Chapter 6

6-1 Eq. (2-36): $S_{ut} = 3.4H_B = 3.4(300) = 1020$ MPa

Eq. (6-10): $S'_e = 0.5S_{ut} = 0.5(1020) = 510$ MPa

Table 6-2: $a = 1.38,\ b = -0.067$

Eq. (6-18): $k_a = aS_{ut}^b = 1.38(1020)^{-0.067} = 0.868$

Eq. (6-19): $k_b = 1.24d^{-0.107} = 1.24(10)^{-0.107} = 0.969$

Eq. (6-17): $S_e = k_a k_b S'_e = (0.868)(0.969)(510) = 429$ MPa *Ans.*

6-2 **(a)** Table A-20: S_{ut} = 80 kpsi

Eq. (6-10): $S'_e = 0.5(80) = 40$ kpsi *Ans.*

(b) Table A-20: S_{ut} = 90 kpsi

Eq. (6-10): $S'_e = 0.5(90) = 45$ kpsi *Ans.*

(c) Aluminum has no endurance limit. *Ans.*

(d) Eq. (6-10): S_{ut} > 200 kpsi, $S'_e = 100$ kpsi *Ans.*

6-3 $S_{ut} = 120$ kpsi, $\sigma_{ar} = 70$ kpsi

Fig. 6-23: $f = 0.82$

Eq. (6-10): $S'_e = S_e = 0.5(120) = 60$ kpsi

Eq. (6-13): $$a = \frac{(f\,S_{ut})^2}{S_e} = \frac{\left[0.82(120)\right]^2}{60} = 161.4 \text{ kpsi}$$

Eq. (6-14): $$b = -\frac{1}{3}\log\left(\frac{f\,S_{ut}}{S_e}\right) = -\frac{1}{3}\log\left(\frac{0.82(120)}{60}\right) = -0.0716$$

Eq. (6-15): $$N = \left(\frac{\sigma_{ar}}{a}\right)^{1/b} = \left(\frac{70}{161.4}\right)^{\frac{1}{-0.0716}} = 117\,000 \text{ cycles} \quad \textit{Ans.}$$

6-4 $S_{ut} = 1600$ MPa, $\sigma_{ar} = 900$ MPa

Fig. 6-23: S_{ut} = 1600 MPa. Off the graph, so estimate f = 0.77.

Eq. (6-10): S_{ut} > 1400 MPa, so S_e = 700 MPa

Eq. (6-13): $$a = \frac{(f\,S_{ut})^2}{S_e} = \frac{\left[0.77(1600)\right]^2}{700} = 2168.3 \text{ MPa}$$

Eq. (6-14): $b = -\frac{1}{3}\log\left(\frac{f\,S_{ut}}{S_e}\right) = -\frac{1}{3}\log\left(\frac{0.77(1600)}{700}\right) = -0.081838$

Eq. (6-15): $N = \left(\frac{\sigma_{ar}}{a}\right)^{1/b} = \left(\frac{900}{2168.3}\right)^{\frac{1}{-0.081838}} = 46\,400 \text{ cycles}$ *Ans.*

6-5 $S_{ut} = 230 \text{ kpsi}, N = 150\,000 \text{ cycles}$

Fig. 6-23, point is off the graph, so estimate: $f = 0.77$

Eq. (6-10): $S_{ut} > 200$ kpsi, so $S'_e = S_e = 100 \text{ kpsi}$

Eq. (6-13): $a = \frac{(f\,S_{ut})^2}{S_e} = \frac{\left[0.77(230)\right]^2}{100} = 313.6 \text{ kpsi}$

Eq. (6-14): $b = -\frac{1}{3}\log\left(\frac{f\,S_{ut}}{S_e}\right) = -\frac{1}{3}\log\left(\frac{0.77(230)}{100}\right) = -0.08274$

Eq. (6-12): $S_f = aN^b = 313.6(150\,000)^{-0.08274} = 117.0 \text{ kpsi}$ *Ans.*

6-6 $S_{ut} = 1100 \text{ MPa} = 160 \text{ kpsi}$

Fig. 6-23: $f = 0.79$

Eq. (6-10): $S'_e = S_e = 0.5(1100) = 550 \text{ MPa}$

Eq. (6-13): $a = \frac{(f\,S_{ut})^2}{S_e} = \frac{\left[0.79(1100)\right]^2}{550} = 1373 \text{ MPa}$

Eq. (6-14): $b = -\frac{1}{3}\log\left(\frac{f\,S_{ut}}{S_e}\right) = -\frac{1}{3}\log\left(\frac{0.79(1100)}{550}\right) = -0.06622$

Eq. (6-12): $S_f = aN^b = 1373(150\,000)^{-0.06622} = 624 \text{ MPa}$ *Ans.*

6-7 $S_{ut} = 150 \text{ kpsi}, S_{yt} = 135 \text{ kpsi}, N = 500 \text{ cycles}$

Fig. 6-23: $f = 0.80$

From Fig. 6-21, we note that below 10^3 cycles on the *S-N* diagram constitutes the low-cycle region. The stress-life approach is not very reliable in this region, but for a rough response to this question, we can write an equation in log-log scale for the line between $(10^0, S_{ut})$ and $(10^3, fS_{ut})$ as

$$S_f = S_{ut} N^{(\log f)/3} = 150(500)^{[\log(0.80)]/3} = 123 \text{ kpsi} \qquad \textit{Ans.}$$

The testing should be done at a completely reversed stress of 123 kpsi, which is below the yield strength, so it is possible. *Ans.*

6-8 $d = 1.5$ in, $S_{ut} = 110$ kpsi

Eq. (6-10): $S'_e = 0.5(110) = 55$ kpsi
Table 6-2: $a = 2.00$, $b = -0.217$
Eq. (6-18): $k_a = aS_{ut}{}^b = 2.00(110)^{-0.217} = 0.721$
Eq. (6-19): $k_b = 0.879\, d^{-0.107} = 0.879(1.5)^{-0.107} = 0.842$
Eq. (6-17): $S_e = k_a k_b S'_e = 0.721(0.842)(55) = 33.4$ kpsi *Ans.*

6-9 For AISI 4340 as-forged steel,

Eq. (6-10): $S_e = 100$ kpsi
Table 6-2: $a = 12.7$, $b = -0.758$
Eq. (6-18): $k_a = 12.7(260)^{-0.758} = 0.188$
Eq. (6-19): $k_b = \left(\dfrac{0.75}{0.30}\right)^{-0.107} = 0.907$

Each of the other modifying factors is unity.

$$S_e = 0.188(0.907)(100) = 17.1 \text{ kpsi} \qquad \textit{Ans.}$$

For AISI 1040:

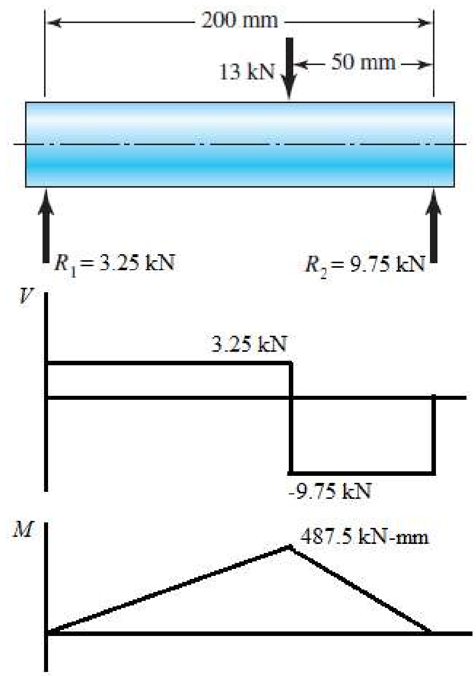
$$S'_e = 0.5(113) = 56.5 \text{ kpsi}$$
$$k_a = 12.7(113)^{-0.758} = 0.353$$
$$k_b = 0.907 \text{ (same as 4340)}$$

Each of the other modifying factors is unity

$$S_e = 0.353(0.907)(56.5) = 18.1 \text{ kpsi} \qquad \textit{Ans.}$$

Not only is AISI 1040 steel a contender, it has a superior endurance strength. *Ans.*

6-10 From Table A-20, S_{ut} = 570 MPa, S_y = 310 MPa. From a free-body diagram analysis, the bearing reaction forces are found to be R_1 = 3.25 kN and R_2 = 9.75 kN. The shear-force and bending-moment diagrams are shown. The critical location is at the section where the bending moment is maximum, on the outer surface where the bending stress is maximum. With a rotating shaft, the bending stress will be completely reversed.



$$\sigma_{\max} = \sigma_{ar} = \frac{Mc}{I} = \frac{487\,500(25/2)}{(\pi/64)(25)^4} = 317.8 \text{ MPa}$$

(a) $$n_y = \frac{S_y}{\sigma_{\max}} = \frac{310}{317.8} = 0.98 \qquad \textit{Ans.}$$

Yielding is predicted, on the outer surface. For some applications, this might not prevent the part from being used, so we will continue checking for fatigue.

(b) Eq. (6-10): $S_e' = 0.5S_{ut} = 0.5(570) = 285$ MPa

Eq. (6-18): $k_a = aS_{ut}^b = 3.04(570)^{-0.217} = 0.767$

Eq. (6-19): $k_b = 1.24d^{-0.107} = 1.24(25)^{-0.107} = 0.879$

Eq. (6-25): $k_c = 1$

Eq. (6-17): $S_e = k_a k_b k_c S_e' = (0.767)(0.879)(1)(285) = 192$ MPa *Ans.*

(c) For completely reversed stress, the fatigue factor of safety can be assessed as the ratio of the endurance limit to the completely reversed stress.

$$n_f = \frac{S_e}{\sigma_{ar}} = \frac{192}{317.8} = 0.60 \qquad \textit{Ans.}$$

Infinite life is not predicted. Use the *S-N* diagram to estimate the life.

(d) Fig. 6-23, or Eq. (6-11): $f = 0.87$

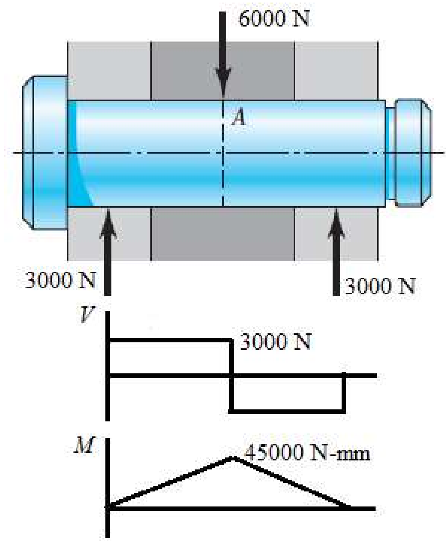
Eq. (6-13): $$a = \frac{(f\,S_{ut})^2}{S_e} = \frac{[0.87(570)]^2}{192} = 1280.8$$

Eq. (6-14): $$b = -\frac{1}{3}\log\left(\frac{f\,S_{ut}}{S_e}\right) = -\frac{1}{3}\log\left(\frac{0.87(570)}{192}\right) = -0.1374$$

Eq. (6-15): $$N = \left(\frac{\sigma_{ar}}{a}\right)^{\frac{1}{b}} = \left(\frac{317.8}{1280.8}\right)^{\frac{1}{-0.1374}} = 25444$$

N = 25000 cycles *Ans.*

6-11 From Table A-20, $S_{ut} = 400$ MPa, $S_y = 220$ MPa. Free-body, shear-force, and bending-moment diagrams are shown.



$$\sigma_{\max} = \frac{32M}{\pi d^3} = \frac{32(45000)}{\pi d^3} = 458366 / d^3$$

The load is repeatedly applied and released, so from Eqs. (6-8) and (6-9),

$$\sigma_m = \sigma_a = \sigma_{\max} / 2 = 229183 / d^3$$

Be sure to confirm that the units are legitimate for stress in MPa and d in mm.

For yielding,

$$n_y = 1.5 = \frac{S_y}{\sigma_{\max}} = \frac{220}{458366 / d^3}$$

$$d = 14.62 \text{ mm}$$

Now check fatigue, opting for the linear Goodman criterion for simplicity of solving for the diameter. First, determine the adjusted endurance limit.

Eq. (6-10): $S_e' = 0.5 S_{ut} = 0.5(400) = 200$ MPa

Eq. (6-18): $k_a = a S_{ut}^b = 3.04(400)^{-0.217} = 0.828$

Estimate the size factor from the diameter determined for yielding. It can be adjusted later.

Eq. (6-19): $k_b = 1.24 d^{-0.107} = 1.24(15)^{-0.107} = 0.93$

Eq. (6-25): $k_c = 1$

Eq. (6-17): $S_e = k_a k_b k_c S_e' = (0.767)(0.879)(1)(285) = 192$ MPa *Ans.*

(c) For completely reversed stress, the fatigue factor of safety can be assessed as the ratio of the endurance limit to the completely reversed stress.

$$n_f = \frac{S_e}{\sigma_{ar}} = \frac{192}{317.8} = 0.60 \qquad \textit{Ans.}$$

Infinite life is not predicted. Use the *S-N* diagram to estimate the life.

(d) Fig. 6-23, or Eq. (6-11): $f = 0.87$

Eq. (6-13): $$a = \frac{(f\, S_{ut})^2}{S_e} = \frac{[0.87(570)]^2}{192} = 1280.8$$

Eq. (6-14): $$b = -\frac{1}{3}\log\left(\frac{f\, S_{ut}}{S_e}\right) = -\frac{1}{3}\log\left(\frac{0.87(570)}{192}\right) = -0.1374$$

Eq. (6-15): $$N = \left(\frac{\sigma_{ar}}{a}\right)^{\frac{1}{b}} = \left(\frac{317.8}{1280.8}\right)^{\frac{1}{-0.1374}} = 25444$$

$$N = 25000 \text{ cycles} \qquad \textit{Ans.}$$

6-12 $D = 1$ in, $d = 0.8$ in, $T = 1800$ lbf·in, and from Table A-20 for AISI 1020 CD, $S_{ut} = 68$ kpsi, and $S_y = 57$ kpsi.

(a) Fig. A-15-15: $\dfrac{r}{d} = \dfrac{0.1}{0.8} = 0.125,\ \dfrac{D}{d} = \dfrac{1}{0.8} = 1.25,\ K_{ts} = 1.40$

Get the notch sensitivity either from Fig. 6-27, or from the curve-fit Eqs. (6-33) and (6-36). Using the equations,

$$\sqrt{a} = 0.190 - 2.51\left(10^{-3}\right)(68) + 1.35\left(10^{-5}\right)(68)^2 - 2.67\left(10^{-8}\right)\left(68^3\right) = 0.07335$$

$$q_s = \frac{1}{1 + \dfrac{\sqrt{a}}{\sqrt{r}}} = \frac{1}{1 + \dfrac{0.07335}{\sqrt{0.1}}} = 0.812$$

Eq. (6-32): $K_{fs} = 1 + q_s\,(K_{ts} - 1) = 1 + 0.812(1.40 - 1) = 1.32$

For a purely reversing torque of $T = 1800$ lbf·in,

$$\tau_a = K_{fs}\frac{Tr}{J} = \frac{K_{fs}16T}{\pi d^3} = \frac{1.32(16)(1800)}{\pi(0.8)^3} = 23\,635 \text{ psi} = 23.6 \text{ kpsi}$$

Eq. (6-10): $S_e' = 0.5(68) = 34$ kpsi

Eq. (6-18): $k_a = 2.00(68)^{-0.217} = 0.80$
Eq. (6-19): $k_b = 0.879(0.8)^{-0.107} = 0.90$
Eq. (6-25): $k_c = 0.59$
Eq. (6-17) (labeling for shear): $S_{se} = 0.80(0.90)(0.59)(34) = 14.4$ kpsi

For purely reversing torsion, use Eq. (6-58) for the ultimate strength in shear.
Eq. (6-58): $S_{su} = 0.67\,S_{ut} = 0.67(68) = 45.6$ kpsi
Fig. 6-23: $f = 0.9$

Adjusting the fatigue strength equations for shear,

Eq. (6-13): $$a = \frac{\left(f\,S_{su}\right)^2}{S_{se}} = \frac{\left[0.9(45.6)\right]^2}{14.4} = 117.0 \text{ kpsi}$$

Eq. (6-14): $$b = -\frac{1}{3}\log\left(\frac{f\,S_{su}}{S_{se}}\right) = -\frac{1}{3}\log\left(\frac{0.9(45.6)}{14.4}\right) = -0.151\,61$$

Eq. (6-15): $$N = \left(\frac{\tau_a}{a}\right)^{\frac{1}{b}} = \left(\frac{23.6}{117.0}\right)^{\frac{1}{-0.151\,61}} = 38.5\left(10^3\right) \text{ cycles} \quad \textit{Ans.}$$

(b) Estimate the ultimate strength at the operating temperature.
Eq. (6-26): $(S_T/S_{RT})_{750^\circ} = 0.98 + 3.5(10^{-4})(750) - 6.3(10^{-7})750^2 = 0.89$
Thus, $\left(S_{ut}\right)_{750^\circ} = \left(S_T/S_{RT}\right)_{750^\circ}\left(S_{ut}\right)_{70^\circ} = 0.89(68) = 60.5$ kpsi

Eq. (6-10): $\left(S_e'\right)_{750^\circ} = 0.5\left(S_{ut}\right)_{750^\circ} = 0.5(60.5) = 30.3$ kpsi

Eq. (6-17): S_{se} = 0.80(0.90)(0.59)(30.3) = 12.9 kpsi

Note that we use k_d = 1 since the ultimate strength has been adjusted for the operating temperature.

Eq. (6-58): $S_{su} = 0.67\left(S_{ut}\right)_{750^\circ} = 0.67\left(60.5\right) = 40.5$ kpsi

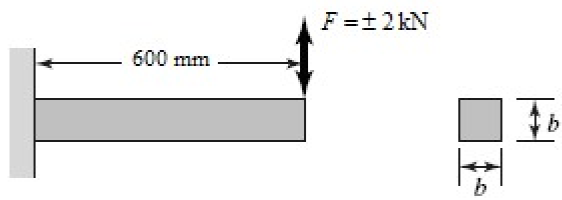
$$a = \frac{\left(f\,S_{su}\right)^2}{S_{se}} = \frac{\left[0.9(40.5)\right]^2}{12.9} = 103.0 \text{ kpsi}$$

$$b = -\frac{1}{3}\log\left(\frac{f\,S_{su}}{S_{se}}\right) = -\frac{1}{3}\log\left(\frac{0.9(40.5)}{12.9}\right) = -0.150\ 37$$

$$N = \left(\frac{\tau_a}{a}\right)^{\frac{1}{b}} = \left(\frac{23.6}{103.0}\right)^{\frac{1}{-0.150\ 37}} = 18.0\left(10^3\right) \text{ cycles} \qquad \textit{Ans.}$$

6-13 $L = 0.6$ m, $F_a = 2$ kN, $n = 1.5$, $N = 10^4$ cycles, $S_{ut} = 770$ MPa, $S_y = 420$ MPa (Table A-20)

First evaluate the fatigue strength.



$$S_e' = 0.5(770) = 385 \text{ MPa}$$
$$k_a = 38.6(770)^{-0.650} = 0.51$$

Since the size is not yet known, assume a typical value of k_b = 0.85 and check later. All other modifiers are equal to one.

Eq. (6-17): S_e = 0.51(0.85)(385) = 167 MPa

Fig. 6-23: f = 0.83

Eq. (6-13): $a = \frac{\left(f\,S_{ut}\right)^2}{S_e} = \frac{\left[0.83(770)\right]^2}{167} = 2446 \text{ MPa}$

Eq. (6-14): $b = -\frac{1}{3}\log\left(\frac{f\,S_{ut}}{S_e}\right) = -\frac{1}{3}\log\left(\frac{0.83(770)}{167}\right) = -0.1943$

Eq. (6-12): $S_f = aN^b = 2446(10^4)^{-0.1943} = 409 \text{ MPa}$

Now evaluate the stress.

$$M_{\max} = (2000 \text{ N})(0.6 \text{ m}) = 1200 \text{ N}\cdot\text{m}$$

$$\sigma_a = \sigma_{\max} = \frac{Mc}{I} = \frac{M\left(b/2\right)}{b(b^3)/12} = \frac{6M}{b^3} = \frac{6\left(1200\right)}{b^3} = \frac{7200}{b^3} \text{ Pa, with } b \text{ in m.}$$

Compare strength to stress and solve for the necessary b.

$$n = \frac{S_f}{\sigma_a} = \frac{409\left(10^6\right)}{7200/b^3} = 1.5$$

$b = 0.0298$ m   Select $b = 30$ mm.

