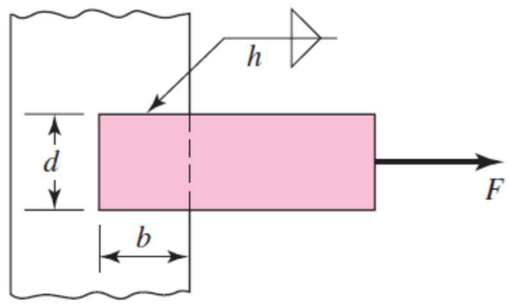
Chapter 9

Figure for Probs. 9-1 to 9-4



9-1 Given, $b = 50$ mm, $d = 50$ mm, $h = 5$ mm, $\tau_{allow} = 140$ MPa.

$$F = 0.707\, hl\tau_{allow} = 0.707(5)[2(50)](140)(10^{-3}) = 49.5 \text{ kN} \qquad Ans.$$

9-2 Given, $b = 2$ in, $d = 2$ in, $h = 5/16$ in, $\tau_{allow} = 25$ kpsi.

$$F = 0.707\, hl\tau_{allow} = 0.707(5/16)[2(2)](25) = 22.1 \text{ kip} \qquad Ans.$$

9-3 Given, $b = 50$ mm, $d = 30$ mm, $h = 5$ mm, $\tau_{allow} = 140$ MPa.

$$F = 0.707\, hl\tau_{allow} = 0.707(5)[2(50)](140)(10^{-3}) = 49.5 \text{ kN} \qquad Ans.$$

9-4 Given, $b = 4$ in, $d = 2$ in, $h = 5/16$ in, $\tau_{allow} = 25$ kpsi.

$$F = 0.707\, hl\tau_{allow} = 0.707(5/16)[2(4)](25) = 44.2 \text{ kip} \qquad Ans.$$

9-5 Prob. 9-1 with E7010 Electrode.

Table 9-6: $f = 14.85\, h$ kip/in $= 14.85$ [5 mm/(25.4 mm/in)] = 2.923 kip/in

$= 2.923(4.45/25.4) = 0.512$ kN/mm

$$F = f\,l = 0.512[2(50)] = 51.2 \text{ kN} \qquad Ans.$$

9-6 Prob. 9-2 with E6010 Electrode.

Table 9-6: $f = 14.85\, h$ kip/in $= 14.85(5/16) = 4.64$ kip/in

$F = f\,l = 4.64[2(2)] = 18.6$ kip *Ans.*

9-7 Prob. 9-3 with E7010 Electrode.

Table 9-6: $f = 14.85\,h$ kip/in $= 14.85$ [5 mm/(25.4 mm/in)] $= 2.923$ kip/in

$= 2.923(4.45/25.4) = 0.512$ kN/mm

$F = f\,l = 0.512[2(50)] = 51.2$ kN *Ans.*

9-8 Prob. 9-4 with E6010 Electrode.

Table 9-6: $f = 14.85\,h$ kip/in $= 14.85(5/16) = 4.64$ kip/in

$F = f\,l = 4.64[2(4)] = 37.1$ kip *Ans.*

9-9 Table A-20:

1018 CD: $S_{ut} = 440$ MPa, $S_y = 370$ MPa
1018 HR: $S_{ut} = 400$ MPa, $S_y = 220$ MPa

Cold-rolled properties degrade to hot-rolled properties in the neighborhood of the weld.
Table 9-4:

$$\begin{aligned}\tau_{\text{all}} &= \min(0.30S_{ut},\ 0.40S_y)\\ &= \min[0.30(400),\ 0.40(220)]\\ &= \min(120,\ 88) = 88\text{ MPa}\end{aligned}$$

for both materials.
Eq. (9-3): $F = 0.707hl\tau_{\text{all}} = 0.707(5)[2(50)](88)(10^{-3}) = 31.1$ kN *Ans.*

9-10 Table A-20:

1020 CD: $S_{ut} = 68$ kpsi, $S_y = 57$ kpsi
1020 HR: $S_{ut} = 55$ kpsi, $S_y = 30$ kpsi

Cold-rolled properties degrade to hot-rolled properties in the neighborhood of the weld.
Table 9-4:

$$\begin{aligned}\tau_{\text{all}} &= \min(0.30S_{ut},\ 0.40S_y)\\ &= \min[0.30(55),\ 0.40(30)]\\ &= \min(16.5,\ 12.0) = 12.0\text{ kpsi}\end{aligned}$$

for both materials.
Eq. (9-3): $F = 0.707hl\tau_{\text{all}} = 0.707(5/16)[2(2)](12.0) = 10.6$ kip *Ans.*

9-11 Table A-20:

1035 HR: S_{ut} = 500 MPa, S_y = 270 MPa

1035 CD: S_{ut} = 550 MPa, S_y = 460 MPa

Cold-rolled properties degrade to hot-rolled properties in the neighborhood of the weld.

Table 9-4:

$$\begin{aligned}\tau_{\text{all}} &= \min(0.30S_{ut},\ 0.40S_y)\\ &= \min[0.30(500),\ 0.40(270)]\\ &= \min(150,\ 108) = 108 \text{ MPa}\end{aligned}$$

for both materials.

Eq. (9-3): $F = 0.707hl\tau_{\text{all}} = 0.707(5)[2(50)](108)(10^{-3}) = 38.2$ kN *Ans.*

9-12 Table A-20:

1035 HR: S_{ut} = 72 kpsi, S_y = 39.5 kpsi

1020 CD: S_{ut} = 68 kpsi, S_y = 57 kpsi, 1020 HR: S_{ut} = 55 kpsi, S_y = 30 kpsi

Cold-rolled properties degrade to hot-rolled properties in the neighborhood of the weld.

Table 9-4:

$$\begin{aligned}\tau_{\text{all}} &= \min(0.30S_{ut},\ 0.40S_y)\\ &= \min[0.30(55),\ 0.40(30)]\\ &= \min(16.5,\ 12.0) = 12.0 \text{ kpsi}\end{aligned}$$

for both materials.

Eq. (9-3): $F = 0.707hl\tau_{\text{all}} = 0.707(5/16)[2(4)](12.0) = 21.2$ kip *Ans.*

9-13

Eq. (9-3): $$\tau = \frac{\sqrt{2}F}{hl} = \frac{\sqrt{2}(100)(10^3)}{5[2(50+50)]} = 141 \text{ MPa}$$ *Ans.*

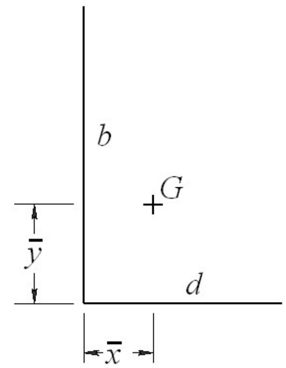
9-14

Eq. (9-3): $$\tau = \frac{\sqrt{2}F}{hl} = \frac{\sqrt{2}(40)}{(5/16)[2(2+2)]} = 22.6 \text{ kpsi}$$ *Ans.*

9-15 Eq. (9-3): $$\tau = \frac{\sqrt{2}F}{hl} = \frac{\sqrt{2}(100)(10^3)}{5[2(50+30)]} = 177 \text{ MPa}$$ *Ans.*

9-16 Eq. (9-3): $$\tau = \frac{\sqrt{2}F}{hl} = \frac{\sqrt{2}(40)}{(5/16)[2(4+2)]} = 15.1 \text{ kpsi}$$ *Ans.*

9-17 A = (throat area)(length) = 0.707h (b + d) *Ans.*



$$\bar{x}\left(b+d\right)=0\left(d\right)+\frac{b}{2}\left(b\right) \quad \Rightarrow \quad \bar{x}=\frac{b^2}{2\left(b+d\right)} \qquad Ans.$$

$$\bar{y}\left(b+d\right)=0\left(b\right)+\frac{d}{2}\left(d\right) \quad \Rightarrow \quad \bar{x}=\frac{d^2}{2\left(b+d\right)} \qquad Ans.$$

For line b:

$$\left(J_u\right)_b=\frac{b^3}{12}+b\left[\left(\frac{b}{2}-\bar{x}\right)^2+\bar{y}^2\right]=\frac{b^3}{12}+b\left(\frac{b^2}{4}-b\bar{x}+\bar{x}^2+\bar{y}^2\right)$$

$$=\frac{b^3}{12}+b\left[\frac{b^2}{4}-\frac{b^3}{2\left(b+d\right)}+\frac{b^4}{4\left(b+d\right)^2}+\frac{d^4}{4\left(b+d\right)^2}\right]=\frac{4b^3\left(b+d\right)^2-6b^4\left(b+d\right)+3b^5+3bd^4}{12\left(b+d\right)^2}$$

Similarly,

$$\left(J_u\right)_d=\frac{4d^3\left(b+d\right)^2-6d^4\left(b+d\right)+3d^5+3b^4d}{12\left(b+d\right)^2}$$

$$J_u=\left(J_u\right)_b+\left(J_u\right)_d$$

$$=\frac{4b^3\left(b+d\right)^2-6b^4\left(b+d\right)+3b^5+3bd^4+4d^3\left(b+d\right)^2-6d^4\left(b+d\right)+3d^5+3b^4d}{12\left(b+d\right)^2}$$

$$=\frac{4b^3\left(b+d\right)^2-6b^4\left(b+d\right)+3b^4\left(b+d\right)+3d^4\left(b+d\right)+4d^3\left(b+d\right)^2-6d^4\left(b+d\right)}{12\left(b+d\right)^2}$$

This reduces to

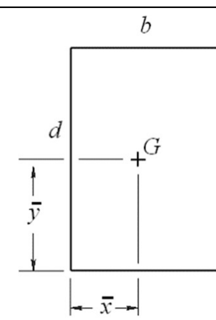
$$J_u=\frac{b^4+4b^3d+d^4+4bd^3}{12\left(b+d\right)} \qquad (1)$$

Add and subtract $6b^2d^2$ to Eq. (1) giving

$$J_u=\frac{\left(b^4+4b^3d+6b^2d^2+4bd^3+d^4\right)-6b^2d^2}{12\left(b+d\right)}$$

$$=\frac{\left(b+d\right)^4-6b^2d^2}{12\left(b+d\right)} \qquad \text{which is the same as Table 9}-1 \qquad Ans.$$

9-18 A = (throat area) (length) = 0.707h (2b + d) *Ans.*



$$\bar{x}\left(2b+d\right)=0\left(d\right)+\frac{d}{2}\left(b\right) \quad \Rightarrow \quad \bar{x}=\frac{b^2}{2b+d} \qquad Ans.$$

$$\bar{y}\left(2b+d\right)=0\left(b\right)+d\left(\frac{d}{2}\right)+d\left(b\right) \quad \Rightarrow \quad \bar{y}=\frac{d\left(b+\frac{d}{2}\right)}{2b+d}=\frac{d}{2} \qquad Ans.$$

For lines b:

$$\left(J_u\right)_b = 2\left\{\frac{b^3}{12} + b\left[\left(\frac{b}{2} - \bar{x}\right)^2 + \bar{y}^2\right]\right\} = \frac{b^3}{6} + 2b\left(\frac{b^2}{4} - b\bar{x} + \bar{x}^2 + \bar{y}^2\right)$$

$$= \frac{b^3}{6} + \frac{b^3}{2} - \frac{2b^4}{2b+d} + \frac{2b^5}{\left(2b+d\right)^2} + \frac{bd^2}{2} = \frac{2b^3}{3} - \frac{2b^4}{2b+d} + \frac{2b^5}{\left(2b+d\right)^2} + \frac{bd^2}{2}$$

Line *d*:

$$\left(J_u\right)_d = \frac{d^3}{12} + d\,\bar{x}^2 = \frac{d^3}{12} + \frac{b^4 d}{\left(2b+d\right)^2}$$

$J_u = (J_u)_b + (J_u)_d$

$$J_u = \frac{2b^3}{3} - \frac{2b^4}{2b+d} + \frac{2b^5}{\left(2b+d\right)^2} + \frac{bd^2}{2} + \frac{d^3}{12} + \frac{b^4 d}{\left(2b+d\right)^2}$$

$$= \frac{8b^3 + 6bd^2 + d^3}{12} + \frac{-2b^4\left(2b+d\right) + 2b^5 + b^4 d}{\left(2b+d\right)^2}$$

$$= \frac{8b^3 + 6bd^2 + d^3}{12} + \frac{-2b^4\left(2b+d\right) + b^4\left(2b+d\right)}{\left(2b+d\right)^2}$$

$$= \frac{8b^3 + 6bd^2 + d^3}{12} - \frac{b^4}{2b+d} \qquad \textit{Ans.}$$

9-19 $b = d = 50$ mm, $c = 150$ mm, $h = 5$ mm, and $\tau_{\text{allow}} = 140$ MPa.

