Three-Axis Magnetometer Calibration based on Optimal Ellipsoidal Fitting under Constraint Condition for Pedestrian Positioning System Using Foot-mounted Inertial Sensor/Magnetometer

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Abstract—**Indoor pedestrian positioning is a technology for indoor pedestrian to provide position information. Global Positioning System (GPS) is the most widely used positioning system in the world at present, the accuracy of GPS in the indoor environment cannot meet the requirements because the GPS signal is extremely attenuated by the architectural shelter. Hence, the indoor positioning system without GPS has become a hot spot for research. The inertial sensor is an autonomous navigation and positioning sensor, which does not rely on external information, the position information of indoor pedestrian can be obtained by calculating the data collected from inertial sensor (accelerometer and gyroscope). It is an effective method to illustrate inertial device on the foot of pedestrian, which is called a pedestrian positioning system based on foot-mounted inertial sensors. Due to the accumulative error increased with time, the positioning system based on inertial sensors cannot provide accurate position information alone during a long time. Hence, the Zero Update (ZUPT) algorithm is proposed to effectively limit the accumulative error of inertial sensor. However, the heading error is unobservable when the velocity is used as the observation, which further limits the positioning accuracy of the indoor pedestrian positioning system based on foot-mounted inertial sensor. The heading information calculated by the output from magnetometer can be an effective observation to assist the foot-mounted inertial sensor. However, magnetometer is susceptible to environmental interference magnetic field, which can reduce the accuracy of the heading information obtained from magnetometer. Hence, the magnetometer calibration is the key to improve the accuracy of observation and achieve high precision indoor pedestrian positioning based on foot-mounted inertial sensor/magnetometer. Three-axis magnetometer interference can be effective suppressed by optimal ellipsoidal fitting based on least square, the algebraic distance equation between measured value and fitted ellipsoid is establish, all magnetometer error parameters can be obtained by calculating the equation. However, the deviation of the measured value from the theoretical value reduce the rank the calculated matrix, many results are obtained rather than the optimal result. To solve this problem, three-axis magnetometer calibration based on optimal ellipsoidal fitting under constrained conditions is proposed. The magnetic interference model is established at first, which can be described by ellipsoid model, then establish the algebraic distance equation between measured value and fitted ellipsoid, the constraint condition can be obtained by constraining the three-dimensional surface to an ellipsoid. The optimal ellipsoidal fitting under the constraint condition is used to calculate magnetometer parameters and compensate the magnetic interference to improve the accuracy of the heading information obtained from magnetometer. The MTi-G710 (integrated accelerometer, gyroscope, magnetometer, barometer, GPS receiver and Beidou receiver) produced by Xsens company is adopted to complete the actual test, which includes Offline turntable test and indoor pedestrian positioning test based on foot-mounted inertial sensor/magnetometer. The ellipsoid fitting effect based on the proposed magnetometer calibration algorithm is obtained in the offline turntable test. In the indoor pedestrian positioning test based on foot-mounted inertial sensor/magnetometer, the pedestrian movement system model and observation model are established, the result of pedestrian is obtained by the data fusion based on foot-mounted inertial sensor/magnetometer. the correctness and effectiveness of the proposed algorithm are verified.**

Keywords—magnotmeter; calibration; ellipsoid fitting; least square

I. INTRODUCTION

Indoor pedestrian positioning is an emerging technology to provide positioning information for pedestrian with dedicated device. It is about eighty percent of human production and living activity carried out in indoor, indoor pedestrian positioning has a broad application prospect. Global positioning system (GPS) is one of the most widely used positioning systems. However, the GPS signal is so weak in indoor to provide accurate positioning information cause the GPS signal is easily blocked by the building [1]. Hence, indoor pedestrian positioning without GPS has become a hot spot of research.

The inertial sensor is an autonomous navigation and positioning sensor, which does not rely on external information, the position information of indoor pedestrian can be obtained by calculating the data collected from inertial sensors (accelerometer and gyroscope) [2]. It is an effective method to illustrate inertial device on the foot of pedestrian, which is called a pedestrian positioning system based on foot-mounted inertial sensors [3]. Due to the accumulative error increased with time, the positioning system based on inertial sensors cannot provide accurate position information alone during a long time. Hence, the Zero Velocity Update (ZUPT) algorithm is proposed to effectively limit the accumulative error of inertial sensor [4]. However, the heading error is unobservable when the velocity is used as the observation, which further limits the positioning accuracy of the indoor pedestrian positioning system based on foot-mounted inertial sensor [5].

Magnetometer is a high-precision equipment, which can provide the magnetic field vector of earth. Combined with the known horizontal attitude, magnetometer can provide a heading information by transforming the coordinates [6]. Magnetometer can cooperate with inertial sensor to form a multi-sensors information fusion positioning system based on inertial sensors and magnetometer to effectively limit the heading error in the pedestrian positioning system based on foot-mounted inertial sensors. However, there are some magnetic anomaly areas inside the earth, and the application of modern electromagnetism make the magnetometer unable to measure the real geomagnetic information [7]. The accuracy of heading information can be reduced caused by the measurement error of the magnetometer.