Since the size factor was guessed, go back and check it now.

Eq. (6-24): $d_e = 0.808\left(hb\right)^{1/2} = 0.808b = 0.808\left(30\right) = 24.2$ mm

Eq. (6-19): $k_b = \left(\dfrac{24.2}{7.62}\right)^{-0.107} = 0.88$

Our guess of 0.85 was slightly conservative, so we will accept the result of

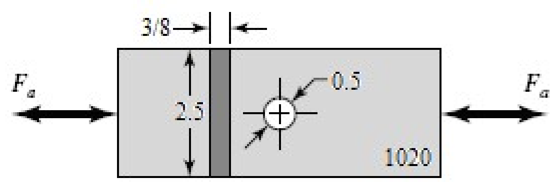
$b = 30$ mm.   *Ans.*

Checking yield,

$$\sigma_{\max} = \frac{7200}{0.030^3}\left(10^{-6}\right) = 267 \text{ MPa}$$

$$n_y = \frac{S_y}{\sigma_{\max}} = \frac{420}{267} = 1.57$$

6-14 Given: w =2.5 in, t = 3/8 in, d = 0.5 in, n_d = 2. From Table A-20, for AISI 1020 CD, S_{ut} = 68 kpsi and S_y = 57 kpsi.



Eq. (6-10): $S'_e = 0.5(68) = 34$ kpsi

Table 6-2: $k_a = 2.00(68)^{-0.217} = 0.80$

Eq. (6-20): k_b = 1 (axial loading)

Eq. (6-25): k_c = 0.85

Eq. (6-17): $S_e = 0.80(1)(0.85)(34) = 23.1$ kpsi

Table A-15-1: $d/w = 0.5/2.5 = 0.2,\ K_t = 2.5$

Get the notch sensitivity either from Fig. 6-26, or from the curve-fit Eqs. (6-33) and (6-35). The relatively large radius is off the graph of Fig. 6-26, so we will assume the curves continue according to the same trend and use the equations to estimate the notch sensitivity.

$$\sqrt{a} = 0.246 - 3.08\left(10^{-3}\right)\left(68\right) + 1.51\left(10^{-5}\right)\left(68\right)^2 - 2.67\left(10^{-8}\right)\left(68^3\right) = 0.09799$$

$$q = \frac{1}{1+\dfrac{\sqrt{a}}{\sqrt{r}}} = \frac{1}{1+\dfrac{0.09799}{\sqrt{0.25}}} = 0.836$$

Eq. (6-32): $K_f = 1 + q(K_t - 1) = 1 + 0.836(2.5 - 1) = 2.25$

$$\sigma_a = K_f \frac{F_a}{A} = \frac{2.25F_a}{(3/8)(2.5-0.5)} = 3F_a$$

Since a finite life was not mentioned, we'll assume infinite life is desired, so the completely reversed stress must stay below the endurance limit.

$$n_f = \frac{S_e}{\sigma_a} = \frac{23.1}{3F_a} = 2$$

$$F_a = 3.85 \text{ kips} \quad \textit{Ans.}$$

6-15 Given: $D = 2$ in, $d = 1.8$ in, $r = 0.1$ in, $M_{\max} = 25\,000$ lbf $\cdot$ in, $M_{\min} = 0$.
From Table A-20, for AISI 1095 HR, S_{ut} = 120 kpsi and S_y = 66 kpsi.

Eq. (6-10): $S'_e = 0.5S_{ut} = 0.5(120) = 60$ kpsi

Eq. (6-18): $k_a = aS_{ut}^b = 2.00(120)^{-0.217} = 0.71$

Eq. (6-23): $d_e = 0.370d = 0.370(1.8) = 0.666$ in

Eq. (6-19): $k_b = 0.879d_e^{-0.107} = 0.879(0.666)^{-0.107} = 0.92$

Eq. (6-25): $k_c = 1$

Eq. (6-17): $S_e = k_a k_b k_c S'_e = (0.71)(0.92)(1)(60) = 39.2$ kpsi

Fig. A-15-14: $D/d = 2/1.8 = 1.11, \quad r/d = 0.1/1.8 = 0.056 \quad \Rightarrow K_t = 2.1$

Get the notch sensitivity either from Fig. 6-26, or from the curve-fit Eqs. (6-33) and (6-35). Using the equations,

$$\sqrt{a} = 0.246 - 3.08\left(10^{-3}\right)(120) + 1.51\left(10^{-5}\right)(120)^2 - 2.67\left(10^{-8}\right)\left(120^3\right) = 0.04770$$

$$q = \frac{1}{1+\frac{\sqrt{a}}{\sqrt{r}}} = \frac{1}{1+\frac{0.04770}{\sqrt{0.1}}} = 0.87$$

Eq. (6-32): $K_f = 1 + q(K_t - 1) = 1 + 0.87(2.1-1) = 1.96$

$$I = (\pi/64)d^4 = (\pi/64)(1.8)^4 = 0.5153 \text{ in}^4$$

$$\sigma_{\max} = \frac{Mc}{I} = \frac{25\,000(1.8/2)}{0.5153} = 43\,664 \text{ psi} = 43.7 \text{ kpsi}$$

$$\sigma_{\min} = 0$$

Eqs. (6-8) and (6-9): $\sigma_m = K_f \dfrac{\sigma_{\max} + \sigma_{\min}}{2} = (1.96)\dfrac{(43.7+0)}{2} = 42.8 \text{ kpsi}$

$$\sigma_a = K_f \left|\frac{\sigma_{\max} - \sigma_{\min}}{2}\right| = (1.96)\left|\frac{(43.7-0)}{2}\right| = 42.8 \text{ kpsi}$$

Eq. (6-41): $n_f = \left(\dfrac{\sigma_a}{S_e} + \dfrac{\sigma_m}{S_{ut}}\right)^{-1} = \left(\dfrac{42.8}{39.2} + \dfrac{42.8}{120}\right)^{-1}$

$$n_f = 0.69 \qquad Ans.$$

A factor of safety less than unity indicates a finite life.

Check for yielding. It is not necessary to include the stress concentration for static yielding of a ductile material.

$$n_y = \frac{S_y}{\sigma_{\max}} = \frac{66}{43.7} = 1.51 \qquad Ans.$$

6-16 From a free-body diagram analysis, the bearing reaction forces are found to be 2.1 kN at the left bearing and 3.9 kN at the right bearing. The critical location will be at the shoulder fillet between the 35 mm and the 50 mm diameters, where the bending moment is large, the diameter is smaller, and the stress concentration exists. The bending moment at this point is M = 2.1(200) = 420 kN·mm. With a rotating shaft, the bending stress will be completely reversed.

$$\sigma_{ar} = \frac{Mc}{I} = \frac{420\,(35/2)}{(\pi/64)(35)^4} = 0.09978 \text{ kN/mm}^2 = 99.8 \text{ MPa}$$

This stress is far below the yield strength of 390 MPa, so yielding is not predicted. Find the stress concentration factor for the fatigue analysis.

Fig. A-15-9: r/d = 3/35 = 0.086, D/d = 50/35 = 1.43, K_t =1.7

Get the notch sensitivity either from Fig. 6-26, or from the curve-fit Eqs. (6-33) and (6-35). Using the equations, with S_{ut} = 470 MPa and r = 3 mm,

$$\sqrt{a} = 1.24 - 2.25\left(10^{-3}\right)(470) + 1.60\left(10^{-6}\right)(470)^2 - 4.11\left(10^{-10}\right)(470)^3 = 0.4933$$

$$q = \frac{1}{1+\dfrac{\sqrt{a}}{\sqrt{r}}} = \frac{1}{1+\dfrac{0.4933}{\sqrt{3}}} = 0.78$$

Eq. (6-32): $K_f = 1 + q(K_t - 1) = 1 + 0.78(1.7 - 1) = 1.55$

Eq. (6-10): $S_e' = 0.5S_{ut} = 0.5(470) = 235$ MPa

Eq. (6-18): $k_a = aS_{ut}^b = 3.04(470)^{-0.217} = 0.80$

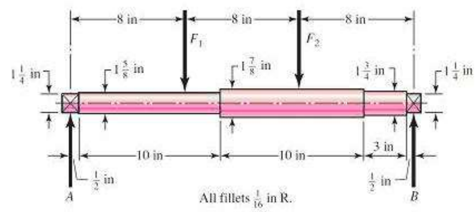
Eq. (6-19): $k_b = 1.24d^{-0.107} = 1.24(35)^{-0.107} = 0.85$

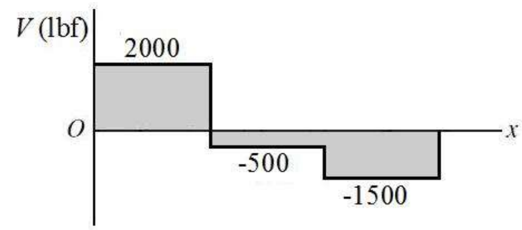
Eq. (6-25): $k_c = 1$

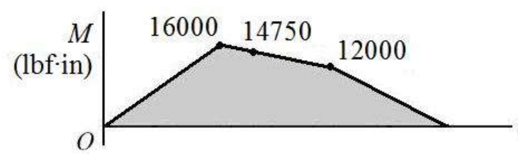
Eq. (6-17): $S_e = k_a k_b k_c S_e' = (0.80)(0.85)(1)(235) = 160$ MPa

$$n_f = \frac{S_e}{K_f \sigma_{ar}} = \frac{160}{1.55(99.8)} = 1.03 \quad \text{Infinite life is predicted.} \qquad \textit{Ans.}$$

6-17 From a free-body diagram analysis, the bearing reaction forces are found to be $R_A = 2000$ lbf and $R_B = 1500$ lbf. The shear-force and bending-moment diagrams are shown. The critical location will be at the shoulder fillet between the 1-5/8 in and the 1-7/8 in diameters, where the bending moment is large, the diameter is smaller, and the stress concentration exists.







$M = 16\,000 - 500\,(2.5) = 14\,750$ lbf · in

With a rotating shaft, the bending stress will be completely reversed.

$$\sigma_{ar} = \frac{Mc}{I} = \frac{14\,750(1.625/2)}{(\pi/64)(1.625)^4} = 35.0 \text{ kpsi}$$

This stress is far below the yield strength of 71 kpsi, so yielding is not predicted.

Fig. A-15-9: $r/d = 0.0625/1.625 = 0.04$, $D/d = 1.875/1.625 = 1.15$, $K_t = 1.95$

Get the notch sensitivity either from Fig. 6-26, or from the curve-fit Eqs. (6-33) and (6-35). Using the equations,

$$\sqrt{a} = 0.246 - 3.08\left(10^{-3}\right)(85) + 1.51\left(10^{-5}\right)(85)^2 - 2.67\left(10^{-8}\right)(85)^3 = 0.07690$$

$$q = \frac{1}{1 + \frac{\sqrt{a}}{\sqrt{r}}} = \frac{1}{1 + \frac{0.07690}{\sqrt{0.0625}}} = 0.76.$$

Eq. (6-32): $K_f = 1 + q(K_t - 1) = 1 + 0.76(1.95 - 1) = 1.72$

Eq. (6-10): $S_e^{'} = 0.5S_{ut} = 0.5(85) = 42.5$ kpsi

Eq. (6-18): $k_a = aS_{ut}^b = 2.00(85)^{-0.217} = 0.76$

Eq. (6-19): $k_b = 0.879d^{-0.107} = 0.879(1.625)^{-0.107} = 0.835$

Eq. (6-25): $k_c = 1$

Eq. (6-17): $S_e = k_a k_b k_c S_e^{'} = (0.76)(0.835)(1)(42.5) = 27.0$ kpsi

$$n_f = \frac{S_e}{K_f \sigma_{\text{ar}}} = \frac{27.0}{1.72(35.0)} = 0.45 \qquad \textit{Ans.}$$

Infinite life is not predicted. Use the *S*-*N* diagram to estimate the life.

Fig. 6-23: $f = 0.87$

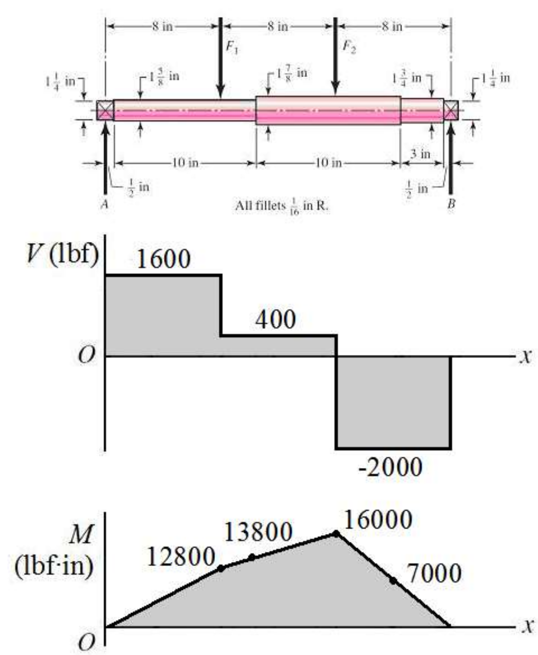
Eq. (6-13): $$a = \frac{(f\,S_{ut})^2}{S_e} = \frac{[0.87(85)]^2}{27.0} = 202.5$$

Eq. (6-14): $$b = -\frac{1}{3}\log\left(\frac{f\,S_{ut}}{S_e}\right) = -\frac{1}{3}\log\left(\frac{0.87(85)}{27.0}\right) = -0.1459$$

Eq. (6-15): $$N = \left(\frac{K_f \sigma_{ar}}{a}\right)^{\frac{1}{b}} = \left(\frac{(1.72)(35.0)}{202.5}\right)^{\frac{1}{-0.1459}} = 4082 \text{ cycles}$$

$N = 4100$ cycles *Ans.*

6-18 From a free-body diagram analysis, the bearing reaction forces are found to be R_A = 1600 lbf and R_B = 2000 lbf. The shear-force and bending-moment diagrams are shown. The critical location will be at the shoulder fillet between the 1-5/8 in and the 1-7/8 in diameters, where the bending moment is large, the diameter is smaller, and the stress concentration exists.



M = 12 800 + 400 (2.5) = 13 800 lbf · in

With a rotating shaft, the bending stress will be completely reversed.

$$\sigma_{ar} = \frac{Mc}{I} = \frac{13\,800(1.625/2)}{(\pi/64)(1.625)^4} = 32.8 \text{ kpsi}$$

This stress is far below the yield strength of 71 kpsi, so yielding is not predicted.

Fig. A-15-9: $r/d = 0.0625/1.625 = 0.04$, $D/d = 1.875/1.625 = 1.15$, $K_t = 1.95$

Get the notch sensitivity either from Fig. 6-26, or from the curve-fit Eqs. (6-33) and (6-35). Using the equations,

$$\sqrt{a} = 0.246 - 3.08\left(10^{-3}\right)(85) + 1.51\left(10^{-5}\right)(85)^2 - 2.67\left(10^{-8}\right)(85)^3 = 0.07690$$

$$q = \frac{1}{1+\frac{\sqrt{a}}{\sqrt{r}}} = \frac{1}{1+\frac{0.07690}{\sqrt{0.0625}}} = 0.76$$

Eq. (6-32): $K_f = 1 + q(K_t - 1) = 1 + 0.76(1.95 - 1) = 1.72$

Eq. (6-10): $S_e' = 0.5S_{ut} = 0.5(85) = 42.5$ kpsi

Eq. (6-18): $k_a = aS_{ut}^b = 2.00(85)^{-0.217} = 0.76$

Eq. (6-19): $k_b = 0.879d^{-0.107} = 0.879(1.625)^{-0.107} = 0.835$

Eq. (6-25): $k_c = 1$

Eq. (6-17): $S_e = k_a k_b k_c S_e' = (0.76)(0.835)(1)(42.5) = 27.0$ kpsi

$$n_f = \frac{S_e}{K_f \sigma_{ar}} = \frac{27.0}{1.72(32.8)} = 0.48 \qquad \textit{Ans.}$$

Infinite life is not predicted. Use the *S-N* diagram to estimate the life.
Fig. 6-23: $f = 0.87$

Eq. (6-13): $$a = \frac{(f\,S_{ut})^2}{S_e} = \frac{[0.87(85)]^2}{27.0} = 202.5$$

Eq. (6-14): $$b = -\frac{1}{3}\log\left(\frac{f\,S_{ut}}{S_e}\right) = -\frac{1}{3}\log\left(\frac{0.87(85)}{27.0}\right) = -0.1459$$

Eq. (6-15): $$N = \left(\frac{K_f \sigma_{ar}}{a}\right)^{\frac{1}{b}} = \left(\frac{(1.72)(32.8)}{202.5}\right)^{\frac{1}{-0.1459}} = 6370 \text{ cycles}$$

$N = 6400$ cycles *Ans.*

6-19 Table A-20: $S_{ut} = 120$ kpsi, $S_y = 66$ kpsi

N = (950 rev/min)(10 hr)(60 min/hr) = 570 000 cycles

One approach is to guess a diameter and solve the problem as an iterative analysis problem. Alternatively, we can estimate the few modifying parameters that are dependent on the diameter and solve the stress equation for the diameter, then iterate to check the estimates. We'll use the second approach since it should require only one iteration, since the estimates on the modifying parameters should be pretty close.

First, we will evaluate the stress. From a free-body diagram analysis, the reaction forces at the bearings are R_1 = 2 kips and R_2 = 6 kips. The critical stress location is in the middle of the span at the shoulder, where the bending moment is high, the shaft diameter is smaller, and a stress concentration factor exists. If the critical location is not obvious, prepare a complete bending moment diagram and evaluate at any potentially critical locations. Evaluating at the critical shoulder,

$$M = 2\ \text{kip}(10\ \text{in}) = 20\ \text{kip}\cdot\text{in}$$

$$\sigma_{ar} = \frac{Mc}{I} = \frac{M(d/2)}{\pi d^4/64} = \frac{32M}{\pi d^3} = \frac{32(20)}{\pi d^3} = \frac{203.7}{d^3}\ \text{kpsi}$$

Now we will get the notch sensitivity and stress concentration factor. The notch sensitivity depends on the fillet radius, which depends on the unknown diameter. For now, let us estimate a value of q = 0.85 from observation of Fig. 6-26, and check it later.

Fig. A-15-9: $D/d = 1.4d/d = 1.4, \quad r/d = 0.1d/d = 0.1, \quad K_t = 1.65$

Eq. (6-32): $K_f = 1 + q(K_t - 1) = 1 + 0.85(1.65 - 1) = 1.55$

Now, evaluate the fatigue strength.

$$S_e' = 0.5(120) = 60\ \text{kpsi}$$

$$k_a = 2.00(120)^{-0.217} = 0.71$$

Since the diameter is not yet known, assume a typical value of k_b = 0.85 and check later. All other modifiers are equal to one.

$$S_e = (0.71)(0.85)(60) = 36.2\ \text{kpsi}$$

Determine the desired fatigue strength from the *S*-*N* diagram.

Fig. 6-23: $f = 0.82$

Eq. (6-13): $a = \dfrac{(f\,S_{ut})^2}{S_e} = \dfrac{[0.82(120)]^2}{36.2} = 267.5$

Eq. (6-14): $b = -\dfrac{1}{3}\log\left(\dfrac{f\,S_{ut}}{S_e}\right) = -\dfrac{1}{3}\log\left(\dfrac{0.82(120)}{36.2}\right) = -0.1448$

Eq. (6-12): $S_f = aN^b = 267.5(570\,000)^{-0.1448} = 39.3\ \text{kpsi}$

Compare strength to stress and solve for the necessary *d*.

$$n_f = \frac{S_f}{K_f \sigma_{ar}} = \frac{39.3}{(1.55)(203.7/d^3)} = 1.6$$

$$d = 2.34 \text{ in}$$

Since the size factor and notch sensitivity were guessed, go back and check them now.

