

ME 5400 Homework set #2

Solve the following systems with all applicable methods:

$$\begin{aligned} 1. \quad & x_1 + 2x_2 - 12x_3 + 8x_4 = 27 \\ & 5x_1 + 4x_2 + 7x_3 - 2x_4 = 5 \\ & -3x_1 + 7x_2 + 9x_3 + 5x_4 = 11 \\ & 6x_1 - 11x_2 - 8x_3 + 3x_4 = 5 \end{aligned}$$

$$\begin{aligned} 2. \quad & 33x_1 + 16x_2 + 72x_3 = 335 \\ & -24x_1 - 10x_2 - 59x_3 = 234 \\ & -8x_1 - 4x_2 - 17x_3 = 81 \end{aligned}$$

$$\begin{aligned} 3. \quad & 1.7x_1 + 2.3x_2 - 1.5x_3 = 2.83 \\ & 1.1x_1 + 1.6x_2 - 1.9x_3 = -0.84 \\ & 2.7x_1 - 2.1x_2 + 1.5x_3 = 2.42 \end{aligned}$$

$$\begin{aligned} 4. \quad & -2x_1 + x_2 = -1 \\ & x_1 - 2x_2 + x_3 = 0 \\ & x_2 - 2x_3 + x_4 = 6 \\ & x_3 - 2x_4 = 0 \end{aligned}$$

$$\begin{aligned} 5. \quad & x_1 + x_2 + x_3 + x_4 + x_5 = 1 \\ & x_1 + 2x_2 + 3x_3 + 4x_4 + 5x_5 = 0 \\ & x_1 + 3x_2 + 6x_3 + 10x_4 + 15x_5 = 0 \\ & x_1 + 4x_2 + 10x_3 + 20x_4 + 35x_5 = 0 \\ & x_1 + 5x_2 + 15x_3 + 35x_4 + 70x_5 = 0 \end{aligned}$$

6. Solve the following systems of equations:

$$\begin{aligned} 400x_1 - 201x_2 &= 200 \\ -800x_1 + 401x_2 &= -200 \end{aligned}$$

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Calculate the condition number of both matrices, comment on the validity of the solution.

7. Find the inverses of Wilson's matrix and Hilbert's matrix, and their condition number. Comment on the validity of your solutions.

$$\text{Wilson's matrix: } \begin{bmatrix} 10 & 7 & 8 & 7 \\ 7 & 5 & 6 & 5 \\ 8 & 6 & 10 & 9 \\ 7 & 5 & 9 & 10 \end{bmatrix}$$

Hilbert's matrix:

$$\begin{bmatrix} 1 & 1/2 & 1/3 & 1/4 \\ 1/2 & 1/3 & 1/4 & 1/5 \\ 1/3 & 1/4 & 1/5 & 1/6 \\ 1/4 & 1/5 & 1/6 & 1/7 \end{bmatrix}$$