface onto a sphere [2]. Without cutting meshes open, it also avoids the variance resulted from different cutting paths among similar shape models. Compared with the spherical parameterization proposed by Praun and Hoppe, which minimizes the stretch between an original mesh and the mapped mesh on a sphere [7], the SNI is based on a conformal map that preserves the angles and local shape. And the SNI does not need mapping from sphere to a polyhedron or unit square in [7], since further classification on the SNI is carried out directly over the sphere without unfolding.

We then use SH decomposition of SNI to facilitate the shape classification process. The feature vector is constituted by SH coefficients C_l^m. In practice, we use the norm of C_l^m in the feature vector and exclude those with $m < 0$ because of symmetries between C_l^m and C_l^{-m}. As $N_x^2 + N_y^2 + N_z^2 = 1$, the SH representation of N_z is regarded as a redundancy and omitted. Therefore, the dimension of the feature vector is $(K+2)(K+1)$, where K is the highest order of SH. The feature vectors of a ball, cube and bunny with $K = 1$ are shown in Table 1. We use a singular value decomposition (SVD) method to compute SH offline and only need to compute SH coefficients C_l^m for online retrieval to shorten response time.

		Ball	Cube	Bunny
N_x	C_0^0	0.00243288	2.62533e-17	0.558153
	C_1^0	0.00178926	6.59502e-17	0.09185
	C_1^1	1.24949	0.8149	0.661967
N_y	C_0^0	0.000376667	5.03471e-17	0.297591
	C_1^0	0.000429478	8.03768e-17	0.0311241
	C_1^1	1.5204	1.19423	1.001

Table 1. The feature vectors of a ball, cube and bunny with K = 1

Residual error is introduced by the truncation of the higher order of SH in practice. The error decreases as the

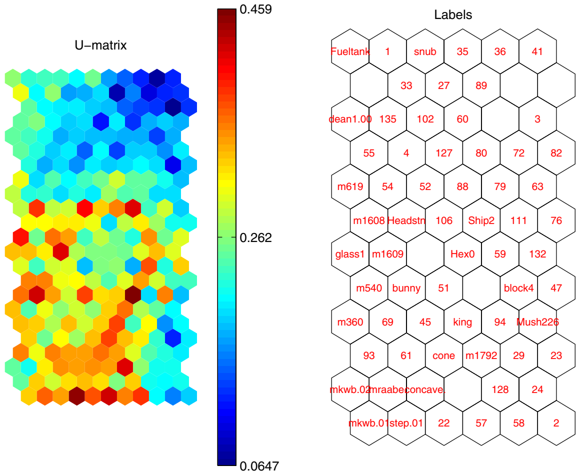


Figure 3. Shape classification result by SOM.

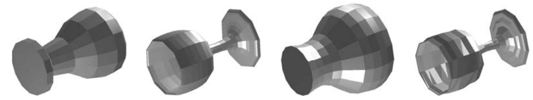
value of K increases. Using finite SH coefficients C_l^m is equivalent to apply a low pass filtering, whose level is controlled by the K value.

Dense meshes contain highly detailed geometric information, which might be redundant for coarse classification. Multi-resolution meshes are desired for a coarse-to-fine classification. We adopt the progressive mesh proposed by Hoppe in the classification [4]. The size of feature vector is scalable to multi-resolution meshes, i.e. shorter feature vector is used in first level classification and longer vector in fine classification. This scalability is achieved by varying the highest SH order, or the K value.

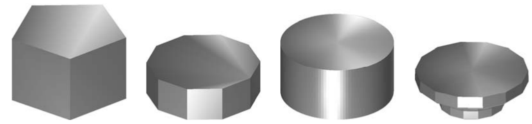
After feature vectors are obtained, we adopt a self-organizing map (SOM) to classify 3D shape models [9]. The SOM is an excellent tool in exploratory phase of data mining, which enables classification without prior knowledge, such as the number of classes. We adopt a U-matrix, the unified distance matrix, to visualize the distance between prototype vectors of neighboring map units.