(a) *Primary shear*, Table 9-1, Case 2 (Note: *b* and *d* are interchanged between problem figure and table figure. Note, also, *F* in kN and τ in MPa):

$$\tau'_y = \frac{V}{A} = \frac{F\left(10^3\right)}{1.414\left(5\right)\left(50\right)} = 2.829F$$

Secondary shear, Table 9-1:

$$J_u = \frac{d\left(3b^2 + d^2\right)}{6} = \frac{50\left[3\left(50^2\right) + 50^2\right]}{6} = 83.33\left(10^3\right)\text{ mm}^3$$

$J = 0.707\, h\, J_u = 0.707(5)(83.33)(10^3) = 294.6(10^3)\text{ mm}^4$

$$\tau''_x = \tau''_y = \frac{Mr_y}{J} = \frac{175F\left(10^3\right)\left(25\right)}{294.6\left(10^3\right)} = 14.85F$$

$$\tau_{\max} = \sqrt{\tau''^2_x + \left(\tau'_y + \tau''_y\right)^2} = F\sqrt{14.85^2 + \left(2.829 + 14.85\right)^2} = 23.1F \quad (1)$$

$$F = \frac{\tau_{\text{allow}}}{23.1} = \frac{140}{23.1} = 6.06 \text{ kN} \quad \textit{Ans.}$$

(**b**) For E7010 from Table 9-6, τ_{allow} = 21 kpsi = 21(6.89) = 145 MPa

1020 HR bar: S_{ut} = 380 MPa, S_y = 210 MPa

1015 HR support: S_{ut} = 340 MPa, S_y = 190 MPa

Table 9-3, E7010 Electrode: S_{ut} = 482 MPa, S_y = 393 MPa

The support controls the design.

Table 9-4: τ_{allow} = min(0.30S_{ut}, 0.40S_y) =min[0.30(340), 0.40(190) = min(102, 76)

= 76 MPa

The allowable load, from Eq. (1) is

$$F = \frac{\tau_{\text{allow}}}{23.1} = \frac{76}{23.1} = 3.29 \text{ kN} \qquad \textit{Ans.}$$

9-20 $b = d$ =2 in, c = 6 in, h = 5/16 in, and τ_{allow} = 25 kpsi.
(**a**) *Primary shear*, Table 9-1(Note: b and d are interchanged between problem figure and table figure. Note, also, F in kip and τ in kpsi):

$$\tau'_y = \frac{V}{A} = \frac{F}{1.414(5/16)(2)} = 1.132F$$

Secondary shear, Table 9-1:

$$J_u = \frac{d(3b^2 + d^2)}{6} = \frac{2\left[3(2^2) + 2^2\right]}{6} = 5.333 \text{ in}^3$$

$$J = 0.707\, h\, J_u = 0.707(5/16)(5.333) = 1.178 \text{ in}^4$$

$$\tau''_x = \tau''_y = \frac{Mr_y}{J} = \frac{7F(1)}{1.178} = 5.942F$$

$$\tau_{\max} = \sqrt{\tau''^2_x + (\tau'_y + \tau''_y)^2} = F\sqrt{5.942^2 + (1.132 + 5.942)^2} = 9.24F \quad (1)$$

$$F = \frac{\tau_{\text{allow}}}{9.24} = \frac{25}{9.24} = 2.71 \text{ kip} \quad \textit{Ans.}$$

(**b**) For E7010 from Table 9-6, $\tau_{\text{allow}} = 21$ kpsi

1020 HR bar: $S_{ut} = 55$ kpsi, $S_y = 30$ kpsi

1015 HR support: $S_{ut} = 50$ kpsi, $S_y = 27.5$ kpsi

Table 9-3, E7010 Electrode: $S_{ut} = 70$ kpsi, $S_y = 57$ kpsi

The support controls the design.

Table 9-4: $\tau_{\text{allow}} = \min(0.30S_{ut}, 0.40S_y) = \min[0.30(50), 0.40(27.5) = \min(15, 11)$
$= 11$ kpsi

The allowable load, from Eq. (1) is

$$F = \frac{\tau_{\text{allow}}}{9.24} = \frac{11}{9.24} = 1.19 \text{ kip} \qquad \textit{Ans.}$$

9-21 $b = 50$ mm, $c = 150$ mm, $d = 30$ mm, $h = 5$ mm, and $\tau_{\text{allow}} = 140$ MPa.
(**a**) *Primary shear*, Table 9-1, Case 2 (Note: b and d are interchanged between problem figure and table figure. Note, also, F in kN and τ in MPa):

$$\tau'_y = \frac{V}{A} = \frac{F(10^3)}{1.414(5)(50)} = 2.829F$$

Secondary shear, Table 9-1:

$$J_u = \frac{d(3b^2 + d^2)}{6} = \frac{50\left[3(30^2) + 50^2\right]}{6} = 43.33(10^3) \text{ mm}^3$$

$$J = 0.707\, h\, J_u = 0.707(5)(43.33)(10^3) = 153.2(10^3) \text{ mm}^4$$

$$\tau''_x = \frac{Mr_y}{J} = \frac{175F(10^3)(15)}{153.2(10^3)} = 17.13F$$

$$\tau''_y = \frac{Mr_x}{J} = \frac{175F(10^3)(25)}{153.2(10^3)} = 28.55F$$

$$\tau_{\max} = \sqrt{\tau''^2_x + (\tau'_y + \tau''_y)^2} = F\sqrt{17.13^2 + (2.829 + 28.55)^2} = 35.8F \quad (1)$$

$$F = \frac{\tau_{\text{allow}}}{35.8} = \frac{140}{35.8} = 3.91 \text{ kN} \quad \textit{Ans.}$$

(**b**) For E7010 from Table 9-6, τ_{allow} = 21 kpsi = 21(6.89) = 145 MPa

1020 HR bar: S_{ut} = 380 MPa, S_y = 210 MPa

1015 HR support: S_{ut} = 340 MPa, S_y = 190 MPa

Table 9-3, E7010 Electrode: S_{ut} = 482 MPa, S_y = 393 MPa

The support controls the design.

Table 9-4: τ_{allow} = min(0.30S_{ut}, 0.40S_y) =min[0.30(340), 0.40(190) = min(102, 76)

= 76 MPa

The allowable load, from Eq. (1) is

$$F = \frac{\tau_{\text{allow}}}{35.8} = \frac{76}{35.8} = 2.12 \text{ kN} \qquad \textit{Ans.}$$

9-22 b = 4 in, c = 6 in, d = 2 in, h = 5/16 in, and τ_{allow} = 25 kpsi.
(**a**) *Primary shear*, Table 9-1(Note: b and d are interchanged between problem figure and table figure. Note, also, F in kip and τ in kpsi):

$$\tau'_y = \frac{V}{A} = \frac{F}{1.414(5/16)(4)} = 0.5658F$$

Secondary shear, Table 9-1:

$$J_u = \frac{d(3b^2 + d^2)}{6} = \frac{4\left[3(2^2) + 4^2\right]}{6} = 18.67 \text{ in}^3$$

J = 0.707 h J_u = 0.707(5/16)(18.67) = 4.125 in^4

$$\tau''_x = \frac{Mr_y}{J} = \frac{8F(1)}{4.125} = 1.939F$$

$$\tau''_y = \frac{Mr_x}{J} = \frac{8F(2)}{4.125} = 3.879F$$

$$\tau_{\max} = \sqrt{\tau''^2_x + (\tau'_y + \tau''_y)^2} = F\sqrt{1.939^2 + (0.5658 + 3.879)^2} = 4.85F \quad (1)$$

$$F = \frac{\tau_{\text{allow}}}{4.85} = \frac{25}{4.85} = 5.15 \text{ kip} \quad Ans.$$

(**b**) For E7010 from Table 9-6, $\tau_{\text{allow}} = 21$ kpsi

1020 HR bar: $S_{ut} = 55$ kpsi, $S_y = 30$ kpsi

1015 HR support: $S_{ut} = 50$ kpsi, $S_y = 27.5$ kpsi

Table 9-3, E7010 Electrode: $S_{ut} = 70$ kpsi, $S_y = 57$ kpsi

The support controls the design.

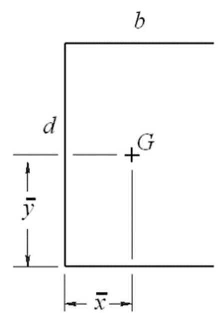
Table 9-4: $\tau_{\text{allow}} = \min(0.30S_{ut}, 0.40S_y) = \min[0.30(50), 0.40(27.5) = \min(15, 11)$

$= 11$ kpsi

The allowable load, from Eq. (1) is

$$F = \frac{\tau_{\text{allow}}}{4.85} = \frac{11}{4.85} = 2.27 \text{ kip} \qquad Ans.$$

9-23 A = (throat area)(length) = $0.707h\,(2b + d)$ *Ans.*

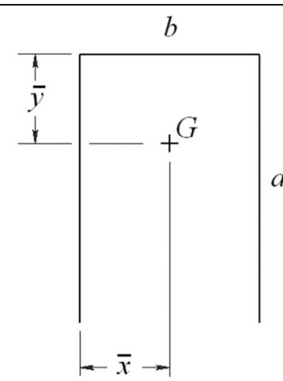


$$\bar{x}(2b+d) = 0(d) + \frac{d}{2}(b) \quad \Rightarrow \quad \bar{x} = \frac{b^2}{2b+d} \qquad Ans.$$

$$\bar{y}(2b+d) = 0(b) + d\left(\frac{d}{2}\right) + d(b) \quad \Rightarrow \quad \bar{y} = \frac{d\left(b+\frac{d}{2}\right)}{2b+d} = \frac{d}{2} \qquad Ans.$$

$$I_u = \frac{d^3}{12} + 2b\left(\frac{d}{2}\right)^2 = \frac{d^2}{12}(6b+d) \qquad Ans.$$

9-24 A = (throat area)(length) = $0.707h\,(b + 2d)$ *Ans.*



$$\bar{x}(b+2d) = 0(d) + \frac{b}{2}(b) + b(d) \quad \Rightarrow \quad \bar{x} = \frac{b\left(\frac{b}{2}+d\right)}{b+2d} = \frac{b}{2} \qquad Ans.$$

$$\bar{y}(b+2d) = 0(b) + \frac{d}{2}(2d) \quad \Rightarrow \quad \bar{y} = \frac{d^2}{b+2d} \qquad Ans.$$

$$I_u = b\bar{y}^2 + \frac{2d^3}{12} + 2d\left(\frac{d}{2} - \bar{y}\right)^2 = \frac{d^3}{6} + \frac{d^3}{2} + b\,\bar{y}^2 - 2d^2\bar{y} + 2d\,\bar{y}^2$$

$$= \frac{2}{3}d^3 - 2d^2\bar{y} + (b + 2d)\bar{y}^2 \qquad \textit{Ans.}$$

9-25 Given, $b = 50$ mm, $c = 150$ mm, $d = 50$ mm, $h = 5$ mm, $\tau_{\text{allow}} = 140$ MPa.