The error of the magnetometer can be divided into system error and environmental interference according to the error characteristics. System error is a kind of inherent error of magnetometer, it can be measured and compensated by turntable calibration. Environmental interference is a kind of environmental magnetic field interference. This paper is devoted to measuring and compensating for the environment magnetic interference, to improve the accuracy of magnetometer measurement and obtain the accuracy heading.

Magnetometer calibration has a long history of research and mature theories. The magnetic interference is the difference between the magnetometer measurement and real geomagnetic field vector. The detailed mathematical model is described in detail in section Ⅲ. Nowadays, there are mainly two methods for calibration of interference to determine error parameters: twelve positions roll method and least square method [8][9]. Twelve position roll method can calibrate the magnetic interference by rotating magnetometer in three-dimensional space, the main shortcomings of this method is the magnetometer in the space twelve rotation position is relatively fixed, which requires a high-precision correction equipment. The least squares method solves the error parameters by minimizing the algebraic distance between the data and the fitted ellipsoid. The principle of least square method is simple and easy to implement. Several methods and algorithms have been proposed to calibrate the magnetometer based on the least squares method. Tsai C L et al introduced an adaptive and learning calibration of magnetometer based on ellipse fitting, error parameters cannot be calibrated completely since only two-dimensional curves are considered [10]. Merayo et al introduced a magnetometer calibration based on least square to estimate the ellipsoid parameters [11]. Wu et al introduced magnetometer calibration based on least square method, which considers the presence of measurement errors [12][13]. However, multiple solutions are obtained because the data deviating from the theoretical value. In order to solve the above problems, an optimal ellipsoidal fitting method based on a condition constraint is presented in this paper, the magnetic interference parameters are obtained by constraining the three-dimensional surface to an ellipsoid based on the least square method.

This paper is organized into following sections. In section Ⅱ, principle of indoor pedestrian positioning based on inertial sensor/magnetometer is explained. Magnetometer interference calibration based on least square is conducted in section Ⅲ. The novel calibration algorithm is detailed in section Ⅳ. Section Ⅴ presents the results of experiments for assessing the performances of the proposed algorithm. Finally, the conclusion and outlook are presented in section Ⅵ.

II. INDOOR PEDESTRIAN POSITONING BASED ON INERTIAL SENSOR/MAGNETOMETER

Indoor pedestrian positioning based on inertial sensor/magnetometer can be described with state equation and observation equation, which can be shown as follows:

$$\begin{aligned} \dot{\mathbf{X}} &= \mathbf{AX} + \boldsymbol{\eta} \\ \mathbf{Z} &= \mathbf{HX} + \boldsymbol{\mu} \end{aligned} \tag{1}$$

Where $\mathbf{X}=\begin{bmatrix}\delta\mathbf{p} & \delta\mathbf{v} & \delta\mathbf{q}\end{bmatrix}^T$ is the state vector, $\delta\mathbf{p} = \begin{bmatrix}\delta p_x & \delta p_y & \delta p_z\end{bmatrix}^T$ is the three-dimensional position error vector, $\delta\mathbf{v} = \begin{bmatrix}\delta v_x & \delta v_y & \delta v_z\end{bmatrix}^T$ is the three-dimensional velocity error vector, $\delta\mathbf{q} = \begin{bmatrix}\delta q_1 & \delta q_2 & \delta q_3\end{bmatrix}^T$ is the quaternion of attitude error. State transition matrix $\mathbf{A}$ is shown as follows:

$$\mathbf{A} = \begin{bmatrix} \mathbf{0}_{3\times3} & \mathbf{0}_{3\times3} & \mathbf{I}_{3\times3} \\ \mathbf{0}_{3\times3} & \mathbf{-[w_m\times]} & \mathbf{0}_{3\times3} \\ \mathbf{0}_{3\times3} & -2C_b^n(\hat{q})[\mathbf{f^b}\times] & \mathbf{0}_{3\times3} \end{bmatrix} \tag{2}$$

Where $[\mathbf{w_m}\times]$ is the symmetry matrix consist of angle velocity vector, $[\mathbf{f^b}\times]$ is the symmetry matrix consist of acceleration vector, $C_b^n(\hat{q})$ is the rotation matrix from body frame to navigation frame. $\boldsymbol{\eta}$ is system noise.

$\mathbf{Z}=\begin{bmatrix}\mathbf{Z}_{\delta\mathrm{v}} & \mathbf{Z}_{\delta\mathbf{q}}\end{bmatrix}^T$ is the observation vector, $\mathbf{Z}_{\delta\mathrm{v}}$ is the three-dimensional velocity error observation, which is obtained from ZUPT, $\mathbf{Z}_{\delta\mathbf{q}}$ is the quaternion of heading error observation, which is obtained from magnetometer, $\boldsymbol{\mu}$ is observation noise. Observation matrix $\mathbf{H}$ is shown as follows:

$$\mathbf{H} = \begin{bmatrix} \mathbf{0}_{3\times3} & \mathbf{0}_{3\times3} & \mathbf{I}_{3\times3} & \mathbf{0}_{3\times3} & \mathbf{0}_{3\times3} \\ \mathbf{0}_{3\times3} & \mathbf{I}_{3\times3} & \mathbf{0}_{3\times3} & \mathbf{0}_{3\times3} & \mathbf{0}_{3\times3} \end{bmatrix} \tag{3}$$

The block diagram of indoor pedestrian positioning system based on inertial sensor/magnetometer is illustrated in Fig.1.