4 Experiments and Discussions

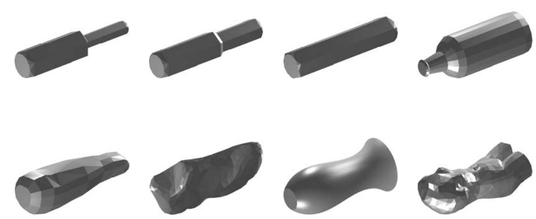
We collect 3D models from various sources on the Internet, with acknowledgment to SAMPL in Ohio State University, Princeton Shape Benchmark, Vranić's 3D Model Database and Stanford 3D Scanning Repository. Unfortunately, current 3D model benchmark is not applicable to our approach due to the limit of genus-zero objects at present. We extract feature vectors of 214 models with the highest SH order $K = 16$, and use a SOM Matlab toolbox from CIS Helsinki University of Technology to get the result of 12×6 prototypes in Fig. 3. The left side of Fig. 3 is a U-matrix marked by different colors, while the right side are prototypes with different labels. Blank label means no feature vector presents in the prototype. As the U-matrix displays distance between prototype vectors, feature vectors with smaller distance means more similarity between the according 3D shape models. And similar shape models should be clustered into the same or close prototypes. By checking into prototypes, we find our method picks up similar 3D models very well as shown in Fig. 4.



(a) Models in prototype labeled 'glass1'



(b) Models in prototype labeled '23'

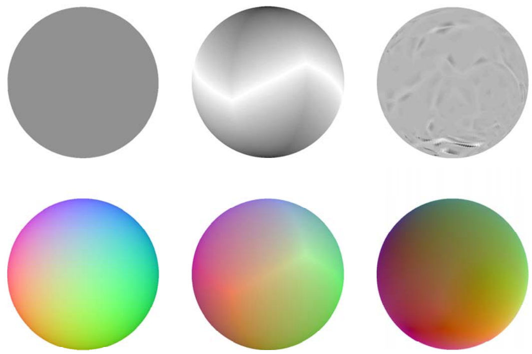


(c) Models in prototype labeled 'Hex0'

Figure 4. The 3D models in the prototypes.

To compare the results of different feature vectors, we generate spherical curvature images (SCI) and spherical geometry images (SGI) in a way similar to SNI as shown in Fig. 5. Curvature inside a mesh polygon is computed by interpolating curvature at polygon vertices. And SGI is computed by interpolating normalized $\{X, Y, Z\}$ coordinates at polygon vertices. The method using SCIs requires dense meshes and classifies cubes and balls into one prototype, whose SCIs resemble with symmetries along X, Y, Z axes and large areas of constant curvature value. In contrast, methods using SGIs and SNIs do not depend so much on mesh resolution and can discriminate cubes and balls correctly. The method using SGIs yields occasional "bad" classification compared with that using SNI. For example, a glass is found in the prototype of cubes because its SGI is not distinctive from those of cubes.

The result of pose alignment can affect feature vectors and final classification results. For example, for cuboids of $1 : 1 : 1$ ratio in different initial poses, the PCA method gives inconsistent rotations as shown in Fig. 6(a). Cuboids with $5 : 1 : 1$ ratio are given inconsistent rotations along X axes in Fig. 6(b). Only those with different ratio along X, Y, Z axes in Fig. 6(c) are registered consistently. The pose variance of objects after registration decreases with asymmetries along X, Y, Z axes increase, which is an artifact of registration using PCA methods. Nevertheless, the effect of rotation on feature vectors is limited as shown in



(a) Ball (b) Cube (c) Bunny

Figure 5. SCI (top) and SGI (bottom).

the experiments. The feature vectors of seven cubes are clustered into the prototypes labeled as '1', '33', and '135', which are very close according to the U-matrix in Fig. 3.

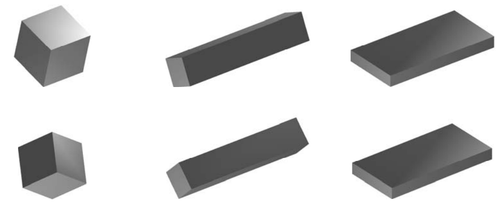
As for multi-resolution representation, we have generated different resolution meshes of same objects using Hoppe's algorithms [4], such as bunnies in Fig. 7(a) and 7(b). The multi-resolution meshes of same objects are clustered into same prototypes. It demonstrates that our method is robust to meshe resolution.