Primary shear (F in kN, τ in MPa, A in mm^2):

$$\tau'_y = \frac{V}{A} = \frac{F(10^3)}{1.414(5)(50+50)} = 1.414F$$

Secondary shear:

Table 9-1: $$J_u = \frac{(b+d)^3}{6} = \frac{(50+50)^3}{6} = 166.7(10^3)\text{ mm}^3$$

$J = 0.707\,h\,J_u = 0.707(5)166.7(10^3) = 589.2(10^3)$ mm^4

$$\tau''_x = \tau''_y = \frac{Mr_y}{J} = \frac{175F(10^3)(25)}{589.2(10^3)} = 7.425F$$

Maximum shear:

$$\tau_{\max} = \sqrt{\tau''^2_x + (\tau'_y + \tau''_y)^2} = F\sqrt{7.425^2 + (1.414 + 7.425)^2} = 11.54F$$

$$F = \frac{\tau_{allow}}{11.54} = \frac{140}{11.54} = 12.1\text{ kN} \qquad \textit{Ans.}$$

9-26 Given, $b = 2$ in, $c = 6$ in, $d = 2$ in, $h = 5/16$ in, $\tau_{\text{allow}} = 25$ kpsi.

Primary shear:

$$\tau'_y = \frac{V}{A} = \frac{F}{1.414(5/16)(2+2)} = 0.5658F$$

Secondary shear:

Table 9-1: $$J_u = \frac{(b+d)^3}{6} = \frac{(2+2)^3}{6} = 10.67\text{ in}^3$$

$J = 0.707\,h\,J_u = 0.707(5/16)10.67 = 2.357$ in^4

$$\tau''_x = \tau''_y = \frac{Mr_y}{J} = \frac{7F(1)}{2.357} = 2.970F$$

Maximum shear:

$$\tau_{\max} = \sqrt{\tau_x''^2 + \left(\tau_y' + \tau_y''\right)^2} = F\sqrt{2.970^2 + \left(0.566 + 2.970\right)^2} = 4.618F$$

$$F = \frac{\tau_{allow}}{4.618} = \frac{25}{4.618} = 5.41 \text{ kip} \quad \textit{Ans.}$$

9-27 Given, b = 50 mm, c = 150 mm, d = 30 mm, h = 5 mm, τ_{allow} = 140 MPa.

Primary shear (F in kN, τ in MPa, A in mm^2):

$$\tau_y' = \frac{V}{A} = \frac{F\left(10^3\right)}{1.414\left(5\right)\left(50+30\right)} = 1.768F$$

Secondary shear:

Table 9-1: $$J_u = \frac{\left(b+d\right)^3}{6} = \frac{\left(50+30\right)^3}{6} = 85.33\left(10^3\right) \text{ mm}^3$$

$J = 0.707\, h\, J_u = 0.707(5)85.33(10^3) = 301.6(10^3)$ mm^4

$$\tau_x'' = \frac{Mr_y}{J} = \frac{175F\left(10^3\right)(15)}{301.6\left(10^3\right)} = 8.704F$$

$$\tau_y'' = \frac{Mr_x}{J} = \frac{175F\left(10^3\right)(25)}{301.6\left(10^3\right)} = 14.51F$$

Maximum shear:

$$\tau_{\max} = \sqrt{\tau_x''^2 + \left(\tau_y' + \tau_y''\right)^2} = F\sqrt{8.704^2 + \left(1.768 + 14.51\right)^2} = 18.46F$$

$$F = \frac{\tau_{allow}}{18.46} = \frac{140}{18.46} = 7.58 \text{ kN} \quad \textit{Ans.}$$

9-28 Given, b = 4 in, c = 6 in, d = 2 in, h = 5/16 in, τ_{allow} = 25 kpsi.

Primary shear:

$$\tau_y' = \frac{V}{A} = \frac{F}{1.414\left(5/16\right)\left(4+2\right)} = 0.3772F$$

Secondary shear:

Table 9-1: $$J_u = \frac{\left(b+d\right)^3}{6} = \frac{\left(4+2\right)^3}{6} = 36 \text{ in}^3$$

$J = 0.707\, h\, J_u = 0.707(5/16)36 = 7.954$ in^4

$$\tau_x'' = \frac{Mr_y}{J} = \frac{8F(1)}{7.954} = 1.006F$$

$$\tau''_y = \frac{Mr_x}{J} = \frac{8F(2)}{7.954} = 2.012F$$

Maximum shear:

$$\tau_{\max} = \sqrt{\tau_x''^2 + \left(\tau'_y + \tau''_y\right)^2} = F\sqrt{1.006^2 + \left(0.3772 + 2.012\right)^2} = 2.592F$$

$$F = \frac{\tau_{allow}}{2.592} = \frac{25}{2.592} = 9.65\text{ kip} \quad \textit{Ans.}$$

9-29 Given: b = 50 mm, c = 150 mm, d = 50 mm, h = 5 mm, τ_{allow} = 140 MPa.

Primary shear (F in kN): $\tau'_x = \tau'_y = \dfrac{V}{A} = \dfrac{F\left(10^3\right)\sin 45^\circ}{1.414(5)(50+50)} = F$

Secondary shear: $M = 0.707\,F\,(10^3)(175 - 25) = 106.05(10^3)\,F$

Table 9-1: $J_u = (b+d)^3/6 = (50+50)^3/6 = 166.7\left(10^3\right)\ \text{mm}^3$

$$J = 0.707hJ_u = 0.707(5)166.7\left(10^3\right) = 589.3\left(10^3\right)\ \text{mm}^4$$

$$\tau''_x = \tau''_y = \frac{Mr_y}{J} = \frac{106.5\left(10^3\right)F(25)}{589.3\left(10^3\right)} = 4.50F$$

Upper right end of weld:

$$\tau_{\max} = \sqrt{\left(\tau'_x + \tau''_x\right)^2 + \left(\tau'_y + \tau''_y\right)^2} = \sqrt{2\left(F + 4.5F\right)^2} = 7.778F$$

$$\tau_{\max} = \tau_{\text{allow}} \quad \Rightarrow \quad F = \frac{140}{7.778} = 18.0\ \text{kN} \qquad \textit{Ans.}$$

9-30 Given: b = 2 in, c = 6 in, d = 2 in, $h = \frac{5}{16}$ in, τ_{allow} = 25 kpsi.

Primary shear (F in kip) : $\tau'_x = \tau'_y = \dfrac{V}{A} = \dfrac{0.707F}{1.414\left(\frac{5}{16}\right)(2+2)} = 0.4F$

Secondary shear: $M = 0.707\,F\,(7 - 1) = 4.242\,F$

Table 9-1: $J_u = (b+d)^3/6 = (2+2)^3/6 = 10.667\ \text{in}^3$

$$J = 0.707hJ_u = 0.707\left(\tfrac{5}{16}\right)10.667 = 2.357\ \text{in}^4$$

$$\tau''_x = \tau''_y = \frac{Mr_y}{J} = \frac{4.242F(1)}{2.357} = 1.80F$$

Upper right end of weld:

$$\tau_{\max} = \sqrt{\left(\tau'_x + \tau''_x\right)^2 + \left(\tau'_y + \tau''_y\right)^2} = \sqrt{2\left(0.4F + 1.80F\right)^2} = 3.111F$$

$$\tau_{\max} = \tau_{\text{allow}} \quad \Rightarrow \quad F = \frac{25}{3.111} = 8.04\text{ kip} \qquad \textit{Ans.}$$

9-31 Given: $b = 50$ mm, $c = 150$ mm, $d = 30$ mm, $h = 5$ mm, $\tau_{\text{allow}} = 140$ MPa.

Primary shear (F in kN): $\tau'_x = \tau'_y = \dfrac{V}{A} = \dfrac{F\left(10^3\right)\sin 45^\circ}{1.414(5)(50+30)} = 1.25F$

Secondary shear: $M = 0.707\,F\,(10^3)(175 - 15) = 113.12\,(10^3)\,F$

Table 9-1: $J_u = (b+d)^3/6 = (50+30)^3/6 = 85.33\left(10^3\right)$ mm^3

$$J = 0.707hJ_u = 0.707(5)85.33\left(10^3\right) = 301.65\left(10^3\right) \text{ mm}^4$$

$$\tau''_x = \frac{Mr_y}{J} = \frac{113.12\left(10^3\right)F(15)}{301.65\left(10^3\right)} = 5.625F$$

$$\tau''_y = \frac{Mr_x}{J} = \frac{113.12\left(10^3\right)F(25)}{301.65\left(10^3\right)} = 9.375F$$

Upper right end of weld:

$$\tau_{\max} = \sqrt{\left(\tau'_x+\tau''_x\right)^2+\left(\tau'_y+\tau''_y\right)^2} = F\sqrt{\left[(1.25+5.625)^2+(1.25+9.375)^2\right]} = 12.66F$$

$$\tau_{\max} = \tau_{\text{allow}} \quad \Rightarrow \quad F = \frac{140}{12.66} = 11.1 \text{ kN} \qquad \textit{Ans.}$$

9-32 Given: $b = 4$ in, $c = 6$ in, $d = 2$ in, $h = \frac{5}{16}$ in, $\tau_{\text{allow}} = 25$ kpsi.

Primary shear (F in kip) : $\tau'_x = \tau'_y = \dfrac{V}{A} = \dfrac{0.707F}{1.414\left(\frac{5}{16}\right)(4+2)} = 0.2667F$

Secondary shear: $M = 0.707\,F\,(8 - 1) = 4.949\,F$

Table 9-1: $J_u = (b+d)^3/6 = (4+2)^3/6 = 36$ in^3

$$J = 0.707hJ_u = 0.707\left(\tfrac{5}{16}\right)36 = 7.954 \text{ in}^4$$

$$\tau''_x = \frac{Mr_y}{J} = \frac{4.949F(1)}{7.954} = 0.6222F$$

$$\tau''_y = \frac{Mr_x}{J} = \frac{4.949F(2)}{7.954} = 1.244F$$

Upper right end of weld:

$$\tau_{\max} = \sqrt{\left(\tau'_x+\tau''_x\right)^2+\left(\tau'_y+\tau''_y\right)^2} = F\sqrt{\left[(0.2667+0.6222)^2+(0.2667+1.244)^2\right]} = 1.753F$$

$$\tau_{\max} = \tau_{\text{allow}} \quad \Rightarrow \quad F = \frac{25}{1.753} = 14.3 \text{ kip} \qquad \textit{Ans.}$$

9-33 Given, $b = 50$ mm, $d = 50$ mm, $h = 5$ mm, E6010 electrode.

$$A = 0.707(5)(50 + 50 + 50) = 530.3 \text{ mm}^2$$

Member endurance limit: From Table A-20 for AISI 1010 HR, $S_{ut} = 320$ MPa.

Eq. (6-18) and Table 6-2: $k_a = 54.9(320)^{-0.758} = 0.693$

$k_b = 1$ (uniform shear), $k_c = 0.59$ (torsion, shear), $k_d = 1$

Eqs. (6-10) and (6-17): $S_e = 0.693(1)(0.59)(1)(0.5)(320) = 65.4$ MPa

Electrode endurance: E6010, Table 9-3, $S_{ut} = 427$ MPa

Eq. (6-18) and Table 6-2: $k_a = 54.9(427)^{-0.758} = 0.557$

As before, $k_b = 1$ (direct shear), $k_c = 0.59$ (torsion, shear), $k_d = 1$

$$S_e = 0.557(1)(0.59)(1)(0.5)(427) = 70.2 \text{ MPa}$$

The members and electrode are basically of equal strength. We will use $S_e = 65.4$ MPa.
For a factor of safety of 1, and with $K_{fs} = 2.7$ (Table 9-5)

$$F = \frac{\tau_{\text{allow}} A}{K_{fs}} = \frac{65.4(530.3)}{2.7} = 12.8(10^3) \text{ N} = 12.8 \text{ kN} \qquad Ans.$$

9-34 Given, $b = 2$ in, $d = 2$ in, $h = 5/16$ in, E6010 electrode.

$$A = 0.707(5/16)(2 + 2 + 2) = 1.326 \text{ in}^2$$

Member endurance limit: From Table A-20 for AISI 1010 HR, $S_{ut} = 47$ kpsi.

