Chapter 5

5-1 S_y = 350 MPa.

MSS: $\sigma_1 - \sigma_3 = S_y/n \quad \Rightarrow \quad n = \dfrac{S_y}{(\sigma_1 - \sigma_3)}$

DE: $\sigma' = \left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2} = \left(\sigma_x^2 - \sigma_x\sigma_y + \sigma_y^2 + 3\tau_{xy}^2\right)^{1/2}$

$$n = \frac{S_y}{\sigma'}$$

(a) MSS: σ_1 = 100 MPa, σ_2 = 100 MPa, σ_3 = 0

$$n = \frac{350}{100 - 0} = 3.5 \quad \textit{Ans.}$$

DE: $\sigma' = (100^2 - 100(100) + 100^2)^{1/2} = 100 \text{ MPa}, \quad n = \dfrac{350}{100} = 3.5 \quad \textit{Ans.}$

(b) MSS: σ_1 = 100 MPa, σ_2 = 50 MPa, σ_3 = 0

$$n = \frac{350}{100 - 0} = 3.5 \quad \textit{Ans.}$$

DE: $\sigma' = (100^2 - 100(50) + 50^2)^{1/2} = 86.6 \text{ MPa}, \quad n = \dfrac{350}{86.6} = 4.04 \quad \textit{Ans.}$

(c) $\sigma_A,\ \sigma_B = \dfrac{100}{2} \pm \sqrt{\left(\dfrac{100}{2}\right)^2 + (-75)^2} = 140,\ -40 \text{ MPa}$

$$\sigma_1 = 140,\ \sigma_2 = 0,\ \sigma_3 = -40 \text{ MPa}$$

MSS: $n = \dfrac{350}{140 - (-40)} = 1.94 \quad \textit{Ans.}$

DE: $\sigma' = \left[100^2 + 3\left(-75^2\right)\right]^{1/2} = 164 \text{ MPa}, \quad n = \dfrac{350}{164} = 2.13 \quad \textit{Ans.}$

(d) $\sigma_A, \sigma_B = \dfrac{-50 - 75}{2} \pm \sqrt{\left(\dfrac{-50 + 75}{2}\right)^2 + (-50)^2} = -11.0, -114.0 \text{ MPa}$

$$\sigma_1 = 0,\ \sigma_2 = -11.0,\ \sigma_3 = -114.0 \text{ MPa}$$

MSS: $n = \dfrac{350}{0 - (-114.0)} = 3.07 \quad \textit{Ans.}$

DE: $\sigma' = [(-50)^2 - (-50)(-75) + (-75)^2 + 3(-50)^2]^{1/2} = 109.0 \text{ MPa}$

$$n = \frac{350}{109.0} = 3.21 \quad \textit{Ans.}$$

(e) $\sigma_A, \sigma_B = \dfrac{100 + 20}{2} \pm \sqrt{\left(\dfrac{100 - 20}{2}\right)^2 + \left(-20\right)^2} = 104.7,\ 15.3 \text{ MPa}$

$$\sigma_1 = 104.7,\ \sigma_2 = 15.3,\ \sigma_3 = 0 \text{ MPa}$$

MSS: $$n = \frac{350}{104.7 - 0} = 3.34 \quad \textit{Ans.}$$

DE: $$\sigma' = \left[100^2 - 100(20) + 20^2 + 3\left(-20\right)^2\right]^{1/2} = 98.0 \text{ MPa}$$

$$n = \frac{350}{98.0} = 3.57 \quad \textit{Ans.}$$

5-2 S_y = 350 MPa.

MSS: $\sigma_1 - \sigma_3 = S_y/n \quad \Rightarrow \quad n = \dfrac{S_y}{(\sigma_1 - \sigma_3)}$

DE: $\sigma' = \left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2} = \dfrac{S_y}{n} \quad \Rightarrow \quad n = \dfrac{S_y}{\sigma'}$

(a) MSS: $$\sigma_1 = 100 \text{ MPa},\ \sigma_3 = 0 \ \Rightarrow \ n = \frac{350}{100 - 0} = 3.5 \quad \textit{Ans.}$$

DE: $$n = \frac{350}{[100^2 - (100)(100) + 100^2]^{1/2}} = 3.5 \quad \textit{Ans.}$$

(b) MSS: $$\sigma_1 = 100\ ,\ \sigma_3 = -100 \text{ MPa} \ \Rightarrow \ n = \frac{350}{100 - (-100)} = 1.75 \quad \textit{Ans.}$$

DE: $$n = \frac{350}{\left[100^2 - (100)(-100) + \left(-100\right)^2\right]^{1/2}} = 2.02 \quad \textit{Ans.}$$

(c) MSS: $$\sigma_1 = 100 \text{ MPa},\ \sigma_3 = 0 \ \Rightarrow \ n = \frac{350}{100 - 0} = 3.5 \quad \textit{Ans.}$$

DE: $$n = \frac{350}{\left[100^2 - (100)(50) + 50^2\right]^{1/2}} = 4.04 \quad \textit{Ans.}$$

(d) MSS: $$\sigma_1 = 100,\ \sigma_3 = -50 \text{ MPa} \ \Rightarrow \ n = \frac{350}{100 - (-50)} = 2.33 \quad \textit{Ans.}$$

DE: $$n = \frac{350}{\left[100^2 - (100)(-50) + \left(-50\right)^2\right]^{1/2}} = 2.65 \quad \textit{Ans.}$$

(e) MSS: $$\sigma_1 = 0,\ \sigma_3 = -100 \text{ MPa} \ \Rightarrow \ n = \frac{350}{0 - (-100)} = 3.5 \quad \textit{Ans.}$$

DE: $$n = \frac{350}{\left[\left(-50\right)^2 - (-50)(-100) + \left(-100\right)^2\right]^{1/2}} = 4.04 \quad \textit{Ans.}$$

5-3 From Table A-20, S_y = 37.5 kpsi

MSS: $\sigma_1 - \sigma_3 = S_y/n \quad \Rightarrow \quad n = \dfrac{S_y}{(\sigma_1 - \sigma_3)}$

DE: $\sigma' = \left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2} = \left(\sigma_x^2 - \sigma_x\sigma_y + \sigma_y^2 + 3\tau_{xy}^2\right)^{1/2}$

$n = \dfrac{S_y}{\sigma'}$

(a) MSS: $\sigma_1 = 25$ kpsi, $\sigma_3 = 0 \quad \Rightarrow \quad n = \dfrac{37.5}{25-0} = 1.5$ *Ans.*

DE: $n = \dfrac{37.5}{\left[25^2 - (25)(15) + 15^2\right]^{1/2}} = 1.72$ *Ans.*

(b) MSS: $\sigma_1 = 15$ kpsi, $\sigma_3 = -15 \quad \Rightarrow \quad n = \dfrac{37.5}{15-(-15)} = 1.25$ *Ans.*

DE: $n = \dfrac{37.5}{\left[15^2 - (15)(-15) + (-15)^2\right]^{1/2}} = 1.44$ *Ans.*

(c) $\sigma_A,\ \sigma_B = \dfrac{20}{2} \pm \sqrt{\left(\dfrac{20}{2}\right)^2 + (-10)^2} = 24.1,\ -4.1$ kpsi

$\sigma_1 = 24.1,\ \sigma_2 = 0,\ \sigma_3 = -4.1$ kpsi

MSS: $n = \dfrac{37.5}{24.1-(-4.1)} = 1.33$ *Ans.*

DE: $\sigma' = \left[20^2 + 3\left(-10^2\right)\right]^{1/2} = 26.5$ kpsi $\quad \Rightarrow \quad n = \dfrac{37.5}{26.5} = 1.42$ *Ans.*

(d) $\sigma_A, \sigma_B = \dfrac{-12+15}{2} \pm \sqrt{\left(\dfrac{-12-15}{2}\right)^2 + (-9)^2} = 17.7, -14.7$ kpsi

$\sigma_1 = 17.7,\ \sigma_2 = 0,\ \sigma_3 = -14.7$ kpsi

MSS: $n = \dfrac{37.5}{17.7-(-14.7)} = 1.16$ *Ans.*

DE: $\sigma' = \left[(-12)^2 - (-12)(15) + 15^2 + 3(-9)^2\right]^{1/2} = 28.1$ kpsi

$n = \dfrac{37.5}{28.1} = 1.33$ *Ans.*

(e) $\sigma_A, \sigma_B = \dfrac{-24-24}{2} \pm \sqrt{\left(\dfrac{-24+24}{2}\right)^2 + \left(-15\right)^2} = -9,\ -39$ kpsi

$\sigma_1 = 0,\ \sigma_2 = -9,\ \sigma_3 = -39$ kpsi

MSS: $n = \dfrac{37.5}{0-(-39)} = 0.96$ *Ans.*

DE: $$\sigma' = \left[(-24)^2 - (-24)(-24) + (-24)^2 + 3(-15)^2 \right]^{1/2} = 35.4 \text{ kpsi}$$

$$n = \frac{37.5}{35.4} = 1.06 \quad \textit{Ans.}$$

5-4 From Table A-20, $S_y = 47$ kpsi.

MSS: $\sigma_1 - \sigma_3 = S_y / n \quad \Rightarrow \quad n = \dfrac{S_y}{(\sigma_1 - \sigma_3)}$

DE: $\sigma' = \left(\sigma_A^2 - \sigma_A \sigma_B + \sigma_B^2\right)^{1/2} = \dfrac{S_y}{n} \quad \Rightarrow \quad n = \dfrac{S_y}{\sigma'}$

(a) MSS: $\sigma_1 = 30 \text{ kpsi}, \ \sigma_3 = 0 \ \Rightarrow \ n = \dfrac{47}{30 - 0} = 1.57 \quad \textit{Ans.}$

DE: $$n = \frac{47}{[30^2 - (30)(30) + 30^2]^{1/2}} = 1.57 \quad \textit{Ans.}$$

(b) MSS: $\sigma_1 = 30\ , \ \sigma_3 = -30 \text{ kpsi} \ \Rightarrow \ n = \dfrac{47}{30 - (-30)} = 0.78 \quad \textit{Ans.}$

DE: $$n = \frac{47}{\left[30^2 - (30)(-30) + (-30)^2\right]^{1/2}} = 0.90 \quad \textit{Ans.}$$

(c) MSS: $\sigma_1 = 30 \text{ kpsi}, \ \sigma_3 = 0 \ \Rightarrow \ n = \dfrac{47}{30 - 0} = 1.57 \quad \textit{Ans.}$

DE: $$n = \frac{47}{\left[30^2 - (30)(15) + 15^2\right]^{1/2}} = 1.81 \quad \textit{Ans.}$$

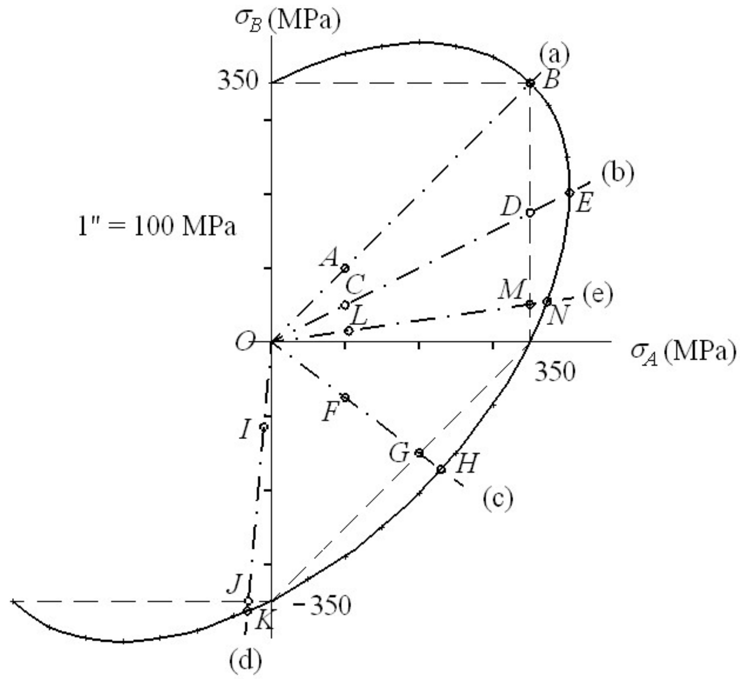
(d) MSS: $\sigma_1 = 0, \ \sigma_3 = -30 \text{ kpsi} \ \Rightarrow \ n = \dfrac{47}{0 - (-30)} = 1.57 \quad \textit{Ans.}$

DE: $$n = \frac{47}{\left[(-30)^2 - (-30)(-15) + (-15)^2\right]^{1/2}} = 1.81 \quad \textit{Ans.}$$

(e) MSS: $\sigma_1 = 10, \ \sigma_3 = -50 \text{ kpsi} \ \Rightarrow \ n = \dfrac{47}{10 - (-50)} = 0.78 \quad \textit{Ans.}$

DE: $$n = \frac{47}{\left[(-50)^2 - (-50)(10) + 10^2\right]^{1/2}} = 0.84 \quad \textit{Ans.}$$

5-5 Note: The drawing in this manual may not be to the scale of original drawing. The measurements were taken from the original drawing.