The SH representation is also used by Kazhdan et al. in their voxelized model with $64 \times 64 \times 64$ grids [5]. Fig. 7(c) shows the voxelized bunny of 15377 cubes from the bunny of 4000 triangles in Fig. 7(a). Though using a much larger data size, Fig. 7(c) loses many fine details of Fig. 7(a) before SH representation, which is also addressed in [10]. Based on a surface based model, the proposed method needs much smaller data size to present at least same level of surface details.

5 Conclusion and Future Work

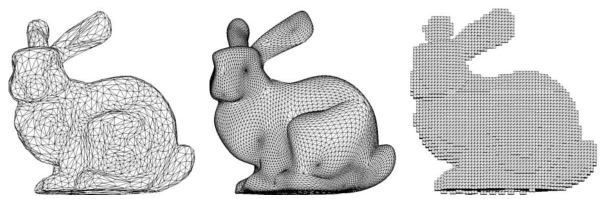
In this paper we propose a new approach for 3D shape classification based on spherical normal images (SNI). The SNI incorporates local features by conformal mapping over a unit sphere and is unique to each shape without ambiguity. And it preserves surface details unlike voxelized models. We also use spherical harmonics (SH) to facilitate the shape classification process. And we use the SVD method to compute SH offline to shorten the response time of online retrieval. The experimental results show that the method using SNI can discriminate the collected shapes very well and performs better than that using spherical curvature images and spherical geometry images. The SNI based method is also robust to mesh resolution and pose variance.

To apply our method to non-zero genus objects, we need to convert them to genus-zero objects first. Our approach is not limited to specific classifiers. To discriminate shapes with training samples, we can also adopt classifiers such as support vector machines (SVM).



(a) 1 : 1 : 1 (b) 5 : 1 : 1 (c) 10 : 5 : 1

Figure 6. The pose alignment results of cuboids.



(a) Original Surface (b) High Resolution (c) Voxelized Model

Figure 7. Bunnies with different resolutions.

References

[1] D.-Y. Chen, X.-P. Tian, Y.-T. Shen, and M. Ouhyoung. On visual similarity based 3D model retrieval. In *Eurographics Computer Graphics Forum*, volume 22, pages 223–233, 2003.

[2] X. Gu, S. J. Gortler, and H. Hoppe. Geometry images. In *Proceedings of SIGGRAPH*, pages 355–361, 2002.

[3] X. Gu and S. T. Yau. Computing conformal structures of surfaces. In *Communication of Information and Systems*, volume 2, pages 121–146, 2002.

[4] H. Hoppe. Progressive meshes. In *SIGGRAPH 96*, pages 99–108, 1996.

[5] M. Kazhdan, T. Funkhouser, and S. Rusinkiewicz. Rotation invariant spherical harmonic representation of 3D shape descriptors. In *Eurographics Symposium on Geometry Processing(2003)*, pages 156–165, 2003.

[6] H. Laga, H. Takahashi, and M. Nakajima. Geometry image matching for similarity estimation of 3D shapes. In *Proceedings of the Computer Graphics International (CGI'04)*, pages 490–496, 2004.

[7] E. Praun and H. Hoppe. Spherical parameterization and remeshing. In *Proceedings of ACM SIGGRAPH*, pages 340–349, 2003.

[8] R. B. Schudy and D. H. Ballard. Towards an anatomical model of heart motion as seen in 4-D cardiac ultrasound data. In *Proceedings of the 6th Conference on Computer Applications in Radiology and Computer-Aided Analysis of Radiological Images*, 1979.

[9] J. Vesanto and E. Alhoniemi. Clustering of the self-organizing map. *IEEE Transactions on Neural Networks*, 11:586–600, 2000.

[10] D. V. Vranić. An improvement of rotation invariant 3D shape descriptor based on functions on concentric spheres. In *ICIP 2003*, volume 3, pages 757–760, 2003.