Eq. (6-18) and Table 6-2: $k_a = 12.7(47)^{-0.758} = 0.686$

$k_b = 1$ (uniform shear), $k_c = 0.59$ (torsion, shear), $k_d = 1$

Eqs. (6-10) and (6-17): $S_e = 0.686(1)(0.59)(1)(0.5)(47) = 9.51$ kpsi

Electrode endurance: E6010, Table 9-3, $S_{ut} = 62$ kpsi

Eq. (6-18) and Table 6-2: $k_a = 12.7(62)^{-0.758} = 0.556$
As before, $k_b = 1$ (uniform shear), $k_c = 0.59$ (torsion, shear), $k_d = 1$

$$S_e = 0.556(1)(0.59)(1)(0.5)(62) = 10.2 \text{ kpsi}$$

For a factor of safety of 1, with $K_{fs} = 2.7$ (Table 9-5), using $S_e = 9.51$ kpsi

$$F = \frac{\tau_{\text{allow}} A}{K_{fs}} = \frac{9.51(1.326)}{2.7} = 4.67 \text{ kip} \qquad Ans.$$

9-35 Given, b = 50 mm, d = 30 mm, h = 5 mm, E7010 electrode.

$$A = 0.707(5)(50 + 50 + 30) = 459.6 \text{ mm}^2$$

Member endurance limit: From Table A-20 for AISI 1010 HR, S_{ut} = 320 MPa.

Eq. (6-18) and Table 6-2: $k_a = 54.9(320)^{-0.758} = 0.693$

k_b = 1 (direct shear), k_c = 0.59 (torsion, shear), k_d = 1

Eqs. (6-10) and (6-17): $S_e = 0.693(1)(0.59)(1)(0.5)(320) = 65.4$ MPa

Electrode endurance: E6010, Table 9-3, S_{ut} = 482 MPa

Eq. (6-18) and Table 6-2: $k_a = 54.9(482)^{-0.758} = 0.508$

As before, k_b = 1 (direct shear), k_c = 0.59 (torsion, shear), k_d = 1

$$S_e = 0.508(1)(0.59)(1)(0.5)(482) = 72.2 \text{ MPa}$$

For a factor of safety of 1,with K_{fs} = 2.7 (Table 9-5), and using S_e =65.4 MPa

$$F = \frac{\tau_{\text{allow}} A}{K_{fs}} = \frac{65.4(459.6)}{2.7} = 11.1\left(10^3\right) \text{ N} = 11.1 \text{ kN} \qquad Ans.$$

9-36 Given, b = 4 in, d = 2 in, h = 5/16 in, E7010 electrode.

$$A = 0.707(5/16)(4 + 4 + 2) = 2.209 \text{ in}^2$$

Member endurance limit: From Table A-20 for AISI 1010 HR, S_{ut} = 47 kpsi.

Eq. (6-18) and Table 6-2: $k_a = 12.7(47)^{-0.758} = 0.686$

k_b = 1 (direct shear), k_c = 0.59 (torsion, shear), k_d = 1

Eqs. (6-10) and (6-17): $S_e = 0.686(1)(0.59)(1)(0.5)(47) = 9.51$ kpsi

Electrode endurance: E7010, Table 9-3, S_{ut} = 70 kpsi

Eq. (6-18) and Table 6-2: $k_a = 12.7(70)^{-0.758} = 0.507$

As before, k_b = 1 (direct shear), k_c = 0.59 (torsion, shear), k_d = 1

$$S_e = 0.507(1)(0.59)(1)(0.5)(70) = 10.5 \text{ kpsi}$$

For a factor of safety of 1, with K_{fs} = 2.7 (Table 9-5), and using S_e = 9.51 kpsi

$$F = \frac{\tau_{\text{allow}} A}{K_{fs}} = \frac{9.51(2.209)}{2.7} = 7.78 \text{ kip} \qquad Ans.$$

9-37 *Primary shear*: $\tau' = 0$ (why?)
Secondary shear:

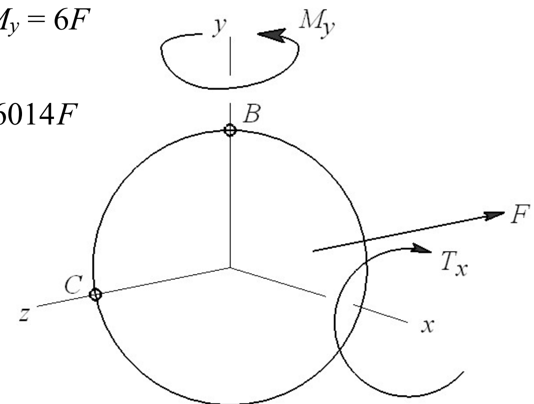
Table 9-1: $J_u = 2\pi r^3 = 2\pi(1.5)^3 = 21.21 \text{ in}^3$

$$J = 0.707\, h\, J_u = 0.707(1/4)(21.21) = 3.749 \text{ in}^4$$

2 welds: $$\tau'' = \frac{Mr}{2J} = \frac{8F(1.5)}{2(3.749)} = 1.600F$$

$$\tau'' = \tau_{\text{allow}} \quad \Rightarrow \quad 1.600F = 20 \quad \Rightarrow \quad F = 12.5 \text{ kip} \qquad Ans.$$

9-38 Direct shear $V_z = F$, Torsion $T_x = 8F$, Bending $M_y = 6F$
B: Direct shear:



$$(\tau)_V = \tau'_z = \frac{V_z}{A} = \frac{F}{1.414\pi hr} = \frac{F}{1.414\pi\left(\frac{1}{4}\right)1.5} = 0.6014F$$

Torsion, Table 9-1:
$J_u = 2\pi r^3 = 2\pi\,(1.5)^3 = 21.21 \text{ in}^3$
$J = 0.707h\, J_u = 0.707\,(\frac{1}{4})\, 21.21 = 3.749 \text{ in}^4$

$$(\tau)_T = \tau''_z = \frac{T_x r}{J} = \frac{8F(1.5)}{3.749} = 3.201F$$

$$\tau_{\max} = \tau'_z + \tau''_z = 0.6014F + 3.201F = 3.802F$$

$$F = \frac{\tau_{\text{allow}}}{3.802} = \frac{20}{3.802} = 5.26 \text{ kip} \qquad Ans.$$

C: Direct shear, torsion, and bending.
Bending, Table 9-2:

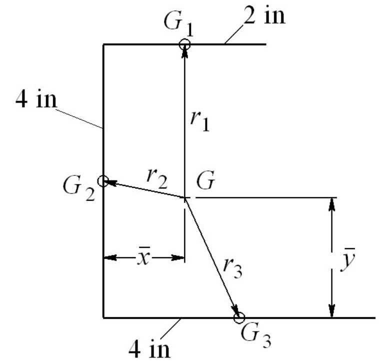
$I_u = \pi r^3 = \pi\,(1.5)^3 = 10.603 \text{ in}^3$, $I = 0.707h\, I_u = 0.707(\frac{1}{4})\, 10.603 = 1.874 \text{ in}^4$.

$$(\tau)_M = \frac{Mc}{I} = \frac{6F(1.5)}{1.874} = 4.803F$$

$$\tau_{\max} = \sqrt{(\tau)_V^2 + (\tau)_T^2 + (\tau)_M^2} = \sqrt{(0.6014F)^2 + (3.201F)^2 + (4.803F)^2} = 5.803F$$

$$F = \frac{\tau_{\text{allow}}}{5.803} = \frac{20}{5.803} = 3.45 \text{ kip} \qquad Ans.$$

9-39 $l = 2 + 4 + 4 = 10$ in



$$\bar{x} = \frac{2(1)+4(0)+4(2)}{10} = 1 \text{ in}$$

$$\bar{y} = \frac{2(4)+4(2)+4(0)}{10} = 1.6 \text{ in}$$

$$M = FR = F\,(10 - 1) = 9\,F$$

$$r_1 = \sqrt{(1-1)^2 + (4-1.6)^2} = 2.4 \text{ in}$$

$$r_2 = \sqrt{1^2 + (2-1.6)^2} = 1.077 \text{ in}$$

$$r_3 = \sqrt{(2-1)^2 + 1.6^2} = 1.887 \text{ in}$$

$$J_{G_1} = \frac{1}{12}(0.707)(5/16)(2^3) = 0.1473 \text{ in}^4$$

$$J_{G_2} = J_{G_3} = \frac{1}{12}(0.707)(5/16)(4^3) = 1.178 \text{ in}^4$$

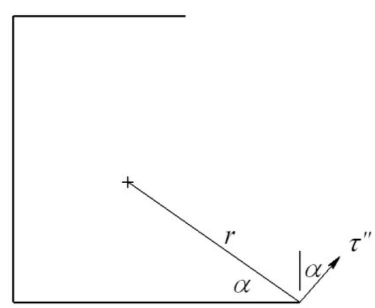
$$J = \sum_{i=1}^{3}\left(J_i + A_i r_{G_i}^2\right)$$

$$= 0.1473 + 0.707(5/16)(2)(2.4^2) + 1.178 + 0.707(5/16)(4)(1.077^2)$$

$$+ 1.178 + 0.707(5/16)(4)(1.887^2) = 9.220 \text{ in}^4$$

$$\alpha = \tan^{-1}\left(\frac{1.6}{4-1}\right) = 28.07^\circ$$

$$r = \sqrt{1.6^2 + (4-1)^2} = 3.4 \text{ in}$$



Primary shear (τ in kpsi, F in kip) :

$$\tau' = \frac{V}{A} = \frac{F}{0.707(5/16)(10)} = 0.4526F$$

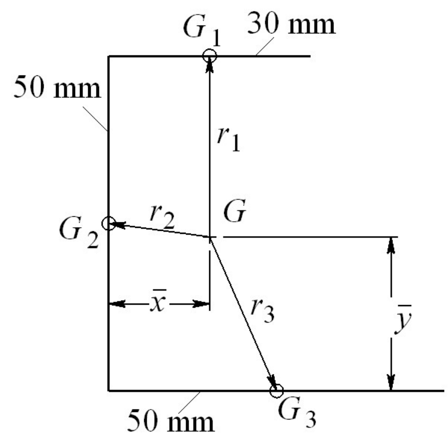
Secondary shear:

$$\tau'' = \frac{Mr}{J} = \frac{9F(3.4)}{9.220} = 3.319F$$

$$\tau_{\max} = \sqrt{\left(3.319F \sin 28.07^\circ\right)^2 + \left(3.319F \cos 28.07^\circ + 0.4526F\right)^2}$$
$$= 3.724F$$

$\tau_{\max} = \tau_{\text{allow}} \quad \Rightarrow \quad 3.724\,F = 25 \quad \Rightarrow \quad F = 6.71$ kip *Ans.*

9-40 $l = 30 + 50 + 50 = 130$ mm



$$\bar{x} = \frac{30(15)+50(0)+50(25)}{130} = 13.08 \text{ mm}$$
$$\bar{y} = \frac{30(50)+50(25)+50(0)}{130} = 21.15 \text{ mm}$$

$M = FR = F(200 - 13.08)$
$= 186.92\,F$ (M in N·m, F in kN)