(**a**) MSS and DE:

$$n = \frac{OB}{OA} = \frac{4.95"}{1.41"} = 3.51 \quad Ans.$$

(**b**) MSS:

$$n = \frac{OD}{OC} = \frac{3.91"}{1.12"} = 3.49 \quad Ans.$$

DE:

$$n = \frac{OE}{OC} = \frac{4.51"}{1.12"} = 4.03 \quad Ans.$$

(**c**) MSS:

$$n = \frac{OG}{OF} = \frac{2.50"}{1.25"} = 2.00 \quad Ans.$$

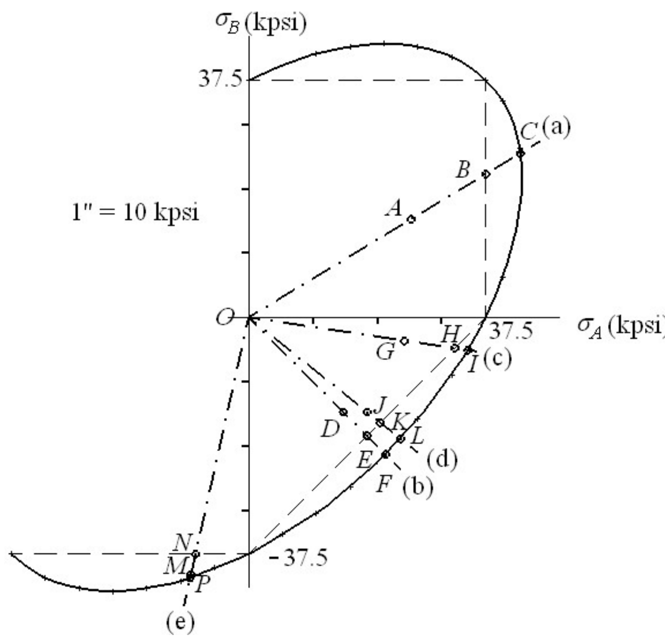
DE: $n = \frac{OH}{OF} = \frac{2.86"}{1.25"} = 2.29 \quad Ans.$

(**d**) MSS: $n = \frac{OJ}{OI} = \frac{3.51"}{1.15"} = 3.05 \quad Ans.$, DE: $n = \frac{OK}{OI} = \frac{3.65"}{1.15"} = 3.17 \quad Ans.$

(**e**) MSS: $n = \frac{OM}{OL} = \frac{3.54"}{1.06"} = 3.34 \quad Ans.$, DE: $n = \frac{ON}{OL} = \frac{3.77"}{1.06"} = 3.56 \quad Ans.$

5-6 Note: The drawing in this manual may not be to the scale of original drawing. The measurements were taken from the original drawing.

(**a**) σ_A = 25 kpsi, σ_B = 15 kpsi



MSS:

$$n = \frac{OB}{OA} = \frac{4.37"}{2.92"} = 1.50 \quad \textit{Ans.}$$

DE:

$$n = \frac{OC}{OA} = \frac{5.02"}{2.92"} = 1.72 \quad \textit{Ans.}$$

(**b**) σ_A = 15 kpsi, σ_B = − 15 kpsi

MSS:

$$n = \frac{OE}{OD} = \frac{2.66"}{2.12"} = 1.25 \quad \textit{Ans.}$$

DE:

$$n = \frac{OF}{OD} = \frac{3.05"}{2.12"} = 1.44 \quad \textit{Ans.}$$

(**c**) $$\sigma_A,\ \sigma_B = \frac{20}{2} \pm \sqrt{\left(\frac{20}{2}\right)^2 + (-10)^2} = 24.1,\ -4.1 \text{ kpsi}$$

MSS: $n = \dfrac{OH}{OG} = \dfrac{3.25"}{2.43"} = 1.34$ *Ans.* DE: $n = \dfrac{OI}{OG} = \dfrac{3.46"}{2.43"} = 1.42$ *Ans.*

(**d**) $$\sigma_A, \sigma_B = \frac{-12+15}{2} \pm \sqrt{\left(\frac{-12-15}{2}\right)^2 + (-9)^2} = 17.7, -14.7 \text{ MPa}$$

MSS: $n = \dfrac{OK}{OJ} = \dfrac{2.67"}{2.30"} = 1.16$ *Ans.* DE: $n = \dfrac{OL}{OJ} = \dfrac{3.06"}{2.30"} = 1.33$ *Ans.*

(**e**) $$\sigma_A, \sigma_B = \frac{-24-24}{2} \pm \sqrt{\left(\frac{-24+24}{2}\right)^2 + \left(-15\right)^2} = -9,\ -39 \text{ kpsi}$$

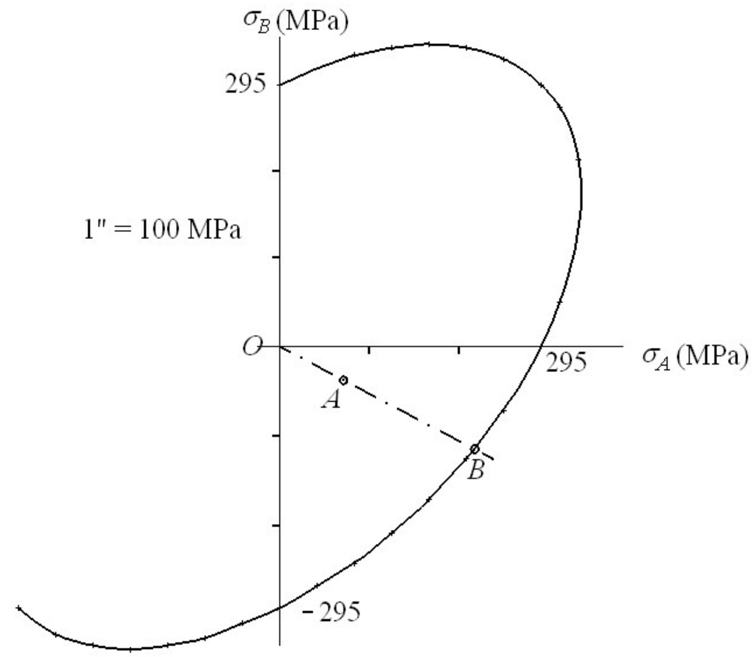
MSS: $n = \dfrac{ON}{OM} = \dfrac{3.85"}{4.00"} = 0.96$ *Ans.* DE: $n = \dfrac{OP}{OM} = \dfrac{4.23"}{4.00"} = 1.06$ *Ans.*

5-7 S_y = 295 MPa, σ_A = 75 MPa, σ_B = – 35 MPa,

(**a**) $$n = \frac{S_y}{\left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2}} = \frac{295}{\left[75^2 - 75(-35) + (-35)^2\right]^{1/2}} = 3.03 \quad Ans.$$

(**b**) Note: The drawing in this manual may not be to the scale of original drawing. The measurements were taken from the original drawing.

$$n = \frac{OB}{OA} = \frac{2.50"}{0.83"} = 3.01 \quad Ans.$$

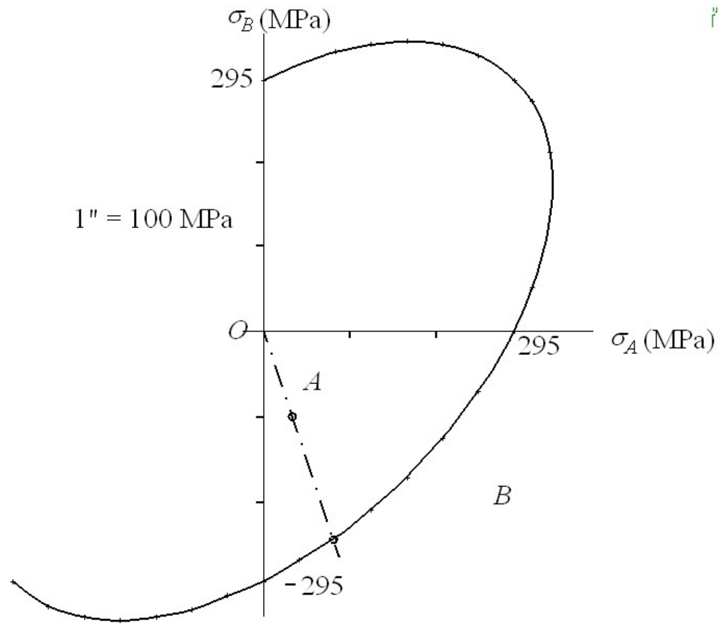


5-8 S_y = 295 MPa, σ_A = 30 MPa, σ_B = − 100 MPa,

(**a**) $$n = \frac{S_y}{\left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2}} = \frac{295}{\left[30^2 - 30(-100) + (-100)^2\right]^{1/2}} = 2.50 \quad Ans.$$

(**b**) Note: The drawing in this manual may not be to the scale of original drawing. The measurements were taken from the original drawing.

$$n = \frac{OB}{OA} = \frac{2.50"}{0.83"} = 3.01 \quad Ans.$$

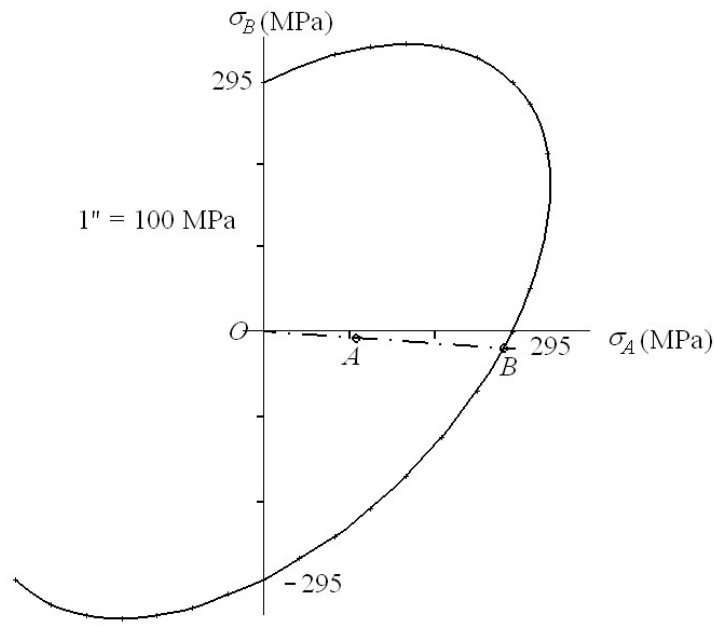


5-9 $S_y = 295$ MPa, $\sigma_A,\ \sigma_B = \dfrac{100}{2} \pm \sqrt{\left(\dfrac{100}{2}\right)^2 + (-25)^2} = 105.9,\ -5.9$ MPa

(**a**) $n = \dfrac{S_y}{\left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2}} = \dfrac{295}{\left[105.9^2 - 105.9(-5.9) + (-5.9)^2\right]^{1/2}} = 2.71$ *Ans.*

(**b**) Note: The drawing in this manual may not be to the scale of original drawing. The measurements were taken from the original drawing.

$n = \dfrac{OB}{OA} = \dfrac{2.87"}{1.06"} = 2.71$ *Ans.*

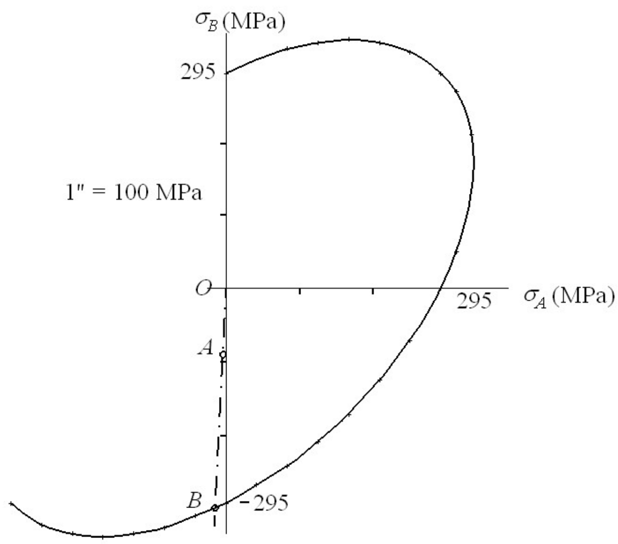


5-10 $S_y = 295$ MPa, $\sigma_A, \sigma_B = \frac{-30-65}{2} \pm \sqrt{\left(\frac{-30+65}{2}\right)^2 + 40^2} = -3.8, -91.2$ MPa

(**a**) $n = \frac{S_y}{\left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2}} = \frac{295}{\left[(-3.8)^2 - (-3.8)(-91.2) + (-91.2)^2\right]^{1/2}} = 3.30$ *Ans.*

(**b**) Note: The drawing in this manual may not be to the scale of original drawing. The measurements were taken from the original drawing.

