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A 3D SHAPE DESCRIPTOR: 4D HYPERSPHERICAL HARMONICS "AN EXPLORATION INTO THE FOURTH DIMENSION"

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ABSTRACT

Shape matching remains a challenging problem. Most search engines on the internet use textual description to match images. More sophisticated systems use *shape descriptors* that are automatically constructed from the original 3D shape. In this paper, we propose a novel shape descriptor based on four dimensional (4D) hyperspherical harmonics. Shape descriptor using 3D spherical harmonics present the benefits of being insensitive to noise, orientation, scale, and translation. However, the radii cuts introduce a disadvantage of failing to recognize inner rotations. We address this problem by mapping 3D objects onto the 4D unit hypersphere and applying 4D hyperspherical harmonic decomposition to get the shape descriptor. The 4D hyperspherical harmonics have the same advantages of the 3D spherical harmonics and remove the disadvantage of the 3D spherical harmonics that is associated with the inner radii cuts.

KEY WORDS

shape descriptor, hyperspherical harmonics

1 INTRODUCTION

The advancement of 3D scanning device technology and the daily use of the World Wide Web have made the index of available 3D shapes expand vastly over the years. As the demand for computer graphics increases, the methods of search and retrieval of shape database must become more robust and effective. 3D shape retrieval has wide applications in target recognition, medical image analysis, entertainment, and architectural industries. It also has the potential to greatly improve many fields of research such as mechanical engineering and molecular biology. The challenge is to convert different 3D object representations, such as surface mesh and volumetric data into a compact computational representation, the so-called shape descriptor, so that different 3D objects can be efficiently compared and matched. The shape descriptors are usually in the form of vectors that can be compared by the distance of two points within the space. These numerical representations serve as search characteristics during the retrieval process. Good shape descriptors must be insensitive to noise, orientation, scale, and translation. They must be fast to compute, small in size, and easy to compare. A variety of methods have been proposed in the past, which can be classified into three main categories or are some combination of the three groups [1].

This research focuses on the global matching method using orthogonal basis functions. We use the basic idea of harmonics and apply them to hypersphere. Hyperspherical harmonics have been used in solving multi-body problems in physics, but to the best of our knowledge, this research is the first step toward mapping 3D objects onto 4D unit hypersphere and using hyperspherical harmonics as shape descriptor. The organization of this paper is as follows. In section 2, we briefly describe the mapping of 2D image onto 3D unit sphere and the properties of 3D spherical harmonics. In section 3, we introduce hyperspherical harmonics and our shape descriptor's implementation. In section 4, the experimental results and the analysis are presented. Section 5 concludes the paper and gives details about future work.

2 3D SPHERICAL HARMONICS

Spherical harmonic representation as a rotational invariant 3D shape descriptor was proposed by Michael Kazhdan. This descriptor transforms rotationally dependent shapes into rotationally invariant descriptors. Spherical harmonic representation provides better shape matching results than those obtained by rotation normalization [2]. The main goals of the spherical harmonics descriptor are to provide better matching results while reducing both the space for storage and the time for comparison. Kazhdan reported that pose alignment via principal component analysis (PCA) hampers the performance of descriptors. Spherical harmonic decomposition is a generalization of Fourier transform on the unit sphere, which uses basis functions of different frequencies. The main idea of this approach is to describe a function on the unit sphere in terms of the amount of energy it contains at different frequencies [2]. These values do not change when the function is rotated, hence rotation invariant.

Besides invariance to rotation, spherical harmonic descriptors offer several other advantages. Precision-recall is a method of determining the retrieval accuracy of a shape descriptor. As reported, spherical harmonic descriptors currently have the best precision-recall of the available 3D shape descriptors.

2.1 Limitation of Spherical Harmonics

Although spherical harmonic representation has proven to be successful, there is still room for improvement. There is a full dimension of information lost in going from a spherical function to its harmonic representation. The descriptors are unchanged if different rotations are applied to different frequency components of a spherical function. For each frequency component, the spherical harmonic representation only stores the energy in that component [2].

The process of spherical harmonics leads to unique sources of error not seen in other methods. The spherical harmonics of a function continue infinitely. Much like the rounding of decimals, spherical harmonic descriptors must arbitrarily decide which degree of harmonic decomposition to end analysis at. Because spherical harmonics are applied over functions on 3D spherical surface, a 3D object must be severed into many shells with varying radii to be represented by spherical harmonics. The number of possible radii cuts is also finite in real applications. The error caused by harmonic truncation is compounded over every radii cut, and the error caused by using finitely many radii cuts is obvious. These radii cuts also introduce the problem of a descriptor that cannot acknowledge inner rotations. Ideally objects would have the same harmonic representation after outer rotation over the entire shape. With inner rotations (see Figure 2), the 3D spherical harmonic descriptors for two objects are the same, when it obviously should not be.

