Quasi-Constant Envelope Phase Shift Keying

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Abstract —

Quasi-constant envelope PSK is proposed to overcome the nonlinearity caused by fully saturated RF power amplifiers in wireless and satellite communications. It is shown that when the simplest linear receiver is employed, they can achieve near optimal performance compared with linear BPSK in AWGN channel. Quasi-constant envelope OQPSK (QCEOQPSK) is presented as a design example. Therefore, quasi-constant envelope PSK can help to achieve near optimal performance and significant terminal cost reduction in wireless and satellite communication systems.

I. Introduction

Recently the practice in broadband wireless and satellite communications has found many new challenges to modulation. A broadband wireless or satellite network has to support a very large number of user terminals. To achieve high system capacity and performance, broadband wireless and satellite networks require modulation methods to have high bandwidth efficiency and ideal bit error rate at low implementation complexity. As data rates are much higher in broadband wireless and satellite networks, a user terminal has to transmit at much higher power levels than in the existing wireless communication systems [2]-[4]. To reach high transmission power and high power efficiency in mobile terminals, power amplifiers working in the fully saturated region have to be employed [1]. They cause severe nonlinear distortion to the signal. When the bandwidth is hundreds of megahertz or larger, the linear RF power amplifiers can also cause significant distortion to signals [5].

Mass production of inexpensive user terminals is also a top challenge to the broadband wireless and satellite communications industry [2]. The same situation is true in sensor networks [3]. As each broadband wireless communication network has to support a very large number of users, modulation methods are required to achieve implementation complexity as low as possible to reduce terminal cost and make mass terminal production possible [2]-[4]. The implementation complexity must be handled very carefully considering the severe nonlinear distortion from power amplifiers.

This paper proposes quasi-constant envelope PSK for wireless and satellite communications systems using fully saturated power amplifiers.

II. System Model

Consider the communications system employing PSK. The baseband signal at the input of the radio is

$$s_b(t) = s_I(t) + js_Q(t) \tag{1}$$

where $s_I(t) = \sum_k d_{2k}p(t-kT_s)$ is the signal to be transmitted by the in-phase channel and $s_Q(t) = \sum_k d_{2k+1}p(t-kT_s-\tau_0)$ is the signal to be transmitted by the quadrature phase channel, $\{d_i|d_i \in \{-1,+1\}\}$ is the information sequence to be transmitted, $p(t)$ is the shaping function, and T_s is the symbol time. The quadrature channel data can be delayed by half a symbol time $\tau_0 = 0.5T_s$ if OQPSK is employed [4]. Otherwise, $\tau_0 = 0$.

The radio upconverts the baseband signal to a passband signal centered at the carrier frequency, which is the input signal to the power amplifier and can be written as

$$s_1(t) = s_I(t)\cos(2\pi f_c t + \phi_0) - s_Q(t)\sin(2\pi f_c t + \phi_0) \tag{2}$$

where f_c is the carrier frequency and ϕ_0 is the initial carrier phase. The power amplifier in the radio amplifies the signal to the desired transmission power. The output of the radio is transmitted through the AWGN channel.

We consider a radio employing fully saturated RF power amplifier. The power amplifier completely removes the amplitude information of its input signal. The phase of the output signal is the same as the phase of the input signal. Let $s_i(t)$ be the input signal to the power amplifier. The output signal $s_o(t)$ of the fully saturated power amplifier can be written as

$$s_o(t) = \begin{cases} 1 & s_i(t) > 0 \\ 0 & s_i(t) = 0 \\ -1 & s_i(t) < 0 \end{cases} \tag{3}$$

Let the envelope of the input signal to the power amplifier be

$$\sqrt{s_I^2(t) + s_Q^2(t)} = \sqrt{P}(1 - y(t)) \tag{4}$$

where P is the average power and $y(t)$ is the instantaneous deviation of the signal envelope. The function $y(t)$ is a random process determined by the information sequence and the shaping pulse. The baseband equivalent signal for the transmitted signal can be written as

$$s_l(t) = \sqrt{P}(s_b(t) + n_d(t)) \tag{5}$$

where

$$n_d(t) = \frac{1 - \sqrt{s_I^2(t) + s_Q^2(t)}}{\sqrt{s_I^2(t) + s_Q^2(t)}} s_b(t) \tag{6}$$

is the distortion caused by the fully saturated power amplifier. The distortion is a random process varying with the information sequence and shaping pulse $p(t)$.