Eq. (6-19): $k_b = 0.91 d^{-0.157} = 0.91(2.34)^{-0.157} = 0.80$

This is a little lower than our initial guess.
From Fig. 6-26 with $r = d/10 = 0.234$ in, we are off the graph, but it appears our guess for q of 0.85 is low. Assuming the trend of the graph continues, we'll choose $q = 0.91$ and iterate the problem with the new values of k_b and q.
Intermediate results are $S_e = 34.1$ kpsi, $S_f = 37.2$ kpsi, and $K_f = 1.59$. This gives

$$n_f = \frac{S_f}{K_f \sigma_{ar}} = \frac{37.2}{(1.59)(203.7/d^3)} = 1.6$$

$d = 2.41$ in *Ans.*

A quick check of k_b and q show that our estimates are still reasonable for this diameter.

6-20 $S_e = 40$ kpsi, $S_y = 60$ kpsi, $S_{ut} = 80$ kpsi, $\tau_m = 15$ kpsi, $\sigma_a = 25$ kpsi, $\sigma_m = \tau_a = 0$

Obtain von Mises stresses for the alternating, mean, and maximum stresses.

$$\sigma'_a = \left(\sigma_a^2 + 3\tau_a^2\right)^{1/2} = \left[25^2 + 3(0)^2\right]^{1/2} = 25.00 \text{ kpsi}$$

$$\sigma'_m = \left(\sigma_m^2 + 3\tau_m^2\right)^{1/2} = \left[0^2 + 3(15)^2\right]^{1/2} = 25.98 \text{ kpsi}$$

$$\sigma'_{\max} = \left(\sigma_{\max}^2 + 3\tau_{\max}^2\right)^{1/2} = \left[(\sigma_a + \sigma_m)^2 + 3(\tau_a + \tau_m)^2\right]^{1/2}$$

$$= \left[25^2 + 3(15^2)\right]^{1/2} = 36.06 \text{ kpsi}$$

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{60}{36.06} = 1.66 \qquad \textit{Ans.}$$

(a) Goodman, Equation (6-41)

$$n_f = \frac{1}{(25.00/40) + (25.98/80)} = 1.05 \qquad \textit{Ans.}$$

(b) Gerber, Equation (6-48)

$$n_f = \frac{1}{2}\left(\frac{80}{25.98}\right)^2 \left(\frac{25.00}{40}\right)\left[-1 + \sqrt{1 + \left(\frac{2(25.98)(40)}{80(25.00)}\right)^2}\right] = 1.31 \qquad \textit{Ans.}$$

(c) Morrow
Estimate the fatigue strength coefficient.
Eq. (6-44): $\sigma'_f = S_{ut} + 50 = 80 + 50 = 130$ kpsi

Eq. (6-46): $n_f = \dfrac{1}{(25.00/40) + (25.98/130)} = 1.21$ *Ans.*

6-21 $S_e = 40$ kpsi, $S_y = 60$ kpsi, $S_{ut} = 80$ kpsi, $\tau_m = 20$ kpsi, $\sigma_a = 10$ kpsi, $\sigma_m = \tau_a = 0$
Obtain von Mises stresses for the alternating, mean, and maximum stresses.

$$\sigma'_a = \left(\sigma_a^2 + 3\tau_a^2\right)^{1/2} = \left[10^2 + 3\left(0\right)^2\right]^{1/2} = 10.00 \text{ kpsi}$$

$$\sigma'_m = \left(\sigma_m^2 + 3\tau_m^2\right)^{1/2} = \left[0^2 + 3\left(20\right)^2\right]^{1/2} = 34.64 \text{ kpsi}$$

$$\sigma'_{\max} = \left(\sigma_{\max}^2 + 3\tau_{\max}^2\right)^{1/2} = \left[\left(\sigma_a + \sigma_m\right)^2 + 3\left(\tau_a + \tau_m\right)^2\right]^{1/2}$$

$$= \left[10^2 + 3\left(20^2\right)\right]^{1/2} = 36.06 \text{ kpsi}$$

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{60}{36.06} = 1.66 \quad \textit{Ans.}$$

(a) Goodman, Equation (6-41)

$$n_f = \frac{1}{(10.00/40) + (34.64/80)} = 1.46 \quad \textit{Ans.}$$

(b) Gerber, Equation (6-48)

$$n_f = \frac{1}{2}\left(\frac{80}{34.64}\right)^2\left(\frac{10.00}{40}\right)\left\{-1 + \sqrt{1 + \left(\frac{2(34.64)(40)}{80(10.00)}\right)^2}\right\} = 1.74 \quad \textit{Ans.}$$

(c) Morrow
Estimate the fatigue strength coefficient.
Eq. (6-44): $\sigma'_f = S_{ut} + 50 = 80 + 50 = 130$ kpsi

Eq. (6-46): $n_f = \dfrac{1}{(10.00/40) + (34.64/130)} = 1.94$ *Ans.*

6-22 $S_e = 40$ kpsi, $S_y = 60$ kpsi, $S_{ut} = 80$ kpsi, $\tau_a = 10$ kpsi, $\tau_m = 15$ kpsi, $\sigma_a = 12$ kpsi, $\sigma_m = 0$
Obtain von Mises stresses for the alternating, mean, and maximum stresses.

$$\sigma'_a = \left(\sigma_a^2 + 3\tau_a^2\right)^{1/2} = \left[12^2 + 3\left(10\right)^2\right]^{1/2} = 21.07 \text{ kpsi}$$

$$\sigma'_m = \left(\sigma_m^2 + 3\tau_m^2\right)^{1/2} = \left[0^2 + 3\left(15\right)^2\right]^{1/2} = 25.98 \text{ kpsi}$$

$$\sigma'_{\max} = \left(\sigma_{\max}^2 + 3\tau_{\max}^2\right)^{1/2} = \left[\left(\sigma_a + \sigma_m\right)^2 + 3\left(\tau_a + \tau_m\right)^2\right]^{1/2}$$

$$= \left[\left(12 + 0\right)^2 + 3\left(10 + 15\right)^2\right]^{1/2} = 44.93 \text{ kpsi}$$

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{60}{44.93} = 1.34 \quad \textit{Ans.}$$

(a) Goodman, Equation (6-41)

$$n_f = \frac{1}{(21.07/40) + (25.98/80)} = 1.17 \quad \textit{Ans.}$$

(b) Gerber, Equation (6-48)

$$n_f = \frac{1}{2}\left(\frac{80}{25.98}\right)^2\left(\frac{21.07}{40}\right)\left\{-1 + \sqrt{1 + \left(\frac{2(25.98)(40)}{80(21.07)}\right)^2}\right\} = 1.47 \quad \textit{Ans.}$$

(c) Morrow

Estimate the fatigue strength coefficient.

Eq. (6-44): $\sigma'_f = S_{ut} + 50 = 80 + 50 = 130$ kpsi

Eq. (6-46): $n_f = \dfrac{1}{(21.07/40) + (25.98/130)} = 1.38 \quad \textit{Ans.}$

6-23 $S_e = 40$ kpsi, $S_y = 60$ kpsi, $S_{ut} = 80$ kpsi, $\tau_a = 30$ kpsi, $\sigma_m = \sigma_a = \tau_a = 0$

Obtain von Mises stresses for the alternating, mean, and maximum stresses.

$$\sigma'_a = \left(\sigma_a^2 + 3\tau_a^2\right)^{1/2} = \left[0^2 + 3\left(30\right)^2\right]^{1/2} = 51.96 \text{ kpsi}$$

$$\sigma'_m = \left(\sigma_m^2 + 3\tau_m^2\right)^{1/2} = 0 \text{ kpsi}$$

$$\sigma'_{\max} = \left(\sigma_{\max}^2 + 3\tau_{\max}^2\right)^{1/2} = \left[\left(\sigma_a + \sigma_m\right)^2 + 3\left(\tau_a + \tau_m\right)^2\right]^{1/2}$$

$$= \left[3\left(30\right)^2\right]^{1/2} = 51.96 \text{ kpsi}$$

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{60}{51.96} = 1.15 \quad \textit{Ans.}$$

(a) through (c)

With a mean stress of zero, the Goodman, Gerber, and Morrow criteria all simplify to the same simple comparison of the alternating stress to the endurance limit,

$$n_f = \frac{S_e}{\sigma'_a} = \frac{40}{51.96} = 0.77 \quad \textit{Ans.}$$

Since infinite life is not predicted, estimate a life from the *S*-*N* diagram. Since $\sigma'_m = 0$, the stress state is completely reversed and the *S*-*N* diagram is applicable for σ'_a.

Fig. 6-23: $f = 0.875$

Eq. (6-13): $$a = \frac{(f\,S_{ut})^2}{S_e} = \frac{[0.875(80)]^2}{40} = 122.5$$

Eq. (6-14): $$b = -\frac{1}{3}\log\left(\frac{f\,S_{ut}}{S_e}\right) = -\frac{1}{3}\log\left(\frac{0.875(80)}{40}\right) = -0.08101$$

Eq. (6-15): $$N = \left(\frac{\sigma_{ar}}{a}\right)^{1/b} = \left(\frac{51.96}{122.5}\right)^{\frac{1}{-0.08101}} = 39\,600 \text{ cycles} \qquad \textit{Ans.}$$

6-24 $S_e = 40$ kpsi, $S_y = 60$ kpsi, $S_{ut} = 80$ kpsi, $\tau_a = 15$ kpsi, $\sigma_m = 15$ kpsi, $\tau_m = \sigma_a = 0$

Obtain von Mises stresses for the alternating, mean, and maximum stresses.

$$\sigma'_a = \left(\sigma_a^2 + 3\tau_a^2\right)^{1/2} = \left[0^2 + 3(15)^2\right]^{1/2} = 25.98 \text{ kpsi}$$

$$\sigma'_m = \left(\sigma_m^2 + 3\tau_m^2\right)^{1/2} = \left[15^2 + 3(0)^2\right]^{1/2} = 15.00 \text{ kpsi}$$

$$\sigma'_{\max} = \left(\sigma_{\max}^2 + 3\tau_{\max}^2\right)^{1/2} = \left[(\sigma_a + \sigma_m)^2 + 3(\tau_a + \tau_m)^2\right]^{1/2}$$

$$= \left[(15)^2 + 3(15)^2\right]^{1/2} = 30.00 \text{ kpsi}$$

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{60}{30} = 2.00 \qquad \textit{Ans.}$$

(a) Goodman, Eq. (6-41)

$$n_f = \frac{1}{(25.98/40) + (15.00/80)} = 1.19 \qquad \textit{Ans.}$$

(b) Gerber, Eq. (6-48)

$$n_f = \frac{1}{2}\left(\frac{80}{15.00}\right)^2\left(\frac{25.98}{40}\right)\left\{-1 + \sqrt{1 + \left(\frac{2(15.00)(40)}{80(25.98)}\right)^2}\right\} = 1.43 \qquad \textit{Ans.}$$

(c) Morrow

Estimate the fatigue strength coefficient.

Eq. (6-44): $\sigma'_f = S_{ut} + 50 = 80 + 50 = 130$ kpsi

Eq. (6-46): $$n_f = \frac{1}{(25.98/40) + (15.00/130)} = 1.31 \qquad \textit{Ans.}$$

6-25 Given: $F_{\max} = 28$ kN, $F_{\min} = -28$ kN . From Table A-20, for AISI 1040 CD, $S_{ut} = 590$ MPa, $S_y = 490$ MPa,

Check for yielding

$$\sigma_{\max} = \frac{F_{\max}}{A} = \frac{28\,000}{10(25-6)} = 147.4 \text{ N/mm}^2 = 147.4 \text{ MPa}$$

$$n_y = \frac{S_y}{\sigma_{\max}} = \frac{490}{147.4} = 3.32 \qquad \textit{Ans.}$$

Determine the fatigue factor of safety based on infinite life

Eq. (6-10): $S_e' = 0.5(590) = 295$ MPa

Eq. (6-18): $k_a = aS_{ut}^b = 3.04(590)^{-0.217} = 0.76$

Eq. (6-20): $k_b = 1$ (axial)

Eq. (6-25): $k_c = 0.85$

Eq. (6-17): $S_e = k_a k_b k_c S_e' = (0.76)(1)(0.85)(295) = 190.6$ MPa

Fig. 6-26: $q = 0.83$

Fig. A-15-1: $d/w = 0.24,\ K_t = 2.44$

$$K_f = 1 + q(K_t - 1) = 1 + 0.83(2.44 - 1) = 2.20$$

$$\sigma_a = K_f \left| \frac{F_{\max} - F_{\min}}{2A} \right| = 2.2 \left| \frac{28\,000 - (-28\,000)}{2(10)(25-6)} \right| = 324.2 \text{ MPa}$$

$$\sigma_m = K_f \frac{F_{\max} + F_{\min}}{2A} = 0$$

Note, since $\sigma_m = 0$, the stress is completely reversing, and

$$n_f = \frac{S_e}{\sigma_a} = \frac{190.6}{324.2} = 0.59 \qquad \textit{Ans.}$$

Since infinite life is not predicted, estimate the life from the *S-N* diagram. With $\sigma_m = 0$, the stress state is completely reversed, and the *S-N* diagram is applicable for σ_a.

Fig. 6-23: $f = 0.87$

Eq. (6-13): $a = \dfrac{(f\,S_{ut})^2}{S_e} = \dfrac{[0.87(590)]^2}{190.6} = 1382$

Eq. (6-14): $b = -\dfrac{1}{3}\log\left(\dfrac{f\,S_{ut}}{S_e}\right) = -\dfrac{1}{3}\log\left(\dfrac{0.87(590)}{190.6}\right) = -0.1434$

Eq. (6-15): $N = \left(\dfrac{\sigma_{ar}}{a}\right)^{1/b} = \left(\dfrac{324.2}{1382}\right)^{\frac{1}{-0.1434}} = 24\,613$ cycles

$N = 25\,000$ cycles *Ans.*

6-26 $S_{ut} = 590$ MPa, $S_y = 490$ MPa, $F_{\max} = 28$ kN, $F_{\min} = 12$ kN

Check for yielding

$$\sigma_{\max} = \frac{F_{\max}}{A} = \frac{28\,000}{10(25-6)} = 147.4 \text{ N/mm}^2 = 147.4 \text{ MPa}$$

$$n_y = \frac{S_y}{\sigma_{\max}} = \frac{490}{147.4} = 3.32 \quad \textit{Ans.}$$

Determine the fatigue factor of safety based on infinite life.
From Prob. 6-25: $S_e = 190.6$ MPa, $K_f = 2.2$

$$\sigma_a = K_f \left| \frac{F_{\max} - F_{\min}}{2A} \right| = 2.2 \left| \frac{28\,000 - (12\,000)}{2(10)(25-6)} \right| = 92.63 \text{ MPa}$$

$$\sigma_m = K_f \frac{F_{\max} + F_{\min}}{2A} = 2.2 \left[\frac{28\,000 + 12\,000}{2(10)(25-6)} \right] = 231.6 \text{ MPa}$$

Goodman criteria, Equation (6-41):

$$n_f = \left(\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} \right)^{-1} = \left(\frac{92.63}{190.6} + \frac{231.6}{590} \right)^{-1}$$

$$n_f = 1.14 \quad \textit{Ans.}$$

Gerber criteria, Equation (6-48):

$$n_f = \frac{1}{2} \left(\frac{S_{ut}}{\sigma_m} \right)^2 \frac{\sigma_a}{S_e} \left[-1 + \sqrt{1 + \left(\frac{2\sigma_m S_e}{S_{ut}\sigma_a} \right)^2} \right]$$

$$= \frac{1}{2} \left(\frac{590}{231.6} \right)^2 \frac{92.63}{190.6} \left[-1 + \sqrt{1 + \left(\frac{2(231.6)(190.6)}{590(92.63)} \right)^2} \right]$$

$$n_f = 1.42 \quad \textit{Ans.}$$

Morrow criteria:

Estimate the fatigue strength coefficient.

Eq. (6-44): $\sigma'_f = S_{ut} + 345 = 590 + 345 = 935$ MPa

Eq. (6-46): $$n_f = \left(\frac{\sigma_a}{S_e} + \frac{\sigma_m}{\sigma'_f} \right)^{-1} = \left(\frac{92.63}{190.6} + \frac{231.6}{935} \right)^{-1}$$

$$n_f = 1.36 \quad \textit{Ans.}$$

The results are consistent with Fig. 6-36, where for a mean stress that is about half of the yield strength, the Goodman line should predict failure significantly before the other two.

6-27 $S_{ut} = 590$ MPa, $S_y = 490$ MPa

From Prob. 6-25: $S_e = 190.6$ MPa, $K_f = 2.2$

(a) $F_{\max} = 28$ kN, $F_{\min} = 0$ kN

Check for yielding

$$\sigma_{\max} = \frac{F_{\max}}{A} = \frac{28\,000}{10(25-6)} = 147.4 \text{ N/mm}^2 = 147.4 \text{ MPa}$$

$$n_y = \frac{S_y}{\sigma_{\max}} = \frac{490}{147.4} = 3.32 \qquad \textit{Ans.}$$

$$\sigma_a = K_f \left| \frac{F_{\max} - F_{\min}}{2A} \right| = 2.2 \left| \frac{28\,000 - 0}{2(10)(25-6)} \right| = 162.1 \text{ MPa}$$

$$\sigma_m = K_f \frac{F_{\max} + F_{\min}}{2A} = 2.2 \left[\frac{28\,000 + 0}{2(10)(25-6)} \right] = 162.1 \text{ MPa}$$

For the Goodman criteria, Eq. (6-41):

$$n_f = \left(\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} \right)^{-1} = \left(\frac{162.1}{190.6} + \frac{162.1}{590} \right)^{-1} = 0.89 \qquad \textit{Ans.}$$

Since infinite life is not predicted, estimate a life from the *S-N* diagram. First, find an equivalent completely reversed stress. Using the Goodman criterion,

Eq. (6-58): $$\sigma_{ar} = \frac{\sigma_a}{1 - (\sigma_m / S_{ut})} = \frac{162.1}{1 - (162.1/590)} = 223.5 \text{ MPa}$$

Fig. 6-23: $f = 0.87$

Eq. (6-13): $$a = \frac{(f\,S_{ut})^2}{S_e} = \frac{[0.87(590)]^2}{190.6} = 1382$$

Eq. (6-14): $$b = -\frac{1}{3}\log\left(\frac{f\,S_{ut}}{S_e} \right) = -\frac{1}{3}\log\left(\frac{0.87(590)}{190.6} \right) = -0.1434$$

Eq. (6-15): $$N = \left(\frac{\sigma_{ar}}{a} \right)^{1/b} = \left(\frac{223.5}{1382} \right)^{\frac{1}{-0.1434}} = 329\,000 \text{ cycles} \qquad \textit{Ans.}$$

(b) $F_{\max} = 28$ kN, $F_{\min} = 12$ kN

The maximum load is the same as in part (a), so

$$\sigma_{\max} = 147.4 \text{ MPa}$$

$$n_y = 3.32 \qquad Ans.$$

Factor of safety based on infinite life:

$$\sigma_a = K_f \left| \frac{F_{\max} - F_{\min}}{2A} \right| = 2.2 \left| \frac{28\,000 - 12\,000}{2(10)(25-6)} \right| = 92.63 \text{ MPa}$$

$$\sigma_m = K_f \frac{F_{\max} + F_{\min}}{2A} = 2.2 \left[\frac{28\,000 + 12\,000}{2(10)(25-6)} \right] = 231.6 \text{ MPa}$$

Eq. (6-41): $$n_f = \left(\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} \right)^{-1} = \left(\frac{92.63}{190.6} + \frac{231.6}{590} \right)^{-1} = 1.14 \qquad Ans.$$

(c) $F_{\max} = 12$ kN, $F_{\min} = -28$ kN

The compressive load is the largest, so check it for yielding.

$$\sigma_{\min} = \frac{F_{\min}}{A} = \frac{-28\,000}{10(25-6)} = -147.4 \text{ MPa}$$

$$n_y = \frac{S_{yc}}{\sigma_{\min}} = \frac{-490}{-147.4} = 3.32 \qquad Ans.$$

Factor of safety based on infinite life:

$$\sigma_a = K_f \left| \frac{F_{\max} - F_{\min}}{2A} \right| = 2.2 \left| \frac{12\,000 - (-28\,000)}{2(10)(25-6)} \right| = 231.6 \text{ MPa}$$

$$\sigma_m = K_f \frac{F_{\max} + F_{\min}}{2A} = 2.2 \left[\frac{12\,000 + (-28\,000)}{2(10)(25-6)} \right] = -92.63 \text{ MPa}$$

For $\sigma_m < 0$, Eq. (6-42): $$n_f = \frac{S_e}{\sigma_a} = \frac{190.6}{231.6} = 0.82 \qquad Ans.$$

Since infinite life is not predicted, estimate a life from the *S-N* diagram. For a negative mean stress, we shall assume the equivalent completely reversed stress is the same as the actual alternating stress, consistent with the horizontal fatigue line in Fig. 6-34. Get a and b from part (a).

