ME 5400 Homework Assignment 1

1. Compare, contrast and comment on the solutions of all the real roots of the following equations using (a) bisection, (b) false position, (c) modified false position, (d) fixed-point iteration, (e) Newton-Raphson, (f) secant and (g) modified secant methods:

    1.1 $f(x) = x^3 - 9x^2 - 5x + 20$
    1.2 $f(x) = 23 - 50x + 35x^2 - 10x^3 + x^4$
    1.3 $f(x) = 4\sin(3x) - \cos(x/2); \ 0 < x < 3.2$
    1.4 $f(x) = (0.8 - 0.5x) / x \ ; \ x > 0$
    1.5 $f(x) = x \cos(x) \ e^{x/6} \ ; \ -3 < x < 3$

2. Find the *maximum* of $f(x) = x^4 - 6x^3 + 5x^2 + 12x + 12$ between $x=0$ and $x=3$ using (a) interval halving and (b) the modified secant method. Describe your solution methods and comment on the results.

3. The function $f(x) = x^4 - 6x^3 + 13x^2 - 12x + 4$ has two double roots, at $x = 1, 2$. Solve for the root at $x = 2$ using the standard Newton-Raphson method and the two multiple-root modifications to N-R discussed in the notes, using an initial guess of $x_0 = 1.7$. Comment on your results.

4. Use and comment on Müller's method to find all of the real roots of
    a. $f(x) = x^4 - 16x^3 + 86x^2 - 176x + 104$
    b. $f(x) = x^4 - 5x^2 + 3$

5. Use and comment on Bairstow's method to find all of the real and complex roots of
    a. $f(x) = -2 + 6.2x - 4x^2 + 0.7x^3$
    b. $f(x) = 9.34 - 21.97x + 16.3x^2 - 3.704x^3$
    c. $f(x) = x^4 - 3x^3 + 5x^2 - x - 9$
    d. $f(x) = x^3 - x^2 + 3x - 3$
    e. $f(x) = 2x^4 + 6x^2 + 12$

6. Find all the points (x, y) in the interval $-1 < x < 4$ that solve the two equations below, using the multivariate Newton-Raphson method

$$y = x^2 - 3x - 2$$
$$y = 5\cos(2x - 0.4) - 1$$