Define the signal-to-distortion power ratio at the power amplifier output as

$$\gamma\{p(t)\} = E\{\frac{P_s}{P_d}\} \tag{7}$$

where

$$P_s = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} |s_b(t)|^2 dt$$

$$P_d = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} |n_d(t)|^2 dt$$

and the ensemble average is taken in the sample space for all of the possible information sequences. The signal-to-distortion power ratio $\gamma\{p(t)\}$ can serve as the fidelity measure for the fully saturated power amplifier. Here the fully saturated power amplifier is treated as a tenary quantizer.

When the envelope of the input signal to a fully saturated power amplifier has small variation, i.e., $\max|y(t)| << 1$, the distortion can be written as

$$n_d(t) = s_b(t)y(t)\sum_{n=0}^{\infty} y^n(t). \qquad (8)$$

Therefore, the signal-to-distortion power ratio can be maximized by minimizing the power of $y(t)$ through choosing the appropriate shaping pulse $\{p(t)\}$.

The modulation method using PSK is called the *quasi-constant envelope PSK* if it satisfies $1 << \gamma < \infty$ when measured at the output of the fully saturated power amplifier.

The receiver is the linear PSK receiver [4]. The filter in the receiver is matched only to the shaping filter in the transmitter.

III. Optimal Filtering

For the system under study, after decades of research, it was widely believed that pulse shaping could not help much. It seemed impossible to achieve near-optimum linear BPSK bit error performance when the simplest linear demodulator is employed [6] The following theorem shows that a special group of shaping pulses can help to tolerate fully saturated power amplifiers.

Theorem *In the communication system employing linear modulation with $p(t)$ as the pulse shaping function, fully saturated power amplifier through AWGN channel and linear demodulation filter matched to $p(t)$, the demodulation performance approaches the BPSK performance theory, i.e.,*

$$\lim_{\gamma\{p(t)\}\to\infty} P_b\{p(t)\} = Q\left(\sqrt{\frac{E_b}{\sigma^2}}\right) \qquad (9)$$

where $\frac{E_b}{\sigma^2}$ is the signal-to-noise power ratio per bit, if and only if
(1) $\gamma\{p(t)\}\to\infty$;
(2) The modulation filter $p(t)$ and the demodulation filter $h(t)$ satisfy the Nyquist pulse shaping criterion, i.e.,

$$x(t = kT_s) = \begin{cases} 1 & k = 0 \\ 0 & k \neq 0 \end{cases}$$

where

$$x(t) = \int_{-\infty}^{\infty} p(\tau)h(t-\tau)d\tau. \qquad (10)$$

In (9), the variance σ^2 is the variance of the AWGN only.

Proof: Let $a(t)$ be the impulse response function of the fully saturated power amplifier. The output of the power amplifier can be written as

$$s_a(t) = \int_{-\infty}^{\infty} s_b(\tau)a(t-\tau)d\tau. \qquad (11)$$

When the first condition is true, we have

$$a(t) = a_0(t) + a_n(t) \qquad (12)$$

where $a_0(t)$ is an ideal linear filter and $a_n(t)$ causes distortion. The matched filter output is

$$r(t) = \int_{-\infty}^{\infty} (s_a(\tau) + n(\tau))h(t-\tau)d\tau. \qquad (13)$$

which can be rewritten as

$$r(t) = y(t) + n_1(t) \qquad (14)$$

where $y(t) = \sum_k d_{2k}x(t-kT_s) + \sum_k d_{2k}g(t-kT_s) + j[\sum_k d_{2k+1}x(t-kT_s-\tau_0)+\sum_k d_{2k+1}g(t-kT_s-\tau_0)]$, $g(t) = \int_{\infty}^{-\infty} x(\tau)a_n(t-\tau)d\tau$, and $n_1(t) = \int_{\infty}^{-\infty} n(\tau)h(t-\tau)d\tau$ is the Gaussian noise with zero mean and variance $\sigma^2 = \frac{N_0}{2}$. When the second condition is true, the bit error rate of coherent demodulation is bounded by

$$P_b\{p(t)\} \leq Q\left(\sqrt{\frac{E_b}{\sigma_d^2+\sigma^2}}\right) \qquad (15)$$

where σ_d^2 is the power of the distortion. For a given signal-to-noise power ratio, i.e., E_b/σ^2 fixed, when the first condition is true, we have (9).