Our proposed method of using hyperspherical harmonic decomposition as a shape descriptor will map 3D objects to the domain of 4D hypersphere and remove the step of radii cuts. Thus the descriptor is sensitive to inner rotations,. The proposed descriptor will no longer be a function of radius, but a function of hyperspherical harmonic frequency only.

3 4D HYPERSPHERICAL HARMONICS

Hyperspherical methods have long been a valuable analytical and computational tool for understanding n-body quantum systems [3,4]. They have also been applied to problems in molecular, nuclear, and atomic physics.

Our choice to use hyperspherical harmonics was based on the need to address the inner radius cut issue. Harmonics taken over the entire shape would produce better results than those produced by finitely many radii cuts. It is known that harmonics may be performed over the n^{th} dimensional sphere, as the idea has been generalized by physicists. A test of the theory was performed based on the assumption that if 2D areas can be mapped to the 3D unit sphere and subsequently spherical analysis may be performed, then a 3D volume may be mapped to the 4D unit hypersphere and subsequently hyperspherical analysis may be performed.

3.1 Theory Testing

Verification of rotational invariance and invertibility were tested via the mapping of 2D image to 3D unit sphere. It was done in MatLab, using Yet Another Wavelet Toolbox to compute spherical harmonics [5]. After the mapping onto the unit sphere, the coordinates become the longitudinal and latitudinal angles. The results of the testing showed that rotation of a 2D shape was represented by a latitudinal phase shift (see Figure 1).

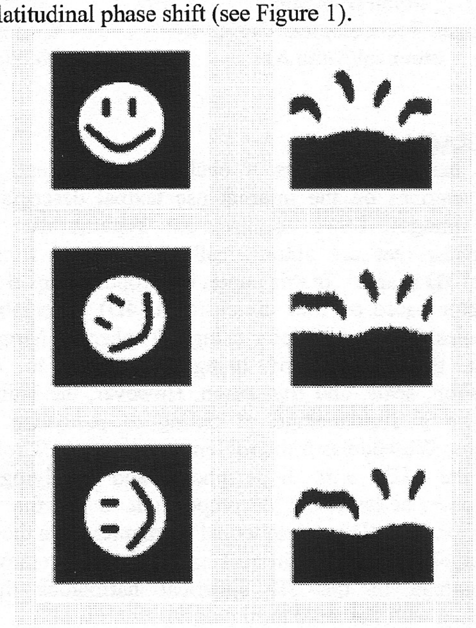

Figure 1: Original 2D shape (upper left), 60° rotation of original shape (middle left), 90° rotation of original shape (bottom left), and longitudinal and latitudinal angles of 3D spherical coordinates (right).

Such a phase shift corresponds to a 3D rotation of the shape function on the unit sphere. And the 3D spherical harmonic descriptors that could be applied to represent the mapped data are invariant to such rotations. The original 2D image can be recovered by inverse mapping of the function over the unit sphere. Figure 4 shows the results. The mapping is thusly safe to use.

After having successful results using 2D-to-3D mapping, we think that 3D volume-to-4D hypersphere mapping was worth pursuing and we are ready to apply hyperspherical harmonic analysis.

3.2 Implementation

The 3D volumetric representation was obtained by running a surface voxelization algorithm over the entire mesh. A solid voxelization algorithm may be applied as well, but it has the limit to work only with closed surface object. We chose surface voxelization for more flexibility.

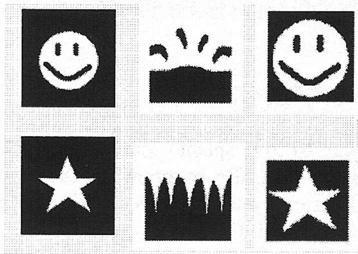

Figure 2: Original 2D shape (left), longitudinal and latitudinal angles of 3D spherical coordinates (middle), then it is inverted back to original shape (right).

Once voxelization is performed, each voxel is considered to be a single point in the original continuous space with a value of true or false. An angular grid is set up for the 4D unit hypersphere on the range of $[0, \pi]$, $[0, \pi]$, $[0, 2\pi]$. Each entry of the grid is mapped backwards to the 3D continuous coordinate system and takes on the truth value of that mapped 3D point.

Now we have an angular voxel grid with the 3D object mapped onto the 4D unit hypersphere. The entire sample space of the angular voxel grid is converted into the corresponding continuous angles, then harmonics are performed on them (up to an arbitrary upper limit). This gives a complex matrix A containing the harmonic values over the sample space of points. The matrix b is filled in with truth values of the corresponding point. Coefficients for each harmonic value are found by solving the linear equation Ax=b for x.

In 3D harmonic methods, a rotationally invariant descriptor is found by representing a spherical function by the size of its projections onto each fundamental frequency [6]. Effectively, this equation is the square root of the sum of the complex m-value norms for each frequency l:

$$f \to \{\|\pi_l(f)\|\}_{l=0}^{\infty} = \left\{\sqrt{\sum_{|m|\le l}\|f_{l,m}\|^2}\right\}_{l=0}^{\infty}$$

To maintain rotational invariance in four dimensions, we decided to generalize said formula. First a three dimensional descriptor is found as above, then the same process is performed using lambda as the fundamental frequency and the spherical l function as the m-value.