Eq. (6-15): $$N = \left(\frac{\sigma_{ar}}{a} \right)^{1/b} = \left(\frac{231.6}{1382} \right)^{\frac{1}{-0.1434}} = 257\,000 \text{ cycles} \qquad Ans.$$

6-28 Eq. (2-36): $S_{ut} = 0.5(400) = 200$ kpsi

Eq. (6-10): $S_e' = 0.5(200) = 100$ kpsi

Eq. (6-18): $k_a = aS_{ut}^b = 11.0(200)^{-0.650} = 0.35$

Eq. (6-24): $d_e = 0.37d = 0.37(0.375) = 0.1388$ in

Eq. (6-19): $k_b = 0.879d_e^{-0.107} = 0.879(0.1388)^{-0.107} = 1.09$

Since we have used the equivalent diameter method to get the size factor, and in doing so introduced greater uncertainties, we will choose not to use a size factor greater than one. Let $k_b = 1$.

Eq. (6-17): $S_e = (0.35)(1)(100) = 35.0$ kpsi

$$F_a = \frac{40-20}{2} = 10 \text{ lb} \qquad F_m = \frac{40+20}{2} = 30 \text{ lb}$$

$$\sigma_a = \frac{32M_a}{\pi d^3} = \frac{32(10)(12)}{\pi(0.375)^3} = 23.18 \text{ kpsi}$$

$$\sigma_m = \frac{32M_m}{\pi d^3} = \frac{32(30)(12)}{\pi(0.375)^3} = 69.54 \text{ kpsi}$$

(a) Goodman criterion, Eq. (6-41):

$$\frac{1}{n_f} = \frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{23.18}{35.0} + \frac{69.54}{200}$$

$$n_f = 0.99 \qquad \textit{Ans.}$$

Since infinite life is not predicted, estimate a life from the *S-N* diagram. First, find an equivalent completely reversed stress. Using the Goodman criterion,

Eq. (6-58): $$\sigma_{ar} = \frac{\sigma_a}{1-(\sigma_m / S_{ut})} = \frac{23.18}{1-(69.54/200)} = 35.54 \text{ kpsi}$$

Fig. 6-23: $f = 0.78$

Eq. (6-13): $$a = \frac{(f\,S_{ut})^2}{S_e} = \frac{[0.78(200)]^2}{35} = 695.3$$

Eq. (6-14): $$b = -\frac{1}{3}\log\left(\frac{f\,S_{ut}}{S_e}\right) = -\frac{1}{3}\log\left(\frac{0.78(200)}{35.0}\right) = -0.2164$$

Eq. (6-15): $$N = \left(\frac{\sigma_{ar}}{a}\right)^{1/b} = \left(\frac{35.54}{695.3}\right)^{\frac{1}{-0.2164}} = 929\,000 \text{ cycles} \qquad \textit{Ans.}$$

(b) Gerber criterion, Eq. (6-48):

$$n_f = \frac{1}{2}\left(\frac{S_{ut}}{\sigma_m}\right)^2 \frac{\sigma_a}{S_e}\left[-1+\sqrt{1+\left(\frac{2\sigma_m S_e}{S_{ut}\sigma_a}\right)^2}\right]$$

$$= \frac{1}{2}\left(\frac{200}{69.54}\right)^2 \frac{23.18}{35.0}\left[-1+\sqrt{1+\left(\frac{2(69.54)(35.0)}{200(23.18)}\right)^2}\right]$$

$$= 1.23 \quad \text{Infinite life is predicted} \qquad \textit{Ans.}$$

6-29 $E = 207.0$ GPa

(a) $I = \frac{1}{12}(20)(4^3) = 106.7\ \text{mm}^4$

$$y = \frac{Fl^3}{3EI} \quad \Rightarrow \quad F = \frac{3EIy}{l^3}$$

$$F_{\min} = \frac{3(207)(10^9)(106.7)(10^{-12})(2)(10^{-3})}{140^3(10^{-9})} = 48.3\ \text{N} \qquad \textit{Ans.}$$

$$F_{\max} = \frac{3(207)(10^9)(106.7)(10^{-12})(6)(10^{-3})}{140^3(10^{-9})} = 144.9\ \text{N} \qquad \textit{Ans.}$$

(b) Get the fatigue strength information.

Eq. (2-36): S_{ut} = =3.4H_B = 3.4(490) = 1666 MPa

From problem statement: $S_y = 0.9S_{ut}$ = 0.9(1666) = 1499 MPa

Eq. (6-10): $S'_e = 700$ MPa

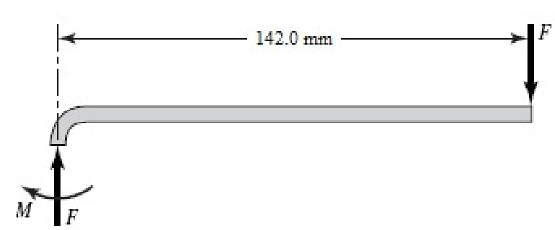
Eq. (6-18): $k_a = 1.38(1666)^{-0.067} = 0.84$

Eq. (6-24): $d_e = 0.808[20(4)]^{1/2} = 7.23$ mm

Eq. (6-19): $k_b = 1.24(7.23)^{-0.107} = 1.00$

Eq. (6-17): $S_e = 0.84(1)(700) = 588$ MPa

This is a relatively thick curved beam, so use the method in Sect. 3-18 to find the stresses. The maximum bending moment will be to the centroid of the section as shown.



$M = 142F$ N·mm, $A = 4(20) = 80\ \text{mm}^2$,
$h = 4$ mm, $r_i = 4$ mm, $r_o = r_i + h = 8$ mm,
$r_c = r_i + h/2 = 6$ mm

Table 3-4: $$r_n = \frac{h}{\ln(r_o/r_i)} = \frac{4}{\ln(8/4)} = 5.7708\ \text{mm}$$

$$e = r_c - r_n = 6 - 5.7708 = 0.2292 \text{ mm}$$
$$c_i = r_n - r_i = 5.7708 - 4 = 1.7708 \text{ mm}$$
$$c_o = r_o - r_n = 8 - 5.7708 = 2.2292 \text{ mm}$$

Get the stresses at the inner and outer surfaces from Eq. (3-76) with the axial stresses added. The signs have been set to account for tension and compression as appropriate.

$$\sigma_i = -\frac{Mc_i}{Aer_i} - \frac{F}{A} = -\frac{(142F)(1.7708)}{80(0.2292)(4)} - \frac{F}{80} = -3.441F \text{ MPa}$$
$$\sigma_o = \frac{Mc_o}{Aer_o} - \frac{F}{A} = \frac{(142F)(2.2292)}{80(0.2292)(8)} - \frac{F}{80} = 2.145F \text{ MPa}$$
$$(\sigma_i)_{\min} = -3.441(144.9) = -498.6 \text{ MPa}$$
$$(\sigma_i)_{\max} = -3.441(48.3) = -166.2 \text{ MPa}$$
$$(\sigma_o)_{\min} = 2.145(48.3) = 103.6 \text{ MPa}$$
$$(\sigma_o)_{\max} = 2.145(144.9) = 310.8 \text{ MPa}$$
$$(\sigma_i)_a = \left|\frac{-166.2 - (-498.6)}{2}\right| = 166.2 \text{ MPa}$$
$$(\sigma_i)_m = \frac{-166.2 + (-498.6)}{2} = -332.4 \text{ MPa}$$
$$(\sigma_o)_a = \left|\frac{310.8 - 103.6}{2}\right| = 103.6 \text{ MPa}$$
$$(\sigma_o)_m = \frac{310.8 + 103.6}{2} = 207.2 \text{ MPa}$$

To check for yielding, we note that the largest stress is –498.6 MPa (compression) on the inner radius. This is considerably less than the estimated yield strength of 1499 MPa, so yielding is not predicted.

Check for fatigue on both inner and outer radii since one has a compressive mean stress and the other has a tensile mean stress.

Inner radius:

Since $\sigma_m < 0$, Eq. (6-42): $n_f = \dfrac{S_e}{\sigma_a} = \dfrac{588}{166.2} = 3.54$

Outer radius:

Since $\sigma_m > 0$, using the Goodman line, Eq. (6-41),

$$n_f = \left(\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} \right)^{-1} = \left(\frac{103.6}{588} + \frac{207.2}{1666} \right)^{-1}$$

$$n_f = 3.33$$

Infinite life is predicted at both inner and outer radii. The outer radius is critical, with a fatigue factor of safety of n_f = 3.33. *Ans.*

6-30 From Table A-20, for AISI 1018 CD, $S_{ut} = 64$ kpsi, $S_y = 54$ kpsi

Eq. (6-10): $S_e' = 0.5(64) = 32$ kpsi

Eq. (6-18): $k_a = 2.00(64)^{-0.217} = 0.81$

Eq. (6-19): $k_b = 1$ (axial)

Eq. (6-25): $k_c = 0.85$

Eq. (6-17): $S_e = (0.81)(1)(0.85)(32) = 22.0$ kpsi

Fillet:

Fig. A-15-5: $D/d = 3.5/3 = 1.17$, $r/d = 0.25/3 = 0.083$, $K_t = 1.85$

Use Fig. 6-26 or Eqs. (6-33) and (6-35) for q. Estimate a little high since it is off the graph. $q = 0.85$

$$K_f = 1 + q(K_t - 1) = 1 + 0.85(1.85 - 1) = 1.72$$

$$\sigma_{\max} = \frac{F_{\max}}{w_2 h} = \frac{5}{3.0(0.5)} = 3.33 \text{ kpsi}$$

$$\sigma_{\min} = \frac{-16}{3.0(0.5)} = -10.67 \text{ kpsi}$$

$$\sigma_a = K_f \left| \frac{\sigma_{\max} - \sigma_{\min}}{2} \right| = 1.72 \left| \frac{3.33 - (-10.67)}{2} \right| = 12.0 \text{ kpsi}$$

$$\sigma_m = K_f \left(\frac{\sigma_{\max} + \sigma_{\min}}{2} \right) = 1.72 \left(\frac{3.33 + (-10.67)}{2} \right) = -6.31 \text{ kpsi}$$

$$n_y = \left| \frac{S_y}{\sigma_{\min}} \right| = \left| \frac{54}{-10.67} \right| = 5.06 \quad \therefore \text{Does not yield.}$$

Since the mean stress is negative, use Eq. (6-42).

$$n_f = \frac{S_e}{\sigma_a} = \frac{22.0}{12.0} = 1.83$$

Hole:

Fig. A-15-1: $d / w_1 = 0.4/3.5 = 0.11 \quad \therefore K_t = 2.68$

Use Fig. 6-26 or Eqs. (6-33) and (6-35) for q. Estimate a little high since it is off the graph, $q = 0.85$

$$K_f = 1 + 0.85(2.68 - 1) = 2.43$$

$$\sigma_{\max} = \frac{F_{\max}}{h(w_1 - d)} = \frac{5}{0.5(3.5 - 0.4)} = 3.226 \text{ kpsi}$$

$$\sigma_{\min} = \frac{F_{\min}}{h(w_1 - d)} = \frac{-16}{0.5(3.5 - 0.4)} = -10.32 \text{ kpsi}$$

$$\sigma_a = K_f \left| \frac{\sigma_{\max} - \sigma_{\min}}{2} \right| = 2.43 \left| \frac{3.226 - (-10.32)}{2} \right| = 16.5 \text{ kpsi}$$

$$\sigma_m = K_f \left(\frac{\sigma_{\max} + \sigma_{\min}}{2} \right) = 2.43 \left(\frac{3.226 + (-10.32)}{2} \right) = -8.62 \text{ kpsi}$$

$$n_y = \left| \frac{S_y}{\sigma_{\min}} \right| = \left| \frac{54}{-10.32} \right| = 5.23 \quad \therefore \text{does not yield}$$

Since the mean stress is negative, use Eq. (6-42).

$$n_f = \frac{S_e}{\sigma_a} = \frac{22.0}{16.5} = 1.33$$

Thus the design is controlled by the threat of fatigue at the hole with a minimum factor of safety of $n_f = 1.33$. *Ans.*

6-31 $S_{ut} = 64$ kpsi, $S_y = 54$ kpsi

Eq. (6-10): $S_e' = 0.5(64) = 32$ kpsi

Eq. (6-18): $k_a = 2.00(64)^{-0.217} = 0.81$

Eq. (6-19): $k_b = 1$ (axial)

Eq. (6-25): $k_c = 0.85$

Eq. (6-17): $S_e = (0.81)(1)(0.85)(32) = 22.0$ kpsi

Fillet:

Fig. A-15-5: $D / d = 2.5/1.5 = 1.67, \quad r / d = 0.25/1.5 = 0.17, \quad K_t \approx 2.1$

Use Fig. 6-26 or Eqs. (6-33) and (6-35) for q. Estimate a little high since it is off the graph. $q = 0.85$

$$K_f = 1 + q(K_t - 1) = 1 + 0.85(2.1 - 1) = 1.94$$

$$\sigma_{\max} = \frac{F_{\max}}{w_2 h} = \frac{16}{1.5(0.5)} = 21.3 \text{ kpsi}$$

$$\sigma_{\min} = \frac{-4}{1.5(0.5)} = -5.33 \text{ kpsi}$$

$$\sigma_a = K_f \left| \frac{\sigma_{\max} - \sigma_{\min}}{2} \right| = 1.94 \left| \frac{21.3 - (-5.33)}{2} \right| = 25.8 \text{ kpsi}$$

$$\sigma_m = K_f \left(\frac{\sigma_{\max} + \sigma_{\min}}{2} \right) = 1.94 \left(\frac{21.3 + (-5.33)}{2} \right) = 15.5 \text{ kpsi}$$

$$n_y = \frac{S_y}{\sigma_{\max}} = \frac{54}{21.3} = 2.54 \quad \therefore \text{Does not yield.}$$

Using Goodman criteria, Eq. (6-41),

$$n_f = \left(\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} \right)^{-1} = \left(\frac{25.8}{22.0} + \frac{15.5}{64} \right)^{-1} = 0.71$$

Hole:
Fig. A-15-1: $d / w_1 = 0.4 / 2.5 = 0.16 \quad \therefore K_t = 2.55$
Use Fig. 6-26 or Eqs. (6-33) and (6-35) for q. Estimate a little high since it is off the graph. $q = 0.85$

$$K_f = 1 + 0.85(2.55 - 1) = 2.32$$

$$\sigma_{\max} = \frac{F_{\max}}{h(w_1 - d)} = \frac{16}{0.5(2.5 - 0.4)} = 15.2 \text{ kpsi}$$

$$\sigma_{\min} = \frac{F_{\min}}{h(w_1 - d)} = \frac{-4}{0.5(2.5 - 0.4)} = -3.81 \text{ kpsi}$$

$$\sigma_a = K_f \left| \frac{\sigma_{\max} - \sigma_{\min}}{2} \right| = 2.32 \left(\frac{15.2 - (-3.81)}{2} \right) = 22.1 \text{ kpsi}$$

$$\sigma_m = K_f \left(\frac{\sigma_{\max} + \sigma_{\min}}{2} \right) = 2.32 \left(\frac{15.2 + (-3.81)}{2} \right) = 13.2 \text{ kpsi}$$

$$n_y = \frac{S_y}{\sigma_{\max}} = \frac{54}{15.2} = 3.55 \quad \therefore \text{Does not yield.}$$

Using Goodman criteria, Eq. (6-41),

$$n_f = \left(\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} \right)^{-1} = \left(\frac{22.1}{22.0} + \frac{13.2}{64} \right)^{-1} = 0.83$$

Thus the design is controlled by the threat of fatigue at the fillet with a minimum factor of safety of $n_f = 0.71$ *Ans.*

6-32 $S_{ut} = 64$ kpsi, $S_y = 54$ kpsi

From Prob. 6-30, the fatigue factor of safety at the hole is $n_f = 1.33$. To match this at the fillet,

$$n_f = \frac{S_e}{\sigma_a} \quad \Rightarrow \quad \sigma_a = \frac{S_e}{n_f} = \frac{22.0}{1.33} = 16.5 \text{ kpsi}$$

where S_e is unchanged from Prob. 6-30. The only aspect of σ_a that is affected by the fillet radius is the fatigue stress concentration factor. Obtaining σ_a in terms of K_f,

$$\sigma_a = K_f \left| \frac{\sigma_{\max} - \sigma_{\min}}{2} \right| = K_f \left| \frac{3.33 - (-10.67)}{2} \right| = 7.00 K_f$$

Equating to the desired stress, and solving for K_f,

$$\sigma_a = 7.00 K_f = 16.5 \qquad \Rightarrow \qquad K_f = 2.36$$

Assume since we are expecting to get a smaller fillet radius than the original, that q will be back on the graph of Fig. 6-26, so we will estimate $q = 0.8$.

$$K_f = 1 + 0.80(K_t - 1) = 2.36 \quad \Rightarrow \quad K_t = 2.7$$

From Fig. A-15-5, with $D / d = 3.5/3 = 1.17$ and $K_t = 2.6$, find r / d. Choosing $r / d = 0.03$, and with $d = w_2 = 3.0$,

$$r = 0.03 w_2 = 0.03(3.0) = 0.09 \text{ in}$$

At this small radius, our estimate for q is too high. From Fig. 6-26, with $r = 0.09$, q should be about 0.75. Iterating, we get $K_t = 2.8$. This is at a difficult range on Fig. A-15-5 to read the graph with any confidence, but we'll estimate $r / d = 0.02$, giving $r = 0.06$ in. This is a very rough estimate, but it clearly demonstrates that the fillet radius can be relatively sharp to match the fatigue factor of safety of the hole. *Ans.*

6-33 $S_y = 60$ kpsi, $S_{ut} = 110$ kpsi

Inner fiber where $r_c = 3/4$ in

$$r_o = \frac{3}{4} + \frac{3}{16(2)} = 0.84375$$

$$r_i = \frac{3}{4} - \frac{3}{32} = 0.65625$$

Table 3-4,

$$r_n = \frac{h}{\ln \dfrac{r_o}{r_i}} = \frac{3/16}{\ln \dfrac{0.84375}{0.65625}} = 0.74608 \text{ in}$$

$$e = r_c - r_n = 0.75 - 0.74608 = 0.00392 \text{ in}$$
$$c_i = r_n - r_i = 0.74608 - 0.65625 = 0.08983$$
$$A = \left(\frac{3}{16}\right)\left(\frac{3}{16}\right) = 0.035156 \text{ in}^2$$

Eq. (3-65),

$$\sigma_i = \frac{Mc_i}{Aer_i} = \frac{-T(0.08983)}{(0.035156)(0.00392)(0.65625)} = -993.3T$$

where T is in lbf·in and σ_i is in psi.

$$\sigma_m = \frac{1}{2}(-993.3)T = -496.7T$$
$$\sigma_a = 496.7T$$

Eq. (6-10): $S_e' = 0.5(110) = 55$ kpsi

Eq. (6-18): $k_a = 2.00(110)^{-0.217} = 0.72$

Eq. (6-24): $d_e = 0.808\left[(3/16)(3/16)\right]^{1/2} = 0.1515$ in

Eq. (6-19): $k_b = 0.879(0.1515)^{-0.107} = 1.08$ (round to 1)