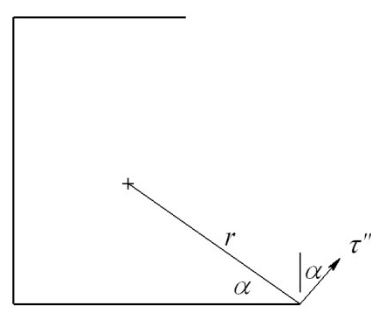
$$r_1 = \sqrt{(15-13.08)^2 + (50-21.15)^2} = 28.92 \text{ mm}$$
$$r_2 = \sqrt{13.08^2 + (25-21.15)^2} = 13.63 \text{ mm}$$
$$r_3 = \sqrt{(25-13.08)^2 + 21.15^2} = 24.28 \text{ mm}$$

$$J_{G_1} = \frac{1}{12}(0.707)(5)\left(30^3\right) = 7.954\left(10^3\right) \text{ mm}^4$$
$$J_{G_2} = J_{G_3} = \frac{1}{12}(0.707)(5)\left(50^3\right) = 36.82\left(10^3\right) \text{ mm}^4$$

$$J = \sum_{i=1}^{3}\left(J_i + A_i r_{G_i}^2\right)$$
$$= 7.954\left(10^3\right) + 0.707(5)(30)\left(28.92^2\right) + 36.82\left(10^3\right) + 0.707(5)(50)\left(13.63^2\right)$$
$$+ 36.82\left(10^3\right) + 0.707(5)(50)\left(24.28^2\right) = 307.3\left(10^3\right) \text{ mm}^4$$

$$\alpha = \tan^{-1}\left(\frac{21.15}{50-13.08}\right) = 29.81^\circ$$

$$r = \sqrt{21.15^2 + (50-13.08)^2} = 42.55 \text{ mm}$$



Primary shear (τ in MPa, F in kN) :

$$\tau' = \frac{V}{A} = \frac{F\left(10^3\right)}{0.707(5)(130)} = 2.176F$$

Secondary shear:

$$\tau'' = \frac{Mr}{J} = \frac{186.92F\left(10^3\right)(42.55)}{307.3\left(10^3\right)} = 25.88F$$

$$\tau_{\max} = \sqrt{\left(25.88F\sin 29.81^\circ\right)^2 + \left(25.88F\cos 29.81^\circ + 2.176F\right)^2}$$
$$= 27.79F$$

$\tau_{\max} = \tau_{\text{allow}} \quad \Rightarrow \quad 27.79\,F = 140 \quad \Rightarrow \quad F = 5.04$ kN *Ans.*

9-41

Weld Pattern	Figure of merit	Rank
1.	$\text{fom}' = \frac{J_u}{lh} = \frac{a^3/12}{ah} = \frac{a^2}{12h} = 0.0833\left(\frac{a^2}{h}\right)$	**5**
2.	$\text{fom}' = \frac{a\left(3a^2 + a^2\right)}{6(2a)h} = \frac{a^2}{3h} = 0.3333\left(\frac{a^2}{h}\right)$	**1**
3.	$\text{fom}' = \frac{(2a)^4 - 6a^2a^2}{12(a+a)2ah} = \frac{5a^2}{24h} = 0.2083\left(\frac{a^2}{h}\right)$	**4**
4.	$\text{fom}' = \frac{1}{3ah}\left(\frac{8a^3 + 6a^3 + a^3}{12} - \frac{a^4}{2a+a}\right) = 0.3056\left(\frac{a^2}{h}\right)$	**2**
5.	$\text{fom}' = \frac{(2a)^3}{6h}\frac{1}{4a} = \frac{8a^3}{24ah} = 0.3333\left(\frac{a^2}{h}\right)$	**1**
6.	$\text{fom}' = \frac{2\pi(a/2)^3}{\pi ah} = \frac{a^3}{4ah} = 0.25\left(\frac{a^2}{h}\right)$	**3**

9-42

Weld Pattern	Figure of merit	Rank
1.	$\text{fom}' = \dfrac{I_u}{lh} = \dfrac{\left(a^3/12\right)}{ah} = 0.0833\left(\dfrac{a^2}{h}\right)$	**6**
2.	$\text{fom}' = \dfrac{\left(a^3/6\right)}{2ah} = 0.0833\left(\dfrac{a^2}{h}\right)$	**6**
3.	$\text{fom}' = \dfrac{\left(aa^2/2\right)}{2ah} = 0.25\left(\dfrac{a^2}{h}\right)$	**1**
4.*	$\text{fom}' = \dfrac{\left(a^2/12\right)\left(6a+a\right)}{3ah} = \dfrac{7a^2}{36h} = 0.1944\left(\dfrac{a^2}{h}\right)$	**2**
5. & 7.	$\bar{x} = \dfrac{a}{2}, \quad \bar{y} = \dfrac{a^2}{a+2a} = \dfrac{a}{3}$	
	$I_u = \dfrac{2a^3}{3} - 2a^2\dfrac{a}{3} + \left(a+2a\right)\left(\dfrac{a}{3}\right)^2 = \dfrac{a^3}{3}$	
	$\text{fom}' = \dfrac{I_u}{lh} = \dfrac{\left(a^3/3\right)}{3ah} = \dfrac{1}{9}\left(\dfrac{a^2}{h}\right) = 0.1111\left(\dfrac{a^2}{h}\right)$	**5**
6. & 8.	$\text{fom}' = \dfrac{\left(a^2/6\right)\left(3a+a\right)}{4ah} = \dfrac{1}{6}\left(\dfrac{a^2}{h}\right) = 0.1667\left(\dfrac{a^2}{h}\right)$	**3**
9.	$\text{fom}' = \dfrac{\pi\left(a/2\right)^3}{\pi ah} = \dfrac{a^2}{8h} = 0.125\left(\dfrac{a^2}{h}\right)$	**4**

*Note. Because this section is not symmetric with the vertical axis, out-of-plane deflection may occur unless special precautions are taken. See the topic of "shear center" in books with more advanced treatments of mechanics of materials.

9-43 Attachment and member (1018 HR), S_y = 220 MPa and S_{ut} = 400 MPa.

The member and attachment are weak compared to the properties of the lowest electrode.

Decision Specify the E6010 electrode

Controlling property, Table 9-4: τ_{all} = min[0.3(400), 0.4(220)] = min(120, 88) = 88 MPa

For a static load, the parallel and transverse fillets are the same. Let the length of a bead be l = 75 mm, and n be the number of beads.

$$\tau = \frac{F}{n(0.707)hl} = \tau_{all}$$

$$nh = \frac{F}{0.707 l \tau_{\text{all}}} = \frac{100(10^3)}{0.707(75)(88)} = 21.43$$

where h is in millimeters. Make a table

Number of beads, n	Leg size, h (mm)
1	21.43
2	10.71
3	7.14
4	5.36 $\rightarrow$ 6 mm

Decision Specify h = 6 mm on all four sides.

Weldment specification:

Pattern: All-around square, four beads each side, 75 mm long

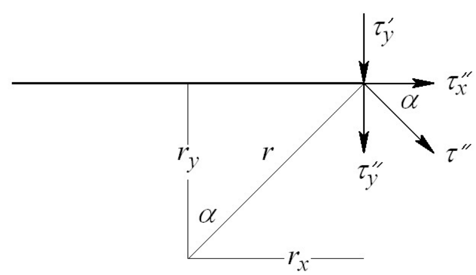
Electrode: E6010

Leg size: h = 6 mm

9-44 *Decision*: Choose a parallel fillet weldment pattern. By so-doing, we've chosen an optimal pattern (see Prob. 9-41) and have thus reduced a synthesis problem to an analysis problem:

Table 9-1, case 2, rotated 90°: $A = 1.414hd = 1.414(h)(75) = 106.05h$ mm^2

Primary shear



$$\tau'_y = \frac{V}{A} = \frac{12(10^3)}{106.05h} = \frac{113.2}{h}$$

Secondary shear:

$$J_u = \frac{d(3b^2 + d^2)}{6}$$

$$= \frac{75[3(75^2) + 75^2]}{6} = 281.3(10^3)\ \text{mm}^3$$

$$J = 0.707(h)(281.3)(10^3) = 198.8(10^3)h\ \text{mm}^4$$

With $\alpha = 45°$,

$$\tau_x'' = \frac{Mr\cos 45^\circ}{J} = \frac{Mr_y}{J} = \frac{12\left(10^3\right)(187.5)(37.5)}{198.8\left(10^3\right)h} = \frac{424.4}{h} = \tau_y''$$

$$\tau_{\max} = \sqrt{\tau_x''^2 + \left(\tau_y' + \tau_y''\right)^2} = \frac{1}{h}\sqrt{424.4^2 + (113.2 + 424.4)^2} = \frac{684.9}{h}$$

Attachment and member (1018 HR): S_y = 220 MPa, S_{ut} = 400 MPa
Decision: Use E60XX electrode which is stronger

$$\tau_{\text{all}} = \min[0.3(400),\ 0.4(220)] = 88 \text{ MPa}$$
$$\tau_{\max} = \tau_{\text{all}} = \frac{684.9}{h} = 88 \text{ MPa}$$
$$h = \frac{684.9}{88} = 7.78 \text{ mm}$$

Decision: Specify 8 mm leg size
Weldment Specifications:
Pattern: Parallel horizontal fillet welds
Electrode: E6010
Type: Fillet
Length of each bead: 75 mm
Leg size: 8 mm

9-45 Problem 9-44 solves the problem using parallel horizontal fillet welds, each 75 mm long obtaining a leg size rounded up to 8 mm.
For this problem, since the width of the plate is fixed and the length has not been determined, we will explore reducing the leg size by using two vertical beads 75 mm long and two horizontal beads such that the beads have a leg size of 6 mm.

Decision: Use a rectangular weld bead pattern with a leg size of 6 mm (case 5 of Table 9-1 with b unknown and d = 75 mm).
Materials:
Attachment and member (1018 HR): S_y = 220 MPa, S_{ut} = 400 MPa
From Table 9-4, AISC welding code,

$$\tau_{\text{all}} = \min[0.3(400), 0.4(220)] = \min(120, 88) = 88 \text{ MPa}$$

Select a stronger electrode material from Table 9-3.
Decision: Specify E6010

Solving for b: In Prob. 9-44, every term was linear in the unknown h. This made solving for h relatively easy. In this problem, the terms will not be linear in b, and so we will use an iterative solution with a spreadsheet.

Throat area and other properties from Table 9-1:

$$A = 1.414(6)(b + 75) = 8.484(b + 75) \qquad (1)$$

$$J_u = \frac{(b+75)^3}{6}, \; J = 0.707\,(6)\,J_u = 0.707(b+75)^3 \quad (2)$$

Primary shear (τ in MPa, h in mm):

$$\tau'_y = \frac{V}{A} = \frac{12(10^3)}{A} \quad (3)$$

Secondary shear (See Prob. 9-44 solution for the definition of α) :

$$\tau'' = \frac{Mr}{J}$$

$$\tau''_x = \tau'' \cos\alpha = \frac{Mr}{J}\cos\alpha = \frac{Mr_y}{J} = \frac{12(10^3)(150 + b/2)(37.5)}{0.707(b+75)^3} \quad (4)$$

$$\tau''_y = \tau'' \sin\alpha = \frac{Mr}{J}\sin\alpha = \frac{Mr_x}{J} = \frac{12(10^3)(150 + b/2)(b/2)}{0.707(b+75)^3} \quad (5)$$

$$\tau_{\max} = \sqrt{\tau'^2_y + (\tau''_x + \tau''_y)^2} \quad (6)$$

Enter Eqs. (1) to (6) into a spreadsheet and iterate for various values of *b*. A portion of the spreadsheet is shown below.

b (mm)	A (mm^2)	J (mm^4)	τ'_y (Mpa)	τ''_y (Mpa)	τ''_x (Mpa)	$\tau_{\max}$ (Mpa)	
41	984.144	1103553.5	12.19334	69.5254	38.00722	90.12492	
42	992.628	1132340.4	12.08912	67.9566	38.05569	88.63156	
43	1001.112	1161623.6	11.98667	66.43718	38.09065	87.18485	< 88 Mpa
44	1009.596	1191407.4	11.88594	64.96518	38.11291	85.7828	

We see that $b \geq 43$ mm meets the strength goal.

Weldment Specifications:
Pattern: Horizontal parallel weld tracks 43 mm long, vertical parallel weld tracks 75 mm long
Electrode: E6010
Leg size: 6 mm

9-46 *Materials:*

Member and attachment (1018 HR): $S_y = 32$ kpsi, $S_{ut} = 58$ kpsi

Table 9-4: $\tau_{\text{all}} = \min[0.3(58), 0.4(32)] = 12.8$ kpsi

Decision: Use E6010 electrode. From Table 9-3: $S_y = 50$ kpsi, $S_{ut} = 62$ kpsi,

$$\tau_{\text{all}} = \min[0.3(62),\, 0.4(50)] = 20 \text{ kpsi}$$

Decision: Since 1018 HR is weaker than the E6010 electrode, use $\tau_{\text{all}} = 12.8$ kpsi

Decision: Use an all-around square weld bead track.