If the first condition is not true, then Eq. (12) does not hold. If the second condition is not true, the inequality (15) does not hold. Therefore, both conditions are necessary for (9) to hold. The proof ends.

The equality in (15) holds if the signal and distortion is independent. The dependence between the signal and the distortion becomes negligibly weak when the first condition in the theorem is true. In real communications systems practice, the AWGN has much higher power than that of the distortion, when looked at the matched filter output. For example, Direct PC and Direct TV systems require $E_b/\sigma^2 \leq 3$ dB, while γ can be higher than 21 dB as shown in Section V.

One can see that the theorem holds for communication systems employing M-ary PSK with $M \leq 4$. This includes BPSK, QPSK and its variations, such as OQPSK, $\frac{\pi}{4}$ QPSK, MSK and GMSK. Investigation has shown that the theorem can be applied to 8PSK. More study is needed for M-ary PSK when $M \geq 16$.

For communication systems employing PSK and fully saturated power amplifier through AWGN channel, the modulation method which can achieve optimal or near optimal demodulation performance is a member of the quasi-constant envelope PSK modulation.

The theorem says that when the fully saturated power amplifier is employed for AWGN channel, near optimal demodulation performance can be achieved by employing the same filter for both the pulse shaping filter and the matched filter. Both the quasi-constant envelope modulated signal and the Nyquist filter be constructed in this way. One can see that filtering is critical in both the transmitter and the receiver, when nonlinearity is introduced by the transmitter. There are more functions $\{p(t)\}$, which satisfy either the first condition or the second condition in the theorem, respectively. There are few functions to satisfy both conditions.

IV. Power Spectral Density

The autocorrelation function of the baseband equivalent transmitted signal is

$$\phi(t, t+\tau) = E\{s_l(t)s_l^*(t+\tau)\} \quad (16)$$

Substituting (5) into (16), we have

$$\phi(t, t+\tau) = \phi_l(t, t+\tau) + \phi_d(t, t+\tau) \quad (17)$$

where

$$\phi_l(t, t+\tau) = E\{s_b(t)s_b^*(t+\tau) \quad (18)$$

is the autocorrelation function of the linear modulation, and $\phi_d(t, t+\tau) = E\{n_d(t)s_b^*(t+\tau) + s_b(t)n_d^*(t+\tau)\} + E\{n_d(t)n_d^*(t+\tau)\}$ is the correlation contributed by the distortion. Substituting (5) into the previous equation, we have

$$\begin{aligned}\phi_d(t, t+\tau) &= E\{s_b(t)s_b^*(t+\tau)\frac{y(t)}{1-y(t)}\} \\ &+E\{s_b(t)s_b^*(t+\tau)\frac{y^*(t+\tau)}{1-y^*(t+\tau)}\} \\ &+E\{s_b(t)s_b^*(t+\tau)\frac{y(t)}{1-y(t)}\frac{y^*(t+\tau)}{1-y^*(t+\tau)}\}.\end{aligned}$$

This equation shows how the envelope variation $y(t)$ of the input signal affects the auto-correlation of the distortion $n_d(t)$ at the fully saturated power amplifier output. A challenging task is to find a useful analytical form for $y(t)$.

The function $\phi(t, t+\tau)$ is periodic in the t variable with period T_s. Assuming that the sequence of information symbols is wide-sense stationary, the baseband signal $s_l(t)$ is a periodically stationary process in the wide sense. Averaging $\phi(t, t+\tau)$ over a symbol period, we have

$$\bar{\phi}(\tau) = \frac{1}{T_s}\int_{-0.5T_s}^{0.5T_s} \phi(t, t+\tau)dt \quad (19)$$