Eq. (6-18): $S_e = (0.72)(1)(55) = 39.6$ kpsi

For a compressive mean component, from Eq. (6-42), $\sigma_a = S_e / n_f$. Thus,

$$0.4967T = \frac{39.6}{3}$$
$$T = 26.6 \text{ lbf} \cdot \text{in}$$

Outer fiber where $r_c = 2.5$ in

$$r_o = 2.5 + \frac{3}{32} = 2.59375$$
$$r_i = 2.5 - \frac{3}{32} = 2.40625$$
$$r_n = \frac{3/16}{\ln \dfrac{2.59375}{2.40625}} = 2.49883$$
$$e = 2.5 - 2.49883 = 0.00117 \text{ in}$$
$$c_o = 2.59375 - 2.49883 = 0.09492 \text{ in}$$

$$\sigma_o = \frac{Mc_o}{Aer_o} = \frac{T(0.09492)}{(0.035156)(0.00117)(2.59375)} = 889.7T \text{ psi}$$

$$\sigma_m = \sigma_a = \frac{1}{2}(889.7T) = 444.9T \text{ psi}$$

(a) Using Eq. (6-41), for Goodman, we have

$$n_f = \left(\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}}\right)^{-1} = \left(\frac{0.4449T}{39.6} + \frac{0.4449T}{110}\right)^{-1} = 3$$

$T = 21.8$ lbf · in *Ans.*

(b) For Morrow, estimate the fatigue strength coefficient from Eq. (6-44),

$$\sigma'_f = S_{ut} + 50 = 110 + 50 = 160 \text{ kpsi}$$

Eq. (6-46): $$n_f = \left(\frac{\sigma_a}{S_e} + \frac{\sigma_m}{\sigma'_f}\right)^{-1} = \left(\frac{0.4449T}{39.6} + \frac{0.4449T}{160}\right)^{-1} = 3$$

$T = 23.8$ lbf · in *Ans.*

(c) To guard against yield, use T of part (b) and the inner stress.

$$n_y = \frac{S_y}{\sigma_i} = \frac{60}{0.9933(23.8)} = 2.54 \quad \textit{Ans.}$$

6-34 From Prob. 6-33, $S_e = 39.6$ kpsi, $S_y = 60$ kpsi, and $S_{ut} = 110$ kpsi

(a) Assuming the beam is straight,

$$\sigma_{\max} = \frac{Mc}{I} = \frac{M(h/2)}{bh^3/12} = \frac{6M}{bh^2} = \frac{6T}{(3/16)^3} = 910.2T$$

Using Eq. (6-41), for Goodman, we have

$$n_f = \left(\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}}\right)^{-1} = \left(\frac{0.4551T}{39.6} + \frac{0.4551T}{110}\right)^{-1} = 3$$

$T = 21.3$ lbf · in *Ans.*

(b)) For Morrow, estimate the fatigue strength coefficient from Eq. (6-44),

$$\sigma'_f = S_{ut} + 50 = 110 + 50 = 160 \text{ kpsi}$$

Eq. (6-46): $$n_f = \left(\frac{\sigma_a}{S_e} + \frac{\sigma_m}{\sigma'_f}\right)^{-1} = \left(\frac{0.4551T}{39.6} + \frac{0.4551T}{160}\right)^{-1} = 3$$

$T = 23.3$ lbf · in *Ans.*

(c) $n_y = \dfrac{S_y}{\sigma_{\max}} = \dfrac{60}{0.9102(23.3)} = 2.83$ *Ans.*

6-35 $K_{f,\text{bend}} = 1.4,\ K_{f,\text{axial}} = 1.1,\ K_{f,\text{tors}} = 2.0,\ S_y = 300\text{ MPa},\ S_{ut} = 400\text{ MPa},\ S_e = 160\text{ MPa}$

Bending: $\sigma_m = 0,\ \sigma_a = 60\text{ MPa}$

Axial: $\sigma_m = 20\text{ MPa},\ \sigma_a = 0$

Torsion: $\tau_m = 35\text{ MPa},\ \tau_a = 35\text{ MPa}$

Eqs. (6-66) and (6-67):

$$\sigma'_a = \sqrt{[1.4(60)+0]^2 + 3[2.0(35)]^2} = 147.5\text{ MPa}$$

$$\sigma'_m = \sqrt{[0+1.1(20)]^2 + 3[2.0(35)]^2} = 123.2\text{ MPa}$$

Check for yielding, using the conservative $\sigma'_{\max} = \sigma'_a + \sigma'_m$,

$$n_y = \frac{S_y}{\sigma'_a + \sigma'_m} = \frac{300}{147.5 + 123.2} = 1.11 \quad \text{Yielding is not predicted. } \textit{Ans.}$$

Using Goodman, Eq. (6-41),

$$n_f = \left(\frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}}\right)^{-1} = \left(\frac{147.5}{160} + \frac{123.2}{400}\right)^{-1}$$

$$n_f = 0.81 \quad \textit{Ans.}$$

Finite life is predicted. To use the Walker criterion for estimating an equivalent completely reversed stress, estimate the material fitting parameter for steels with Eq. (6-57).

$$\gamma = -0.0002S_{ut} + 0.8818 = -0.0002(400) + 0.8818 = 0.8018$$

Eq. (6-61): $\sigma_{ar} = (\sigma_m + \sigma_a)^{1-\gamma}\,\sigma_a^{\gamma} = (123.2 + 147.5)^{1-0.8018}\,147.5^{0.8018} = 166.4\text{ MPa}$

Fig. 6-23: Off the chart, so use $f = 0.9$

Eq. (6-13): $a = \dfrac{(f\,S_{ut})^2}{S_e} = \dfrac{[0.9(400)]^2}{160} = 810$

Eq. (6-14): $b = -\dfrac{1}{3}\log\left(\dfrac{f\,S_{ut}}{S_e}\right) = -\dfrac{1}{3}\log\left(\dfrac{0.9(400)}{160}\right) = -0.1174$

Eq. (6-15): $N = \left(\dfrac{\sigma_{ar}}{a}\right)^{1/b} = \left(\dfrac{166.4}{810}\right)^{\frac{1}{-0.1174}} = 716\,000\text{ cycles}$ *Ans.*

6-36 $K_{f,\text{bend}} = 1.4$, $K_{f,\text{tors}} = 2.0$, $S_y = 300$ MPa, $S_{ut} = 400$ MPa, $S_e = 160$ MPa

Bending: $\sigma_{\max} = 150$ MPa, $\sigma_{\min} = -40$ MPa, $\sigma_m = 55$ MPa, $\sigma_a = 95$ MPa

Torsion: $\tau_m = 90$ MPa, $\tau_a = 9$ MPa

Eqs. (6-66) and (6-67):

$$\sigma'_a = \sqrt{\left[1.4(95)\right]^2 + 3\left[2.0(9)\right]^2} = 136.6 \text{ MPa}$$

$$\sigma'_m = \sqrt{\left[1.4(55)\right]^2 + 3\left[2.0(90)\right]^2} = 321.1 \text{ MPa}$$

Using Goodman, Eq. (6-41),

$$n_f = \left(\frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}}\right)^{-1} = \left(\frac{136.6}{160} + \frac{321.1}{400}\right)^{-1} = 0.60 \qquad \textit{Ans.}$$

Check for yielding, using the conservative $\sigma'_{\max} = \sigma'_a + \sigma'_m$,

$$n_y = \frac{S_y}{\sigma'_a + \sigma'_m} = \frac{300}{136.6 + 321.1} = 0.66 \qquad \textit{Ans.}$$

Since the conservative yield check indicates yielding, we will check more carefully with $\sigma'_{\max}$ obtained directly from the maximum stresses, using the distortion energy failure theory, without stress concentrations. Note that this is exactly the method used for static failure in Ch. 5.

$$\sigma'_{\max} = \sqrt{\left(\sigma_{\max}\right)^2 + 3\left(\tau_{\max}\right)^2} = \sqrt{\left(150\right)^2 + 3\left(90 + 9\right)^2} = 227.8 \text{ MPa}$$

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{300}{227.8} = 1.32 \qquad \textit{Ans.}$$

Since yielding is not predicted, and infinite life is not predicted, we would like to estimate a life from the *S-N* diagram.

To use the Walker criterion for estimating an equivalent completely reversed stress, estimate the material fitting parameter for steels with Eq. (6-57).

$$\gamma = -0.0002 S_{ut} + 0.8818 = -0.0002(400) + 0.8818 = 0.8018$$

Eq. (6-61): $\sigma_{ar} = \left(\sigma_m + \sigma_a\right)^{1-\gamma} \sigma_a^{\gamma} = \left(321.1 + 136.6\right)^{1-0.8018} 136.6^{0.8018} = 173.6$ MPa

Fig. 6-23: Off the chart, so use $f = 0.9$

Eq. (6-13): $a = \dfrac{(f\,S_{ut})^2}{S_e} = \dfrac{\left[0.9(400)\right]^2}{160} = 810$

Eq. (6-14): $b = -\frac{1}{3}\log\left(\frac{f\,S_{ut}}{S_e}\right) = -\frac{1}{3}\log\left(\frac{0.9(400)}{160}\right) = -0.1174$

Eq. (6-15): $N = \left(\frac{\sigma_{ar}}{a}\right)^{1/b} = \left(\frac{173.6}{810}\right)^{\frac{1}{-0.1174}} = 499\ 000$ cycles *Ans.*

6-37 Table A-20: $S_{\mathrm{ut}} = 64$ kpsi, $S_{\mathrm{y}} = 54$ kpsi

From Prob. 3-79, the critical stress element experiences $\sigma = 15.3$ kpsi and $\tau = 4.43$ kpsi. The bending is completely reversed due to the rotation, and the torsion is steady, giving $\sigma_a = 15.3$ kpsi, $\sigma_m = 0$ kpsi, $\tau_a = 0$ kpsi, $\tau_m = 4.43$ kpsi. Obtain von Mises stresses for the alternating, mean, and maximum stresses.

$$\sigma'_a = \left(\sigma_a^2 + 3\tau_a^2\right)^{1/2} = \left[15.3^2 + 3\left(0\right)^2\right]^{1/2} = 15.3 \text{ kpsi}$$

$$\sigma'_m = \left(\sigma_m^2 + 3\tau_m^2\right)^{1/2} = \left[0^2 + 3\left(4.43\right)^2\right]^{1/2} = 7.67 \text{ kpsi}$$

$$\sigma'_{\max} = \left(\sigma_{\max}^2 + 3\tau_{\max}^2\right)^{1/2} = \left[15.3^2 + 3\left(4.43\right)^2\right]^{1/2} = 17.11 \text{ kpsi}$$

Check for yielding, using the distortion energy failure theory.

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{54}{17.11} = 3.16$$

Obtain the modifying factors and endurance limit.

Eq. (6-10): $S'_e = 0.5\left(64\right) = 32$ kpsi

Eq. (6-18): $k_a = 2.00(64)^{-0.217} = 0.81$

Eq. (6-19): $k_b = 0.879(1.25)^{-0.107} = 0.86$

Eq. (6-17): $S_e = 0.81(0.86)(32) = 22.3$ kpsi

Using Goodman, Eq. (6-41),

$$n_f = \left(\frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}}\right)^{-1} = \left(\frac{15.3}{22.3} + \frac{7.67}{64}\right)^{-1} = 1.24 \qquad \textit{Ans.}$$

6-38 Table A-20: $S_{\mathrm{ut}} = 440$ MPa, $S_{\mathrm{y}} = 370$ MPa

From Prob. 3-80, the critical stress element experiences $\sigma = 263$ MPa and $\tau = 57.7$ MPa. The bending is completely reversed due to the rotation, and the torsion is steady, giving $\sigma_a = 263$ MPa, $\sigma_m = 0$, $\tau_a = 0$ MPa, $\tau_m = 57.7$ MPa. Obtain von Mises stresses for the alternating, mean, and maximum stresses.

$$\sigma'_a = \left(\sigma_a^2 + 3\tau_a^2\right)^{1/2} = \left[263^2 + 3(0)^2\right]^{1/2} = 263 \text{ MPa}$$

$$\sigma'_m = \left(\sigma_m^2 + 3\tau_m^2\right)^{1/2} = \left[0^2 + 3(57.7)^2\right]^{1/2} = 99.9 \text{ MPa}$$

$$\sigma'_{\max} = \left(\sigma_{\max}^2 + 3\tau_{\max}^2\right)^{1/2} = \left[263^2 + 3(57.7)^2\right]^{1/2} = 281 \text{ MPa}$$

Check for yielding, using the distortion energy failure theory.

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{370}{281} = 1.32$$

Obtain the modifying factors and endurance limit.

Eq. (6-10): $S'_e = 0.5(440) = 220 \text{ MPa}$

Eq. (6-18): $k_a = 3.04(440)^{-0.217} = 0.81$

Eq. (6-19): $k_b = 1.24(30)^{-0.107} = 0.86$

Eq. (6-17): $S_e = 0.81(0.86)(220) = 153 \text{ MPa}$

Using Goodman,

Eq. (6-41): $$n_f = \left(\frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}}\right)^{-1} = \left(\frac{263}{153} + \frac{99.9}{440}\right)^{-1}$$

$n_f = 0.51$ Infinite life is not predicted. *Ans.*

6-39 Table A-20: $S_{ut} = 64$ kpsi, $S_y = 54$ kpsi

From Prob. 3-81, the critical stress element experiences $\sigma = 21.5$ kpsi and $\tau = 5.09$ kpsi. The bending is completely reversed due to the rotation, and the torsion is steady, giving $\sigma_a = 21.5$ kpsi, $\sigma_m = 0$ kpsi, $\tau_a = 0$ kpsi, $\tau_m = 5.09$ kpsi. Obtain von Mises stresses for the alternating, mean, and maximum stresses.

$$\sigma'_a = \left(\sigma_a^2 + 3\tau_a^2\right)^{1/2} = \left[21.5^2 + 3(0)^2\right]^{1/2} = 21.5 \text{ kpsi}$$

$$\sigma'_m = \left(\sigma_m^2 + 3\tau_m^2\right)^{1/2} = \left[0^2 + 3(5.09)^2\right]^{1/2} = 8.82 \text{ kpsi}$$

$$\sigma'_{\max} = \left(\sigma_{\max}^2 + 3\tau_{\max}^2\right)^{1/2} = \left[21.5^2 + 3(5.09)^2\right]^{1/2} = 23.24 \text{ kpsi}$$

Check for yielding, using the distortion energy failure theory.

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{54}{23.24} = 2.32$$

Obtain the modifying factors and endurance limit.

Eq. (6-10): $S'_e = 0.5(64) = 32 \text{ kpsi}$

Eq. (6-18): $k_a = 2.00(64)^{-0.217} = 0.81$

Eq. (6-19): $k_b = 0.879(1)^{-0.107} = 0.88$

Eq. (6-17): $S_e = 0.81(0.88)(32) = 22.8 \text{ kpsi}$

Using Goodman, Eq. (6-41),

$$n_f = \left(\frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}} \right)^{-1} = \left(\frac{21.5}{22.8} + \frac{8.82}{64} \right)^{-1} = 0.93 \qquad \textit{Ans.}$$

6-40 Table A-20: $S_{\text{ut}} = 440 \text{ MPa}$, $S_{\text{y}} = 370 \text{ MPa}$

From Prob. 3-82, the critical stress element experiences $\sigma = 72.9$ MPa and $\tau = 20.3$ MPa. The bending is completely reversed due to the rotation, and the torsion is steady, giving $\sigma_a = 72.9$ MPa, $\sigma_m = 0$ MPa, $\tau_a = 0$ MPa, $\tau_m = 20.3$ MPa. Obtain von Mises stresses for the alternating, mean, and maximum stresses.

$$\sigma'_a = \left(\sigma_a^2 + 3\tau_a^2\right)^{1/2} = \left[72.9^2 + 3(0)^2\right]^{1/2} = 72.9 \text{ MPa}$$

$$\sigma'_m = \left(\sigma_m^2 + 3\tau_m^2\right)^{1/2} = \left[0^2 + 3(20.3)^2\right]^{1/2} = 35.2 \text{ MPa}$$

$$\sigma'_{\max} = \left(\sigma_{\max}^2 + 3\tau_{\max}^2\right)^{1/2} = \left[72.9^2 + 3(20.3)^2\right]^{1/2} = 80.9 \text{ MPa}$$

Check for yielding, using the distortion energy failure theory.

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{370}{80.9} = 4.57$$

Obtain the modifying factors and endurance limit.

Eq. (6-10): $S'_e = 0.5(440) = 220 \text{ MPa}$

Eq. (6-18): $k_a = 3.04(440)^{-0.217} = 0.81$

Eq. (6-19): $k_b = 1.24(20)^{-0.107} = 0.90$

Eq. (6-17): $S_e = 0.81(0.90)(220) = 160.4 \text{ MPa}$

Using Goodman, Eq. (6-41),

$$n_f = \left(\frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}} \right)^{-1} = \left(\frac{72.9}{160.4} + \frac{35.2}{440} \right)^{-1} = 1.87 \qquad \textit{Ans.}$$

6-41 Table A-20: $S_{\text{ut}} = 64 \text{ kpsi}$, $S_{\text{y}} = 54 \text{ kpsi}$

From Prob. 3-83, the critical stress element experiences σ = 35.2 kpsi and τ = 7.35 kpsi. The bending is completely reversed due to the rotation, and the torsion is steady, giving σ_a = 35.2 kpsi, σ_m = 0 kpsi, τ_a = 0 kpsi, τ_m = 7.35 kpsi. Obtain von Mises stresses for the alternating, mean, and maximum stresses.

$$\sigma_a' = \left(\sigma_a^2 + 3\tau_a^2\right)^{1/2} = \left[35.2^2 + 3\left(0\right)^2\right]^{1/2} = 35.2 \text{ kpsi}$$

$$\sigma_m' = \left(\sigma_m^2 + 3\tau_m^2\right)^{1/2} = \left[0^2 + 3\left(7.35\right)^2\right]^{1/2} = 12.7 \text{ kpsi}$$

$$\sigma_{\max}' = \left(\sigma_{\max}^2 + 3\tau_{\max}^2\right)^{1/2} = \left[35.2^2 + 3\left(7.35\right)^2\right]^{1/2} = 37.4 \text{ kpsi}$$

Check for yielding, using the distortion energy failure theory.

$$n_y = \frac{S_y}{\sigma_{\max}'} = \frac{54}{37.4} = 1.44$$

Obtain the modifying factors and endurance limit.

Eq. (6-10): $S_e' = 0.5(64) = 32 \text{ kpsi}$

Eq. (6-18): $k_a = 2.00(64)^{-0.217} = 0.81$

Eq. (6-19): $k_b = 0.879(1.25)^{-0.107} = 0.86$

Eq. (6-17): $S_e = 0.81(0.86)(32) = 22.3 \text{ kpsi}$

Using Goodman, Eq. (6-41),

$$n_f = \left(\frac{\sigma_a'}{S_e} + \frac{\sigma_m'}{S_{ut}}\right)^{-1} = \left(\frac{35.2}{22.3} + \frac{12.7}{64}\right)^{-1} = 0.56 \qquad \textit{Ans.}$$

6-42 Table A-20: $S_{\text{ut}} = 440$ MPa, $S_{\text{y}} = 370$ MPa

From Prob. 3-84, the critical stress element experiences σ = 333.9 MPa and τ = 126.3 MPa. The bending is completely reversed due to the rotation, and the torsion is steady, giving σ_a = 333.9 MPa, σ_m = 0 MPa, τ_a = 0 MPa, τ_m = 126.3 MPa. Obtain von Mises stresses for the alternating, mean, and maximum stresses.

$$\sigma_a' = \left(\sigma_a^2 + 3\tau_a^2\right)^{1/2} = \left[333.9^2 + 3\left(0\right)^2\right]^{1/2} = 333.9 \text{ MPa}$$

$$\sigma_m' = \left(\sigma_m^2 + 3\tau_m^2\right)^{1/2} = \left[0^2 + 3\left(126.3\right)^2\right]^{1/2} = 218.8 \text{ MPa}$$

$$\sigma_{\max}' = \left(\sigma_{\max}^2 + 3\tau_{\max}^2\right)^{1/2} = \left[333.9^2 + 3\left(126.3\right)^2\right]^{1/2} = 399.2 \text{ MPa}$$

Check for yielding, using the distortion energy failure theory.

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{370}{399.2} = 0.93$$

The sample fails by yielding, infinite life is not predicted. *Ans.*

The fatigue analysis will be continued only to obtain the requested fatigue factor of safety, though the yielding failure will dictate the life.

Obtain the modifying factors and endurance limit.