$l_1 = 6 + a = 6 + 6.25 = 12.25$ in

Throat area and other properties from Table 9-1:

$$A = 1.414h(b+d) = 1.414(h)(6+6) = 16.97h$$

Primary shear

$$\tau'_y = \frac{V}{A} = \frac{F}{A} = \frac{20\left(10^3\right)}{16.97h} = \frac{1179}{h} \text{ psi}$$

Secondary shear

$$J_u = \frac{(b+d)^3}{6} = \frac{(6+6)^3}{6} = 288 \text{ in}^3$$

$$J = 0.707h(288) = 203.6h \text{ in}^4$$

$$\tau''_x = \tau''_y = \frac{Mr_y}{J} = \frac{20\left(10^3\right)(6.25+3)(3)}{203.6h} = \frac{2726}{h} \text{ psi}$$

$$\tau_{\max} = \sqrt{\tau''^2_x + (\tau'_y + \tau''_y)^2} = \frac{1}{h}\sqrt{2726^2 + (1179+2726)^2} = \frac{4762}{h} \text{ psi}$$

Relate stress to strength

$$\tau_{\max} = \tau_{\text{all}} \quad \Rightarrow \quad \frac{4762}{h} = 12.8\left(10^3\right) \quad \Rightarrow \quad h = \frac{4762}{12.8\left(10^3\right)} = 0.372 \text{ in}$$

Decision:

Specify $3/8$ in leg size

Specifications:

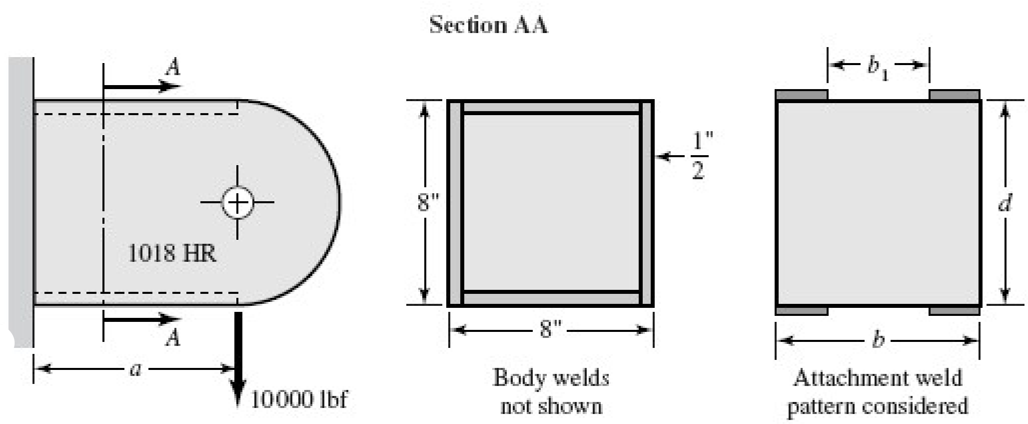
Pattern: All-around square weld bead track
Electrode: E6010
Type of weld: Fillet
Weld bead length: 24 in
Leg size: $3/8$ in
Attachment length: 12.25 in

9-47 This is a good analysis task to test a student's understanding.

(1) Solicit information related to a priori decisions.
(2) Solicit design variables *b* and *d*.
(3) Find *h* and round and output all parameters on a single screen. Allow return to Step 1 or Step 2.
(4) When the iteration is complete, the final display can be the bulk of your adequacy assessment.

Such a program can teach too.

9-48 The objective of this design task is to have the students teach themselves that the weld patterns of Table 9-2 can be added or subtracted to obtain the properties of a contemplated weld pattern. The instructor can control the level of complication. We have left the presentation of the drawing to you. Here is *one* possibility. Study the problem's opportunities, and then present this (or your sketch) with the problem assignment.



Use b_1 as the design variable. Express properties as a function of b_1. From Table 9-3, case 3:

$$A = 1.414h(b - b_1)$$

$$I_u = \frac{bd^2}{2} - \frac{b_1 d^2}{2} = \frac{(b - b_1)d^2}{2}$$

$$I = 0.707hI_u$$

$$\tau' = \frac{V}{A} = \frac{F}{1.414h(b - b_1)}$$

$$\tau'' = \frac{Mc}{I} = \frac{Fa(d/2)}{0.707hI_u}$$

Parametric study

Let $a = 10$ in, $b = 8$ in, $d = 8$ in, $b_1 = 2$ in, $\tau_{\text{all}} = 12.8$ kpsi, $l = 2(8 - 2) = 12$ in

$$A = 1.414h(8-2) = 8.48h \text{ in}^2$$
$$I_u = (8-2)(8^2/2) = 192 \text{ in}^3$$
$$I = 0.707(h)(192) = 135.7h \text{ in}^4$$
$$\tau' = \frac{10\,000}{8.48h} = \frac{1179}{h} \text{ psi}$$
$$\tau'' = \frac{10\,000(10)(8/2)}{135.7h} = \frac{2948}{h} \text{ psi}$$
$$\tau_{\max} = \frac{1}{h}\sqrt{1179^2 + 2948^2} = \frac{3175}{h} = 12\,800 \text{ psi}$$

from which $h = 0.248$ in. Do not round off the leg size – something to learn.

$$\text{fom}' = \frac{I_u}{hl} = \frac{192}{0.248(12)} = 64.5 \text{ in}$$
$$A = 8.48(0.248) = 2.10 \text{ in}^2$$
$$I = 135.7(0.248) = 33.65 \text{ in}^4$$
$$\text{vol} = \frac{h^2}{2}l = \frac{0.248^2}{2}12 = 0.369 \text{ in}^3$$
$$\text{eff} = \frac{I}{\text{vol}} = \frac{33.65}{0.369} = 91.2 \text{ in}$$
$$\tau' = \frac{1179}{0.248} = 4754 \text{ psi}$$
$$\tau'' = \frac{2948}{0.248} = 11\,887 \text{ psi}$$
$$\tau_{\max} = \frac{3175}{0.248} = 12\,800 \text{ psi}$$

Now consider the case of uninterrupted welds,

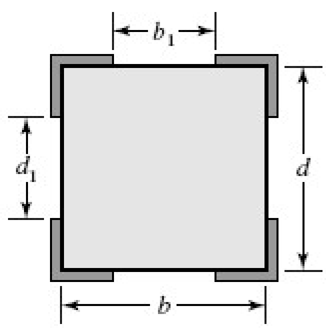
$$b_1 = 0$$
$$A = 1.414(h)(8-0) = 11.31h$$
$$I_u = (8-0)(8^2/2) = 256 \text{ in}^3$$
$$I = 0.707(256)h = 181h \text{ in}^4$$
$$\tau' = \frac{10\,000}{11.31h} = \frac{884}{h}$$
$$\tau'' = \frac{10\,000(10)(8/2)}{181h} = \frac{2210}{h}$$
$$\tau_{\max} = \frac{1}{h}\sqrt{884^2 + 2210^2} = \frac{2380}{h} = \tau_{\text{all}}$$
$$h = \frac{\tau_{\max}}{\tau_{\text{all}}} = \frac{2380}{12\,800} = 0.186 \text{ in}$$

Do not round off h.

$$A = 11.31(0.186) = 2.10 \text{ in}^2$$
$$I = 181(0.186) = 33.67 \text{ in}^4$$
$$\tau' = \frac{884}{0.186} = 4753 \text{ psi}, \quad \text{vol} = \frac{0.186^2}{2} 16 = 0.277 \text{ in}^3$$
$$\tau'' = \frac{2210}{0.186} = 11\,882 \text{ psi}$$
$$\text{fom}' = \frac{I_u}{hl} = \frac{256}{0.186(16)} = 86.0 \text{ in}$$
$$\text{eff} = \frac{I}{(h^2/2)l} = \frac{33.67}{(0.186^2/2)16} = 121.7 \text{ in}$$

Conclusions: To meet allowable stress limitations, I and A do not change, nor do τ and σ. To meet the shortened bead length, h is increased proportionately. However, volume of bead laid down increases as h^2. The uninterrupted bead is superior. In this example, we did not round h and as a result we learned something. Our measures of merit are also sensitive to rounding. When the design decision is made, rounding to the next larger standard weld fillet size will decrease the merit.

Had the weld bead gone around the corners, the situation would change. Here is a follow up task analyzing an alternative weld pattern.



9-49 From Table 9-2

For the box $A = 1.414h(b + d)$

Subtracting b_1 from b and d_1 from d

$$A = 1.414h\left(b - b_1 + d - d_1\right)$$
$$I_u = \frac{d^2}{6}(3b + d) - \frac{d_1^3}{6} - \frac{b_1 d^2}{2} = \frac{1}{2}\left(b - b_1\right)d^2 + \frac{1}{6}\left(d^3 - d_1^3\right)$$

Length of bead $\quad l = 2(b - b_1 + d - d_1)$

$$\text{fom} = I_u / hl$$

9-50 Computer programs will vary.

9-51 τ_{all} = 12 kpsi. Use Fig. 9-17(*a*) for general geometry, but employ $\overline{\underline{}}$ beads and then $||$ beads.

Horizontal parallel weld bead pattern
b = 3 in, d = 6 in

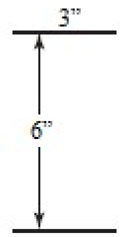


Table 9-2: $A = 1.414hb = 1.414(h)(3) = 4.24h \text{ in}^2$

$$I_u = \frac{bd^2}{2} = \frac{3(6)^2}{2} = 54 \text{ in}^3$$
$$I = 0.707hI_u = 0.707(h)(54) = 38.2h \text{ in}^4$$
$$\tau' = \frac{10}{4.24h} = \frac{2.358}{h} \text{ kpsi}$$
$$\tau'' = \frac{Mc}{I} = \frac{10(10)(6/2)}{38.2h} = \frac{7.853}{h} \text{ kpsi}$$

$$\tau_{\max} = \sqrt{\tau'^2 + \tau''^2} = \frac{1}{h}\sqrt{2.358^2 + 7.853^2} = \frac{8.199}{h} \text{ kpsi}$$

Equate the maximum and allowable shear stresses.

$$\tau_{\max} = \tau_{\text{all}} = \frac{8.199}{h} = 12$$

from which $h = 0.683$ in. It follows that

$$I = 38.2(0.683) = 26.1 \text{ in}^4$$

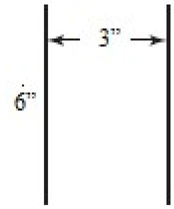
The volume of the weld metal is

$$\text{vol} = \frac{h^2 l}{2} = \frac{(0.683)^2(3+3)}{2} = 1.40 \text{ in}^3$$

The effectiveness, $(\text{eff})_{\text{H}}$, is

$$(\text{eff})_{\text{H}} = \frac{I}{\text{vol}} = \frac{26.1}{1.4} = 18.6 \text{ in} \qquad \textit{Ans.}$$

Vertical parallel weld beads



$b = 3$ in
$d = 6$ in

From Table 9-2, case 2

$$A = 1.414hd = 1.414(h)(6) = 8.48h \text{ in}^2$$

$$I_u = \frac{d^3}{6} = \frac{6^3}{6} = 72 \text{ in}^3$$

$$I = 0.707hI_u = 0.707(h)(72) = 50.9h$$

$$\tau' = \frac{10}{8.48h} = \frac{1.179}{h} \text{ psi}$$

$$\tau'' = \frac{Mc}{I} = \frac{10(10)(6/2)}{50.9h} = \frac{5.894}{h} \text{ psi}$$

$$\tau_{\max} = \sqrt{\tau'^2 + \tau''^2} = \frac{1}{h}\sqrt{1.179^2 + 5.894^2} = \frac{6.011}{h} \text{ kpsi}$$

Equating $\tau_{\max}$ to τ_{all} gives $h = 0.501$ in. It follows that

$$I = 50.9(0.501) = 25.5 \text{ in}^4$$

$$\text{vol} = \frac{h^2 l}{2} = \frac{0.501^2}{2}(6+6) = 1.51 \text{ in}^3$$

$$(\text{eff})_V = \frac{I}{\text{vol}} = \frac{25.5}{1.51} = 16.7 \text{ in} \qquad \textit{Ans.}$$

9-52 $F = 0$, $T = 15$ kip·in.

