ME 549 Homework #6

1. The motion of a damped spring-mass system is described by the ordinary differential equation:

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = 0$$

where x = displacement from the equilibrium position (m), t = time (s), m = mass (kg) and c = damping coefficient (N-s/m). The damping coefficient takes on three values: 5 (underdamped), 40 (critically damped) and 200 (overdamped). The spring constant k = 22 N/m, the mass is 20 kg. The initial velocity is zero and the initial displacement is 1 m. Solve this equation using a 4th order Runge-Kutta approach over the time period $0 \le t \le 25$ s. Plot the displacement vs. time for each of the three values of the damping coefficient on the same curve. Comment.

2. Solve Problem 1 with an initial velocity = 2 m/s and initial displacement of 0.5 m.

3. Solve for the response of the following system with the same mass, spring and damping parameters as in Problem 1, zero initial velocity and displacement:

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = f(t)$$

where $f(t)$ = 120 N when $1.0 \text{ s} \le t \le 1.4$ s, zero otherwise. Comment.

4. Solve for the response of the system of Problem 3, with zero initial velocity and displacement, where

$f(t) = A \sin[\pi(t-t_0)/T]$ when $t_0 \le t \le (t_0 + T)$, zero otherwise
A = 1200 N, T = 5 ms, t_0=1 s