$n = \frac{OB}{OA} = \frac{3.00"}{0.90"} = 3.33$ *Ans.*

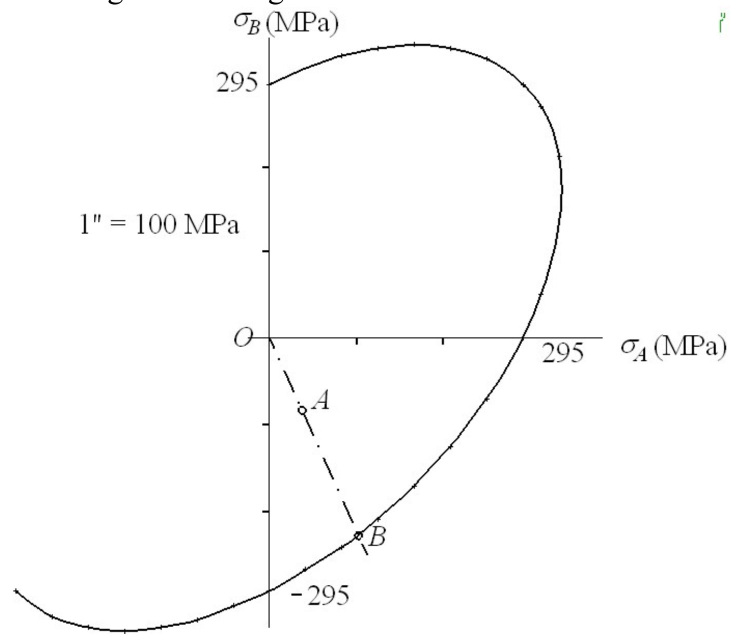


5-11 S_y = 295 MPa, $\sigma_A, \sigma_B = \frac{-80+30}{2} \pm \sqrt{\left(\frac{-80-30}{2}\right)^2 + (-10)^2} = 30.9, -80.9$ MPa

(**a**) $n = \frac{S_y}{\left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2}} = \frac{295}{\left[30.9^2 - 30.9(-80.9) + (-80.9)^2\right]^{1/2}} = 2.95$ *Ans.*

(**b**) Note: The drawing in this manual may not be to the scale of original drawing. The measurements were taken from the original drawing.

$$n = \frac{OB}{OA} = \frac{2.55''}{0.87''} = 2.93 \quad Ans.$$



5-12 S_{yt} = 60 kpsi, S_{yc} = 75 kpsi. Eq. (5-26) for yield is

$$n = \left(\frac{\sigma_1}{S_{yt}} - \frac{\sigma_3}{S_{yc}}\right)^{-1}$$

(**a**) $\sigma_1 = 25$ kpsi, $\sigma_3 = 0 \quad \Rightarrow \quad n = \left(\frac{25}{60} - \frac{0}{75}\right)^{-1} = 2.40$ *Ans.*

(**b**) $\sigma_1 = 15,\ \sigma_3 = -15$ kpsi $\quad \Rightarrow \quad n = \left(\frac{15}{60} - \frac{-15}{75}\right)^{-1} = 2.22$ *Ans.*

(**c**) $\sigma_A,\ \sigma_B = \dfrac{20}{2} \pm \sqrt{\left(\dfrac{20}{2}\right)^2 + (-10)^2} = 24.1,\ -4.1 \text{ kpsi}$,

$$\sigma_1 = 24.1,\ \sigma_2 = 0,\ \sigma_3 = -4.1 \text{ kpsi} \quad \Rightarrow \quad n = \left(\frac{24.1}{60} - \frac{-4.1}{75}\right)^{-1} = 2.19 \qquad Ans.$$

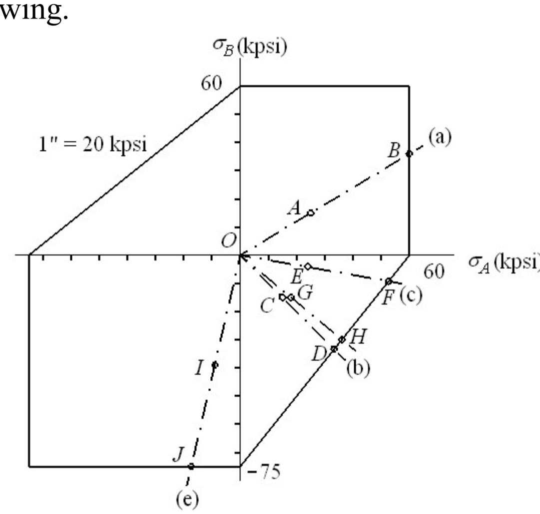
(**d**) $\sigma_A, \sigma_B = \dfrac{-12+15}{2} \pm \sqrt{\left(\dfrac{-12-15}{2}\right)^2 + (-9)^2} = 17.7, -14.7 \text{ kpsi}$

$$\sigma_1 = 17.7,\ \sigma_2 = 0,\ \sigma_3 = -14.7 \text{ kpsi} \quad \Rightarrow \quad n = \left(\frac{17.7}{60} - \frac{-14.7}{75}\right)^{-1} = 2.04 \qquad Ans.$$

(**e**) $\sigma_A, \sigma_B = \dfrac{-24-24}{2} \pm \sqrt{\left(\dfrac{-24+24}{2}\right)^2 + (-15)^2} = -9,\ -39 \text{ kpsi}$

$$\sigma_1 = 0,\ \sigma_2 = -9,\ \sigma_3 = -39 \text{ kpsi} \quad \Rightarrow \quad n = \left(\frac{0}{60} - \frac{-39}{75}\right)^{-1} = 1.92 \qquad Ans.$$

5-13 Note: The drawing in this manual may not be to the scale of original drawing. The measurements were taken from the original drawing.



(**a**) $\sigma_A = 25,\ \sigma_B = 15$ kpsi

$$n = \frac{OB}{OA} = \frac{3.49"}{1.46"} = 2.39 \quad Ans.$$

(**b**) $\sigma_A = 15,\ \sigma_B = -15$ kpsi

$$n = \frac{OD}{OC} = \frac{2.36"}{1.06"} = 2.23 \quad Ans.$$

(**c**)

$$\sigma_A,\ \sigma_B = \frac{20}{2} \pm \sqrt{\left(\frac{20}{2}\right)^2 + (-10)^2} = 24.1,\ -4.1 \text{ kpsi}$$

$$n = \frac{OF}{OE} = \frac{2.67"}{1.22"} = 2.19 \quad Ans.$$

(**d**)

$$\sigma_A, \sigma_B = \frac{-12+15}{2} \pm \sqrt{\left(\frac{-12-15}{2}\right)^2 + (-9)^2} = 17.7, -14.7 \text{ kpsi}$$

$$n = \frac{OH}{OG} = \frac{2.34"}{1.15"} = 2.03 \quad Ans.$$

(**e**)

$$\sigma_A, \sigma_B = \frac{-24-24}{2} \pm \sqrt{\left(\frac{-24+24}{2}\right)^2 + (-15)^2} = -9, \ -39 \text{ kpsi}$$

$$n = \frac{OJ}{OI} = \frac{3.85"}{2.00"} = 1.93 \quad Ans.$$

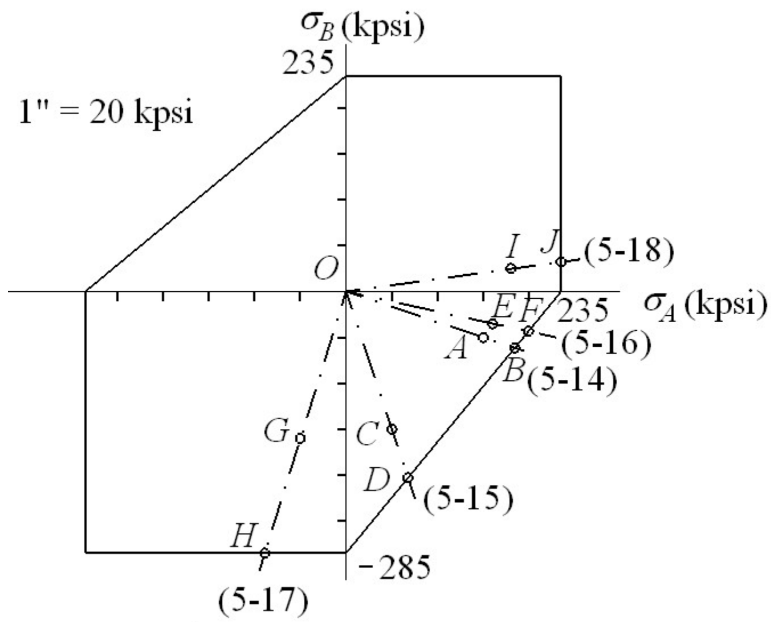
5-14 Since $\varepsilon_f > 0.05$, and $S_{yt} \neq S_{yc}$, the Coulomb-Mohr theory for ductile materials will be used.

(**a**) From Eq. (5-26),

$$n = \left(\frac{\sigma_1}{S_{yt}} - \frac{\sigma_3}{S_{yc}}\right)^{-1} = \left(\frac{150}{235} - \frac{-50}{285}\right)^{-1} = 1.23 \quad Ans.$$

(**b**) Plots for Problems 5-14 to 5-18 are found here. Note: The drawing in this manual may not be to the scale of original drawing. The measurements were taken from the original drawing.

$$n = \frac{OB}{OA} = \frac{1.94"}{1.58"} = 1.23 \quad Ans.$$



5-15 (**a**) From Eq. (5-26),

$$n = \left(\frac{\sigma_1}{S_{yt}} - \frac{\sigma_3}{S_{yc}}\right)^{-1} = \left(\frac{50}{235} - \frac{-150}{285}\right)^{-1} = 1.35 \quad Ans.$$

(**b**) The plot for this problem is found on the page for Prob. 5-14. Note: The drawing in this manual may not be to the scale of original drawing. The measurements were taken from the original drawing.

$$n = \frac{OD}{OC} = \frac{2.14"}{1.58"} = 1.35 \quad Ans.$$

5-16 $$\sigma_A,\ \sigma_B = \frac{125}{2} \pm \sqrt{\left(\frac{125}{2}\right)^2 + (-75)^2} = 160,\ -35 \text{ kpsi}$$

(**a**) From Eq. (5-26),

$$n = \left(\frac{\sigma_1}{S_{yt}} - \frac{\sigma_3}{S_{yc}}\right)^{-1} = \left(\frac{160}{235} - \frac{-35}{285}\right)^{-1} = 1.24 \quad Ans.$$

(**b**) The plot for this problem is found on the page for Prob. 5-14. Note: The drawing in this manual may not be to the scale of original drawing. The measurements were taken from the original drawing.

$$n = \frac{OF}{OE} = \frac{2.04"}{1.64"} = 1.24 \quad Ans.$$

5-17 $$\sigma_A,\ \sigma_B = \frac{-80-125}{2} \pm \sqrt{\left(\frac{-80+125}{2}\right)^2 + 50^2} = -47.7,\ -157.3 \text{ kpsi}$$

(**a**) From Eq. (5-26),

$$n = \left(\frac{\sigma_1}{S_{yt}} - \frac{\sigma_3}{S_{yc}}\right)^{-1} = \left(\frac{0}{235} - \frac{-157.3}{285}\right)^{-1} = 1.81 \quad Ans.$$

(**b**) The plot for this problem is found on the page for Prob. 5-14. Note: The drawing in this manual may not be to the scale of original drawing. The measurements were taken from the original drawing.

$$n = \frac{OH}{OG} = \frac{2.99"}{1.64"} = 1.82 \quad Ans.$$

5-18 $$\sigma_A,\ \sigma_B = \frac{125+80}{2} \pm \sqrt{\left(\frac{125-80}{2}\right)^2 + (-75)^2} = 180.8,\ 24.2 \text{ kpsi}$$

(**a**) From Eq. (5-26),

$$n=\left(\frac{\sigma_1}{S_{yt}}-\frac{\sigma_3}{S_{yc}}\right)^{-1}=\left(\frac{180.8}{235}-\frac{0}{285}\right)^{-1}=1.30 \quad Ans.$$

(**b**) The plot for this problem is found on the page for Prob. 5-14. Note: The drawing in this manual may not be to the scale of original drawing. The measurements were taken from the original drawing.