Table 9-1: $J_u = 2\pi r^3 = 2\pi(1)^3 = 6.283$ in^3, $J = 0.707(1/4)\,6.283 = 1.111$ in^4

$$\tau_{\max} = \frac{Tr}{J} = \frac{15(1)}{1.111} = 13.5 \text{ kpsi} \qquad \textit{Ans.}$$

9-53 $F = 2$ kip, $T = 0$.

Table 9-2: $A = 1.414\,\pi h r = 1.414\,\pi(1/4)(1) = 1.111$ in^2
$I_u = \pi r^3 = \pi(1)^3 = 3.142$ in^3, $I = 0.707(1/4)\,3.142 = 0.5553$ in^4

$$\tau' = \frac{V}{A} = \frac{2}{1.111} = 1.80 \text{ kpsi}$$

$$\tau'' = \frac{Mr}{I} = \frac{2(6)(1)}{0.5553} = 21.6 \text{ kpsi}$$

$$\tau_{\max} = (\tau'^2 + \tau''^2)^{1/2} = (1.80^2 + 21.6^2)^{1/2} = 21.7 \text{ kpsi} \quad \textit{Ans.}$$

9-54 $F = 2$ kip, $T = 15$ kip·in.

Bending:

Table 9-2: $A = 1.414\ \pi h\, r = 1.414\ \pi (1/4)(1) = 1.111 \text{ in}^2$

$$I_u = \pi r^3 = \pi (1)^3 = 3.142 \text{ in}^3, \quad I = 0.707(1/4)\ 3.142 = 0.5553 \text{ in}^4$$

$$\tau' = \frac{V}{A} = \frac{2}{1.111} = 1.80 \text{ kpsi}$$

$$(\tau'')_M = \frac{Mr}{I} = \frac{2(6)(1)}{0.5553} = 21.6 \text{ kpsi}$$

Torsion:

Table 9-1: $J_u = 2\pi r^3 = 2\pi (1)^3 = 6.283 \text{ in}^3, \quad J = 0.707(1/4)\ 6.283 = 1.111 \text{ in}^4$

$$(\tau'')_T = \frac{Tr}{J} = \frac{15(1)}{1.111} = 13.5 \text{ kpsi}$$

$$\tau_{\max} = \sqrt{\tau'^2 + (\tau'')_M^2 + (\tau'')_T^2} = \sqrt{1.80^2 + 21.6^2 + 13.5^2} = 25.5 \text{ kpsi} \quad \textit{Ans.}$$

9-55 $F = 2$ kip, $T = 15$ kip·in.

Bending:

Table 9-2: $A = 1.414\ \pi h\, r = 1.414\ \pi h\,(1) = 4.442h \text{ in}^2$

$$I_u = \pi r^3 = \pi (1)^3 = 3.142 \text{ in}^3, \quad I = 0.707\, h\,(3.142) = 2.221h \text{ in}^4$$

$$\tau' = \frac{V}{A} = \frac{2}{4.442h} = \frac{0.4502}{h} \text{ kpsi}$$

$$(\tau'')_M = \frac{Mr}{I} = \frac{2(6)(1)}{2.221h} = \frac{5.403}{h} \text{ kpsi}$$

Torsion:

Table 9-1: $J_u = 2\pi r^3 = 2\pi(1)^3 = 6.283$ in^3, $J = 0.707\,h\,(6.283) = 4.442$ in^4

$$(\tau'')_T = \frac{Tr}{J} = \frac{15(1)}{4.442h} = \frac{3.377}{h} \text{ kpsi}$$

$$\tau_{\max} = \sqrt{\tau'^2 + (\tau'')_M^2 + (\tau'')_T^2} = \sqrt{\left(\frac{0.4502}{h}\right)^2 + \left(\frac{5.403}{h}\right)^2 + \left(\frac{3.377}{h}\right)^2} = \frac{6.387}{h} \text{ kpsi}$$

$$\tau_{\max} = \tau_{\text{all}} \quad \Rightarrow \quad \frac{6.387}{h} = 20 \quad \Rightarrow \quad h = 0.319 \text{ in} \quad \textit{Ans.}$$

Should specify a $\frac{3}{8}$ in weld. *Ans.*

9-56 $h = 9$ mm, $d = 200$ mm, $b = 25$ mm

From Table 9-2, case 2:

$A = 1.414(9)(200) = 2.545(10^3)$ mm^2

$$I_u = \frac{d^3}{6} = \frac{200^3}{6} = 1.333(10^6) \text{ mm}^3$$

$I = 0.707h\,I_u = 0.707(9)(1.333)(10^6) = 8.484(10^6)$ mm^4

$$\tau' = \frac{F}{A} = \frac{25(10^3)}{2.545(10^3)} = 9.82 \text{ MPa}$$

$M = 25(150) = 3750$ N·m

$$\tau'' = \frac{Mc}{I} = \frac{3750(100)}{8.484(10^6)}(10^3) = 44.20 \text{ MPa}$$

$$\tau_{\max} = \sqrt{\tau'^2 + \tau''^2} = \sqrt{9.82^2 + 44.20^2} = 45.3 \text{ MPa} \quad \textit{Ans.}$$

9-57 $h = 0.25$ in, $b = 2.5$ in, $d = 5$ in.

Table 9-2, case 5: $A = 0.707h\,(b + 2d) = 0.707(0.25)[2.5 + 2(5)] = 2.209$ in^2

$$\bar{y} = \frac{d^2}{b+2d} = \frac{5^2}{2.5+2(5)} = 2 \text{ in}$$

$$I_u = \frac{2d^3}{3} - 2d^2\bar{y} + (b+2d)\bar{y}^2$$

$$= \frac{2(5^3)}{3} - 2(5^2)(2) + [2.5+2(5)](2^2) = 33.33 \text{ in}^3$$

$$I = 0.707\, h\, I_u = 0.707(1/4)(33.33) = 5.891 \text{ in}^4$$

Primary shear:

$$\tau' = \frac{F}{A} = \frac{2}{2.209} = 0.905 \text{ kpsi}$$

Secondary shear (the critical location is at the bottom of the bracket):

$$y = 5 - 2 = 3 \text{ in}$$

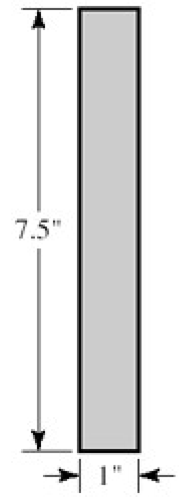
$$\tau'' = \frac{My}{I} = \frac{2(5)(3)}{5.891} = 5.093 \text{ kpsi}$$

$$\tau_{\max} = \sqrt{\tau'^2 + \tau''^2} = \sqrt{0.905^2 + 5.093^2} = 5.173 \text{ kpsi}$$

$$n = \frac{\tau_{\text{all}}}{\tau_{\max}} = \frac{18}{5.173} = 3.48 \qquad \textit{Ans.}$$

9-58 The largest possible weld size is 1/16 in. This is a small weld and thus difficult to accomplish. The bracket's load-carrying capability is not known. There are geometry problems associated with sheet metal folding, load-placement and location of the center of twist. This is not available to us. We will identify the strongest possible weldment. Use a rectangular, weld-all-around pattern – Table 9-2, case 6:

$$A = 1.414\,h(b+d) = 1.414(1/16)(1+7.5)$$
$$= 0.7512 \text{ in}^2$$
$$\bar{x} = b/2 = 0.5 \text{ in}$$
$$\bar{y} = d/2 = 7.5/2 = 3.75 \text{ in}$$
$$I_u = \frac{d^2}{6}(3b+d) = \frac{7.5^2}{6}[3(1)+7.5] = 98.44 \text{ in}^3$$
$$I = 0.707hI_u = 0.707(1/16)(98.44) = 4.350 \text{ in}^4$$
$$M = (3.75+0.5)W = 4.25W$$
$$\tau' = \frac{V}{A} = \frac{W}{0.7512} = 1.331W$$
$$\tau'' = \frac{Mc}{I} = \frac{4.25W(7.5/2)}{4.350} = 3.664W$$
$$\tau_{\max} = \sqrt{\tau'^2 + \tau''^2} = W\sqrt{1.331^2 + 3.664^2} = 3.90W$$



Material properties: The allowable stress given is low. Let's demonstrate that.
For the 1020 CD bracket, use HR properties of S_y = 30 kpsi and S_{ut} = 55. The 1030 HR support, S_y = 37.5 kpsi and S_{ut} = 68. The E6010 electrode has strengths of S_y = 50 and S_{ut} = 62 kpsi.

Allowable stresses:

1020 HR: τ_{all} = min[0.3(55), 0.4(30)] = min(16.5, 12) = 12 kpsi

1020 HR: τ_{all} = min[0.3(68), 0.4(37.5)] = min(20.4, 15) = 15 kpsi

E6010: τ_{all} = min[0.3(62), 0.4(50)] = min(18.6, 20) = 18.6 kpsi

Since Table 9-6 gives 18.0 kpsi as the allowable shear stress, use this lower value.
Therefore, the allowable shear stress is

$$\tau_{all} = \min(14.4, 12, 18.0) = 12 \text{ kpsi}$$

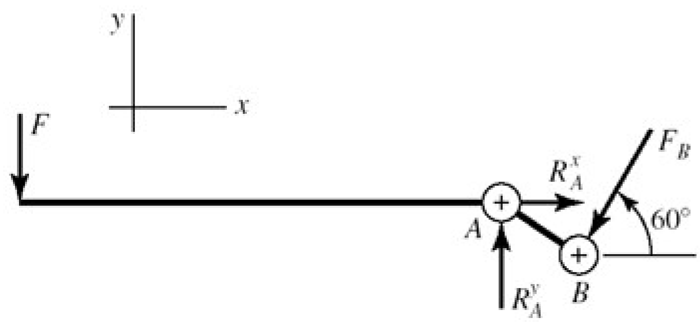
However, the allowable stress in the problem statement is 1.5 kpsi which is low from the weldment perspective. The load associated with this strength is

$$\tau_{\max} = \tau_{all} = 3.90W = 1500$$
$$W = \frac{1500}{3.90} = 385 \text{ lbf}$$

If the welding can be accomplished (1/16 leg size is a small weld), the weld strength is 12 000 psi and the load associated with this strength is W = 12 000/3.90 = 3077 lbf. Can the bracket carry such a load?
There are geometry problems associated with sheet metal folding. Load placement is important and the center of twist has not been identified. Also, the load-carrying capability of the top bend is unknown.

These uncertainties may require the use of a different weld pattern. Our solution provides the best weldment and thus insight for comparing a welded joint to one which employs screw fasteners.

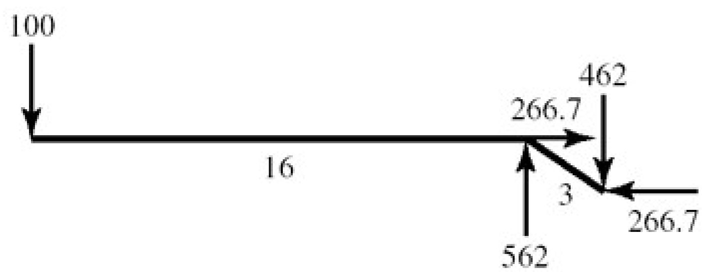
9-59



$$F = 100 \text{ lbf}, \quad \tau_{\text{all}} = 3 \text{ kpsi}$$
$$F_B = 100(16 / 3) = 533.3 \text{ lbf}$$
$$F_B^x = -533.3 \cos 60^\circ = -266.7 \text{ lbf}$$
$$F_B^y = -533.3 \cos 30^\circ = -462 \text{ lbf}$$

It follows that $R_A^y = 562$ lbf and $R_A^x = 266.7$ lbf, $R_A = 622$ lbf

$$M = 100(16) = 1600 \text{ lbf} \cdot \text{in}$$



The OD of the tubes is 1 in. From Table 9-1, case 6:

$$A = 2\left[1.414(\pi h r)\right] = 2(1.414)(\pi h)(1 / 2) = 4.442h \text{ in}^2$$
$$J_u = 2\pi r^3 = 2\pi(1 / 2)^3 = 0.7854 \text{ in}^3$$
$$J = 2(0.707)hJ_u = 1.414(0.7854)h = 1.111h \text{ in}^4$$

The weld only carries the torsional load between the handle and tube *A*. Consequently, the primary shear in the weld is zero, and the maximum shear stress is comprised entirely of the secondary shear.