Eq. (6-10): $S'_e = 0.5(440) = 220 \text{ MPa}$

Eq. (6-18): $k_a = 3.04(440)^{-0.217} = 0.81$

Eq. (6-19): $k_b = 1.24(50)^{-0.107} = 0.82$

Eq. (6-17): $S_e = 0.81(0.82)(220) = 146.1 \text{ MPa}$

Using Goodman, Eq. (6-41),

$$n_f = \left(\frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}}\right)^{-1} = \left(\frac{333.9}{146.1} + \frac{218.8}{440}\right)^{-1} = 0.36 \qquad \textit{Ans.}$$

6-43 Table A-20: $S_{ut} = 64$ kpsi, $S_y = 54$ kpsi

From Prob. 3-85, the critical stress element experiences completely reversed bending stress due to the rotation, and steady torsional and axial stresses.

$$\sigma_{a,\text{bend}} = 9.495 \text{ kpsi}, \qquad \sigma_{m,\text{bend}} = 0 \text{ kpsi}$$

$$\sigma_{a,\text{axial}} = 0 \text{ kpsi}, \qquad \sigma_{m,\text{axial}} = -0.362 \text{ kpsi}$$

$$\tau_a = 0 \text{ kpsi}, \qquad \tau_m = 11.07 \text{ kpsi}$$

Obtain von Mises stresses for the alternating, mean, and maximum stresses.

$$\sigma'_a = \left(\sigma_a^2 + 3\tau_a^2\right)^{1/2} = \left[(9.495)^2 + 3(0)^2\right]^{1/2} = 9.495 \text{ kpsi}$$

$$\sigma'_m = \left(\sigma_m^2 + 3\tau_m^2\right)^{1/2} = \left[(-0.362)^2 + 3(11.07)^2\right]^{1/2} = 19.18 \text{ kpsi}$$

$$\sigma'_{\max} = \left(\sigma_{\max}^2 + 3\tau_{\max}^2\right)^{1/2} = \left[(-9.495 - 0.362)^2 + 3(11.07)^2\right]^{1/2} = 21.56 \text{ kpsi}$$

Check for yielding, using the distortion energy failure theory.

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{54}{21.56} = 2.50$$

Obtain the modifying factors and endurance limit.

Eq. (6-10): $S'_e = 0.5(64) = 32$ kpsi

Eq. (6-18): $k_a = 2.00(64)^{-0.217} = 0.81$

Eq. (6-19): $k_b = 0.879(1.13)^{-0.107} = 0.87$

Eq. (6-17): $S_e = 0.81(0.87)(32) = 22.6$ kpsi

Using Goodman, Eq. (6-41),

$$n_f = \left(\frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}}\right)^{-1} = \left(\frac{9.495}{22.6} + \frac{19.18}{64}\right)^{-1} = 1.39 \qquad \textit{Ans.}$$

6-44 Table A-20: $S_{ut} = 64$ kpsi, $S_y = 54$ kpsi

From Prob. 3-87, the critical stress element experiences completely reversed bending stress due to the rotation, and steady torsional and axial stresses.

$$\sigma_{a,\text{bend}} = 33.99 \text{ kpsi}, \qquad \sigma_{m,\text{bend}} = 0 \text{ kpsi}$$

$$\sigma_{a,\text{axial}} = 0 \text{ kpsi}, \qquad \sigma_{m,\text{axial}} = -0.153 \text{ kpsi}$$

$$\tau_a = 0 \text{ kpsi}, \qquad \tau_m = 7.847 \text{ kpsi}$$

Obtain von Mises stresses for the alternating, mean, and maximum stresses.

$$\sigma'_a = \left(\sigma_a^2 + 3\tau_a^2\right)^{1/2} = \left[(33.99)^2 + 3(0)^2\right]^{1/2} = 33.99 \text{ kpsi}$$

$$\sigma'_m = \left(\sigma_m^2 + 3\tau_m^2\right)^{1/2} = \left[(-0.153)^2 + 3(7.847)^2\right]^{1/2} = 13.59 \text{ kpsi}$$

$$\sigma'_{\max} = \left(\sigma_{\max}^2 + 3\tau_{\max}^2\right)^{1/2} = \left[(-33.99 - 0.153)^2 + 3(7.847)^2\right]^{1/2} = 36.75 \text{ kpsi}$$

Check for yielding, using the distortion energy failure theory.

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{54}{36.75} = 1.47$$

Obtain the modifying factors and endurance limit.

Eq. (6-10): $S'_e = 0.5(64) = 32$ kpsi

Eq. (6-18): $k_a = 2.00(64)^{-0.217} = 0.81$

Eq. (6-19): $k_b = 0.879(0.88)^{-0.107} = 0.89$

Eq. (6-17): $S_e = 0.81(0.89)(32) = 23.1$ kpsi

Using Goodman, Eq. (6-41),

$$n_f = \left(\frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}}\right)^{-1} = \left(\frac{33.99}{23.1} + \frac{13.59}{64}\right)^{-1} = 0.59 \qquad \textit{Ans.}$$

6-45 Table A-20: $S_{\text{ut}} = 440$ MPa, $S_{\text{y}} = 370$ MPa

From Prob. 3-88, the critical stress element experiences $\sigma = 68.6$ MPa and $\tau = 37.7$ MPa. The bending is completely reversed due to the rotation, and the torsion is steady, giving $\sigma_a = 68.6$ MPa, $\sigma_m = 0$ MPa, $\tau_a = 0$ MPa, $\tau_m = 37.7$ MPa. Obtain von Mises stresses for the alternating, mean, and maximum stresses.

$$\sigma'_a = \left(\sigma_a^2 + 3\tau_a^2\right)^{1/2} = \left[68.6^2 + 3\left(0\right)^2\right]^{1/2} = 68.6 \text{ MPa}$$

$$\sigma'_m = \left(\sigma_m^2 + 3\tau_m^2\right)^{1/2} = \left[0^2 + 3\left(37.7\right)^2\right]^{1/2} = 65.3 \text{ MPa}$$

$$\sigma'_{\max} = \left(\sigma_{\max}^2 + 3\tau_{\max}^2\right)^{1/2} = \left[68.6^2 + 3\left(37.7\right)^2\right]^{1/2} = 94.7 \text{ MPa}$$

Check for yielding, using the distortion energy failure theory.

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{370}{94.7} = 3.91$$

Obtain the modifying factors and endurance limit.

Eq. (6-10): $S'_e = 0.5(440) = 220$ MPa

Eq. (6-18): $k_a = 3.04(440)^{-0.217} = 0.81$

Eq. (6-19): $k_b = 1.24(30)^{-0.107} = 0.86$

Eq. (6-17): $S_e = 0.81(0.86)(220) = 153$ MPa

Using Goodman, Eq. (6-41),

$$n_f = \left(\frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}}\right)^{-1} = \left(\frac{68.6}{153} + \frac{65.3}{440}\right)^{-1} = 1.68 \qquad \textit{Ans.}$$

6-46 Table A-20: $S_{ut} = 64$ kpsi, $S_y = 54$ kpsi

From Prob. 3-90, the critical stress element experiences $\sigma = 3.46$ kpsi and $\tau = 0.882$ kpsi. The bending is completely reversed due to the rotation, and the torsion is steady, giving $\sigma_a = 3.46$ kpsi, $\sigma_m = 0$, $\tau_a = 0$ kpsi, $\tau_m = 0.882$ kpsi. Obtain von Mises stresses for the alternating, mean, and maximum stresses.

$$\sigma'_a = \left(\sigma_a^2 + 3\tau_a^2\right)^{1/2} = \left[3.46^2 + 3\left(0\right)^2\right]^{1/2} = 3.46 \text{ kpsi}$$

$$\sigma'_m = \left(\sigma_m^2 + 3\tau_m^2\right)^{1/2} = \left[0^2 + 3\left(0.882\right)^2\right]^{1/2} = 1.53 \text{ kpsi}$$

$$\sigma'_{\max} = \left(\sigma_{\max}^2 + 3\tau_{\max}^2\right)^{1/2} = \left[3.46^2 + 3\left(0.882\right)^2\right]^{1/2} = 3.78 \text{ kpsi}$$

Check for yielding, using the distortion energy failure theory.

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{54}{3.78} = 14.3$$

Obtain the modifying factors and endurance limit.

Eq. (6-10): $S'_e = 0.5(64) = 32 \text{ kpsi}$

Eq. (6-18): $k_a = 2.00(64)^{-0.217} = 0.81$

Eq. (6-19): $k_b = 0.879(1.375)^{-0.107} = 0.85$

Eq. (6-17): $S_e = 0.81(0.85)(32) = 22.0 \text{ kpsi}$

Using Goodman,

Eq. (6-41): $$n_f = \left(\frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}}\right)^{-1} = \left(\frac{3.46}{22.0} + \frac{1.53}{64}\right)^{-1}$$

$$n_f = 5.5 \qquad \textit{Ans.}$$

6-47 Table A-20: $S_{ut} = 64$ kpsi, $S_y = 54$ kpsi

From Prob. 3-91, the critical stress element experiences σ = 16.3 kpsi and τ = 5.09 kpsi. Since the load is applied and released repeatedly, this gives $\sigma_{\max}$ = 16.3 kpsi, $\sigma_{\min}$ = 0 kpsi, $\tau_{\max}$ = 5.09 kpsi, $\tau_{\min}$ = 0 kpsi. Consequently, $\sigma_m = \sigma_a$ = 8.15 kpsi, $\tau_m = \tau_a$ = 2.55 kpsi.

For bending, from Eqs. (6-33) and (6-35),

$$\sqrt{a} = 0.246 - 3.08\left(10^{-3}\right)\left(64\right) + 1.51\left(10^{-5}\right)\left(64\right)^2 - 2.67\left(10^{-8}\right)\left(64\right)^3 = 0.10373$$

$$q = \frac{1}{1 + \frac{\sqrt{a}}{\sqrt{r}}} = \frac{1}{1 + \frac{0.10373}{\sqrt{0.1}}} = 0.75$$

Eq. (6-32): $K_f = 1 + q(K_t - 1) = 1 + 0.75(1.5 - 1) = 1.38$

For torsion, from Eqs. (6-33) and (6-36),

$$\sqrt{a} = 0.190 - 2.51\left(10^{-3}\right)\left(64\right) + 1.35\left(10^{-5}\right)\left(64\right)^2 - 2.67\left(10^{-8}\right)\left(64\right)^3 = 0.07800$$

$$q = \frac{1}{1 + \frac{\sqrt{a}}{\sqrt{r}}} = \frac{1}{1 + \frac{0.07800}{\sqrt{0.1}}} = 0.80$$

Eq. (6-32): $K_{fs} = 1 + q_s(K_{ts} - 1) = 1 + 0.80(2.1 - 1) = 1.88$

Obtain von Mises stresses for the alternating and mean stresses from Eqs. (6-66) and (6-67).

$$\sigma'_a = \left\{\left[(1.38)(8.15)\right]^2 + 3\left[(1.88)(2.55)\right]^2\right\}^{1/2} = 13.98 \text{ kpsi}$$

$$\sigma'_m = \sigma'_a = 13.98 \text{ kpsi}$$

Check for yielding, using the conservative $\sigma'_{\max} = \sigma'_a + \sigma'_m$,

$$n_y = \frac{S_y}{\sigma'_a + \sigma'_m} = \frac{54}{13.98 + 13.98} = 1.93$$

Obtain the modifying factors and endurance limit.

Eq. (6-10): $S'_e = 0.5(64) = 32 \text{ kpsi}$

Eq. (6-18): $k_a = aS_{ut}^b = 2.00(64)^{-0.217} = 0.81$

Eq. (6-23): $d_e = 0.370d = 0.370(1) = 0.370 \text{ in}$

Eq. (6-19): $k_b = 0.879 d_e^{-0.107} = 0.879(0.370)^{-0.107} = 0.98$

Eq. (6-17): $S_e = (0.81)(0.98)(32) = 25.4 \text{ kpsi}$

Using Goodman,

Eq. (6-41): $$n_f = \left(\frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}}\right)^{-1} = \left(\frac{13.98}{25.4} + \frac{13.98}{64}\right)^{-1}$$

$$n_f = 1.3 \quad \textit{Ans.}$$

6-48 Table A-20: $S_{ut} = 64$ kpsi, $S_y = 54$ kpsi

From Prob. 3-92, the critical stress element experiences $\sigma = 16.4$ kpsi and $\tau = 4.46$ kpsi. Since the load is applied and released repeatedly, this gives $\sigma_{\max} = 16.4$ kpsi, $\sigma_{\min} = 0$ kpsi, $\tau_{\max} = 4.46$ kpsi, $\tau_{\min} = 0$ kpsi. Consequently, $\sigma_m = \sigma_a = 8.20$ kpsi, $\tau_m = \tau_a = 2.23$ kpsi.

For bending, from Eqs. (6-33) and (6-35),

$$\sqrt{a} = 0.246 - 3.08\left(10^{-3}\right)(64) + 1.51\left(10^{-5}\right)(64)^2 - 2.67\left(10^{-8}\right)(64)^3 = 0.10373$$

$$q = \frac{1}{1 + \frac{\sqrt{a}}{\sqrt{r}}} = \frac{1}{1 + \frac{0.10373}{\sqrt{0.1}}} = 0.75$$

Eq. (6-32): $K_f = 1 + q(K_t - 1) = 1 + 0.75(1.5 - 1) = 1.38$

For torsion, from Eqs. (6-33) and (6-36),

$$\sqrt{a} = 0.190 - 2.51\left(10^{-3}\right)\left(64\right) + 1.35\left(10^{-5}\right)\left(64\right)^2 - 2.67\left(10^{-8}\right)\left(64\right)^3 = 0.07800$$

$$q = \frac{1}{1 + \frac{\sqrt{a}}{\sqrt{r}}} = \frac{1}{1 + \frac{0.07800}{\sqrt{0.1}}} = 0.80$$

Eq. (6-32): $K_{fs} = 1 + q_s(K_{ts} - 1) = 1 + 0.80(2.1 - 1) = 1.88$

Obtain von Mises stresses for the alternating and mean stresses from Eqs. (6-66) and (6-67).

$$\sigma'_a = \left\{\left[\left(1.38\right)\left(8.20\right)\right]^2 + 3\left[\left(1.88\right)\left(2.23\right)\right]^2\right\}^{1/2} = 13.45 \text{ kpsi}$$

$$\sigma'_m = \sigma'_a = 13.45 \text{ kpsi}$$

Check for yielding, using the conservative $\sigma'_{\max} = \sigma'_a + \sigma'_m$,

$$n_y = \frac{S_y}{\sigma'_a + \sigma'_m} = \frac{54}{13.45 + 13.45} = 2.01$$

Obtain the modifying factors and endurance limit.

Eq. (6-10): $S'_e = 0.5(64) = 32 \text{ kpsi}$

Eq. (6-18): $k_a = aS_{ut}^b = 2.00(64)^{-0.217} = 0.81$

Eq. (6-23): $d_e = 0.370d = 0.370(1) = 0.370 \text{ in}$

Eq. (6-19): $k_b = 0.879 d_e^{-0.107} = 0.879(0.370)^{-0.107} = 0.98$

Eq. (6-17): $S_e = (0.81)(0.98)(32) = 25.4 \text{ kpsi}$

Using Goodman,

Eq. (6-41): $n_f = \left(\frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}}\right)^{-1} = \left(\frac{13.45}{25.4} + \frac{13.45}{64}\right)^{-1}$

$n_f = 1.35 \quad$ *Ans.*

6-49 Table A-20: $S_{ut} = 64$ kpsi, $S_y = 54$ kpsi

From Prob. 3-93, the critical stress element experiences repeatedly applied bending, axial, and torsional stresses of $\sigma_{x,\text{bend}} = 20.2$ kpsi, $\sigma_{x,\text{axial}} = 0.1$ kpsi, and $\tau = 5.09$ kpsi.. Since the axial stress is practically negligible compared to the bending stress, we will simply combine the two and not treat the axial stress separately for stress concentration factor and load factor. This gives $\sigma_{\max} = 20.3$ kpsi, $\sigma_{\min} = 0$ kpsi, $\tau_{\max} = 5.09$ kpsi, $\tau_{\min} = 0$ kpsi. Consequently, $\sigma_m = \sigma_a = 10.15$ kpsi, $\tau_m = \tau_a = 2.55$ kpsi.

For bending, from Eqs. (6-33) and (6-35),

$$\sqrt{a} = 0.246 - 3.08\left(10^{-3}\right)(64) + 1.51\left(10^{-5}\right)(64)^2 - 2.67\left(10^{-8}\right)(64)^3 = 0.10373$$

$$q = \frac{1}{1 + \dfrac{\sqrt{a}}{\sqrt{r}}} = \frac{1}{1 + \dfrac{0.10373}{\sqrt{0.1}}} = 0.75$$

Eq. (6-32): $K_f = 1 + q(K_t - 1) = 1 + 0.75(1.5 - 1) = 1.38$

For torsion, from Eqs. (6-33) and (6-36),

$$\sqrt{a} = 0.190 - 2.51\left(10^{-3}\right)(64) + 1.35\left(10^{-5}\right)(64)^2 - 2.67\left(10^{-8}\right)(64)^3 = 0.07800$$

$$q = \frac{1}{1 + \dfrac{\sqrt{a}}{\sqrt{r}}} = \frac{1}{1 + \dfrac{0.07800}{\sqrt{0.1}}} = 0.80$$

Eq. (6-32): $K_{fs} = 1 + q_s(K_{ts} - 1) = 1 + 0.80(2.1 - 1) = 1.88$

Obtain von Mises stresses for the alternating and mean stresses from Eqs. (6-66) and (6-67).

$$\sigma'_a = \left\{\left[(1.38)(10.15)\right]^2 + 3\left[(1.88)(2.55)\right]^2\right\}^{1/2} = 16.28 \text{ kpsi}$$

$$\sigma'_m = \sigma'_a = 16.28 \text{ kpsi}$$

Check for yielding, using the conservative $\sigma'_{\max} = \sigma'_a + \sigma'_m$,

$$n_y = \frac{S_y}{\sigma'_a + \sigma'_m} = \frac{54}{16.28 + 16.28} = 1.66$$

Obtain the modifying factors and endurance limit.

Eq. (6-10): $S'_e = 0.5(64) = 32 \text{ kpsi}$

Eq. (6-18): $k_a = aS_{ut}^b = 2.00(64)^{-0.217} = 0.81$

Eq. (6-23): $d_e = 0.370d = 0.370(1) = 0.370 \text{ in}$

Eq. (6-19): $k_b = 0.879d_e^{-0.107} = 0.879(0.370)^{-0.107} = 0.98$

Eq. (6-17): $S_e = (0.81)(0.98)(32) = 25.4 \text{ kpsi}$

Using Goodman,

Eq. (6-41): $n_f = \left(\dfrac{\sigma'_a}{S_e} + \dfrac{\sigma'_m}{S_{ut}}\right)^{-1} = \left(\dfrac{16.28}{25.4} + \dfrac{16.28}{64}\right)^{-1}$

$n_f = 1.12$ *Ans.*

6-50 Table A-20: $S_{ut} = 64$ kpsi, $S_y = 54$ kpsi

From Prob. 3-94, the critical stress element on the neutral axis in the middle of the longest side of the rectangular cross section experiences a repeatedly applied shear stress of τ_{max} = 14.3 kpsi, τ_{min} = 0 kpsi. Thus, $\tau_m = \tau_a$ = 7.15 kpsi. Since the stress is entirely shear, it is convenient to check for yielding using the standard Maximum Shear Stress theory.

$$n_y = \frac{S_y / 2}{\tau_{\max}} = \frac{54/2}{14.3} = 1.89$$

Find the modifiers and endurance limit.

Eq. (6-10): $S'_e = 0.5(64) = 32$ kpsi

Eq. (6-18): $k_a = aS_{ut}^b = 2.00(64)^{-0.217} = 0.81$

The size factor for a rectangular cross section loaded in torsion is not readily available. An equivalent diameter based on the 95 percent stress area is not readily obtained, since the stress situation in this case is nonlinear, as described in Section 3-12. Noting that the maximum stress occurs at the middle of the longest side, or with a radius from the center of the cross section equal to half of the shortest side, we will simply choose an equivalent diameter equal to the length of the shortest side.

$d_e = 0.25$ in

Eq. (6-19): $k_b = 0.879 d_e^{-0.107} = 0.879(0.25)^{-0.107} = 1.02$

We will round down to k_b = 1.

