Instructions for submitting homework assignment reports:

- Introduce each problem with a *brief* description of the methods to be used, in your own words, stressing the characteristics of the method that will be exploited in the problem
- List the assumptions, tolerances, starting values, etc. for the problem solution
- As much as possible, present the results in graphical or tabular form, and never with more significant digits than the tolerances or error limits used in the method
- Always list the complete result for the methods used – the numerical result, number of iterations, step size, order of the approximations, etc. – so that the result can be duplicated if necessary
- Always comment on any unusual, odd or significant behavior of the methods used when solving the problem
- At the end of the report, list any, and all, references used in preparing the report. Remember that you must reference any idea, quote or concept that is not original to you.

Example:

Compare, contrast and comment on the solutions of all the real roots of the following equation using (a) bisection, (b) false position, (c) fixed-point iteration, (d) Newton-Raphson and (e) secant methods:

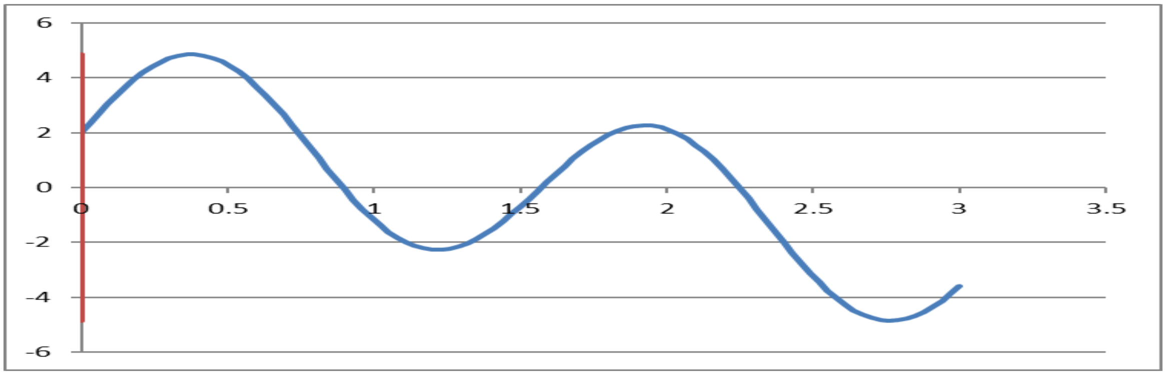
$$f(x) = 3\sin(4x) + 2\cos(x) \; ; \; 0 < x < 3$$

Solution:

The numerical methods used in this problem are:

- **Bisection** is a closed method that divides the distance between the upper and lower bounds of an interval in half with each iteration, always keeping the root between the bounds. Performance is reliable but often slow.
- **False position** is a closed method that uses similar triangles to estimate the location of the root between upper and lower bounds. Performance is usually better than Bisection, but can be one-sided, leading to reduced performance.
- **Fixed-Point Iteration** is an easy-to-program open method that solves the related equation $g(x)=x$. This method cannot converge if $|g'(x)|>1$, and therefore is often unreliable.
- **Newton-Raphson** is a high performance, open root-finding method that requires both an analytic function and its first derivative.
- **Secant** is a variation of Newton-Raphson that uses a forward difference approximation of the first derivative. Performance is often similar to Newton-Raphson without the requirements of an analytical function and its derivative.

The graph of this function in the domain $0 < x < 3$ is shown below.



Using Newton-Raphson, the three real roots in this domain are (to 6 significant digits)

$x_1 = 0.893195$

$x_2 = 1.57079$

$x_3 = 2.24839$

	x_1	x_2	x_3
Bisection	$x_{lo} = 0.5$ $x_{hi} = 1$ 20 iterations	$x_{lo} = 1.5$ $x_{hi} = 1.7$ 17 iterations	$x_{lo} = 2$ $x_{hi} = 2.5$ 18 Iterations
False Position	$x_{lo} = 0.5$ $x_{hi} = 1$ 4 iterations (a)	$x_{lo} = 1.5$ $x_{hi} = 1.7$ 5 iterations	$x_{lo} = 2$ $x_{hi} = 2.5$ 6 iterations (a)
Fixed-Point Iteration	$x_0 = 1$ (b)	$x_0 = 1.5$ (c)	$x_0 = 2.5$ (d)
Newton-Raphson	$x_0 = 1$ 4 iterations	$x_0 = 1.5$ 3 iterations	$x_0 = 2.5$ 4 iterations
Secant	$x_0 = 1$ 5 iterations	$x_0 = 1.5$ 4 iterations	$x_0 = 2.5$ 5 iterations

Notes/comments:

a. One-sided performance, only low bound observed to move
b. Did not converge, $|g'(x_0)| = 8.52$
c. Did not converge, $|g'(x_0)| = 10.53$
d. Did not converge, $|g'(x_0)| = 10.27$