$$n=\frac{OJ}{OI}=\frac{2.37"}{1.83"}=1.30 \quad Ans.$$

5-19 S_{ut} = 30 kpsi, S_{uc} = 90 kpsi

BCM: Eqs. (5-31), MM: Eqs. (5-32)

(**a**) σ_A = 25 kpsi, σ_B = 15 kpsi

BCM : Eq. (5-31*a*), $n=\frac{S_{ut}}{\sigma_A}=\frac{30}{25}=1.2 \quad Ans.$

MM: Eq. (5-32*a*), $n=\frac{S_{ut}}{\sigma_A}=\frac{30}{25}=1.2 \quad Ans.$

(**b**) σ_A = 15 kpsi, σ_B = −15 kpsi,

BCM: Eq. (5-31*a*), $n=\left(\frac{15}{30}-\frac{-15}{90}\right)^{-1}=1.5 \quad Ans.$

MM: $\sigma_A \ge 0 \ge \sigma_B$, and $|\sigma_B/\sigma_A| \le 1$, Eq. (5-32a), $n=\frac{S_{ut}}{\sigma_A}=\frac{30}{15}=2.0 \quad Ans.$

(**c**) $$\sigma_A,\ \sigma_B=\frac{20}{2}\pm\sqrt{\left(\frac{20}{2}\right)^2+(-10)^2}=24.14,\ -4.14 \text{ kpsi}$$

BCM: Eq. (5-31*b*), $n=\left(\frac{24.14}{30}-\frac{-4.14}{90}\right)^{-1}=1.18 \quad Ans.$

MM: $\sigma_A \ge 0 \ge \sigma_B$, and $|\sigma_B/\sigma_A| \le 1$, Eq. (5-32*a*), $n=\frac{S_{ut}}{\sigma_A}=\frac{30}{24.14}=1.24 \quad Ans.$

(**d**) $$\sigma_A,\ \sigma_B=\frac{-15+10}{2}\pm\sqrt{\left(\frac{-15-10}{2}\right)^2+(-15)^2}=17.03,\ -22.03 \text{ kpsi}$$

BCM: Eq. (5-31*b*), $n = \left(\dfrac{17.03}{30} - \dfrac{-22.03}{90} \right)^{-1} = 1.23$ *Ans.*

MM: $\sigma_A \geq 0 \geq \sigma_B$, and $|\sigma_B / \sigma_A| \geq 1$, Eq. (5-32*b*),

$$n = \left[\frac{(S_{uc} - S_{ut})\sigma_A}{S_{uc} S_{ut}} - \frac{\sigma_B}{S_{uc}} \right]^{-1} = \left[\frac{(90-30)17.03}{90(30)} - \frac{-22.03}{90} \right]^{-1} = 1.60 \quad Ans.$$

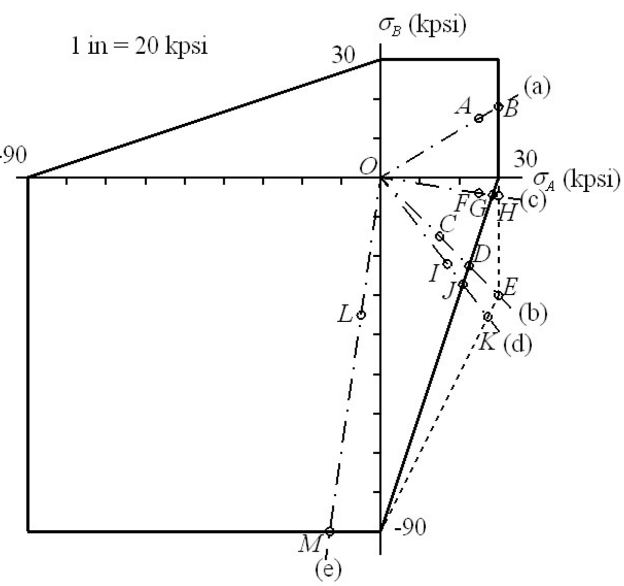
(**e**) $\sigma_A,\ \sigma_B = \dfrac{-20-20}{2} \pm \sqrt{\left(\dfrac{-20+20}{2}\right)^2 + (-15)^2} = -5,\ -35$ kpsi

BCM: Eq. (5-31*c*), $n = -\dfrac{S_{uc}}{\sigma_B} = -\dfrac{90}{-35} = 2.57$ *Ans.*

MM: Eq. (5-32*c*), $n = -\dfrac{S_{uc}}{\sigma_B} = -\dfrac{90}{-35} = 2.57$ *Ans.*

5-20 Note: The drawing in this manual may not be to the scale of original drawing. The measurements were taken from the original drawing.

(**a**) $\sigma_A = 25$, $\sigma_B = 15$ kpsi



BCM & MM:

$n = \dfrac{OB}{OA} = \dfrac{1.74"}{1.46"} = 1.19$ *Ans.*

(**b**) $\sigma_A = 15$, $\sigma_B = -15$ kpsi

BCM: $n = \dfrac{OC}{OD} = \dfrac{1.59"}{1.06"} = 1.5$ *Ans.*

MM: $n = \dfrac{OE}{OC} = \dfrac{2.12"}{1.06"} = 2.0$ *Ans.*

(**c**) $\sigma_A,\ \sigma_B = \dfrac{20}{2} \pm \sqrt{\left(\dfrac{20}{2}\right)^2 + (-10)^2}$

$= 24.14,\ -4.14$ kpsi

BCM: $n = \dfrac{OG}{OF} = \dfrac{1.44"}{1.22"} = 1.18$ *Ans.*

MM: $n = \dfrac{OH}{OF} = \dfrac{1.52"}{1.22"} = 1.25$ *Ans.*

(d) $\sigma_A,\ \sigma_B = \dfrac{-15+10}{2} \pm \sqrt{\left(\dfrac{-15-10}{2}\right)^2 + (-15)^2} = 17.03,\ -22.03 \text{ kpsi}$

BCM: $n = \dfrac{OJ}{OI} = \dfrac{1.72"}{1.39"} = 1.24 \quad Ans.$

MM: $n = \dfrac{OK}{OI} = \dfrac{2.24"}{1.39"} = 1.61 \quad Ans.$

(e) $\sigma_A,\ \sigma_B = \dfrac{-20-20}{2} \pm \sqrt{\left(\dfrac{-20+20}{2}\right)^2 + (-15)^2} = -5,\ -35 \text{ kpsi}$

BCM and MM: $n = \dfrac{OM}{OL} = \dfrac{4.55"}{1.77"} = 2.57 \quad Ans.$

5-21 From Table A-24, S_{ut} = 31 kpsi, S_{uc} = 109 kpsi

BCM: Eqs. (5-31), MM: Eqs. (5-32)

(**a**) σ_A = 15, σ_B = 10 kpsi.

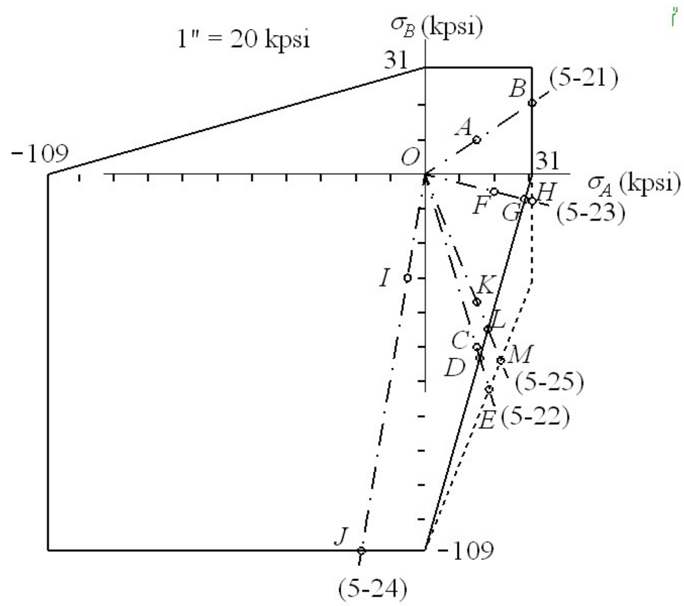
BCM: Eq. (5-31*a*), $n = \dfrac{S_{ut}}{\sigma_A} = \dfrac{31}{15} = 2.07 \quad Ans.$

MM: Eq. (5-32*a*), $n = \dfrac{S_{ut}}{\sigma_A} = \dfrac{31}{15} = 2.07 \quad Ans.$

(**b**), (**c**) The plot is shown below is for Probs. 5-21 to 5-25. Note: The drawing in this manual may not be to the scale of original drawing. The measurements were taken from the original drawing.

BCM and MM:

$n = \dfrac{OB}{OA} = \dfrac{1.86"}{0.90"} = 2.07 \quad Ans.$



5-22 S_{ut} = 31 kpsi, S_{uc} = 109 kpsi

BCM: Eq. (5-31), MM: Eqs. (5-32)

(**a**) σ_A = 15, σ_B = – 50 kpsi, $|\sigma_B / \sigma_A| > 1$

BCM: Eq. (5-31*b*), $n = \left(\dfrac{15}{31} - \dfrac{-50}{109} \right)^{-1} = 1.06$ *Ans.*

MM: Eq. (5-32*b*), $n = \left[\dfrac{(S_{uc} - S_{ut})\sigma_A}{S_{uc}S_{ut}} - \dfrac{\sigma_B}{S_{uc}} \right]^{-1} = \left[\dfrac{(109-31)15}{109(31)} - \dfrac{-50}{109} \right]^{-1} = 1.24$ *Ans.*

(**b**), (**c**) The plot is shown in the solution to Prob. 5-21.

BCM: $n = \dfrac{OD}{OC} = \dfrac{2.78"}{2.61"} = 1.07$ *Ans.*

MM: $n = \dfrac{OE}{OC} = \dfrac{3.25"}{2.61"} = 1.25$ *Ans.*

5-23 From Table A-24, S_{ut} = 31 kpsi, S_{uc} = 109 kpsi

BCM: Eq. (5-31), MM: Eqs. (5-32)

$$\sigma_A,\ \sigma_B = \frac{15}{2} \pm \sqrt{\left(\frac{15}{2}\right)^2 + (-10)^2} = 20,\ -5 \text{ kpsi}$$

(**a**) BCM: Eq. (5-32*b*), $n = \left(\dfrac{20}{31} - \dfrac{-5}{109} \right)^{-1} = 1.45$ *Ans.*

MM: Eq. (5-32*a*), $n = \dfrac{S_{ut}}{\sigma_A} = \dfrac{31}{20} = 1.55$ *Ans.*

(**b**), (**c**) The plot is shown in the solution to Prob. 5-21.

BCM: $n = \dfrac{OG}{OF} = \dfrac{1.48"}{1.03"} = 1.44$ *Ans.*

MM: $n = \dfrac{OH}{OF} = \dfrac{1.60"}{1.03"} = 1.55$ *Ans.*

5-24 From Table A-24, S_{ut} = 31 kpsi, S_{uc} = 109 kpsi

BCM: Eq. (5-31), MM: Eqs. (5-32)

$$\sigma_A,\ \sigma_B = \frac{-10-25}{2} \pm \sqrt{\left(\frac{-10+25}{2}\right)^2 + (-10)^2} = -5,\ -30 \text{ kpsi}$$

(**a**) BCM: Eq. (5-31*c*), $n = -\dfrac{S_{uc}}{\sigma_B} - \dfrac{109}{-30} = 3.63 \quad Ans.$

MM: Eq. (5-32*c*), $\quad n = -\dfrac{S_{uc}}{\sigma_B} = -\dfrac{109}{-30} = 3.63 \quad Ans.$

(**b**), (**c**) The plot is shown in the solution to Prob. 5-21.

BCM and MM: $\quad n = \dfrac{OJ}{OI} = \dfrac{5.53''}{1.52''} = 3.64 \quad Ans.$

5-25 From Table A-24, S_{ut} = 31 kpsi, S_{uc} = 109 kpsi

BCM: Eq. (5-31), MM: Eqs. (5-32)

$$\sigma_A,\ \sigma_B = \frac{-35+13}{2} \pm \sqrt{\left(\frac{-35-13}{2}\right)^2 + (-10)^2} = 15,\ -37 \text{ kpsi}$$

(**a**) BCM: Eq. (5-31*b*), $\quad n = \left(\dfrac{15}{31} - \dfrac{-37}{109}\right)^{-1} = 1.21 \quad Ans.$

MM: Eq. (5-32*b*), $\quad n = \left[\dfrac{(S_{uc} - S_{ut})\sigma_A}{S_{uc}S_{ut}} - \dfrac{\sigma_B}{S_{uc}}\right]^{-1} = \left[\dfrac{(109-31)15}{109(31)} - \dfrac{-37}{109}\right]^{-1} = 1.46 \quad Ans.$

(**b**), (**c**) The plot is shown in the solution to Prob. 5-21.