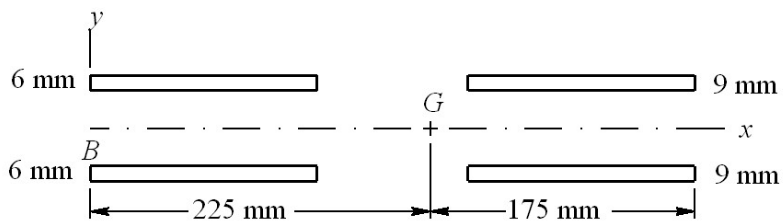
$$\tau_{max} = \tau'' = \frac{Tc}{J} = \frac{Mc}{J} = \frac{1600(0.5)}{1.111h} = \frac{720.1}{h}\text{psi}$$

$$\tau_{max} = \tau_{all} = \frac{720.1}{h} = 3000$$

$$h = \frac{720.1}{3000} = 0.240 \rightarrow 1/4 \text{ in}$$

Decision: Use 1/4 in fillet welds *Ans.*

9-60



For the pattern in bending shown, find the centroid G of the weld group.

$$\bar{x} = \frac{75(6)(150)+325(9)(150)}{(6)(150)+(9)(150)} = 225 \text{ mm}$$

$$I_{6\text{mm}} = 2\left(I_G + A\bar{x}^2\right)_{6\text{mm}}$$

$$= 2\left[\frac{0.707(6)(150^3)}{12} + 0.707(6)(150)(225-75)^2\right] = 31.02\left(10^6\right) \text{ mm}^4$$

$$I_{9\text{mm}} = 2\left[\frac{0.707(9)(150^3)}{12} + 0.707(9)(150)(175-75)^2\right] = 22.67\left(10^6\right) \text{ mm}^4$$

$$I = I_{6\text{ mm}} + I_{9\text{ mm}} = (31.02 + 22.67)(10^6) = 53.69(10^6) \text{ mm}^4$$

The critical location is at B. With τ in MPa, and F in kN

$$\tau' = \frac{V}{A} = \frac{F\left(10^3\right)}{2\left[0.707(6+9)(150)\right]} = 0.3143F$$

$$\tau'' = \frac{Mc}{I} = \frac{200F\left(10^3\right)(225)}{53.69\left(10^6\right)} = 0.8381F$$

$$\tau_{\max} = \sqrt{\tau'^2 + \tau''^2} = F\sqrt{0.3143^2 + 0.8381^2} = 0.8951F$$

Materials:

1015 HR (Table A-20): S_y = 190 MPa, E6010 Electrode(Table 9-3): S_y = 345 MPa

Eq. (5-21): $\tau_{\text{all}} = 0.577(190) = 109.6$ MPa

$$F = \frac{\tau_{\text{all}}/n}{0.8951} = \frac{109.6/2}{0.8951} = 61.2 \text{ kN} \qquad \textit{Ans.}$$

9-61 In the textbook, Fig. Problem 9-61*b* is a free-body diagram of the bracket. Forces and moments that act on the welds are equal, but of opposite sense.

(a) $M = 1200(0.366) = 439$ lbf · in *Ans.*

(b) $F_y = 1200 \sin 30° = 600$ lbf *Ans.*

(c) $F_x = 1200 \cos 30° = 1039$ lbf *Ans.*

(d) From Table 9-2, case 6:

$$A = 1.414(0.25)(0.25 + 2.5) = 0.972 \text{ in}^2$$

$$I_u = \frac{d^2}{6}(3b + d) = \frac{2.5^2}{6}[3(0.25) + 2.5] = 3.39 \text{ in}^3$$

The second area moment about an axis through *G* and parallel to *z* is

$$I = 0.707hI_u = 0.707(0.25)(3.39) = 0.599 \text{ in}^4 \qquad \textit{Ans.}$$

(e) Refer to Fig. Problem 9-61*b*. The shear stress due to F_y is

$$\tau_1 = \frac{F_y}{A} = \frac{600}{0.972} = 617 \text{ psi}$$

The shear stress along the throat due to F_x is

$$\tau_2 = \frac{F_x}{A} = \frac{1039}{0.972} = 1069 \text{ psi}$$

The resultant of τ_1 and τ_2 is in the throat plane

$$\tau' = \sqrt{\tau_1^2 + \tau_2^2} = \sqrt{617^2 + 1069^2} = 1234 \text{ psi}$$

The bending of the throat gives

$$\tau'' = \frac{Mc}{I} = \frac{439(1.25)}{0.599} = 916 \text{ psi}$$

The maximum shear stress is

$$\tau_{max} = \sqrt{\tau'^2 + \tau''^2} = \sqrt{1234^2 + 916^2} = 1537 \text{ psi} \quad \textit{Ans.}$$

(f) *Materials:*

1018 HR Member: $S_y = 32$ kpsi, $S_{ut} = 58$ kpsi (Table A-20)

E6010 Electrode: $S_y = 50$ kpsi (Table 9-3)

$$n = \frac{S_{sy}}{\tau_{max}} = \frac{0.577S_y}{\tau_{max}} = \frac{0.577(32)}{1.537} = 12.0 \quad \textit{Ans.}$$

(g) Bending in the attachment near the base. The cross-sectional area is approximately equal to *bh*.

$$A_1 \doteq bh = 0.25(2.5) = 0.625 \text{ in}^2$$

$$\tau_{xy} = \frac{F_x}{A_1} = \frac{1039}{0.625} = 1662 \text{ psi}$$

$$\frac{I}{c} = \frac{bd^2}{6} = \frac{0.25(2.5)^2}{6} = 0.260 \text{ in}^3$$

At location A,

$$\sigma_y = \frac{F_y}{A_1} + \frac{M}{I/c}$$

$$\sigma_y = \frac{600}{0.625} + \frac{439}{0.260} = 2648 \text{ psi}$$

The von Mises stress σ' is

$$\sigma' = \sqrt{\sigma_y^2 + 3\tau_{xy}^2} = \sqrt{2648^2 + 3(1662)^2} = 3912 \text{ psi}$$

Thus, the factor of safety is,

$$n = \frac{S_y}{\sigma'} = \frac{32}{3.912} = 8.18 \quad \textit{Ans.}$$

The clip on the mooring line bears against the side of the 1/2-in hole. If the clip fills the hole

$$\sigma = \frac{F}{td} = \frac{-1200}{0.25(0.50)} = -9600 \text{ psi}$$

$$n = -\frac{S_y}{\sigma'} = -\frac{32(10^3)}{-9600} = 3.33 \quad \textit{Ans.}$$

Further investigation of this situation requires more detail than is included in the task statement.

(h) In shear fatigue, the weakest constituent of the weld melt is 1018 HR with $S_{ut} = 58$ kpsi, Eq. (6-10), gives

$$S'_e = 0.5S_{ut} = 0.5(58) = 29.0 \text{ kpsi}$$

Eq. (6-18): $k_a = 12.7(58)^{-0.758} = 0.585$

For uniform shear stress on the throat, assume $k_b = 1$.

Eq.(6-25): $k_c = 0.59$

From Eq. (6-17), the endurance strength in shear is

$$S_{se} = 0.585(1)(0.59)(29.0) = 10.0 \text{ kpsi}$$

From Table 9-5, the shear stress-concentration factor is $K_{fs} = 2.7$. The loading is repeatedly-applied

$$\tau_a = \tau_m = K_{fs}\frac{\tau_{max}}{2} = 2.7\frac{1.537}{2} = 2.07 \text{ kpsi}$$

Eq. (6-48): Gerber factor of safety n_f, adjusted for shear, with $S_{su} = 0.67S_{ut}$

$$n_f = \frac{1}{2}\left(\frac{S_{su}}{\tau_m}\right)^2 \frac{\tau_a}{S_{se}}\left[-1 + \sqrt{1 + \left(\frac{2\tau_m S_{se}}{S_{su}\tau_a}\right)}\right]$$

$$= \frac{1}{2}\left[\frac{0.67(58)}{2.07}\right]^2\left(\frac{2.07}{10.0}\right)\left\{-1 + \sqrt{1 + \left[\frac{2(2.07)(10.0)}{0.67(58)(2.07)}\right]^2}\right\} = 4.55 \quad \textit{Ans.}$$

Attachment metal should be checked for bending fatigue.

9-62 (**a**) Use $b = d = 4$ in. Since $h = 5/8$ in, the primary shear is

$$\tau' = \frac{F}{1.414(5/8)(4)} = 0.2829F$$

The secondary shear calculations, for a moment arm of 14 in give

$$J_u = \frac{4[3(4^2) + 4^2]}{6} = 42.67 \text{ in}^3$$

$$J = 0.707hJ_u = 0.707(5/8)42.67 = 18.85 \text{ in}^4$$

$$\tau''_x = \tau''_y = \frac{Mr_y}{J} = \frac{14F(2)}{18.85} = 1.485F$$

Thus, the maximum shear and allowable load are:

$$\tau_{max} = F\sqrt{1.485^2 + (0.2829 + 1.485)^2} = 2.309F$$

$$F = \frac{\tau_{all}}{2.309} = \frac{25}{2.309} = 10.8 \text{ kip} \quad Ans.$$

The load for part (a) has increased by a factor of 10.8/2.71 = 3.99 *Ans*.

(**b**) From Prob. 9-20*b*, $\tau_{all} = 11$ kpsi

$$F_{all} = \frac{\tau_{all}}{2.309} = \frac{11}{2.309} = 4.76 \text{ kip}$$

The allowable load in part (b) has increased by a factor of 4.76/1.19 = 4 *Ans.*

9-63 Purchase the hook having the design shown in Fig. Problem 9-63*b*. Referring to text Fig. 9-29*a*, this design reduces peel stresses.

9-64 **(a)**

$$\bar{\tau} = \frac{1}{l}\int_{-l/2}^{l/2} \frac{P\omega\cosh(\omega x)}{4b\sinh(\omega l/2)}dx = A_1\int_{-l/2}^{l/2}\cosh(\omega x)\,dx = \frac{A_1}{\omega}\sinh(\omega x)\Big|_{-l/2}^{l/2}$$

$$= \frac{A_1}{\omega}[\sinh(\omega l/2) - \sinh(-\omega l/2)] = \frac{A_1}{\omega}[\sinh(\omega l/2) - (-\sinh(\omega l/2))]$$

$$= \frac{2A_1\sinh(\omega l/2)}{\omega} = \frac{P\omega}{4bl\sinh(\omega l/2)}[2\sinh(\omega l/2)] = \frac{P}{2bl} \quad Ans.$$

(b)
$$\tau(l/2) = \frac{P\omega\cosh(\omega l/2)}{4b\sinh(\omega l/2)} = \frac{P\omega}{4b\tanh(\omega l/2)} \quad Ans.$$

(c)
$$K = \frac{\tau(l/2)}{\bar{\tau}} = \frac{P\omega}{4b\tanh(\omega l/2)}\left(\frac{2bl}{P}\right) = \frac{\omega l/2}{\tanh(\omega l/2)} \quad Ans.$$

For computer programming, it can be useful to express the hyperbolic tangent in terms of exponentials:

$$K = \frac{\omega l}{2}\,\frac{\exp(\omega l/2) - \exp(-\omega l/2)}{\exp(\omega l/2) + \exp(-\omega l/2)} \quad Ans.$$

9-65 This is a computer programming exercise. All programs will vary.