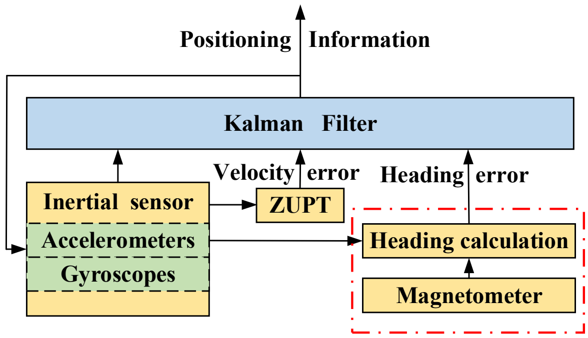

Fig.1 The block diagram of indoor pedestrian positioning system based on inertial sensor/magnetometer

III. Magnetic Interference Calibration Based on Least Square Method

A. Mgnetic interference model

According to scientists on the magnetic field and magnetometer research, environmental magnetic field interference has two components: hard magnetic interference and soft magnetic interference [14]. Hard magnetic interference is the device contains residual magnetic and equipment running current flow through the interference magnetic field. Hard magnetic interference is a constant disturbance. Soft magnetic interference is external magnetic field such as mobile phones and other electronic devices to produce interference magnetic field. Soft magnetic interference cannot be expressed in a fixed form. Environmental magnetic interference mathematical model is shown as follows:

$$\mathbf{H}_m=\mathbf{F}\mathbf{H}_e+\mathbf{H}_p = \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix}\mathbf{H}_e + \begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix} \quad (4)$$

Where $\mathbf{H}_m$ is the three-axis component of real geomagnetic field, $\mathbf{H}_e$ is the three-axis measurement of magnetometer. $\mathbf{F}$ is soft magnetic interference matrix, every element in F does not interfere with each other. $\mathbf{H}_p$ is hard magnetic interference. Hence, the measurement and calibration of the interference is transformed into the solution of 12 unknown error parameters.

Equation (4) can be transformed into the following form:

$$\mathbf{H}_e = \mathbf{F}^{-1}\times(\mathbf{H}_m-\mathbf{H}_p)=\mathbf{G}\times(\mathbf{H}_m-\mathbf{H}_p) \quad (5)$$

$$\|\mathbf{H}_e\|^2=\mathbf{H}_e^{\mathrm{T}}\times\mathbf{H}_e=(\mathbf{H}_m-\mathbf{H}_p)^{\mathrm{T}}\mathbf{G}^{\mathrm{T}}\mathbf{G}(\mathbf{H}_m-\mathbf{H}_p) \quad (6)$$

Equation (6) can be transformed into the following form：

$$\mathbf{H}_m^{\mathrm{T}}\frac{\mathbf{G}^{\mathrm{T}}\mathbf{G}}{\|\mathbf{H}_e\|^2}\mathbf{H}_m-2\frac{\mathbf{H}_p\mathbf{G}^{\mathrm{T}}\mathbf{G}}{\|\mathbf{H}_e\|^2}\mathbf{H}_m+\frac{\mathbf{H}_p^{\mathrm{T}}\mathbf{G}^{\mathrm{T}}\mathbf{G}\mathbf{H}_p}{\|\mathbf{H}_e\|^2}-1=0 \quad (7)$$

In order to calibrate the magnetic interference and restore the real three-axis geomagnetic field information, then obtain the accurate heading information, ellipsoid model can be adopted to calculate and calibrate the magnetic interference. The proof is given as follows.

According to the study of Michel Moulin, the magnetometer output in the coordinate is sphere when magnetometer makes a full rotation around a point [15]. The sphere center is offset, and the sphere deforms into an ellipsoid, the mathematical model of ellipsoid is shown as follows:

$$ax^2+by^2+cz^2+dxy+exz+fyz+px+qy+rz+s=0 \quad (8)$$

Where x,y,z are the three-axis position of ellipsoid point, a,b,c,d,e,f,p,q,r,s are the ellipsoid parameters, which are interrelated with magnetic interference.

The matrix form of equation (8) can be shown as follows:

$$(\mathbf{X}-\mathbf{X_0})^{\mathrm{T}}\mathbf{A}(\mathbf{X}-\mathbf{X_0})=1 \quad (9)$$

Expand equation (9):

$$\mathbf{X}^{\mathrm{T}}\mathbf{A}\mathbf{X}-2\mathbf{X}_0^{\mathrm{T}}\mathbf{A}\mathbf{X}+\mathbf{X}_0^{\mathrm{T}}\mathbf{A}\mathbf{X}_0=1 \quad (10)$$

Where $\mathbf{X}=\begin{bmatrix} x & y & z\end{bmatrix}^T$, the rotation of ellipsoid standard form is denoted as $\mathbf{A}=\begin{bmatrix} a & d/2 & e/2 \\ d/2 & b & f/2 \\ e/2 & f/2 & c \end{bmatrix}$, the center offset is denoted as $\mathbf{X}_0=-\mathbf{A}^{-1}\begin{bmatrix} p/2 & q/2 & r/2\end{bmatrix}^T$.