BCM: $\quad n = \dfrac{OL}{OK} = \dfrac{2.42''}{2.00''} = 1.21 \quad Ans.$

MM: $\quad n = \dfrac{OM}{OK} = \dfrac{2.91''}{2.00''} = 1.46 \quad Ans.$

5-26 S_{ut} = 36 kpsi, S_{uc} = 35 kpsi

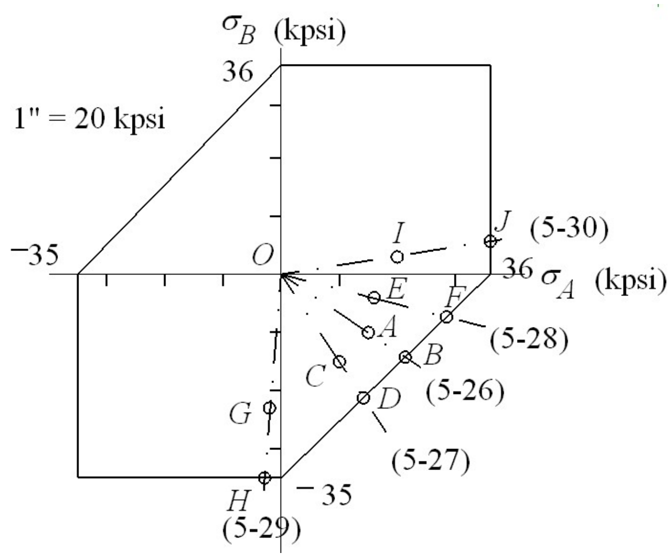
BCM: Eq. (5-31),

(**a**) σ_A = 15, σ_B = − 10 kpsi.

BCM: Eq. (5-31*b*), $\quad n = \left(\dfrac{15}{36} - \dfrac{-10}{35}\right)^{-1} = 1.42 \quad Ans.$

(**b**) The plot is shown below is for Probs. 5-26 to 5-30. Note: The drawing in this manual may not be to the scale of original drawing. The measurements were taken from the original drawing.

$$n = \frac{OB}{OA} = \frac{1.28''}{0.90''} = 1.42 \quad Ans.$$



5-27 S_{ut} = 36 kpsi, S_{uc} = 35 kpsi

BCM: Eq. (5-31),

(**a**) σ_A = 15, σ_B = − 15 kpsi.

BCM: Eq. (5-31*b*), $n = \left(\frac{10}{36} - \frac{-15}{35}\right)^{-1} = 1.42 \quad Ans.$

(**b**) The plot is shown in the solution to Prob. 5-26.

$$n = \frac{OD}{OC} = \frac{1.28''}{0.90''} = 1.42 \quad Ans.$$

5-28 S_{ut} = 36 kpsi, S_{uc} = 35 kpsi

BCM: Eq. (5-31),

(**a**) $\sigma_A,\ \sigma_B = \frac{12}{2} \pm \sqrt{\left(\frac{12}{2}\right)^2 + (-8)^2} = 16,\ -4$ kpsi

BCM: Eq. (5-31*b*), $n = \left(\frac{16}{36} - \frac{-4}{35}\right)^{-1} = 1.79 \quad Ans.$

(**b**) The plot is shown in the solution to Prob. 5-26.

$$n = \frac{OF}{OE} = \frac{1.47"}{0.82"} = 1.79 \quad Ans.$$

5-29 S_{ut} = 36 kpsi, S_{uc} = 35 kpsi

BCM: Eq. (5-31),

(**a**) $\sigma_A,\ \sigma_B = \frac{-10-15}{2} \pm \sqrt{\left(\frac{-10+15}{2}\right)^2 + 10^2} = -2.2,\ -22.8 \text{ kpsi}$

BCM: Eq. (5-31*c*), $n = -\frac{35}{-22.8} = 1.54 \quad Ans.$

(**b**) The plot is shown in the solution to Prob. 5-26.

$$n = \frac{OH}{OG} = \frac{1.76"}{1.15"} = 1.53 \quad Ans.$$

5-30 S_{ut} = 36 kpsi, S_{uc} = 35 kpsi

BCM: Eq. (5-31),

(**a**) $\sigma_A,\ \sigma_B = \frac{15+8}{2} \pm \sqrt{\left(\frac{15-8}{2}\right)^2 + (-8)^2} = 20.2,\ 2.8 \text{ kpsi}$

BCM: Eq. (5-31*a*), $n = \frac{36}{20.2} = 1.78 \quad Ans.$

(**b**) The plot is shown in the solution to Prob. 5-26.

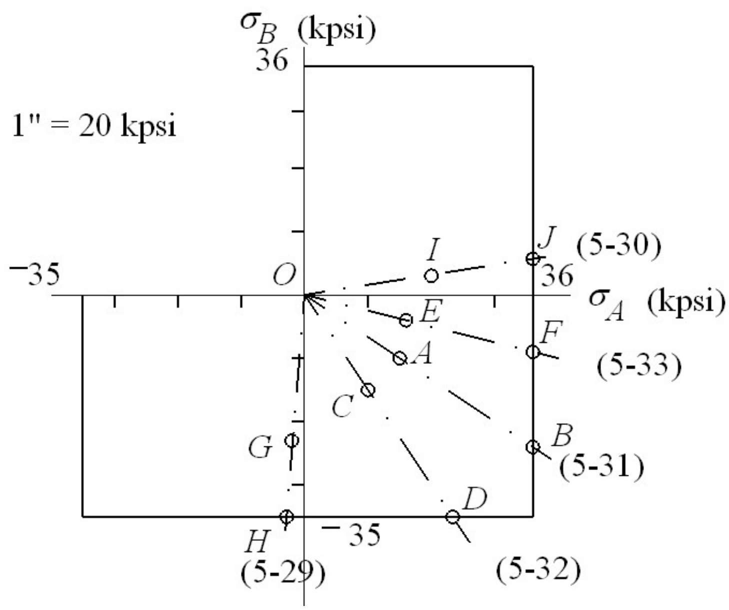
$$n = \frac{OJ}{OI} = \frac{1.82"}{1.02"} = 1.78 \quad Ans.$$

5-31 S_{ut} = 36 kpsi, S_{uc} = 35 kpsi. MM: Use Eq. (5-32). For this problem, MM reduces to the MNS theory.

(**a**) σ_A = 15, σ_B = – 10 kpsi. Eq. (5-32*a*), $n = \frac{S_{ut}}{\sigma_A} = \frac{36}{15} = 2.4 \quad Ans.$

(**b**) The plot on the next page is for Probs. 5-31 to 5-35. Note: The drawing in this manual may not be to the scale of original drawing. The measurements were taken from the original drawing.

$$n = \frac{OB}{OA} = \frac{2.16''}{0.90''} = 2.43 \quad Ans.$$



5-32 S_{ut} = 36 kpsi, S_{uc} = 35 kpsi. MM: Use Eq. (5-32).For this problem, MM reduces to the MNS theory.

(**a**) σ_A = 10, σ_B = – 15 kpsi. Eq. (5-32b) is not valid and must use Eq, (5-32c),

$$n = -\frac{S_{uc}}{\sigma_B} = -\frac{35}{-15} = 2.33 \quad Ans.$$

(**b**) The plot is shown in the solution to Prob. 5-31.

$$n = \frac{OD}{OC} = \frac{2.10''}{0.90''} = 2.33 \quad Ans.$$

5-33 S_{ut} = 36 kpsi, S_{uc} = 35 kpsi. MM: Use Eq. (5-32).For this problem, MM reduces to the MNS theory.

(**a**) $\sigma_A,\ \sigma_B = \frac{12}{2} \pm \sqrt{\left(\frac{12}{2}\right)^2 + (-8)^2} = 16,\ -4 \text{ kpsi}$

$$n = \frac{S_{ut}}{\sigma_A} = \frac{36}{16} = 2.25 \quad Ans.$$

(**b**) The plot is shown in the solution to Prob. 5-31.

$$n = \frac{OF}{OE} = \frac{1.86''}{0.82''} = 2.27 \quad Ans.$$

5-34 S_{ut} = 36 kpsi, S_{uc} = 35 kpsi. MM: Use Eq. (5-32).For this problem, MM reduces to the MNS theory.

(**a**) $\sigma_A,\ \sigma_B = \dfrac{-10-15}{2} \pm \sqrt{\left(\dfrac{-10+15}{2}\right)^2 + 10^2} = -2.2,\ -22.8$ kpsi

$$n = -\frac{S_{uc}}{\sigma_B} = -\frac{35}{-22.8} = 1.54 \quad Ans.$$

(**b**) The plot is shown in the solution to Prob. 5-31.

$$n = \frac{OH}{OG} = \frac{1.76"}{1.15"} = 1.53 \quad Ans.$$

5-35 S_{ut} = 36 kpsi, S_{uc} = 35 kpsi. MM: Use Eq. (5-32).For this problem, MM reduces to the MNS theory.

(**a**) $\sigma_A,\ \sigma_B = \dfrac{15+8}{2} \pm \sqrt{\left(\dfrac{15-8}{2}\right)^2 + (-8)^2} = 20.2,\ 2.8$ kpsi

$$n = \frac{S_{ut}}{\sigma_A} = \frac{36}{20.2} = 1.78 \quad Ans.$$

(**b**) The plot is shown in the solution to Prob. 5-31.

$$n = \frac{OJ}{OI} = \frac{1.82"}{1.02"} = 1.78 \quad Ans.$$

5-36 Given: AISI 1006 CD steel, F = 0.55 kN, P = 4.0 kN, and T = 25 N·m. From Table A-20, S_y =280 MPa. Apply the DE theory to stress elements A and B

A: $$\sigma_x = \frac{4P}{\pi d^2} = \frac{4(4)10^3}{\pi(0.015^2)} = 22.6(10^6)\text{ Pa} = 22.6\text{ MPa}$$

$$\tau_{xy} = \frac{16T}{\pi d^3} - \frac{4}{3}\frac{V}{A} = \frac{16(25)}{\pi(0.015^3)} - \frac{4}{3}\left[\frac{0.55(10^3)}{(\pi/4)0.015^2}\right] = 33.6(10^6)\text{ Pa} = 33.6\text{ MPa}$$

$$\sigma' = \left[22.6^2 + 3(33.6^2)\right]^{1/2} = 62.4\text{ MPa}$$

$$n = \frac{280}{62.4} = 4.49 \qquad Ans.$$

B: $$\sigma_x = \frac{32Fl}{\pi d^3} + \frac{4P}{\pi d^2} = \frac{32(0.55)10^3(0.1)}{\pi(0.015^3)} + \frac{4(4)10^3}{\pi(0.015^2)} = 189(10^6)\text{ Pa} = 189\text{ MPa}$$

$$\tau_{xy} = \frac{16T}{\pi d^3} = \frac{16(25)}{\pi(0.015^3)} = 37.7(10^6)\text{ Pa} = 37.7\text{ MPa}$$

$$\sigma' = \left(\sigma_x^2 + 3\tau_{xy}^2\right)^{1/2} = \left[189^2 + 3\left(37.7^2\right)\right]^{1/2} = 200 \text{ MPa}$$

$$n = \frac{S_y}{\sigma'} = \frac{280}{200} = 1.4 \qquad Ans.$$

5-37 From Prob. 3-45, the critical location is at the top of the beam at $x = 27$ in from the left end, where there is only a bending stress of $\sigma = -7\,456$ psi. Thus, $\sigma' = 7\,456$ psi and

$(S_y)_{\min} = n\sigma' = 2(7\,456) = 14\,912$ psi

Choose $(S_y)_{\min} = 15$ kpsi *Ans.*

5-38 From Table A-20 for 1020 CD steel, $S_y = 57$ kpsi. From Eq. (3-42)

$$T = \frac{63\,025H}{n} \qquad (1)$$

where n is the shaft speed in rev/min. From Eq. (5-3), for the MSS theory,

$$\tau_{\max} = \frac{S_y}{2n_d} = \frac{16T}{\pi d^3} \qquad (2)$$

where n_d is the design factor. Substituting Eq. (1) into Eq. (2) and solving for d gives

$$d = \left[\frac{32\left(63\,025\right)Hn_d}{n\pi S_y}\right]^{1/3} \qquad (3)$$

Substituting $H = 20$ hp, $n_d = 3$, $n = 1750$ rev/min, and $S_y = 57(10^3)$ psi results in

$$d_{\min} = \left[\frac{32\left(63\,025\right)20\left(3\right)}{1750\pi\left(57\right)10^3}\right]^{1/3} = 0.728 \text{ in} \qquad Ans.$$