Eq. (6-25): $k_c = 0.59$

Eq. (6-17): $S_{se} = 0.81(1)(0.59)(32) = 15.3$ kpsi

Since the stress is entirely shear, we choose to use a load factor k_c = 0.59, and convert the ultimate strength to a shear value rather than using the combination loading method of Sec. 6-16. From Eq. (6-58), $S_{su} = 0.67S_u$ = 0.67 (64) = 42.9 kpsi.

Using Goodman,

Eq. (6-41): $$n_f = \left(\frac{\tau_a}{S_{se}} + \frac{\tau_m}{S_{su}} \right)^{-1} = \left(\frac{7.15}{15.3} + \frac{7.15}{42.9} \right)^{-1} = 1.58 \qquad \textit{Ans.}$$

6-51 Table A-20: $S_{ut} = 64$ kpsi, $S_y = 54$ kpsi

From Prob. 3-95, the critical stress element experiences σ = 28.0 kpsi and τ = 15.3 kpsi. Since the load is applied and released repeatedly, this gives σ_{max} = 28.0 kpsi, σ_{min} = 0 kpsi, τ_{max} = 15.3 kpsi, τ_{min} = 0 kpsi. Consequently, $\sigma_m = \sigma_a$ = 14.0 kpsi, $\tau_m = \tau_a$ = 7.65 kpsi. From Table A-15-8 and A-15-9,

$$D/d = 1.5/1 = 1.5, \qquad r/d = 0.125/1 = 0.125$$
$$K_{t,\text{bend}} = 1.60, \qquad K_{t,\text{tors}} = 1.39$$

Figs. 6-26 and 6-27: $q_{\text{bend}} = 0.78$, $q_{\text{tors}} = 0.82$
Eq. (6-32):

$$K_{f,\text{bend}} = 1 + q_{\text{bend}}\left(K_{t,\text{bend}} - 1\right) = 1 + 0.78\left(1.60 - 1\right) = 1.47$$
$$K_{f,\text{tors}} = 1 + q_{\text{tors}}\left(K_{t,\text{tors}} - 1\right) = 1 + 0.82\left(1.39 - 1\right) = 1.32$$

Obtain von Mises stresses for the alternating and mean stresses from Eqs. (6-66) and (6-67).

$$\sigma_a' = \left\{\left[(1.47)(14.0)\right]^2 + 3\left[(1.32)(7.65)\right]^2\right\}^{1/2} = 27.0 \text{ kpsi}$$
$$\sigma_m' = \sigma_a' = 27.0 \text{ kpsi}$$

Check for yielding, using the conservative $\sigma'_{\max} = \sigma_a' + \sigma_m'$,

$$n_y = \frac{S_y}{\sigma_a' + \sigma_m'} = \frac{54}{27.0 + 27.0} = 1.00$$

Since stress concentrations are included in this quick yield check, the low factor of safety is acceptable.

Eq. (6-10): $S_e' = 0.5(64) = 32$ kpsi

Eq. (6-18): $k_a = aS_{ut}^b = 2.00(64)^{-0.217} = 0.81$

Eq. (6-23): $d_e = 0.370d = 0.370(1) = 0.370$ in

Eq. (6-19): $k_b = 0.879d_e^{-0.107} = 0.879(0.370)^{-0.107} = 0.98$

Eq. (6-17): $S_e = (0.81)(0.98)(0.5)(64) = 25.4$ kpsi

For the Morrow criterion, estimate the fatigue strength coefficient for steel.
Eq. (6-44): $\sigma_f' = S_{ut} + 50 = 64 + 50 = 114$ kpsi

Eq. (6-46): $$n_f = \left(\frac{\sigma_a'}{S_e} + \frac{\sigma_m'}{\sigma_f'}\right)^{-1} = \left(\frac{27.0}{25.4} + \frac{27.0}{114}\right)^{-1}$$

$$n_f = 0.77 \qquad \textit{Ans.}$$

Since infinite life is not predicted, estimate a life from the *S-N* diagram. First, find an equivalent completely reversed stress, again using Morrow.

Eq. (6-59): $\sigma_{ar} = \dfrac{\sigma'_a}{1-(\sigma'_m/\sigma'_f)} = \dfrac{27.0}{1-(27.0/114)} = 35.4$ kpsi

Fig. 6-23: Off the chart, so use $f = 0.9$

Eq. (6-13): $a = \dfrac{(f\,S_{ut})^2}{S_e} = \dfrac{[0.9(64)]^2}{25.4} = 130.6$

Eq. (6-14): $b = -\dfrac{1}{3}\log\left(\dfrac{f\,S_{ut}}{S_e}\right) = -\dfrac{1}{3}\log\left(\dfrac{0.9(64)}{25.4}\right) = -0.1185$

Eq. (6-15): $N = \left(\dfrac{\sigma_{ar}}{a}\right)^{1/b} = \left(\dfrac{35.4}{130.6}\right)^{\frac{1}{-0.1185}} = 60856 \text{ cycles} \approx 61\,000 \text{ cycles}$ *Ans.*

6-52 Table A-20: $S_{ut} = 64$ kpsi, $S_y = 54$ kpsi

From Prob. 3-96, the critical stress element experiences $\sigma_{x,\text{bend}} = 46.1$ kpsi, $\sigma_{x,\text{axial}} = 0.382$ kpsi and $\tau = 15.3$ kpsi. The axial load is practically negligible, but we'll include it to demonstrate the process. Since the load is applied and released repeatedly, this gives $\sigma_{\max,\text{bend}} = 46.1$ kpsi, $\sigma_{\min,\text{bend}} = 0$ kpsi, $\sigma_{\max,\text{axial}} = 0.382$ kpsi, $\sigma_{\min,\text{axial}} = 0$ kpsi, $\tau_{\max} = 15.3$ kpsi, $\tau_{\min} = 0$ kpsi. Consequently, $\sigma_{m,\text{bend}} = \sigma_{a,\text{bend}} = 23.05$ kpsi, $\sigma_{m,\text{axial}} = \sigma_{a,\text{axial}} = 0.191$ kpsi, $\tau_m = \tau_a = 7.65$ kpsi. From Table A-15-7, A-15-8 and A-15-9,

$$D/d = 1.5/1 = 1.5, \quad r/d = 0.125/1 = 0.125$$
$$K_{t,\text{bend}} = 1.60, \quad K_{t,\text{tors}} = 1.39, \quad K_{t,\text{axial}} = 1.75$$

Eqs. (6-33), (6-35), and (6-36), or Figs. 6-26 and 6-27: $q_{\text{bend}} = q_{\text{axial}} = 0.78$, $q_{\text{tors}} = 0.82$

Eq. (6-32):

$$K_{f,\text{bend}} = 1 + q_{\text{bend}}\left(K_{t,\text{bend}} - 1\right) = 1 + 0.78(1.60-1) = 1.47$$
$$K_{f,\text{axial}} = 1 + q_{\text{axial}}\left(K_{t,\text{axial}} - 1\right) = 1 + 0.78(1.75-1) = 1.59$$
$$K_{f,\text{tors}} = 1 + q_{\text{tors}}\left(K_{t,\text{tors}} - 1\right) = 1 + 0.82(1.39-1) = 1.32$$

Obtain von Mises stresses for the alternating and mean stresses from Eqs. (6-66) and (6-67).

$$\sigma'_a = \left\{\left[(1.47)(23.05) + (1.59)(0.191)\right]^2 + 3\left[(1.32)(7.65)\right]^2\right\}^{1/2} = 38.4 \text{ kpsi}$$
$$\sigma'_m = \left\{\left[(1.47)(23.05) + (1.59)(0.191)\right]^2 + 3\left[(1.32)(7.65)\right]^2\right\}^{1/2} = 38.4 \text{ kpsi}$$

Check for yielding, using the conservative $\sigma'_{\max} = \sigma'_a + \sigma'_m$,

$$n_y = \frac{S_y}{\sigma'_a + \sigma'_m} = \frac{54}{38.4 + 38.4} = 0.70$$

Since the conservative yield check indicates yielding, we will check more carefully with $\sigma'_{\max}$ obtained directly from the maximum stresses, using the distortion energy failure theory, without stress concentrations. Note that this is exactly the method used for static failure in Ch. 5.

$$\sigma'_{\max} = \sqrt{\left(\sigma_{\text{max,bend}} + \sigma_{\text{max,axial}}\right)^2 + 3\left(\tau_{\max}\right)^2} = \sqrt{\left(46.1 + 0.382\right)^2 + 3\left(15.3\right)^2} = 53.5 \text{ kpsi}$$

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{54}{53.5} = 1.01 \qquad \textit{Ans.}$$

This shows that yielding is imminent, and further analysis of fatigue life should not be interpreted as a guarantee of more than one cycle of life.

Eq. (6-10): $S'_e = 0.5(64) = 32 \text{ kpsi}$

Eq. (6-18): $k_a = aS_{ut}^b = 2.00(64)^{-0.217} = 0.81$

Eq. (6-23): $d_e = 0.370d = 0.370(1) = 0.370 \text{ in}$

Eq. (6-19): $k_b = 0.879 d_e^{-0.107} = 0.879(0.370)^{-0.107} = 0.98$

Eq. (6-17): $S_e = (0.81)(0.98)(0.5)(64) = 25.4 \text{ kpsi}$

For the Morrow criterion, estimate the fatigue strength coefficient for steel.

Eq. (6-44): $\sigma'_f = S_{ut} + 50 = 64 + 50 = 114 \text{ kpsi}$

Eq. (6-46): $$n_f = \left(\frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{\sigma'_f}\right)^{-1} = \left(\frac{38.4}{25.4} + \frac{38.4}{114}\right)^{-1}$$

$$n_f = 0.54 \qquad \textit{Ans.}$$

Since infinite life is not predicted, estimate a life from the *S-N* diagram. First, find an equivalent completely reversed stress, again using Morrow.

Eq. (6-59): $$\sigma_{ar} = \frac{\sigma'_a}{1 - (\sigma'_m / \sigma'_f)} = \frac{38.4}{1 - (38.4/114)} = 57.9 \text{ kpsi}$$

Fig. 6-23: Off the chart, so use $f = 0.9$

Eq. (6-13): $$a = \frac{(f\,S_{ut})^2}{S_e} = \frac{\left[0.9(64)\right]^2}{25.4} = 130.6$$

Eq. (6-14): $$b = -\frac{1}{3}\log\left(\frac{f\,S_{ut}}{S_e}\right) = -\frac{1}{3}\log\left(\frac{0.9(64)}{25.4}\right) = -0.1185$$

Eq. (6-15): $$N = \left(\frac{\sigma_{ar}}{a}\right)^{1/b} = \left(\frac{57.9}{130.6}\right)^{\frac{1}{-0.1185}} = 960 \text{ cycles} \qquad \textit{Ans.}$$

6-53 Table A-20: $S_{ut} = 64$ kpsi, $S_y = 54$ kpsi

From Prob. 3-97, the critical stress element experiences $\sigma_{x,\text{bend}} = 55.5$ kpsi, $\sigma_{x,\text{axial}} = 0.382$ kpsi and $\tau = 15.3$ kpsi. The axial load is practically negligible, but we'll include it to demonstrate the process. Since the load is applied and released repeatedly, this gives $\sigma_{\text{max,bend}} = 55.5$ kpsi, $\sigma_{\text{min,bend}} = 0$ kpsi, $\sigma_{\text{max,axial}} = 0.382$ kpsi, $\sigma_{\text{min,axial}} = 0$ kpsi, $\tau_{\text{max}} = 15.3$ kpsi, $\tau_{\text{min}} = 0$ kpsi. Consequently, $\sigma_{m,\text{bend}} = \sigma_{a,\text{bend}} = 27.75$ kpsi, $\sigma_{m,\text{axial}} = \sigma_{a,\text{axial}} = 0.191$ kpsi, $\tau_m = \tau_a = 7.65$ kpsi. From Table A-15-7, A-15-8 and A-15-9,

$$D/d = 1.5/1 = 1.5, \qquad r/d = 0.125/1 = 0.125$$

$$K_{t,\text{bend}} = 1.60, \qquad K_{t,\text{tors}} = 1.39, \qquad K_{t,\text{axial}} = 1.75$$

Eqs. (6-33), (6-35), and (6-36), or Figs. 6-26 and 6-27: $q_{\text{bend}} = q_{\text{axial}} = 0.78$, $q_{\text{tors}} = 0.82$

Eq. (6-32):

$$K_{f,\text{bend}} = 1 + q_{\text{bend}}\left(K_{t,\text{bend}} - 1\right) = 1 + 0.78(1.60 - 1) = 1.47$$

$$K_{f,\text{axial}} = 1 + q_{\text{axial}}\left(K_{t,\text{axial}} - 1\right) = 1 + 0.78(1.75 - 1) = 1.59$$

$$K_{f,\text{tors}} = 1 + q_{\text{tors}}\left(K_{t,\text{tors}} - 1\right) = 1 + 0.82(1.39 - 1) = 1.32$$

Obtain von Mises stresses for the alternating and mean stresses from Eqs. (6-66) and (6-67).

$$\sigma'_a = \left\{\left[(1.47)(27.75) + (1.59)(0.191)\right]^2 + 3\left[(1.32)(7.65)\right]^2\right\}^{1/2} = 44.66 \text{ kpsi}$$

$$\sigma'_m = \left\{\left[(1.47)(27.75) + (1.59)(0.191)\right]^2 + 3\left[(1.32)(7.65)\right]^2\right\}^{1/2} = 44.66 \text{ kpsi}$$

Since these stresses are relatively high compared to the yield strength, we will go ahead and check for yielding using the distortion energy failure theory.

$$\sigma'_{\text{max}} = \sqrt{\left(\sigma_{\text{max,bend}} + \sigma_{\text{max,axial}}\right)^2 + 3\left(\tau_{\text{max}}\right)^2} = \sqrt{(55.5 + 0.382)^2 + 3(15.3)^2} = 61.8 \text{ kpsi}$$

$$n_y = \frac{S_y}{\sigma'_{\text{max}}} = \frac{54}{61.8} = 0.87 \qquad \textit{Ans.}$$

This shows that yielding is predicted. Further analysis of fatigue life is just to be able to report the fatigue factor of safety, though the life will be dictated by the static yielding failure, i.e. $N = 1/2$ cycle. *Ans.*

Eq. (6-10): $S'_e = 0.5(64) = 32$ kpsi

Eq. (6-18): $k_a = aS_{ut}^b = 2.00(64)^{-0.217} = 0.81$

Eq. (6-23): $d_e = 0.370d = 0.370(1) = 0.370$ in

Eq. (6-19): $k_b = 0.879d_e^{-0.107} = 0.879(0.370)^{-0.107} = 0.98$

Eq. (6-17): $S_e = (0.81)(0.98)(0.5)(64) = 25.4$ kpsi

For the Morrow criterion, estimate the fatigue strength coefficient for steel.

Eq. (6-44): $\sigma'_f = S_{ut} + 50 = 64 + 50 = 114$ kpsi

Eq. (6-46): $$n_f = \left(\frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{\sigma'_f} \right)^{-1} = \left(\frac{44.66}{25.4} + \frac{44.66}{114} \right)^{-1}$$

$$n_f = 0.47 \quad Ans.$$

6-54 From Table A-20, for AISI 1040 CD, S_{ut} = 85 kpsi and S_y = 71 kpsi. From the solution to Prob. 6-17 we find the completely reversed stress at the critical shoulder fillet to be σ_{ar} = 35.0 kpsi, producing σ_a = 35.0 kpsi and σ_m = 0 kpsi. This problem adds a steady torque which creates torsional stresses of

$$\tau_m = \frac{Tr}{J} = \frac{2500(1.625/2)}{\pi(1.625^4)/32} = 2967 \text{ psi} = 2.97 \text{ kpsi}, \quad \tau_a = 0 \text{ kpsi}$$

From Table A-15-8 and A-15-9, r/d = 0.0625/1.625 = 0.04, D/d = 1.875/1.625 = 1.15, $K_{t,\text{bend}}$ =1.95, $K_{t,\text{tors}}$ =1.60

Eqs. (6-33), (6-35) and (6-36), or Figs. 6-26 and 6-27: q_{bend} = 0.76, q_{tors} = 0.81

Eq. (6-32):

$$K_{f,\text{bend}} = 1 + q_{\text{bend}}\left(K_{t,\text{bend}} - 1\right) = 1 + 0.76(1.95 - 1) = 1.72$$

$$K_{f,\text{tors}} = 1 + q_{\text{tors}}\left(K_{t,\text{tors}} - 1\right) = 1 + 0.81(1.60 - 1) = 1.49$$

Obtain von Mises stresses for the alternating and mean stresses from Eqs. (6-66) and (6-67).

$$\sigma'_a = \left\{\left[(1.72)(35.0)\right]^2 + 3\left[(1.49)(0)\right]^2\right\}^{1/2} = 60.2 \text{ kpsi}$$

$$\sigma'_m = \left\{\left[(1.72)(0)\right]^2 + 3\left[(1.49)(2.97)\right]^2\right\}^{1/2} = 7.66 \text{ kpsi}$$

Check for yielding, using the conservative $\sigma'_{\max} = \sigma'_a + \sigma'_m$,

$$n_y = \frac{S_y}{\sigma'_a + \sigma'_m} = \frac{71}{60.2 + 7.66} = 1.05$$

From the solution to Prob. 6-17, S_e = 27.0 kpsi. Using Goodman,

Eq. (6-41): $$n_f = \left(\frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}} \right)^{-1} = \left(\frac{60.2}{27.0} + \frac{7.66}{85} \right)^{-1}$$

$$n_f = 0.43 \quad Ans.$$

Since infinite life is not predicted, estimate a life from the *S-N* diagram. First, find an equivalent completely reversed stress. Choosing the Goodman criterion, Eq. (6-58): $\sigma_{ar} = \dfrac{\sigma'_a}{1-(\sigma'_m / S_{ut})} = \dfrac{60.2}{1-(7.66/85)} = 66.2$ kpsi

Fig. 6-23: $f = 0.867$

Eq. (6-13): $a = \dfrac{(f\,S_{ut})^2}{S_e} = \dfrac{[0.867(85)]^2}{27.0} = 201.1$

Eq. (6-14): $b = -\dfrac{1}{3}\log\left(\dfrac{f\,S_{ut}}{S_e}\right) = -\dfrac{1}{3}\log\left(\dfrac{0.867(85)}{27.0}\right) = -0.1454$

Eq. (6-15): $N = \left(\dfrac{\sigma_{ar}}{a}\right)^{1/b} = \left(\dfrac{66.2}{201.1}\right)^{\frac{1}{-0.1454}} = 2084$ cycles

$N = 2100$ cycles *Ans.*

6-55 From the solution to Prob. 6-18 we find the completely reversed stress at the critical shoulder fillet to be $\sigma_{rev} = 32.8$ kpsi, producing $\sigma_a = 32.8$ kpsi and $\sigma_m = 0$ kpsi. This problem adds a steady torque which creates torsional stresses of

$$\tau_m = \frac{Tr}{J} = \frac{2200(1.625/2)}{\pi(1.625^4)/32} = 2611 \text{ psi} = 2.61 \text{ kpsi}, \quad \tau_a = 0 \text{ kpsi}$$

From Table A-15-8 and A-15-9, $r/d = 0.0625/1.625 = 0.04$, $D/d = 1.875/1.625 = 1.15$, $K_{t,\text{bend}} = 1.95$, $K_{t,\text{tors}} = 1.60$

Eqs. (6-33), (6-35) and (6-36), or Figs. 6-26 and 6-27: $q_{\text{bend}} = 0.76$, $q_{\text{tors}} = 0.81$

Eq. (6-32):

$$K_{f,\text{bend}} = 1 + q_{\text{bend}}(K_{t,\text{bend}} - 1) = 1 + 0.76(1.95 - 1) = 1.72$$
$$K_{f,\text{tors}} = 1 + q_{\text{tors}}(K_{t,\text{tors}} - 1) = 1 + 0.81(1.60 - 1) = 1.49$$

Obtain von Mises stresses for the alternating and mean stresses from Eqs. (6-66) and (6-67).

$$\sigma'_a = \left\{\left[(1.72)(32.8)\right]^2 + 3\left[(1.49)(0)\right]^2\right\}^{1/2} = 56.4 \text{ kpsi}$$
$$\sigma'_m = \left\{\left[(1.72)(0)\right]^2 + 3\left[(1.49)(2.61)\right]^2\right\}^{1/2} = 6.74 \text{ kpsi}$$