Compare equation (6) with equation (10), the magnetic interference model is the same as the ellipsoidal model. ellipsoidal model can be adopted to describe and calibrate the magnetic interference.

B. Least square method

The traditional ellipsoid fitting method is least square method to fit the ellipsoid model, the principle of ellipsoid fitting is to make the algebraic distance between data and fitting ellipsoid minimize.

$$\mathbf{F}(\mathbf{N},\mathbf{D}_i)=\sum_{i=1}^{K}(ax_i^2+by_i^2+cz_i^2+dx_iy_i+ex_iz_i+fy_iz_i+px_i+qy_i+rz_i+s)^2 \quad (11)$$

Where $\mathbf{F}(\mathbf{N},\mathbf{D}_i)$ is the sum of the algebraic distance square between data and the fitting ellipsoid.

$\mathbf{N}=(a,b,c,d,e,f,p,q,r,s)'$ are unknown parameters, $\mathbf{D_i}=(x_i^2,y_i^2,z_i^2,x_iy_i,x_iz_i,y_iz_i,x_i,y_i,z_i,1)$ are data combination. In order to calculate the unknown parameters, calculate equation (11) partial derivative and make it to zero, the extremum of equation (13) can be obtained.

$$\frac{\partial \mathbf{F}}{\partial a}=\frac{\partial \mathbf{F}}{\partial b}=\frac{\partial \mathbf{F}}{\partial c}=\frac{\partial \mathbf{F}}{\partial d}=\frac{\partial \mathbf{F}}{\partial e}=\frac{\partial \mathbf{F}}{\partial f}=\frac{\partial \mathbf{F}}{\partial p}=\frac{\partial \mathbf{F}}{\partial q}=\frac{\partial \mathbf{F}}{\partial r}=\frac{\partial \mathbf{F}}{\partial s}=0 \quad (12)$$

Which can be shown as follows:

$$\mathbf{MN}=0 \quad (13)$$

Where $\mathbf{M}=\sum_{i=1}^{K}\mathbf{D}_i\mathbf{D}_i^T$, the unknown parameters can be calculated according to the equation (18), the magnetic interference can be calibrated. However, the solution matrix cannot be solved, and the rank of the matrix is reduced cause the data deviates from the theoretical value, therefore, there are many results and great errors. In order to solve this problem, an optimal ellipsoidal fitting method based on a conditional constraint is presented in this paper, the magnetic interference parameters are obtained by constraining the three-dimensional surface to an ellipsoid based on the least square method.

IV. Magnetic Interference Calibration Based on Condition Constrained Optimal Ellipsoidal Fitting

A. Condition constrained optimal ellipse fitting

According to the ellipse model in two-dimensional, Fitzgibbon et al proposed the condition constrained ellipse fitting based on least square [16]. The ellipse fitting method is shown as follows:

$$\mathbf{F}(\mathbf{N},\mathbf{D})=(\mathbf{N}\cdot\mathbf{D})^2=ax^2+bxy+cy^2+dx+ey+f=0 \quad (14)$$

Where the objective function $\mathbf{F}(\mathbf{N},\mathbf{D})$ is the algebraic distance between the data point and the ellipse, theoretical value of $\mathbf{F}(\mathbf{N},\mathbf{D})$ is zero, the actual value has a deviation. $\mathbf{N}=(a,b,c,d,e,f)'$ are unknown parameters, $\mathbf{D}=(x^2,xy,y^2,x,y,1)$ are the combination of data.

In order to get the optimal solution of constraint solving matrix elements, many experts proposed the constraints of the above equation:

$\|a\|^2=1$ in the traditional method;

Bolles and Fischler proposed $a=1$;

Rosin and Gander proposed $a+c=1$;

Rosin proposed $f=1$;

Bookstein proposed $a^2+\frac{1}{2}b^2+c^2=1$;

but many constraint solutions are still not ideal [14]. Fitzgibbon proposed a constraint to limit the two-dimensional curve to an ellipse, and the solution matrix is guaranteed to be a positive definite matrix.

Equation (14) can be expressed as follows:

$$(\mathbf{X}-\mathbf{X}_0)^T\mathbf{A}(\mathbf{X}-\mathbf{X}_0)=1 \quad (15)$$

Where $\mathbf{X}=\begin{bmatrix}x & y\end{bmatrix}^T$, $\boldsymbol{A}=\begin{bmatrix}a & b/2\\ b/2 & c\end{bmatrix}$, $\mathbf{X}_0=-\mathbf{A}^{-1}\begin{bmatrix}d & e\end{bmatrix}^T$, since $\mathbf{A}$ is a positive definite matrix, $|\mathbf{A}|=ac-b^2/4>0$, which is $4ac-b^2>0$. It is assumed that $4ac-b^2=1$, which can be expressed as follows:

$$\mathbf{N}^T\mathbf{CN}=\mathbf{N}^T\begin{bmatrix}0&2&0&0&0&0\\2&0&0&0&0&0\\0&0&-1&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\end{bmatrix}\mathbf{N}=1 \quad (16)$$

Hence, the constraint can be expressed as follows:

$$\begin{cases}\min\|\mathbf{F}(\mathbf{N},\mathbf{D})\|^2=\|\mathbf{D}_i\mathbf{N}\|^2\\ \mathbf{N}^T\mathbf{CN}=1\end{cases} \quad (17)$$

Where $\mathbf{D}_i$ is the matrix consist of data,

$$\mathbf{D}_i=\begin{bmatrix}x_1^2 & x_1y_1 & y_1^2 & x_1 & y_1 & 1\\ x_2^2 & x_2y_2 & y_2^2 & x_2 & y_2 & 1\\ x_i^2 & x_iy_i & y_i^2 & x_i & y_i & 1\\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots\\ x_K^2 & x_Ky_K & y_K^2 & x_K & y_K & 1\end{bmatrix} \quad (18)$$

Where $i=1,2,3\cdots K$, K is the data length.