5-39 Given: $d = 30$ mm, AISI 1018 steel, $H = 10$ kW, $n = 200$ rev/min.
Table A-20, $S_y = 220$ MPa
Eq. (3-44): $T = 9.55\, H/n = 9.55(10)10^3/200 = 477.5$ N·m

$$\tau_{\max} = \frac{16T}{\pi d^3} = \frac{16\left(477.5\right)}{\pi\left(0.030\right)^3}10^{-6} = 90.07 \text{ MPa}$$

(a) Eq. (5-3): $n = \frac{S_y}{2\tau_{\max}} = \frac{220}{2\left(90.07\right)} = 1.22$ *Ans.*

(b) From Eq. (5-13), $\sigma'_{\max} = \sqrt{3\tau^2_{\max}} = \sqrt{3}\,(90.07) = 156.0 \text{ MPa}$

Eq. (5-19): $n = \dfrac{S_y}{\sigma'_{\max}} = \dfrac{220}{156} = 1.41$ *Ans.*

5-40 Given: d = 20 mm, AISC 1035 HR steel, n_s = 400 rev/min, n_y = 1.5.
Table A-20, S_y = 270 MPa

(a) Eq. (5-30): $\tau_{\max} = \dfrac{S_y}{2n_y} = \dfrac{270}{2(1.5)} = 90 \text{ MPa}$

Substituting Eq. (3-44) in the equation for $\tau_{\max}$ gives

$$\tau_{\max} = \frac{16T}{\pi d^3} = \frac{16}{\pi d^3}\left(9.55\frac{H}{n_s}\right) \quad \Rightarrow \quad H = \frac{\pi d^3 n_s \tau_{\max}}{16(9.55)} = \frac{\pi\left(0.02^3\right)400(90)10^6}{16(9.55)}$$

$$= 5.92\left(10^3\right) \text{ W} = 5.92 \text{ kW} \qquad \textit{Ans.}$$

(b) From Eq. (5-13): $\sigma'_{\max} = \sqrt{3}\ \tau_{\max} = \dfrac{S_y}{n_y} = \dfrac{270}{1.5} \quad \Rightarrow \quad \tau_{\max} = \dfrac{270}{1.5\sqrt{3}} = 103.9 \text{ MPa}$

Thus,

$$H = \frac{\pi d^3 n_s \tau_{\max}}{16(9.55)} = \frac{\pi\left(0.02^3\right)400(103.9)10^6}{16(9.55)} = 6.84\left(10^3\right) \text{ W} = 6.84 \text{ kW} \qquad \textit{Ans}$$

5-41 Table A-20 for AISI 1040 CD steel, S_y = 490 MPa.
From Prob. 3-47,

A: σ_x =79.6 MPa, τ_{xy} = 63.7 MPa. $\tau_{\max} = \sqrt{\left(\dfrac{\sigma}{2}\right)^2 + \tau^2_{xy}} = \sqrt{\left(\dfrac{79.6}{2}\right)^2 + 63.7^2} = 75.1 \text{ MPa.}$

B: $\tau_{\max} = \tau_{zx}$ = 53.1 MPa. C: $\tau_{\max} = \tau_{zx}$ = 116.8 MPa.
Critical case is at point C.

(a) MSS Theory: $n_y = \dfrac{S_y}{2\tau_{\max}} = \dfrac{490}{2(116.8)} = 2.1$ *Ans.*

(b) DE Theory: $n = \dfrac{S_y}{\sqrt{3}\tau_{\max}} = \dfrac{490}{\sqrt{3}(116.8)} = 2.4$ *Ans.*

5-42 Table A-20 for AISI 1040 CD steel, S_y = 490 MPa
From Prob. 3-53, $\sigma_1 = 122.6$ MPa, $\sigma_2 = 0$, $\sigma_3 = -10.2$ MPa, $\tau_{\max} = R = 66.4$ MPa

(a) MSS Theory: $n_y = \dfrac{S_y}{2\tau_{\max}} = \dfrac{490}{2(66.4)} = 3.69$ *Ans.*

(b) DE Theory, Eq. (5-13): $\sigma' = \left[122.6^2 - 122.6(-10.2) + (-10.2)^2\right]^{1/2} = 128 \text{ MPa}$

$$n_y = \frac{S_y}{\sigma'} = \frac{490}{128} = 3.83 \qquad Ans.$$

5-43 Table A-20 for AISI 1020 CD steel, S_y = 390 MPa

From Prob. 3-54, $\sigma_1 = 194.2$ MPa, $\sigma_2 = 0$, $\sigma_3 = -10$ MPa, $\tau_{\max} = 102.1$ MPa

(a) MSS Theory: $n_y = \frac{S_y}{2\tau_{\max}} = \frac{490}{2(102.1)} = 1.91 \qquad Ans.$

(b) DE Theory, Eq. (5-13): $\sigma' = \left[194.2^2 - 194.2(-10) + (-10)^2\right]^{1/2} = 199.4 \text{ MPa}$

$$n_y = \frac{S_y}{\sigma'} = \frac{490}{194.2} = 1.96 \qquad Ans.$$

5-44 Table A-20 for AISI 1035 CD steel, S_y = 67 kpsi

From Prob. 3-55, $\sigma_1 = 45.8$ kpsi, $\sigma_2 = 0$, $\sigma_3 = -0.45$ kpsi, $\tau_{\max} = 23.1$ kpsi

(a) MSS Theory: $n_y = \frac{S_y}{2\tau_{\max}} = \frac{67}{2(23.1)} = 1.45 \qquad Ans.$

(b) DE Theory, Eq. (5-13): $\sigma' = \left[45.8^2 - 45.8(-0.45) + (-0.45)^2\right]^{1/2} = 46.0 \text{ MPa}$

$$n_y = \frac{S_y}{\sigma'} = \frac{67}{46} = 1.46 \qquad Ans.$$

5-45 Table A-20 for AISI 1040 CD steel, S_y = 71 kpsi

From Prob. 3-101, $\sigma_1 = 18.47$ kpsi, $\sigma_2 = -3.60$ kpsi, $\tau_{\max} = 11.03$ kpsi

(a) MSS Theory: $n_y = \frac{S_y}{2\tau_{\max}} = \frac{71}{2(11.03)} = 3.22 \qquad Ans.$

(b) DE Theory, Eq. (5-13): $\sigma' = \left[18.47^2 - 18.47(-3.60) + (-3.60)^2\right]^{1/2} = 20.51 \text{ MPa}$

$$n_y = \frac{S_y}{\sigma'} = \frac{71}{20.51} = 3.46 \qquad Ans.$$

5-46 Table A-20 for AISI 1040 CD steel, S_y = 71 kpsi

From Prob. 3-102, $\sigma_1 = 29.1$ kpsi, $\sigma_2 = -14.2$ kpsi, $\tau_{\max} = 21.7$ kpsi

(a) MSS Theory: $n_y = \frac{S_y}{2\tau_{\max}} = \frac{71}{2(21.7)} = 1.64 \qquad Ans.$

(b) DE Theory, Eq. (5-13): $\sigma' = \left[29.1^2 - 29.1(-14.2) + (-14.2)^2\right]^{1/2} = 38.2 \text{ MPa}$

$$n_y = \frac{S_y}{\sigma'} = \frac{71}{38.2} = 1.86 \qquad Ans.$$

5-47 Table A-21 for AISI 4140 steel Q & T 400° F , S_y = 238 kpsi, F = 15 kip.

$\Sigma M_A = 0 = 3\,R_D - 2\,F \quad \Rightarrow \quad R_D = 2\,(15)/3 = 10$ kip,

$\Sigma F_y = 0 = R_A + R_D - F \quad \Rightarrow \quad R_A = 15 - 10 = 5$ kip

Critical sections are at points B and C where the areas are minimal.

B: d_B = 1.1 in, $M_B = R_A\,(1)$ = 5 kip·in, $V_B = R_A$ = 5 kip, T_B = 7 kip·in

$A_B = (\pi/4)\,1.1^2 = 0.9503$ in^2,

C: d_C = 1.3 in, $M_C = R_D\,(1)$ = 10 kip·in, $V_C = R_D$ = 10 kip, T_C = 7 kip·in

$A_C = (\pi/4)\,1.3^2 = 1.327$ in^2,

Critical locations are at the outer surfaces where bending stresses are maximum, and at the center planes where the transverse shear stresses are maximum. In both cases, there exists the torsional shear stresses.

B: Outer surface:

$$\sigma_B = \frac{32M_B}{\pi d^3} = \frac{32(5)}{\pi(1.1)^3} = 38.26 \text{ kpsi}, \quad \tau_B = \frac{16T_B}{\pi d^3} = \frac{16(7)}{\pi(1.1)^3} = 26.78 \text{ kpsi}$$

$$(\tau_B)_{\max} = \sqrt{\left(\frac{\sigma_B}{2}\right)^2 + \tau_B^2} = \sqrt{\left(\frac{38.26}{2}\right)^2 + 26.78^2} = 32.9 \text{ kpsi}$$

$$\sigma'_B = \sqrt{\sigma_B^2 + 3\tau_B^2} = \sqrt{38.26^2 + 3(26.78)^2} = 60.1 \text{ kpsi}$$

Center plane:

$$(\tau_B)_V = \frac{4V_B}{3A_B} = \frac{4(5)}{3(0.9503)} = 7.02 \text{ kpsi}$$

$$(\tau_B)_{\max} = \tau_B + (\tau_B)_V = 26.78 + 7.02 = 33.8 \text{ kpsi}$$

$$\sigma'_B = \sqrt{3}(\tau_B)_{\max} = \sqrt{3}(33.8) = 58.5 \text{ kpsi}$$

C: Outer surface:

$$\sigma_C = \frac{32M_C}{\pi d^3} = \frac{32(10)}{\pi(1.3)^3} = 46.36 \text{ kpsi}, \quad \tau_C = \frac{16T_C}{\pi d^3} = \frac{16(7)}{\pi(1.3)^3} = 16.23 \text{ kpsi}$$

$$(\tau_C)_{\max} = \sqrt{\left(\frac{\sigma_C}{2}\right)^2 + \tau_C^2} = \sqrt{\left(\frac{46.36}{2}\right)^2 + 16.23^2} = 28.30 \text{ kpsi}$$

$$\sigma'_C = \sqrt{\sigma_C^2 + 3\tau_C^2} = \sqrt{46.36^2 + 3(16.23)^2} = 54.2 \text{ kpsi}$$

Center plane:

$$(\tau_C)_V = \frac{4V_C}{3A_C} = \frac{4(10)}{3(1.327)} = 10.05 \text{ kpsi}$$

$$(\tau_C)_{\max} = \tau_C + (\tau_C)_V = 16.23 + 10.05 = 26.28 \text{ kpsi}$$

$$\sigma'_C = \sqrt{3}(\tau_C)_{\max} = \sqrt{3}(26.28) = 45.5 \text{ kpsi}$$

(a) MSS Theory: Critical location is at point B at the center plane:

$$n_y = \frac{S_y}{2\tau_{\max}} = \frac{238}{2(33.8)} = 3.52 \qquad Ans.$$

(b) DE Theory: Critical location is at point B at the outer surface:

$$n_y = \frac{S_y}{\sigma'_B} = \frac{238}{60.1} = 3.96 \qquad Ans.$$

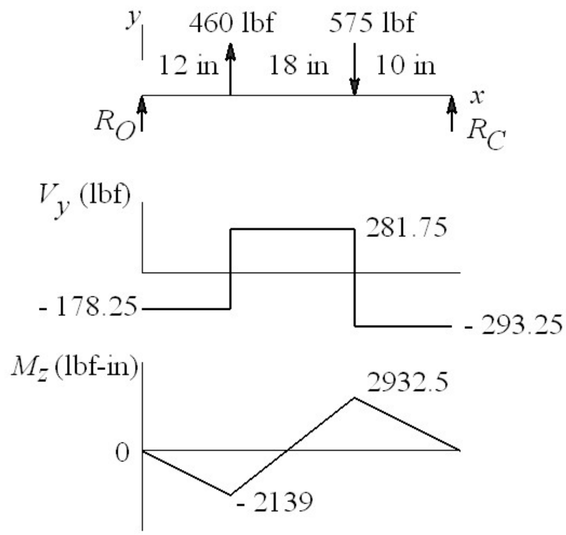
5-48 $\Sigma M_O = 0 = 40\, R_C - 30(575) + 12(460)$

$R_C = 293.25$ lbf

$\Sigma F_y = 0 = R_O + 293.25 + 460 - 575$

$R_O = -\,178.25$ lbf

$M_{\max} = 2.9325$ kip·in



$$\sigma = \frac{32M_{\max}}{\pi d^3} = \frac{32(2.9325)}{\pi(1^3)} = 29.87 \text{ kpsi}$$

$$\tau = \frac{16T}{\pi d^3} = \frac{16(1.5)}{\pi(1^3)} = 7.64 \text{ kpsi}$$

(a) $$\tau_{\max} = \sqrt{\left(\frac{29.87}{2}\right)^2 + 7.64^2} = 16.78 \text{ kpsi}$$

$$n_y = \frac{S_y}{2\tau_{\max}} = \frac{50}{2(16.78)} = 1.49 \qquad Ans.$$

(b) Eq. (5-15): $\sigma' = [29.87^2 + 3\,(7.64)^2]^{1/2} = 32.67$ kpsi

$$n_y = \frac{S_y}{\sigma'} = \frac{50}{32.67} = 1.53 \qquad Ans.$$

5-49 Given: AISI 1010 HR, $n_y = 2$, $L = 0.5$ m, $F = 150$ N, $T = 25$ N·m

Table A-20, $S_y = 180$ MPa.