The power spectral density of the signal $s_l(t)$ is the Fourier transform of $\bar{\phi}(\tau)$, i.e.,

$$\Phi(f) = \int_{-\infty}^{\infty} \bar{\phi}(\tau)e^{-j2\pi f\tau}d\tau \quad (20)$$

which can be rewritten as

$$\Phi(f) = \Phi_l(f) + \Phi_d(f)$$

where

$$\Phi_l(f) = \frac{1}{T_s}\int_{-\infty}^{\infty}\int_{-0.5T_s}^{0.5T_s} \phi_l(t, t+\tau)e^{-j2\pi f\tau}dtd\tau \quad (21)$$

is the power spectral density of the linear modulation, and

$$\Phi_d(f) = \frac{1}{T_s}\int_{-\infty}^{\infty}\int_{-0.5T_s}^{0.5T_s} \phi_d(t, t+\tau)e^{-j2\pi f\tau}dtd\tau \quad (22)$$

is the power spectral density contributed by the distortion.

When linear power amplifier is employed, $n_d(t) = 0$. Then, $\Phi_d(f) = 0$. In other words, there is no distortion to the power spectral density of the linearly modulated signal. When nonlinear power amplifier is employed, $n_d(t) \neq 0$. Then, $\Phi_d(f) \neq 0$ and contributes to the distortion of the power spectral density, including sidelobe regrowth.

When quasi-constant envelope modulation is used, the power of the distortion is minimized. The distortion to power spectral density is also minimized accordingly.

V. Design Example

We have found several PSK example systems satisfying the conditions in Sec. III. One example is provided in the following.

Table I shows the signal-to-distortion power ratio $\gamma\{p(t)\}$ for the system employing OQPSK. The pulse shaping filter is of finite impulse response, which is truncated from the square root raised cosine function. The duration of the filter is L symbols and the roll-off factor is β. It can be seen that the signal-to-distortion power ratio γ can be maximized when the roll-off factor is $\beta = 1.0$. The maximum value of the ratio γ is 21.38 dB. The effect of the shaping pulse length to the ratio γ is negligible. Therefore, the modulator output signal has quasi-constant envelope. The power spectral density of this system was shown in [4].

Table 1: Signal-to-distortion power ratio γ of OQPSK signal amplified by fully saturated power amplifier. The shaping pulse is the square root raised cosine function with the duration of L symbols and roll-off factor β.

β	γ (dB)		
	$L=8$	$L=6$	$L=4$
1.0	21.38	21.33	21.34
0.9	19.86	19.81	19.84
0.8	18.43	18.37	18.40
0.7	17.11	17.06	17.06
0.6	15.92	15.89	15.85
0.5	14.84	14.83	14.80
0.4	13.88	13.87	13.90
0.3	13.03	13.05	13.17

Fig. 1 shows the simulated bit error rate for coherent demodulation. The duration of both the pulse shaping filter and the matched filter is $L = 8$ symbols The ideal BER for BPSK in a traditional AWGN channel is plotted as the reference. It can be seen that at BER$= 10^{-5}$ the SNR degradation is 0.1 dB, when the roll-off factor is $\beta = 1$. The degradation increases when the roll-off factor β decreases. For $\beta = 0.3$, the SNR degradation is 1.2 dB at BER$=5 \cdot 10^{-5}$.

When linear receiver and hard decision are employed, the quasi-constant envelope OQPSK can achieve near optimum demodulation performance compared with BPSK, when the square root raised cosine function is employed as the pulse shaping filter and the matched filter with the roll-off factor $\beta = 1$ and the duration is not less than 4 symbols. For $0.7 \leq \beta \leq 1$, increasing the shaping pulse duration to more than 4 symbols can not bring apparent improvement to BER performance. This is because the fluctuation in the modulated signal envelope is small and the extended shaping pulse makes negligible difference to smooth the signal. For $\beta = 0.3$, increasing the shaping pulse duration can improve BER performance. When β is small, the fluctuation in the modulated signal envelope is large and the extended shaping pulse can contribute to smooth the signal. For practice, we recommend $\beta \in [0.7, 1]$ and the shaping pulse duration to be in the range [6,8] symbols.