The conditional extreme problem can be solved by the Lagrange multiplier method, the constraint can be expressed as follows:

$$\begin{matrix}2\mathbf{D}_i{}^T\mathbf{D}_i\mathbf{N}-2\lambda\mathbf{CN}=0\\ \mathbf{N}^T\mathbf{CN}=1\end{matrix} \quad (19)$$

Since the eigenvalue of $\mathbf{D}_i^T\mathbf{D}_i$ and $\mathbf{C}$ are equal, $\mathbf{C}$ has only one eigenvalue, which is corresponding with optimal solution.

B. Condition constrained optimal ellipsoid fitting

Refer to the ellipse fitting method above, constraint can be added for ellipsoid fitting. The objective equation of magnetometer error ellipsoid model can be expressed as follows:

$$\mathbf{F}(\mathbf{N},\mathbf{D}) = \left(ax^2+by^2+cz^2+dxy+exz+fyz+px+qy+rz+s\right)=0 \tag{20}$$

Where the objective function $\mathbf{F}(\mathbf{N},\mathbf{D})$ is the algebraic distance between the data point and the ellipse, theoretical value of $\mathbf{F}(\mathbf{N},\mathbf{D})$ is zero, the actual value has a deviation. $\mathbf{N}=(a,b,c,d,e,f,p,q,r,s)'$ are unknown parameters, $\mathbf{D}=(x^2,y^2,z^2,xy,xz,yz,x,y,z,1)$ are the combination of data.

It is assumed that

$$\begin{gathered} I=a+b+c \\ J=ab+bc+ac-d^2-e^2-f^2 \\ \mathbf{A}=\begin{vmatrix} a & d/2 & e/2 \\ d/2 & b & f/2 \\ e/2 & f/2 & c \end{vmatrix} \end{gathered} \tag{21}$$

According to the characters of ellipsoid, ellipsoid requires $J>0$ and $I\times\mathbf{A}>0$,The ellipsoid half-length axis is at most twice the shorter half-length axis when $4J-I^2>0$. However, $4J-I^2>0$ is only the sufficient conditions of ellipsoid, there is a positive integer $\alpha>4$ to make $\alpha J-I^2>0$ for any ellipsoid. For the surface equation $I^2\ge 3J$, it is a sphere when equal sign is established. Hence, $3\le\alpha\le 4$ is the best ellipsoid fitting constraint [17].

It is assumed that $kJ-I^2=1$, the constraint of magnetometer ellipsoid model can be shown as follows：

$$\begin{cases} \min\|\mathbf{F}(\mathbf{N},\mathbf{D})\|^2=\min\|\mathbf{D}_i\mathbf{N}\|^2 \\ kJ-I^2=1 \end{cases} \tag{22}$$

Where $\mathbf{N}=(a,b,c,d,e,f,p,q,r,s)'$ are the unknown parameters, data matrix can be shown as follows:

$$\mathbf{D}=\begin{bmatrix} x_1^2 & y_1^2 & z_1^2 & x_1y_1 & x_1z_1 & y_1z_1 & x_1 & y_1 & z_1 & 1 \\ x_2^2 & y_2^2 & z_2^2 & x_2y_2 & x_2z_2 & y_2z_2 & x_2 & y_2 & z_2 & 1 \\ x_3^2 & y_3^2 & z_3^2 & x_3y_3 & x_3z_3 & y_2z_2 & x_3 & y_3 & z_3 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x_K^2 & y_K^2 & z_K^2 & x_Ky_K & x_Kz_K & y_Kz_K & x_K & y_K & z_K & 1 \end{bmatrix} \tag{23}$$

Constraint equation (22) can be express in matrix form as follows:

$$\mathbf{N}^T\mathbf{C}\mathbf{N}=\mathbf{N}^T\begin{bmatrix} \mathbf{C}_1 & \mathbf{0}_{4\times 6} \\ \mathbf{0}_{4\times 6} & \mathbf{0}_{6\times 6} \end{bmatrix}\mathbf{N}=1 \tag{24}$$

Where $\mathbf{C}_1$ can be expressed as follows:

$$\mathbf{C}_1=\begin{bmatrix} -1 & \frac{k}{2}-1 & \frac{k}{2}-1 & 0 & 0 & 0 \\ \frac{k}{2}-1 & -1 & \frac{k}{2}-1 & 0 & 0 & 0 \\ \frac{k}{2}-1 & \frac{k}{2}-1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -k & 0 & 0 \\ 0 & 0 & 0 & 0 & -k & 0 \\ 0 & 0 & 0 & 0 & 0 & -k \end{bmatrix}=1 \tag{25}$$