$M_z = FL = 150\,(0.5) = 75$ N·m

$$\sigma = \frac{32M_z}{\pi d^3} = \frac{32(75)}{\pi d^3} = \frac{2400}{\pi d^3}$$

$$\tau = \frac{16T}{\pi d^3} = \frac{16(25)}{\pi d^3} = \frac{400}{\pi d^3}$$

(a) $$\tau_{\max} = \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} = \sqrt{\left(\frac{1200}{\pi d^3}\right)^2 + \left(\frac{400}{\pi d^3}\right)^2} = \frac{402.63}{d^3} = \frac{S_y}{2n} = \frac{180(10^6)}{2(2)}$$

$$d = \left[\frac{4(402.63)}{180(10^6)}\right]^{1/3} = 0.0208 \text{ m} = 20.8 \text{ mm} \qquad Ans.$$

(b) $\sigma' = \left(\sigma^2 + 3\tau^2\right)^{1/2} = \left[\left(\frac{2400}{\pi d^3}\right)^2 + 3\left(\frac{400}{\pi d^3}\right)^2\right]^{1/2} = \frac{795.14}{d^3} = \frac{S_y}{n_y} = \frac{180(10^6)}{2}$

$$d = \left[\frac{2(795.14)}{180(10^6)}\right]^{1/3} = 0.0207 \text{ m} = 20.7 \text{ mm} \qquad Ans.$$

5-50 From Table A-20, S_y = 54 kpsi. From the solution of Prob. 3-79, in the plane of analysis

σ_1 = 16.5 kpsi, σ_2 = – 1.19 kpsi, and τ_{max} = 8.84 kpsi

The state of stress is *plane stress*. Thus, the three-dimensional principal stresses are

σ_1 = 16.5 kpsi, σ_2 = 0, and σ_3 = – 1.19 kpsi

MSS: From Eq. (5-3), $n = \frac{S_y}{\sigma_1 - \sigma_3} = \frac{54}{16.5 - (-1.19)} = 3.05 \qquad Ans.$

Note: Whenever the two principal stresses of a plane stress state are of opposite sign, the maximum shear stress found in the analysis is the *true* maximum shear stress. Thus, the factor of safety could have been found from

$$n = \frac{S_y}{2\tau_{max}} = \frac{54}{2(8.84)} = 3.05 \qquad Ans.$$

DE: The von Mises stress can be found from the principal stresses or from the stresses found in part (d) of Prob. 3-79. That is,

Eqs. (5-13) and (5-19)

$$n = \frac{S_y}{\sigma'} = \frac{S_y}{\left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2}} = \frac{54}{\left[16.5^2 - 16.5(-1.19) + (-1.19)^2\right]^{1/2}}$$

$$= 3.15 \qquad Ans.$$

or, Eqs. (5-15) and (5-19) using the results of part (d) of Prob. 3-79

$$n = \frac{S_y}{\sigma'} = \frac{S_y}{\left(\sigma^2 + 3\tau^2\right)^{1/2}} = \frac{54}{\left[15.3^2 + 3\left(4.43^2\right)\right]^{1/2}}$$

$$= 3.15 \qquad Ans.$$

5-51 From Table A-20, S_y = 370 MPa. From the solution of Prob. 3-80, in the plane of analysis

σ_1 = 275 MPa, σ_2 = − 12.1 MPa, and τ_{max} = 144 MPa

The state of stress is *plane stress*. Thus, the three-dimensional principal stresses are

σ_1 = 275 MPa, σ_2 = 0, and σ_3 = − 12.1 MPa

MSS: From Eq. (5-3), $n = \dfrac{S_y}{\sigma_1 - \sigma_3} = \dfrac{370}{275 - (-12.1)} = 1.29$ *Ans.*

DE: From Eqs. (5-13) and (5-19)

$$n = \frac{S_y}{\sigma'} = \frac{S_y}{\left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2}} = \frac{370}{\left[275^2 - 275(-12.1) + (-12.1)^2\right]^{1/2}}$$

$= 1.32$ *Ans.*

5-52 From Table A-20, S_y = 54 kpsi. From the solution of Prob. 3-81, in the plane of analysis

σ_1 = 22.6 kpsi, σ_2 = − 1.14 kpsi, and τ_{max} = 11.9 kpsi

The state of stress is *plane stress*. Thus, the three-dimensional principal stresses are

σ_1 = 22.6 kpsi, σ_2 = 0, and σ_3 = − 1.14 kpsi

MSS: From Eq. (5-3), $n = \dfrac{S_y}{\sigma_1 - \sigma_3} = \dfrac{54}{22.6 - (-1.14)} = 2.27$ *Ans.*

DE: From Eqs. (5-13) and (5-19)

$$n = \frac{S_y}{\sigma'} = \frac{S_y}{\left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2}} = \frac{54}{\left[22.6^2 - 22.6(-1.14) + (-1.14)^2\right]^{1/2}}$$

$= 2.33$ *Ans.*

5-53 From Table A-20, S_y = 370 MPa. From the solution of Prob. 3-82, in the plane of analysis

σ_1 = 78.2 MPa, σ_2 = − 5.27 MPa, and τ_{max} = 41.7 MPa

The state of stress is *plane stress*. Thus, the three-dimensional principal stresses are

σ_1 = 78.2 MPa, σ_2 = 0, and σ_3 = − 5.27 MPa

MSS: From Eq. (5-3), $n = \dfrac{S_y}{\sigma_1 - \sigma_3} = \dfrac{370}{78.2 - (-5.27)} = 4.43$ *Ans.*

DE: From Eqs. (5-13) and (5-19)

$$n = \frac{S_y}{\sigma'} = \frac{S_y}{\left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2}} = \frac{370}{\left[78.2^2 - 78.2(-5.27) + (-5.27)^2\right]^{1/2}}$$

$= 4.57$ *Ans.*

5-54 From Table A-20, S_y = 54 kpsi. From the solution of Prob. 3-83, in the plane of analysis

σ_1 = 36.7 kpsi, σ_2 = – 1.47 kpsi, and τ_{max} = 19.1 kpsi

The state of stress is *plane stress*. Thus, the three-dimensional principal stresses are

σ_1 = 36.7 kpsi, σ_2 = 0, and σ_3 = – 1.47 kpsi

MSS: From Eq. (5-3), $n = \dfrac{S_y}{\sigma_1 - \sigma_3} = \dfrac{54}{36.7 - (-1.47)} = 1.41$ *Ans.*

DE: From Eqs. (5-13) and (5-19)

$$n = \frac{S_y}{\sigma'} = \frac{S_y}{\left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2}} = \frac{54}{\left[36.7^2 - 36.7(-1.47) + (-1.47)^2\right]^{1/2}}$$

$= 1.44$ *Ans.*

5-55 From Table A-20, S_y = 370 MPa. From the solution of Prob. 3-84, in the plane of analysis

σ_1 = 376 MPa, σ_2 = – 42.4 MPa, and τ_{max} = 209 MPa

The state of stress is *plane stress*. Thus, the three-dimensional principal stresses are

σ_1 = 376 MPa, σ_2 = 0, and σ_3 = – 42.4 MPa

MSS: From Eq. (5-3), $n = \dfrac{S_y}{\sigma_1 - \sigma_3} = \dfrac{370}{376 - (-42.4)} = 0.88$ *Ans.*

DE: From Eqs. (5-13) and (5-19)

$$n = \frac{S_y}{\sigma'} = \frac{S_y}{\left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2}} = \frac{370}{\left[376^2 - 376(-42.4) + (-42.4)^2\right]^{1/2}}$$

$$= 0.93 \quad Ans.$$

5-56 From Table A-20, S_y = 54 kpsi. From the solution of Prob. 3-85, in the plane of analysis

σ_1 = 7.19 kpsi, σ_2 = – 17.0 kpsi, and τ_{max} = 12.1 kpsi

The state of stress is *plane stress*. Thus, the three-dimensional principal stresses are

σ_1 = 7.19 kpsi, σ_2 = 0, and σ_3 = – 17.0 kpsi

MSS: From Eq. (5-3), $n = \dfrac{S_y}{\sigma_1 - \sigma_3} = \dfrac{54}{7.19 - (-17.0)} = 2.23 \quad Ans.$

DE: From Eqs. (5-13) and (5-19)

$$n = \frac{S_y}{\sigma'} = \frac{S_y}{\left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2}} = \frac{54}{\left[7.19^2 - 7.19(-17.0) + (-17.0)^2\right]^{1/2}}$$

$$= 2.51 \quad Ans.$$

5-57 From Table A-20, S_y = 54 kpsi. From the solution of Prob. 3-87, in the plane of analysis

σ_1 = 1.72 kpsi, σ_2 = – 35.9 kpsi, and τ_{max} = 18.8 kpsi

The state of stress is *plane stress*. Thus, the three-dimensional principal stresses are

σ_1 = 1.72 kpsi, σ_2 = 0, and σ_3 = – 35.9 kpsi

MSS: From Eq. (5-3), $n = \dfrac{S_y}{\sigma_1 - \sigma_3} = \dfrac{54}{1.72 - (-35.9)} = 1.44 \quad Ans.$

DE: From Eqs. (5-13) and (5-19)

$$n = \frac{S_y}{\sigma'} = \frac{S_y}{\left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2}} = \frac{54}{\left[1.72^2 - 1.72(-35.9) + (-35.9)^2\right]^{1/2}}$$

$$= 1.47 \quad Ans.$$

5-58 From Table A-20, S_y = 370 MPa. From the solution of Prob. 3-88,

Bending: σ_B = 68.6 MPa, Torsion: τ_B = 37.7 MPa

For a plane stress analysis it was found that τ_{max} = 51.0 MPa. With combined bending and torsion, the plane stress analysis yields the true τ_{max}.

MSS: From Eq. (5-3), $n = \dfrac{S_y}{2\tau_{max}} = \dfrac{370}{2(51.0)} = 3.63$ *Ans.*

DE: From Eqs. (5-15) and (5-19)

$$n = \frac{S_y}{\sigma'} = \frac{S_y}{\left(\sigma_B^2 + 3\tau_B^2\right)^{1/2}} = \frac{370}{\left[68.6^2 + 3\left(37.7^2\right)\right]^{1/2}}$$

$= 3.91$ *Ans.*

5-59 From Table A-20, S_y = 54 kpsi. From the solution of Prob. 3-90,

Bending: σ_C = 3460 psi, Torsion: τ_C = 882 kpsi

For a plane stress analysis it was found that τ_{max} = 1940 psi. With combined bending and torsion, the plane stress analysis yields the true τ_{max}.