4 RESULTS

We chose to use seven models for testing. These models came from the Princeton Benchmark and are referenced by their filename in the database [7]. Four models are shown in Figure 3. The other three models were created via rotating the m0 model over the x-axis, y-axis, and z-axis by π radians.

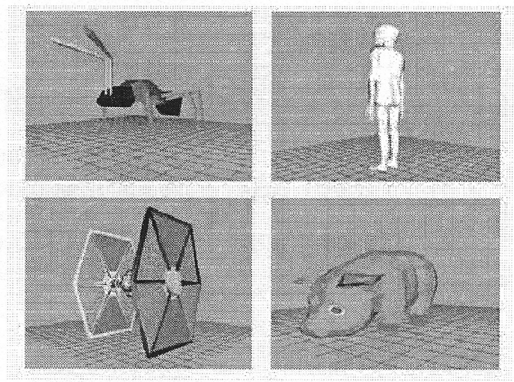

Figure 3: Prerendered images. [m0 (top left), m150 (top right), m1401 (bottom left), m100 (bottom right)].

The models were voxelized (see Figure 4) and the process described in section 3.3 was applied. The analysis was performed up to the fourth harmonic frequency. Ideally, the results would be more accurate as the highest harmonic frequency of decomposition is increased.

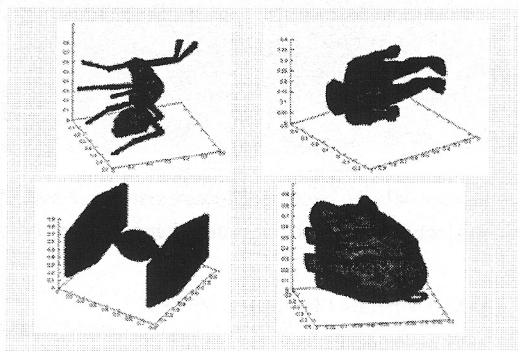

Figure 4: Voxelization results. [m0 (top left), m150 (top right), m1401 (bottom left), m100 (bottom right)].

Table 1 shows the shape descriptors for the m0 rotated and unrotated models. The x-rotation seemed to be the most unlike the others. Although the descriptor is not perfectly invariant to rotation, when comparing the descriptors of the m0 model against the other original models shown in Table 2, it is obvious that the values are significantly different from the values shown in Table 1. This leads us to believe that the descriptor and the math associated is going in the correct direction.

Finally, the Euclidean distance between the original m0 and the other six descriptors was computed. The results are shown in Table 3. It is apparent that a search engine using our descriptor would bring up the rotations as the most similar followed later by the other, unrelated models.

A precision-recall analysis has not yet been performed. At the current time, the x-rotation result gives cause to look more closely at the method and further perfect it.

	$\lambda = 0$	$\lambda = 1$	$\lambda = 2$	$\lambda = 3$	$\lambda = 4$
no rotation	.0015	.0330	.0209	.0788	.0795
x-rotation	.0024	.0309	.0177	.1065	.0788
y-rotation	.0013	.0291	.0115	.0868	.0786
z-rotation	.0014	.0291	.0173	.0778	.0734

Table 1: Shape descriptors of Model m0 in different orientations.

	$\lambda = 0$	$\lambda = 1$	$\lambda = 2$	$\lambda = 3$	$\lambda = 4$
m0	.0015	.0330	.0209	.0788	.0795
m100	.0084	.0850	.0264	.1711	.1595
m150	.0103	.0922	.0310	.2239	.2145
m1401	.0090	.0899	.0299	.2134	.1561

Table 2: Shape descriptors of different models.

	m0
x-rotation	.0280
y-rotation	.0094
z-rotation	.0081
m100	.1330
m150	.2072
m1401	.1654

Table 3: Euclidean distance between shape descriptors.

5. CONCLUSION / FUTURE WORK

In this paper, we proposed a novel shape descriptor using hyperspherical harmonics. Hyperspherical harmonics incorporate global features of a shape by initially taking a surface voxelization of the object and mapping the entire volumetric data onto a 4D hypersphere. Hyperspherical harmonics are not limited to certain types of shapes, it can be perform on points clouds (by skipping voxelization), genus-zero, mesh, or polygon-mesh. The initial volumetric data must be normalized for scale and translation. The major advantage of hyperspherical harmonics is that it is invariant to global rotation and sensitive to inner rotation.

The method discussed in this paper still needs to be refined with proper mathematical analysis. To truly display the capabilities of the descriptor, precision-recall will need to be found and compared against the Princeton Benchmark.

In the future, this method could be generalized to analyze four dimensional data using fifth dimensional harmonic decomposition techniques. Such a generalization could be used to compare measured, time-based heart data against a database of similar data that represents known heart conditions. Such a generalization could be used to categorize any function-based biological scans, including fMRIs.

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