The conditional extreme problem can be solved by the Lagrange multiplier method, the constraint can be expressed as follows:

$$\begin{cases} \mathbf{D}^T\mathbf{D}\mathbf{N}=\lambda\mathbf{C}\mathbf{N} \\ \mathbf{N}^T\mathbf{C}\mathbf{N}=1 \end{cases} \tag{26}$$

According to $\|\mathbf{D}_i\mathbf{N}\|^2=\mathbf{N}^T\mathbf{D}_i{}^T\mathbf{D}_i\mathbf{N}=\lambda\mathbf{N}^T\mathbf{C}\mathbf{N}\ge 0$, and $\mathbf{N}^T\mathbf{C}\mathbf{N}=1$, the necessary eigenvalue is not less than zero. The eigenvalues of matrix $\mathbf{C}$ can be obtained as $\{k-3 \quad -k/2 \quad -k/2 \quad -k \quad -k \quad -k \quad 0 \quad 0 \quad 0 \quad 0\}$. Hence, if and only if $k>3$, $\mathbf{D}^T\mathbf{D}\mathbf{N}=\lambda\mathbf{C}\mathbf{N}$ has only one positive eigenvalue, which is corresponding with optimal solution.

C. Calculation of magnatic interference matrix

The section Ⅲ has proved that the ellipsoid model can completely describe the magnetic interference, and the section B in Ⅳ has provide the ellipsoid parameters based on constraint ellipsoid fitting. The solution of magnetic interference matrix is given as follows. The equation (7) can be changed into the following form:

$$\left(\mathbf{H}_m-\mathbf{H}_p\right)^T\frac{\mathbf{G}^T\mathbf{G}}{\|\mathbf{H}_e\|^2}\left(\mathbf{H}_m-\mathbf{H}_p\right)=1 \tag{27}$$

Where:

$$\mathbf{A}=\frac{\mathbf{G}^T\mathbf{G}}{\|\mathbf{H}_e\|^2}=\begin{bmatrix} a & d/2 & e/2 \\ d/2 & b & f/2 \\ e/2 & f/2 & c \end{bmatrix} \tag{28}$$

$$\mathbf{H}_p = -\mathbf{A}^{-1}\begin{bmatrix} p/2 \\ q/2 \\ r/2 \end{bmatrix} \tag{29}$$

According to equation (28) and equation (29), $\mathbf{A}$ and $\mathbf{H}_p$ can be obtained. The relation of $\mathbf{A}$ and $\mathbf{F}$ is shown as follows:

$$\left(\mathbf{F}^{-1}\right)^T \mathbf{F}^{-1} = \mathbf{G}^T\mathbf{G} = \mathbf{A}\left\|\mathbf{H}_e\right\|^2 \tag{30}$$

Where $\left\|\mathbf{H}_e\right\|^2$ is modulus square of real geomagnetic field, which can be obtained by calculating the local latitude and longitude.

$$\mathbf{A} = \mathbf{QRQ}^T = \mathbf{QPP}^T\mathbf{Q}^T \tag{31}$$

where $\mathbf{R} = \mathbf{PP} = \mathbf{PP}^T$, $\mathbf{R}$ is consists of λ_1, λ_2, λ_3, which are eigenvalues of $\mathbf{A}$, it can be shown as follows:

$$\mathbf{R} = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} \tag{32}$$

The solution of $\mathbf{F}$ is shown as follows:

$$\mathbf{F} = \mathbf{G}^{-1} = \left(\mathbf{H}_e\left(\mathbf{QP}\right)^T\right)^{-1} \tag{33}$$

The magnetic interference can be completely calculated at this point, the real three-axis magnetic field information is shown as follows:

$$\mathbf{H}_e = \mathbf{F}^{-1}\cdot\left(\mathbf{H}_m - \mathbf{H}_p\right) = \mathbf{G}\left(\mathbf{H}_m - \mathbf{H}_p\right) \quad (32).$$

This paper proposed a magnetometer interference calibration method based on optimal ellipsoid fitting with constraint, which limit the three-dimensional surface to an ellipsoid, the uniqueness of the proposed method has proved with mathematical method. The block diagram of magnetometer interference calibration based on condition constrained optimal ellipsoid fitting is illustrated in Fig.2.

V. EXPEREMENT AND RESULT

In this section we present experiment and result to confirm the effectiveness and correctness of the proposed algorithm. The MTi-G710, integrate accelerometer, gyroscope, magnetometer, barometer, GPS receiver and Beidou receiver) produced by Xsens company is adopted to complete the actual test, which includes magnetometer interference calibration test and indoor pedestrian positioning test based on foot-mounted inertial sensor/magnetometer. The diagram of MTi-G710 and indoor pedestrian positioning are illustrated in fig.3.

In the magnetometer interference calibration test, it is assumed that k=4, in this case, the eigenvector corresponding to the only one positive eigenvalue is obtained, which can be used to calculate magnetic interference matrix. In order to minimize the algebraic distance between the experimental data and the fitted ellipsoid, we adopt the principle of selecting data multiple times and solving the minimum algebraic distance to calculate the interference matrix, which is illustrated in fig.4.

