ME 5400 Homework 4

1. Fit the following data with (a) a saturation-growth model, (b) a power equation and (c) a parabola. In each case, plot the data and the equation.
   (0.75,1.32), (2,1.73), (3, 2.13), (4, 2.42), (6, 2.40), (8, 2.72), (8.5, 2.65)

2. Fit the following data with the power model ($y = ax^b$). Use the resulting power equation to predict y at $x = 9$.
   (2.5, 13), (3.5, 11.5), (5, 8.5), (6, 8.2), (7.5, 7.1), (10, 6.3),
   (12.5, 5.2), (15, 4.8), (17.5, 4.5), (20, 4.1)

3. Fit an exponential model to
   (0.4, 832), (0.8, 987), (1.2, 1503), (1.6, 1960), (2, 2710), (2.3, 3330)

4. Estimate the function $\cos(\pi/4)$ using linear interpolation. For each of the interpolations, calculate the percent relative error based on the true value
   a. Interpolate between $\cos(0)$ and $\cos(\pi/2)$
   b. Interpolate between $\cos(\pi/6)$ and $\cos(\pi/3)$

5. Fit a second-order Newton's interpolating polynomial to $\cos(\pi/4)$ based on the data at $x = 0$, $\pi/6$ and $\pi/3$. Compute the true percent relative error.

6. Fit a third-order Newton's interpolating polynomial to estimate $\cos(\pi/4)$ based on the data at $x = 0$, $\pi/6$, $\pi/3$ and $\pi/2$. Compute the true percent relative error.

7. Repeat Problems 5 and 6 using the Lagrange Polynomial.

8. With the following data, estimate $f(4)$ using Newton's interpolating polynomials of order 1 through 4. Choose your base points for good accuracy.
   (1, 3), (2, 6), (3, 18), (5, 96), (7, 290), (8,448)

9. Repeat Problem 8 using Lagrange Polynomials of order 1 through 3.

10. Employ inverse interpolation using cubic splines and bisection to determine the value of x that corresponds to $f(x) = 0.225$ for the following data.
    (2, 0.503), (3, 0.334), (4, 0.250), (5, 0.201), (6, 0.171), (7, 0.1421)

11. a. Generate an appropriate number of points, at an appropriate sampling rate, of the function
    $$f(t) = 3 \sin[\, 2\pi\ (4\ \text{Hz})\ t\ ]$$
    Generate and plot the FFT (magnitude vs frequency in Hz). Describe the characteristics and behavior seen in the plot of the FFT.

    b. Repeat part (a) with the following function
    $$f(t) = 3 \sin[\, 2\pi\ (4\ \text{Hz})\ t\ ] + 2 \cos[\, 2\pi\ (7\ \text{Hz})\ t\ ]$$

c. Repeat part (b) with the following function

$$f(t) = 3 \sin[\, 2\pi\, (4\ \text{Hz})\, t\,] \cos[\, 2\pi\, (7\ \text{Hz})\, t\,]$$