The rate 1/2 convolutional code with the constraint length $K = 6, 7$ is evaluated for communication systems employing the quasi-constant envelope OQPSK and fully saturated RF power amplifiers. The demodulator employs linear filters matched only to linear pulse shaping filters. The decoder is the Viterbi decoder using soft decision. Fig. 2 shows the simulated bit error rate in AWGN channel. The bit error rate of a compatible system employing the ideal linear RF power amplifier is also simulated for comparison. The simulated bit error rate for the system employing the ideal linear RF power amplifier in AWGN channel matches well with the coded performance for BPSK. It can be seen that the fully saturated RF power amplifier causes small SNR degradation. When the bit error rate is 10^{-5}, the degradation is only 0.25 dB. Increasing the constraint length from $K = 6$ to $K = 7$ does not reduce this degradation. Therefore, the SNR degradation caused by the fully saturated RF power amplifier is tolerable in practical broadband wireless and satellite communications systems.

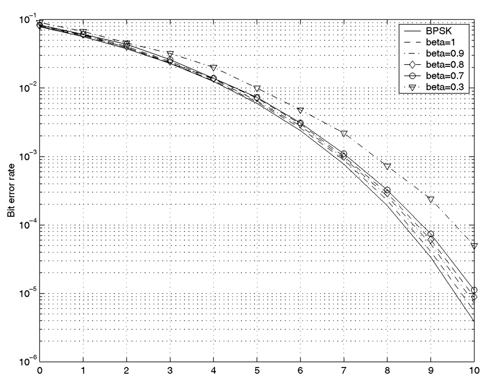

Figure 1: Bit error rate of the quasi-constant envelope OQPSK. The shaping pulse in the transmitter is the square root raised cosine function with the duration as 8 symbols. The RF power amplifier is fully saturated.

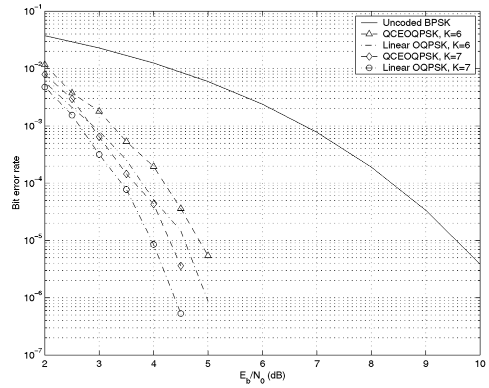

Figure 2: Bit error rate of communication systems employing the quasi-constant OQPSK through fully saturated power amplifiers and AWGN channel. Rate 1/2 convolutional code is employed with the Viterbi decoder. The solid line is the ideal bit error rate for BPSK. The triangle line is for $K = 6$ and the diamond line is for $K = 7$ with fully saturated power amplifier. The dash-dot line is for $K = 6$ and the circle line is for $K = 7$ with linear power amplifier for comparison.

VI. Conclusions

Quasi-constant envelope PSK is proposed to overcome the nonlinearity in wireless and satellite communications systems employing fully saturated power amplifiers. It is shown that they can achieve near optimum BER performance when the simplest linear receiver is employed. Therefore, quasi-constant envelope PSK can help to significantly improve battery life and dramatically reduce terminal cost.

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References

[1] M. D. Weiss, F. H. Raab, and Z. Popovic, "Linearity of X-band class-F power amplifiers in high-efficiency transmitters," *IEEE Trans. Microwave Theory Tech.*, vol. 49, pp. 1174-1179, June 2001.

[2] E. M. Wildauer, M. J. Pender, Jr., and J. E. Ohlson, "Modulation method and system using constant envelope ODSCDMA with low out-of-band emissions for non-linear amplification," US Patent No. 5,903,555, May 11, 1999 (filed Oct. 30, 1996).

[3] J. M. Rabaey, M. Ammer, J. Silva, D. Patel and S. Roundy, "Picoradio supports Ad Hoc ultra-low power wireless networking," *IEEE Computer*, pp. 42-48, July 2000.

[4] Q. Liu and J. Li, "Quasi-constant envelope OQPSK through nonlinear radio and AWGN channel," *Proc. IEEE Military Commun. Conf.*, Oct. 7-10, 2002, Anaheim, CA.

[5] S. Vaughn and R. Sorace, "Demonstration of the TDRS Ka-band transponder," *Proc. IEEE Military Commun. Conf.*, vol. 2, pp. 1055-1065, 22-25 Oct., 2000, Los Angeles, CA.

[6] E. Biglieri, M. Elia, and L. Lopresti, "The optimal linear receiving filter for digital transmission over nonlinear channels," *IEEE Trans. Inform. Theory,* vol. 35, pp. 620-625, May 1989.