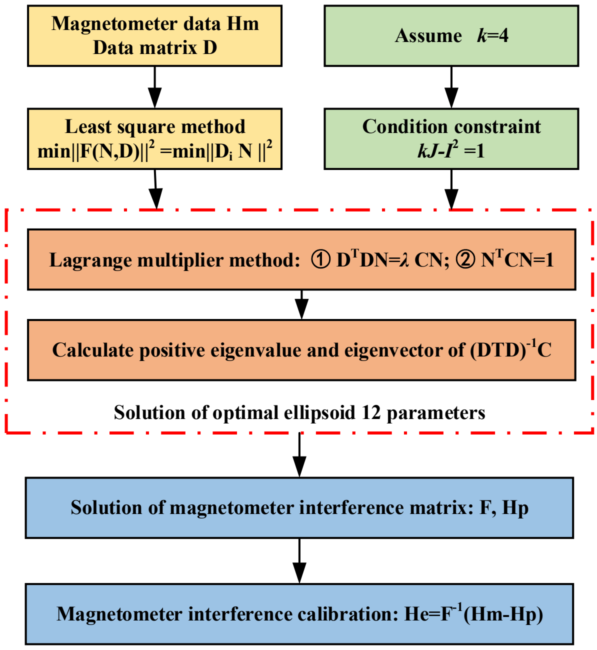

Fig.2 block diagram of magnetometer interference calibration based on condition constrained optimal ellipsoid fitting

Fig.3 diagram of MTi-G710 and indoor pedestrian positioning

The MTi-G710 is rotated around the three-axis in turn, the ellipsoid consist of the magnetometer data before calibration is illustrated in Fig.5. The three-dimensional curve consist of magnetometer data after calibration is illustrated in Fig.6. Compared Fig.5 with Fig.6, the curve in Fig.6 is approximately a sphere. According to the section three, the real geomagnetic filed data rotated around the three-axis is a sphere. Hence, the proposed algorithm effectively calibrates the magnetic interference. However, the curve is not completely a sphere cause the interference model is simple (noise is not considered).

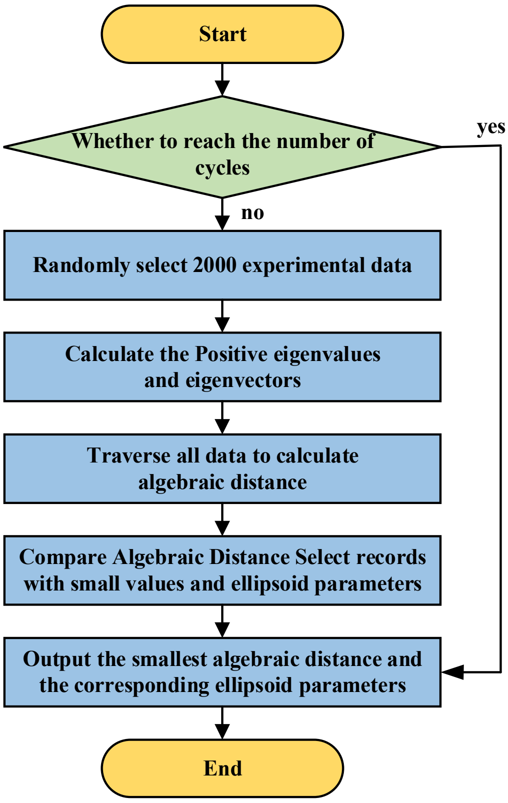

Fig.4 the algorithm flowchart of magnetometer interference calibration based on condition constrained optimal ellipsoid fitting.

We randomly intercept the three-axis data of magnetometer before and after calibration, which is shown in Fig.7. The square modulus of magnetometer data is illustrated in Fig.8. The red curve is magnetometer data after calibration, the blue curve is magnetometer data before calibration. It can be seen that the red curve is more stable after calibration in Fig.7 and Fig.8.

