Simplex Method examples

<u>Example 1</u>

Maximize:	$Z = 150x_1 + 175x_2$	w/ slack variables:	$Z - 150x_1 – 175x_2 = 0$	(1)
Subject to:	$7x_1 + 11x_2 \leq 77$		$7x_1 + 11x_2 + S_1 = 77$	(2)
	$10x_1 + 8x_2 \leq 80$		$10x_1 + 8x_2 + S_2 = 80$	(3)
	$x_1 \leq 9$		$x_1 + S_3 = 9$	(4)
	$x_2 \leq 6$		$x_2 + S_4 = 6$	(5)

Initial non-basic variables (x_1, x_2) appear in the objective function (1). Choose the variable with the largest negative coefficient (x_2) to enter the calculations and check the intercepts in the constraints (2-5) to see which variables leaves the calculations:

S_1: 77/11 = 7

S_2: 80/8 = 10

S_3: parallel to x_2, no intercept

S_4: 6/1 = 6 – smallest non-negative intercept, closest vertex to current point

S_4 is the leaving variable, x_2 is the entering variable. Solve Eq. (5) for x_2, substitute into Eqs. (1-5) above to yield:

$Z – 150x_1 + 175S_4 – 1050 = 0$ (6)

$7x_1 + S_1 -11S_4 = 11$ (7)

$10x_1 + S_2 – 8S_4 = 32$ (8)

$x_1 + S_3 = 9$ (9)

$x_2 + S_4 = 6$ (10)

Again choose the variable with the largest negative coefficient in the objective function (6), this time x_1. Check intercepts:

S_1: 11/7 = 11/7 – smallest non-negative intercept, closest vertex to current point

S_2: 32/10 = 3.2

S_3: 9/1 = 9

S_4: parallel to x_1, no intercept

S_1 is the leaving variable, x_1 is the entering variable. Solve equation (7) for x_1, substitute into Equations (6-10) above to yield:

$Z + 21.4286S_1 – 60.7143S_4 – 1285.71 = 0$ (11)

$x_1 + (1/7)S_1 – (11/7)S_4 = 11/7$ (12)

$$1.42857S_1 + S_2 + 7.71429S_4 = 16.2857 \qquad (13)$$

$$-0.142857S_1 + S_3 + 1.57143S_4 = 7.42857 \qquad (14)$$

$$x_2 + S_4 = 6 \qquad (15)$$

Again choose the variable with the largest negative coefficient in the objective function (11), this time S_4. Check intercepts:

S_1: (11/7)/(-11/7) = -1

S_2: 16.2857/7.71429 = 2.1111 – smallest non-negative intercept

S_3: 7.42857/1.57143 = 4.72727

S_4: 6/1 = 6

S_4 enters, S_2 leaves… solving Eq. (13) for S_4 gives

$$S_4 = -0.185185S_1 – 0.129630S_2 + 2.1111 \qquad (16)$$

Substituting into (11) gives:

$$Z + 32.6720S_1 + 7.870S_2 – 1413.88 = 0 \qquad (17)$$

There are no more negative coefficients, so the method stops. S_1 and S_2 are non-basic variables, $S_1 = S_2 = 0$, and $Z_{max} = 1413.88$. Solving for the remaining variables using Eqs. (12-15) gives:

$x_1 = 4.8889$

$S_3 = 4.1111$

$x_2 = 3.8889$

$S_4 = 2.1111$

Example 2 – text's solution

Maximize:	$Z = 150x_1 + 175x_2$	w/ slack variables:	$Z - 150x_1 - 175x_2 = 0$	(1)
Subject to:	$7x_1 + 11x_2 \leq 77$		$7x_1 + 11x_2 + S_1 = 77$	(2)
	$10x_1 + 8x_2 \leq 80$		$10x_1 + 8x_2 + S_2 = 80$	(3)
	$x_1 \leq 9$		$x_1 + S_3 = 9$	(4)
	$x_2 \leq 6$		$x_2 + S_4 = 6$	(5)

Choose x_1 as the first entering variable. Checking the intercepts gives:

S_1: 77/7 = 11

S_2: 80/10 = 8 – smallest non-negative intercept

S_3: 9/1 = 9

S_4: parallel to x_1

S_2 leaves and x_1 enters. Solve Eq. (3) for x_1 and substitute, giving:

$Z - 55x_2 + 15S_2 - 1200 = 0$ (6)

$5.40x_2 + S_1 - 0.70S_2 = 21$ (7)

$x_1 + 0.80x_2 + 0.10S_2 = 8$ (8)

$-0.80x_2 - 0.10S_2 + S_3 = 1$ (9)

$x_2 + S_4 = 6$ (10)

Choose x_2 to enter, check intercepts:

S_1: 21/5.40 = 3.8889 – smallest non-negative intercept

S_2: 8/(10/8) = 10

S_3: -10/8 = -1.25

S_4: 6/1 = 6

S_1 leaves, x_2 enters. Solve Eq. (7) for x_2, substitute to find:

$Z + 10.1852S_1 + 7.87035S_2 - 1413.89 = 0$

There are no more negative coefficients, so the method stops. S_1 and S_2 are non-basic variables, $S_1 = S_2 = 0$, and $Z_{max} = 1413.89$. Solving for the remaining variables using Eqs. (7-10) gives:

$x_2 = 3.8889$ $x_1 = 4.8889$

$S_3 = 4.1111$ $S_4 = 2.1111$

Example 3 – Problem 15.3 from the text

Maximize: $Z = 1.75x_1 + 1.25x_2$ w/ slack variables: $Z - 1.75x_1 - 1.25x_2 = 0$ (1)

Subject to: $x_1 + 1.1x_2 \leq 8$ $x_1 + 1.1x_2 + S_1 = 8$ (2)

$2.5x_1 + x_2 \leq 9$ $2.5x_1 + x_2 + S_2 = 9$ (3)

Choose x1 as the entering variable, check intercepts:

S_1: 8/1 = 8

S_2: 9/2.5 = 3.6 – smallest non-negative intercept

S2 leaves, x_1 enters. Solve Eq. (3) for x_1 substitute into Eqs. (1-3), yielding

$Z - 0.55\,x_2 + 0.70S_2 - 6.3 = 0$ (4)

$0.70\,x_2 + S_1 - 0.40S_2 = 4.4$ (5)

$x_1 + 0.40x_2 + 0.40S_2 = 3.6$ (6)

Choose x_2 to enter, check intercepts:

S_1: 4.4/0.7 = 6.286 – smallest non-negative intercept

S_2: 3.6/0.4 = 9

S_1 leaves, x_2 enters. Solve Eq. (5) for x_2, substitute into Eqs. (4-6):

$Z + 0.786S_1 + 0.3857S_2 - 9.757 = 0$

There are no more negative coefficients, so the method stops. S_1 and S_2 are non-basic variables, $S_1 = S_2 = 0$, and $Z_{max} = 9.757$. Solving for the remaining variables using Eqs. (4-6) gives:

$x_1 = 1.086$

$x_2 = 6.286$

Example 4 – Problem 15.4 from the text

Maximize: $Z = 6x_1 + 8x_2$ w/ slack variables: $Z - 6x_1 - 8x_2 = 0$ (1)

Subject to: $5x_1 + 2x_2 \leq 40$ $5x_1 + 2x_2 + S_1 = 40$ (2)

$6x_1 + 6x_2 \leq 60$ $6x_1 + 6x_2 + S_2 = 60$ (3)

$2x_1 + 4x_2 \leq 32$ $2x_1 + 4x_2 + S_3 = 32$ (4)

x_2 enters, with intercepts:

S_1: 40/2 = 20

S_2: 60/6 = 10

S_3: 32/4 = 8 – smallest non-negative intercept, S_3 leaves

Solve Eq. (4) for x_1 and substitute, yielding

$Z - 6x_1 + 2S_3 - 64 = 0$ (5)

$4\,x_1 + S_1 - (1/2)S_3 = 24$ (6)

$3x_1 + S_2 - (3/2)S_3 = 12$ (7)

$x_2 + (1/2)x_1 + (1/4)S_3 = 8$ (8)

x_1 enters, with intercepts:

S_1: 24/4 = 6

S_2: 12/3 = 4 – smallest non-negative intercept, S_2 leaves

S_3: 8/(1/2) = 16

Solve Eq. (6) for $x_{2\,,}$ yielding

$Z + (2/3)S_2 + S_3 - 72 = 0$

No negative coefficients remain; the method stops. S_2 and S_3 are non-basic variables, $S_2 = S_3 = 0$, $Z_{max} = 72$. The remaining variables from Eqs (5-8) are

$x_1 = 4$

$x_2 = 6$

$S_1 = 8$

Example 5

Maximize:	$Z = x_1 + x_2$	w/ slack variables:	$Z - x_1 - x_2 = 0$	(1)
Subject to:	$x_1 + x_2 \geq 10$	(surplus variable)	$x_1 + x_2 - S_1 = 10$	(2)
	$x_1 \leq 8$		$x_1 + S_2 = 8$	(3)
	$x_2 \leq 12$		$x_2 + S_3 = 12$	(4)

x_1 enters, check intercepts:

S_1: 10/1 = 10

S_2: 8/1 = 8 – smallest non-negative intercept, S_2 leaves

S_3: parallel to x_1

Which gives

$Z + S_2 - x_2 - 8 = 0$

$x_2 - S_1 - S_2 = 2$

$x_1 + S_2 = 8$

$x_2 + S_3 = 12$

x_2 enters, check intercepts:

S_1: 2/-1 = -2

S_3: 12/1 = 12 – smallest non-negative intercept, S_3 leaves

Giving:

$Z + S_2 + S_3 - 20 = 0$

$- S_1 - S_2 - S_3 = -10$

$x_1 + S_2 = 8$

$x_2 + S_3 = 12$

There are no more negative coefficients in the objective function, so the method stops.

$S_2 = S_3 = 0$

$Z_{max} = 20$

$x_1 = 8$

$x_2 = 12$

$S_1 = 10$