Check for yielding, using the conservative $\sigma'_{\max} = \sigma'_a + \sigma'_m$,

$$n_y = \frac{S_y}{\sigma'_a + \sigma'_m} = \frac{71}{56.4 + 6.74} = 1.12$$

From the solution to Prob. 6-18, $S_e = 27.0$ kpsi. Using Goodman, Eq. (6-41): $n_f = \left(\frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}} \right)^{-1} = \left(\frac{56.4}{27.0} + \frac{6.74}{85} \right)^{-1}$

$n_f = 0.46 \quad Ans.$

Since infinite life is not predicted, estimate a life from the *S-N* diagram. First, find an equivalent completely reversed stress. Choosing the Goodman criterion,

Eq. (6-58): $\sigma_{ar} = \frac{\sigma'_a}{1-(\sigma'_m / S_{ut})} = \frac{56.4}{1-(6.74/85)} = 61.3 \text{ kpsi}$

Fig. 6-23: $f = 0.867$

Eq. (6-13): $a = \frac{(f\,S_{ut})^2}{S_e} = \frac{[0.867(85)]^2}{27.0} = 201.1$

Eq. (6-14): $b = -\frac{1}{3}\log\left(\frac{f\,S_{ut}}{S_e}\right) = -\frac{1}{3}\log\left(\frac{0.867(85)}{27.0}\right) = -0.1454$

Eq. (6-15): $N = \left(\frac{\sigma_{ar}}{a}\right)^{1/b} = \left(\frac{61.3}{201.1}\right)^{\frac{1}{-0.1454}} = 3536 \text{ cycles}$

$N = 3500$ cycles *Ans.*

6-56 $S_{ut} = 55 \text{ kpsi}, S_y = 30 \text{ kpsi}, K_{ts} = 1.6, L = 2 \text{ ft}, F_{min} = 150 \text{ lbf}, F_{max} = 500 \text{ lbf}$

Eqs. (6-33) and (6-36), or Fig. 6-27: $q_s = 0.80$

Eq. (6-32): $K_{fs} = 1 + q_s(K_{ts} - 1) = 1 + 0.80(1.6 - 1) = 1.48$

$$T_{max} = 500(2) = 1000 \text{ lbf}\cdot\text{in}, \quad T_{min} = 150(2) = 300 \text{ lbf}\cdot\text{in}$$

$$\tau_{max} = \frac{16K_{fs}T_{max}}{\pi d^3} = \frac{16(1.48)(1000)}{\pi(0.875)^3} = 11\,251 \text{ psi} = 11.25 \text{ kpsi}$$

$$\tau_{min} = \frac{16K_{fs}T_{min}}{\pi d^3} = \frac{16(1.48)(300)}{\pi(0.875)^3} = 3375 \text{ psi} = 3.38 \text{ kpsi}$$

$$\tau_m = \frac{\tau_{max} + \tau_{min}}{2} = \frac{11.25 + 3.38}{2} = 7.32 \text{ kpsi}$$

$$\tau_a = \frac{\tau_{max} - \tau_{min}}{2} = \frac{11.25 - 3.38}{2} = 3.94 \text{ kpsi}$$

Since the stress is entirely shear, it is convenient to check for yielding using the standard Maximum Shear Stress theory.

$$n_y = \frac{S_y / 2}{\tau_{max}} = \frac{30/2}{11.25} = 1.33$$

Find the modifiers and endurance limit.

Eq. (6-10): $S_e' = 0.5(55) = 27.5$ kpsi

Eq. (6-18): $k_a = 11.0(55)^{-0.650} = 0.81$

Eq. (6-23): $d_e = 0.370(0.875) = 0.324$ in

Eq. (6-19): $k_b = 0.879(0.324)^{-0.107} = 0.99$

Eq. (6-25): $k_c = 0.59$

Eq. (6-17): $S_{se} = 0.81(0.99)(0.59)(27.5) = 13.0$ kpsi

Since the stress is entirely shear, we will use a load factor $k_c = 0.59$, and convert the ultimate strength to a shear value rather than using the combination loading method of Sec. 6-16. From Eq. (6-58), $S_{su} = 0.67S_u = 0.67\ (55) = 36.9$ kpsi.

(a) Goodman,

Eq. (6-41): $$n_f = \left(\frac{\tau_a}{S_{se}} + \frac{\tau_m}{S_{su}}\right)^{-1} = \left(\frac{3.94}{13.0} + \frac{7.32}{36.9}\right)^{-1} = 1.99 \qquad Ans.$$

(b) Gerber

Eq. (6-48): $$n_f = \frac{1}{2}\left(\frac{S_{su}}{\tau_m}\right)^2 \frac{\tau_a}{S_{se}}\left[-1 + \sqrt{1 + \left(\frac{2\tau_m S_{se}}{S_{su}\tau_a}\right)^2}\right]$$

$$n_f = \frac{1}{2}\left(\frac{36.9}{7.32}\right)^2 \left(\frac{3.94}{13.0}\right)\left[-1 + \sqrt{1 + \left(\frac{2(7.32)(13.0)}{36.9(3.94)}\right)^2}\right]$$

$$n_f = 2.49 \qquad Ans.$$

6-57 $S_{ut} = 145$ kpsi, $S_y = 120$ kpsi

From Eqs. (6-33) and (6-35), or Fig. 6-26, with a notch radius of 0.1 in, $q = 0.9$. Thus, with $K_t = 3$ from the problem statement,

$$K_f = 1 + q(K_t - 1) = 1 + 0.9(3 - 1) = 2.80$$

$$\sigma_{\max} = -K_f \frac{4P}{\pi d^2} = \frac{-2.80(4)(P)}{\pi(1.2)^2} = -2.476P$$

$$\sigma_m = -\sigma_a = \frac{1}{2}(-2.476P) = -1.238P$$

$$T_{\max} = \frac{f\,P\left(D + d\right)}{4} = \frac{0.3P\left(6 + 1.2\right)}{4} = 0.54P$$

From Eqs. (6-33) and (6-36), or Fig. 6-27, with a notch radius of 0.1 in, $q_s = 0.92$. Thus, with $K_{ts} = 1.8$ from the problem statement,

$$K_{fs} = 1 + q_s(K_{ts} - 1) = 1 + 0.92(1.8 - 1) = 1.74$$

$$\tau_{\max} = \frac{16K_{fs}T}{\pi d^3} = \frac{16(1.74)(0.54P)}{\pi(1.2)^3} = 2.769P$$

$$\tau_a = \tau_m = \frac{\tau_{\max}}{2} = \frac{2.769P}{2} = 1.385P$$

Eqs. (6-66) and (6-67):

$$\sigma'_a = [\sigma_a{}^2 + 3\tau_a^2]^{1/2} = [(1.238P)^2 + 3(1.385P)^2]^{1/2} = 2.70P$$

$$\sigma'_m = [\sigma_m{}^2 + 3\tau_m^2]^{1/2} = [(-1.238P)^2 + 3(1.385P)^2]^{1/2} = 2.70P$$

Eq. (6-10): $S'_e = 0.5(145) = 72.5$ kpsi

Eq. (6-18): $k_a = 2.00(145)^{-0.217} = 0.68$

Eq. (6-19): $k_b = 0.879(1.2)^{-0.107} = 0.862$

Eq. (6-17): $S_e = (0.68)(0.862)(72.5) = 42.5$ kpsi

Eq. (6-41): $$n_f = \left(\frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}}\right)^{-1} = \left(\frac{2.70P}{42.5} + \frac{2.70P}{145}\right)^{-1} = 3$$

$P = 4.1$ kips *Ans.*

Yield (conservative):

$$n_y = \frac{S_y}{\sigma'_a + \sigma'_m} = \frac{120}{(2.70)(4.1) + (2.70)(4.1)} = 5.4$$ Yielding is not predicted. *Ans.*

6-58 From Prob. 6-57, $K_f = 2.80$, $K_{fs} = 1.74$, $S_e = 42.5$ kpsi

$$\sigma_{\max} = -K_f \frac{4P_{\max}}{\pi d^2} = -2.80\frac{4(18)}{\pi(1.2^2)} = -44.56 \text{ kpsi}$$

$$\sigma_{\min} = -K_f \frac{4P_{\min}}{\pi d^2} = -2.80\frac{4(4.5)}{\pi(1.2)^2} = -11.14 \text{ kpsi}$$

$$T_{\max} = f\,P_{\max}\left(\frac{D+d}{4}\right) = 0.3(18)\left(\frac{6+1.2}{4}\right) = 9.72 \text{ kip}\cdot\text{in}$$

$$T_{\min} = f\,P_{\min}\left(\frac{D+d}{4}\right) = 0.3(4.5)\left(\frac{6+1.2}{4}\right) = 2.43 \text{ kip}\cdot\text{in}$$

$$\tau_{\max} = K_{fs}\frac{16T_{\max}}{\pi d^3} = 1.74\frac{16(9.72)}{\pi(1.2)^3} = 49.85 \text{ kpsi}$$

$$\tau_{\min} = K_{fs}\frac{16T_{\min}}{\pi d^3} = 1.74\frac{16(2.43)}{\pi(1.2)^3} = 12.46 \text{ kpsi}$$

$$\sigma_a = \left|\frac{-44.56-(-11.14)}{2}\right| = 16.71 \text{ kpsi}$$
$$\sigma_m = \frac{-44.56+(-11.14)}{2} = -27.85 \text{ kpsi}$$
$$\tau_a = \frac{49.85-12.46}{2} = 18.70 \text{ kpsi}$$
$$\tau_m = \frac{49.85+12.46}{2} = 31.16 \text{ kpsi}$$

Eqs. (6-66) and (6-67):

$$\sigma_a' = [(\sigma_a/0.85)^2 + 3\tau_a^2]^{1/2} = [(16.71/0.85)^2 + 3(18.70)^2]^{1/2} = 37.89 \text{ kpsi}$$
$$\sigma_m' = [\sigma_m{}^2 + 3\tau_m^2]^{1/2} = [(-27.85)^2 + 3(31.16)^2]^{1/2} = 60.73 \text{ kpsi}$$

Goodman:

Eq. (6-41): $n_f = \left(\frac{\sigma_a'}{S_e} + \frac{\sigma_m'}{S_{ut}}\right)^{-1} = \left(\frac{37.89}{42.5} + \frac{60.73}{145}\right)^{-1}$

$n_f = 0.76$

Since infinite life is not predicted, estimate a life from the *S-N* diagram. First, find an equivalent completely reversed stress (See Ex. 6-12).
Choosing the Goodman criterion,

Eq. (6-58): $\sigma_{ar} = \frac{\sigma_a'}{1-(\sigma_m'/S_{ut})} = \frac{37.89}{1-(60.73/145)} = 65.2 \text{ kpsi}$

Fig. 6-23: $f = 0.8$

Eq. (6-13): $a = \frac{(f\,S_{ut})^2}{S_e} = \frac{[0.8(145)]^2}{42.5} = 316.6$

Eq. (6-14): $b = -\frac{1}{3}\log\left(\frac{f\,S_{ut}}{S_e}\right) = -\frac{1}{3}\log\left(\frac{0.8(145)}{42.5}\right) = -0.1454$

Eq. (6-15): $N = \left(\frac{\sigma_{ar}}{a}\right)^{1/b} = \left(\frac{65.2}{316.6}\right)^{\frac{1}{-0.1454}} = 52\,460 \text{ cycles}$

$N = 52\,500$ cycles *Ans.*

6-59 For AISI 1020 CD, From Table A-20, $S_y = 390$ MPa, $S_{ut} = 470$ MPa. Given: $S_e = 175$ MPa.

First Loading: $\left(\sigma_m\right)_1 = \dfrac{360+160}{2} = 260 \text{ MPa}, \quad \left(\sigma_a\right)_1 = \dfrac{360-160}{2} = 100 \text{ MPa}$

Goodman, Eq. (6-58):

$$\left(\sigma_a\right)_{e1} = \frac{\left(\sigma_a\right)_1}{1-\left(\sigma_m\right)_1/S_{ut}} = \frac{100}{1-260/470} = 223.8 \text{ MPa} > S_e \ \therefore \text{ finite life}$$

Fig. 6-23: Off the graph, so let $f = 0.9$.

$$a = \frac{\left[0.9(470)\right]^2}{175} = 1022.5 \text{ MPa}$$

$$b = -\frac{1}{3}\log\frac{0.9(470)}{175} = -0.127\,767$$

$$N = \left(\frac{223.8}{1022.5}\right)^{-1/0.127767} = 145\ 920 \text{ cycles}$$

Second loading: $\left(\sigma_m\right)_2 = \dfrac{320+(-200)}{2} = 60 \text{ MPa}, \quad \left(\sigma_a\right)_2 = \dfrac{320-(-200)}{2} = 260 \text{ MPa}$

$$\left(\sigma_a\right)_{e2} = \frac{260}{1-60/470} = 298.0 \text{ MPa}$$

(**a**) Miner's method: $N_2 = \left(\dfrac{298.0}{1022.5}\right)^{-1/0.127767} = 15\ 520 \text{ cycles}$

$$\frac{n_1}{N_1} + \frac{n_2}{N_2} = 1 \quad \Rightarrow \quad \frac{80\ 000}{145\ 920} + \frac{n_2}{15\ 520} = 1 \quad \Rightarrow \quad n_2 = 7000 \text{ cycles} \quad \textit{Ans.}$$

(**b**) Manson's method: The number of cycles remaining after the first loading

$N_{\text{remaining}} = 145\ 920 - 80\ 000 = 65\ 920$ cycles

Two data points: 0.9(470) MPa, 10^3 cycles
223.8 MPa, 65 920 cycles

$$\frac{0.9(470)}{223.8} = \frac{a_2\left(10^3\right)^{b_2}}{a_2\left(65\ 920\right)^{b_2}}$$

$$1.8901 = \left(0.015170\right)^{b_2}$$

$$b_2 = \frac{\log 1.8901}{\log 0.015170} = -0.151\ 997$$

$$a_2 = \frac{223.8}{\left(65\ 920\right)^{-0.151\ 997}} = 1208.7 \text{ MPa}$$

$$n_2 = \left(\frac{298.0}{1208.7}\right)^{1/-0.151\ 997} = 10\ 000 \text{ cycles} \qquad \textit{Ans.}$$

6-60 Given: S_e = 50 kpsi, S_{ut} = 140 kpsi, f =0.8. Using Miner's method,

$$a = \frac{\left[0.8(140)\right]^2}{50} = 250.88 \text{ kpsi}$$

$$b = -\frac{1}{3}\log\frac{0.8(140)}{50} = -0.116\ 749$$

$$\sigma_1 = 95 \text{ kpsi}, \qquad N_1 = \left(\frac{95}{250.88}\right)^{1/-0.116\ 749} = 4100 \text{ cycles}$$

$$\sigma_2 = 80 \text{ kpsi}, \qquad N_2 = \left(\frac{80}{250.88}\right)^{1/-0.116\ 749} = 17\ 850 \text{ cycles}$$

$$\sigma_3 = 65 \text{ kpsi}, \qquad N_3 = \left(\frac{65}{250.88}\right)^{1/-0.116\ 749} = 105\ 700 \text{ cycles}$$

$$\frac{0.2N}{4100} + \frac{0.5N}{17\ 850} + \frac{0.3N}{105\ 700} = 1 \quad \Rightarrow \quad N = 12\ 600 \text{ cycles} \quad \textit{Ans.}$$

6-61 Given: S_{ut} = 530 MPa, S_e = 210 MPa, and f = 0.9.

(a) Miner's method

$$a = \frac{\left[0.9(530)\right]^2}{210} = 1083.47 \text{ MPa}$$

$$b = -\frac{1}{3}\log\frac{0.9(530)}{210} = -0.118\ 766$$

$$\sigma_1 = 350 \text{ MPa}, \quad N_1 = \left(\frac{350}{1083.47}\right)^{1/-0.118\ 766} = 13\ 550 \text{ cycles}$$

$$\sigma_2 = 260 \text{ MPa}, \quad N_2 = \left(\frac{260}{1083.47}\right)^{1/-0.118\,766} = 165\,600 \text{ cycles}$$

$$\sigma_3 = 225 \text{ MPa}, \quad N_3 = \left(\frac{225}{1083.47}\right)^{1/-0.118\,766} = 559\,400 \text{ cycles}$$

$$\frac{n_1}{N_1} + \frac{n_2}{N_2} + \frac{n_3}{N_3} = 1$$

$$\frac{5000}{13\,550} + \frac{50\,000}{165\,600} + \frac{n_3}{559\,400} = 184\,100 \text{ cycles} \quad \textit{Ans.}$$

(**b**) Manson's method:

The life remaining after the first series of cycling is N_{R1} = 13 550 − 5000 = 8550 cycles. The two data points required to define $S'_{e,1}$ are [0.9(530), 10^3] and (350, 8550).

$$\frac{0.9(530)}{350} = \frac{a_2\left(10^3\right)^{b_2}}{a_2\left(8550\right)^{b_2}} \quad \Rightarrow \quad 1.3629 = \left(0.11696\right)^{b_2}$$

$$b_2 = \frac{\log(1.362\,9)}{\log(0.116\,96)} = -0.144\,280$$

$$a_2 = \frac{350}{(8550)^{-0.144\,280}} = 1292.3 \text{ MPa}$$

$$N_2 = \left(\frac{260}{1292.3}\right)^{-1/0.144\,280} = 67\,090 \text{ cycles}$$

$$N_{R2} = 67\,090 - 50\,000 = 17\,090 \text{ cycles}$$

$$\frac{0.9(530)}{260} = \frac{a_3\left(10^3\right)^{b_3}}{a_3\left(17\,090\right)^{b_3}} \quad \Rightarrow \quad 1.834\,6 = \left(0.058\,514\right)^{b_2}$$

$$b_3 = \frac{\log(1.834\,6)}{\log(0.058\,514)} = -0.213\,785, \quad a_3 = \frac{260}{(17\,090)^{-0.213\,785}} = 2088.7 \text{ MPa}$$

$$N_3 = \left(\frac{225}{2088.7}\right)^{-1/0.213\,785} = 33\,610 \text{ cycles} \quad \textit{Ans.}$$

6-62 Given: S_e = 45 kpsi, S_{ut} = 85 kpsi, f = 0.86, and σ_a = 35 kpsi and σ_m = 30 kpsi for 12 (10^3) cycles.

Goodman equivalent reversing stress, Eq. (6-58):

$$\sigma_{ar} = \frac{\sigma_a}{1-(\sigma_m / S_{ut})} = \frac{35}{1-(30/85)} = 54.09 \text{ kpsi}$$

Initial cycling

$$a = \frac{[0.86(85)]^2}{45} = 116.00 \text{ kpsi}$$

$$b = -\frac{1}{3}\log\frac{0.86(85)}{45} = -0.070\ 235$$

$$\sigma_1 = 54.09 \text{ kpsi}, \qquad N_1 = \left(\frac{54.09}{116.00}\right)^{1/-0.070\ 235} = 52\ 190 \text{ cycles}$$

(**a**) Miner's method: The number of remaining cycles at 54.09 kpsi is

$$N_{\text{remaining}} = 52\ 190 - 12\ 000 = 40\ 190 \text{ cycles.}$$

The new coefficients are $b' = b$, and $a' = S_f / N^b = 54.09/(40\ 190)^{-0.070\ 235} = 113.89$ kpsi.
The new endurance limit is

$$S'_{e,1} = a'N_e^{b'} = 113.89\left(10^6\right)^{-0.070\ 235} = 43.2 \text{ kpsi} \qquad \textit{Ans.}$$

(**b**) Manson's method: The number of remaining cycles at 54.09 kpsi is

$$N_{\text{remaining}} = 52\ 190 - 12\ 000 = 40\ 190 \text{ cycles.}$$

At 10^3 cycles,

$$S_f = 0.86(85) = 73.1 \text{ kpsi.}$$

The new coefficients are

$$b' = [\log(73.1/54.09)]/\log(10^3/40\ 190) = -0.081\ 540$$

and $a' = \sigma_1/(N_{\text{remaining}})^{b'} = 54.09/(40\ 190)^{-0.081\ 540} = 128.39$ kpsi.
The new endurance limit is

$$S'_{e,1} = a'N_e^{b'} = 128.39\left(10^6\right)^{-0.081\ 540} = 41.6 \text{ kpsi} \qquad \textit{Ans.}$$