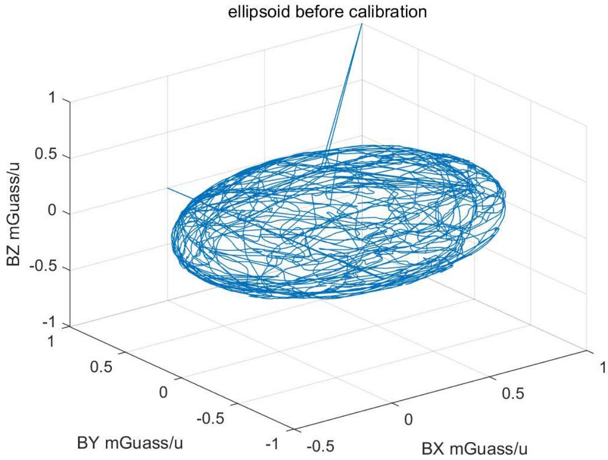

Fig.5 The ellipsoid consists of magnetometer data before calibration

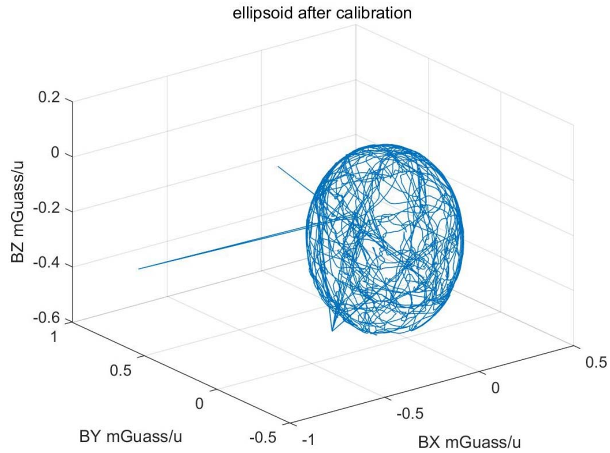

Fig.6 The ellipsoid consists of magnetometer data after calibration

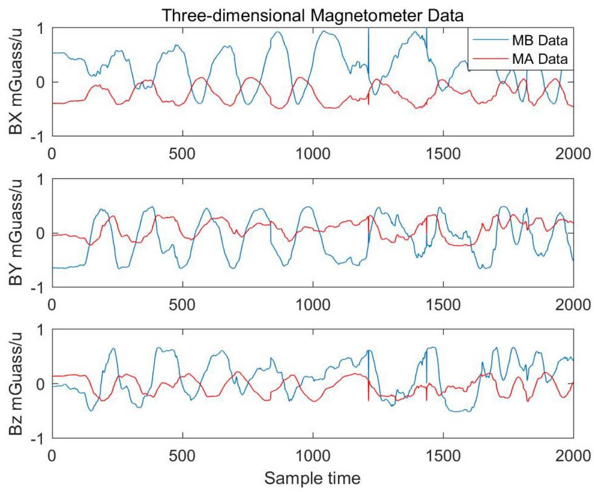

Fig.7 The magnetometer data before and after calibration.

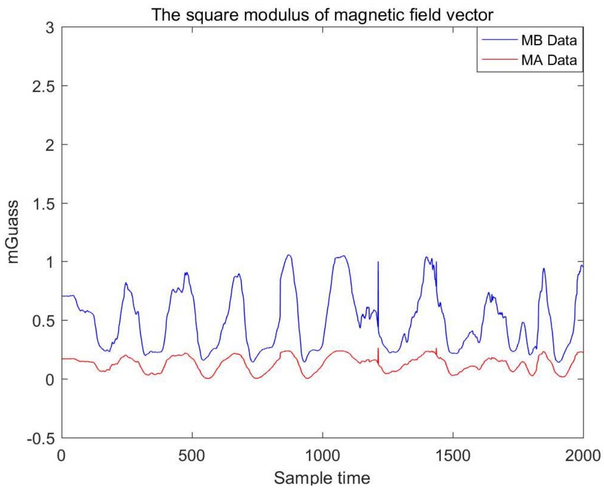

Fig.8 The square modulus of magnetic field vector before and after calibration.

In the indoor pedestrian positioning test, in order to contrast the impact of magnetic interference, the pedestrian walked along a 12 meters straight line in the lab. The results of indoor pedestrian positioning experiment are illustrated in Fig.9. The red box is the starting point, the red line is the real trajectory, the black line is the trajectory of inertial sensor/ZUPT method, the green line is the trajectory of inertial sensor/ZUPT/magnetometer before calibration method, the blue line is the trajectory of inertial sensor/ZUPT/magnetometer after calibration method. Compared with these lines, inertial sensor/ZUPT/magnetometer after calibration method has achieved the best positioning accuracy. However, the interference model is simple, the noise and other conditions are not considered, which can be further improved.

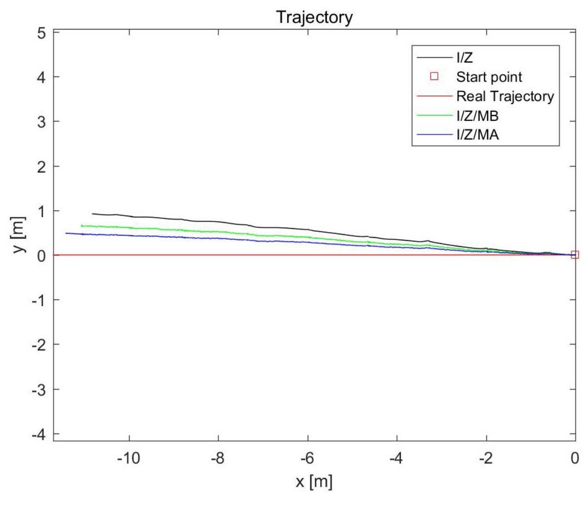

Fig.9 The trajectory of indoor pedestrian positioning experiment.

VI. CONCLUSIONS

A magnetometer interference calibration algorithm based on condition constrained least square optimal ellipsoid method is proposed in this paper to calculate and calibrate the magnetic interference of magnetometer. The effectiveness and correctness of the proposed algorithm is proved by comparing the flattening of ellipsoid before and after calibration and the trajectories of indoor pedestrian positioning in different environment. More research on magnetometer interference calibration are arranged as follows: firstly, the magnetic javascript:;interference model is further optimized (considering the noise), and secondly, according the quick compensation of magnetic interference is required in indoor pedestrian positioning, improve the robustness, realize the three-axis 180 degrees rotation to quickly calibrate the magnetometer interference before indoor pedestrian poisoning experiment.

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