MSS: From Eq. (5-3), $n = \dfrac{S_y}{2\tau_{max}} = \dfrac{54\left(10^3\right)}{2(1940)} = 13.9$ *Ans.*

DE: From Eqs. (5-15) and (5-19)

$$n = \frac{S_y}{\sigma'} = \frac{S_y}{\left(\sigma_C^2 + 3\tau_C^2\right)^{1/2}} = \frac{54\left(10^3\right)}{\left[3460^2 + 3\left(882^2\right)\right]^{1/2}}$$

$= 14.3$ *Ans.*

5-60 From Table A-20, S_y = 54 kpsi. From the solution of Prob. 3-91, in the plane of analysis

σ_1 = 17.8 kpsi, σ_2 = − 1.46 kpsi, and τ_{max} = 9.61 kpsi

The state of stress is *plane stress*. Thus, the three-dimensional principal stresses are

σ_1 = 17.8 kpsi, σ_2 = 0, and σ_3 = − 1.46 kpsi

MSS: From Eq. (5-3), $n = \dfrac{S_y}{\sigma_1 - \sigma_3} = \dfrac{54}{17.8 - (-1.46)} = 2.80$ *Ans.*

DE: From Eqs. (5-13) and (5-19)

$$n = \frac{S_y}{\sigma'} = \frac{S_y}{\left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2}} = \frac{54}{\left[17.8^2 - 17.8(-1.46) + (-1.46)^2\right]^{1/2}}$$

$$= 2.91 \quad Ans.$$

5-61 From Table A-20, S_y = 54 kpsi. From the solution of Prob. 3-92, in the plane of analysis

σ_1 = 17.5 kpsi, σ_2 = – 1.13 kpsi, and τ_{max} = 9.33 kpsi

The state of stress is *plane stress*. Thus, the three-dimensional principal stresses are

σ_1 = 17.5 kpsi, σ_2 = 0, and σ_3 = – 1.13 kpsi

MSS: From Eq. (5-3), $n = \dfrac{S_y}{\sigma_1 - \sigma_3} = \dfrac{54}{17.5 - (-1.13)} = 2.90 \quad Ans.$

DE: From Eqs. (5-13) and (5-19)

$$n = \frac{S_y}{\sigma'} = \frac{S_y}{\left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2}} = \frac{54}{\left[17.5^2 - 17.5(-1.13) + (-1.13)^2\right]^{1/2}}$$

$$= 2.98 \quad Ans.$$

5-62 From Table A-20, S_y = 54 kpsi. From the solution of Prob. 3-93, in the plane of analysis

σ_1 = 21.5 kpsi, σ_2 = – 1.20 kpsi, and τ_{max} = 11.4 kpsi

The state of stress is *plane stress*. Thus, the three-dimensional principal stresses are

σ_1 = 21.5 kpsi, σ_2 = 0, and σ_3 = – 1.20 kpsi

MSS: From Eq. (5-3), $n = \dfrac{S_y}{\sigma_1 - \sigma_3} = \dfrac{54}{21.5 - (-1.20)} = 2.38 \quad Ans.$

DE: From Eqs. (5-13) and (5-19)

$$n = \frac{S_y}{\sigma'} = \frac{S_y}{\left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2}} = \frac{54}{\left[21.5^2 - 21.5(-1.20) + (-1.20)^2\right]^{1/2}}$$

$$= 2.44 \quad Ans.$$

5-63 From Table A-20, S_y = 54 kpsi. From the solution of Prob. 3-94, the concern was failure due to twisting of the flat bar where it was found that τ_{max} = 14.3 kpsi in the middle of the longest side of the rectangular cross section. The bar is also in bending, but the bending stress is zero where τ_{max} exists.

MSS: From Eq. (5-3), $n = \dfrac{S_y}{2\tau_{max}} = \dfrac{54}{2(14.3)} = 1.89$ *Ans.*

DE: From Eqs. (5-15) and (5-19)

$$n = \frac{S_y}{\sigma'} = \frac{S_y}{\left(3\tau_{max}^2\right)^{1/2}} = \frac{54}{\left[3\left(14.3^2\right)\right]^{1/2}} = 2.18 \quad \textit{Ans.}$$

5-64 From Table A-20, S_y = 54 kpsi. From the solution of Prob. 3-95, in the plane of analysis

σ_1 = 34.7 kpsi, σ_2 = – 6.7 kpsi, and τ_{max} = 20.7 kpsi

The state of stress is *plane stress*. Thus, the three-dimensional principal stresses are

σ_1 = 34.7 kpsi, σ_2 = 0, and σ_3 = – 6.7 kpsi

MSS: From Eq. (5-3), $n = \dfrac{S_y}{\sigma_1 - \sigma_3} = \dfrac{54}{34.7-(-6.7)} = 1.30$ *Ans.*

DE: From Eqs. (5-13) and (5-19)

$$n = \frac{S_y}{\sigma'} = \frac{S_y}{\left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2}} = \frac{54}{\left[34.7^2 - 34.7(-6.7) + (-6.7)^2\right]^{1/2}}$$

$$= 1.40 \quad \textit{Ans.}$$

5-65 From Table A-20, S_y = 54 kpsi. From the solution of Prob. 3-96, in the plane of analysis

σ_1 = 51.1 kpsi, σ_2 = – 4.58 kpsi, and τ_{max} = 27.8 kpsi

The state of stress is *plane stress*. Thus, the three-dimensional principal stresses are

σ_1 = 51.1 kpsi, σ_2 = 0, and σ_3 = – 4.58 kpsi

MSS: From Eq. (5-3), $n = \dfrac{S_y}{\sigma_1 - \sigma_3} = \dfrac{54}{51.1-(-4.58)} = 0.97$ *Ans.*

DE: From Eqs. (5-13) and (5-19)

$$n = \frac{S_y}{\sigma'} = \frac{S_y}{\left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2}} = \frac{54}{\left[51.1^2 - 51.1(-4.58) + (-4.58)^2\right]^{1/2}}$$

$$= 1.01 \qquad Ans.$$

5-66 From Table A-20, S_y = 54 kpsi. From the solution of Prob. 3-97, in the plane of analysis

σ_1 = 59.7 kpsi, σ_2 = – 3.92 kpsi, and τ_{max} = 31.8 kpsi

The state of stress is *plane stress*. Thus, the three-dimensional principal stresses are

σ_1 = 59.7 kpsi, σ_2 = 0, and σ_3 = – 3.92 kpsi

MSS: From Eq. (5-3), $n = \dfrac{S_y}{\sigma_1 - \sigma_3} = \dfrac{54}{59.7 - (-3.92)} = 0.85$ *Ans.*

DE: From Eqs. (5-13) and (5-19)

$$n = \frac{S_y}{\sigma'} = \frac{S_y}{\left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2}} = \frac{54}{\left[59.7^2 - 59.7(-3.92) + (-3.92)^2\right]^{1/2}}$$

$$= 0.87 \qquad Ans.$$

5-67 For Prob. 3-95, from Prob. 5-64 solution, with 1018 CD, DE theory yields, n = 1.40.

From Table A-21, for 4140 Q&T @400°F, S_y = 238 kpsi. From Prob. 3-98 solution which considered stress concentrations for Prob. 3-95

σ_1 = 53.0 kpsi, σ_2 = – 8.48 kpsi, and τ_{max} = 30.7 kpsi

DE: From Eqs. (5-13) and (5-19)

$$n = \frac{S_y}{\sigma'} = \frac{S_y}{\left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2}} = \frac{238}{\left[53.0^2 - 53.0(-8.48) + (-8.48)^2\right]^{1/2}}$$

$$= 4.12 \qquad Ans.$$

Using the 4140 versus the 1018 CD, the factor of safety increases by a factor of 4.12/1.40 = 2.94. *Ans.*

5-68 Design Decisions Required:

- Material and condition
- Design factor
- Failure model
- Diameter of pin

Using F = 416 lbf from Ex. 5-3,

$$\sigma_{\max} = \frac{32M}{\pi d^3}$$

$$d = \left(\frac{32M}{\pi \sigma_{\max}}\right)^{\frac{1}{3}}$$

Decision 1: Select the same material and condition of Ex. 5-3 (AISI 1035 steel, S_y = 81 kpsi)

Decision 2: Since we prefer the pin to yield, set n_d a little larger than 1. Further explanation will follow.

Decision 3: Use the Distortion Energy static failure theory.

Decision 4: Initially set n_d = 1

$$\sigma_{\max} = \frac{S_y}{n_d} = \frac{S_y}{1} = 81\,000 \text{ psi}$$

$$d = \left(\frac{32(416)(15)}{\pi(81\,000)}\right)^{\frac{1}{3}} = 0.922 \text{ in}$$

Choose preferred size of $d = 1.000$ in

$$F = \frac{\pi(1)^3(81\,000)}{32(15)} = 530 \text{ lbf}$$

$$n = \frac{530}{416} = 1.27$$

Set design factor to $n_d = 1.27$

Adequacy Assessment:

$$\sigma_{\max} = \frac{S_y}{n_d} = \frac{81\,000}{1.27} = 63\,800 \text{ psi}$$

$$d = \left(\frac{32(416)(15)}{\pi(63\,800)}\right)^{\frac{1}{3}} = 1.00 \text{ in (OK)}$$

$$F = \frac{\pi(1)^3(81\,000)}{32(15)} = 530 \text{ lbf}$$

$$n = \frac{530}{416} = 1.27 \text{ (OK)}$$

5-69 From Table A-20, for a thin walled cylinder made of AISI 1020 CD steel, S_{yt} = 57 kpsi, S_{ut} = 68 kpsi.

Since r/t = 7.5/0.0625 = 120 > 10, the shell can be considered thin-wall. From the solution of Prob. 3-106 the principal stresses are

$$\sigma_1 = \sigma_2 = \frac{pd}{4t} = \frac{p(15)}{4(0.0625)} = 60p, \qquad \sigma_3 = -p$$

From Eq. (5-12)

$$\sigma' = \frac{1}{\sqrt{2}}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]^{1/2}$$

$$= \frac{1}{\sqrt{2}}\left[(60p - 60p)^2 + (60p + p)^2 + (-p - 60p)^2\right]^{1/2} = 61p$$

For yield, $\sigma' = S_y \Rightarrow 61p = 57\,(10^3) \Rightarrow p = 934$ psi *Ans.*

For rupture, $61p = 68 \quad \Rightarrow \quad p = 1.11$ kpsi *Ans.*

5-70 Given: AISI CD 1040 steel, n_y = 2, OD = 50 mm, ID = 42 mm, L = 150 mm.

Table A-20, S_y = 490 MPa

At $r = r_i$ = 21 mm, Eq. (3-51) gives

$$(\sigma_t)_{\max} = p_i \frac{r_o^2 + r_i^2}{r_o^2 - r_i^2} = p_i \frac{25^2 + 21^2}{25^2 - 21^2} = 5.793\, p_i = \sigma_1$$

$$(\sigma_r)_{\max} = -p_i = \sigma_3$$

Closed end, Eq. (3-52) gives

$$\sigma_l = \frac{p_i r_i^2}{r_o^2 - r_i^2} = \frac{p_i (21)^2}{25^2 - 21^2} = 2.397\, p_i = \sigma_2$$

(a) $$\tau_{\max} = \frac{\sigma_1 - \sigma_3}{2} = \frac{5.793 p_i - (-p_i)}{2} = 3.397 p_i$$

$$\tau_{\max} = \frac{S_y}{2n_y} \quad \Rightarrow \quad 3.397 p_i = \frac{490}{2(2)} \quad \Rightarrow \quad p_i = 36.1 \text{ MPa} \qquad \textit{Ans.}$$

(b) Eq. (5-12):

$$\sigma' = \left[\frac{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2}{2}\right]^{1/2}$$

$$= \left[\frac{(5.793-2.397)^2+(2.397+1)^2+(-1-5.793)^2}{2}\right]^{1/2} p_i = 5.883 p_i$$

$$\sigma' = \frac{S_y}{n_y} \quad \Rightarrow \quad 5.883 p_i = \frac{490}{2} \quad \Rightarrow \quad p_i = 41.6 \text{ MPa} \qquad \textit{Ans.}$$

5-71 Given: AISI 1040 CD steel, OD = 50 mm, ID = 42 mm, L = 150 mm, p_i = 40 MPa
Table A-20, S_y = 490 MPa
At $r = r_i$ = 21 mm, Eq. (3-51) gives

$$(\sigma_t)_{\max} = p_i \frac{r_o^2 + r_i^2}{r_o^2 - r_i^2} = 40\frac{25^2+21^2}{25^2-21^2} = 231.74 \text{ MPa} = \sigma_1$$

$$(\sigma_r)_{\max} = -p_i = -40 \text{ MPa} = \sigma_3$$

Closed end, Eq. (3-52) gives

$$\sigma_l = \frac{p_i r_i^2}{r_o^2 - r_i^2} = \frac{40(21)^2}{25^2-21^2} = 95.87 \text{ MPa} = \sigma_2$$

(a) $\tau_{\max} = \dfrac{\sigma_1 - \sigma_3}{2} = \dfrac{231.74-(-40)}{2} = 135.87 \text{ MPa}$

$$n_y = \frac{S_y}{2\tau_{\max}} = \frac{490}{2(135.87)} = 1.80 \qquad \textit{Ans.}$$

(b) Eq. (5-12):

$$\sigma' = \left[\frac{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2}{2}\right]^{1/2}$$

$$= \left[\frac{(231.74-95.87)^2+(95.87+40)^2+(-40-231.74)^2}{2}\right]^{1/2} = 235.3 \text{ MPa}$$

$$n_y = \frac{S_y}{\sigma'} = \frac{490}{235.3} = 2.08 \qquad \textit{Ans.}$$

5-72 For AISI 1020 HR steel, from Tables A-5 and A-20, w = 0.282 lbf/in^3, S_y = 30 kpsi, and ν = 0.292. Then, $\rho = w/g$ = 0.282/386 lbf·s^2/in. For the problem, r_i = 3 in, and r_o = 5 in. Substituting into Eqs. (3-55), gives

$$\sigma_t = \frac{0.282}{386}\omega^2\left(\frac{3+0.292}{8}\right)\left[9+25+\frac{9(25)}{r^2}-\frac{1+3(0.292)}{3+0.292}r^2\right]$$

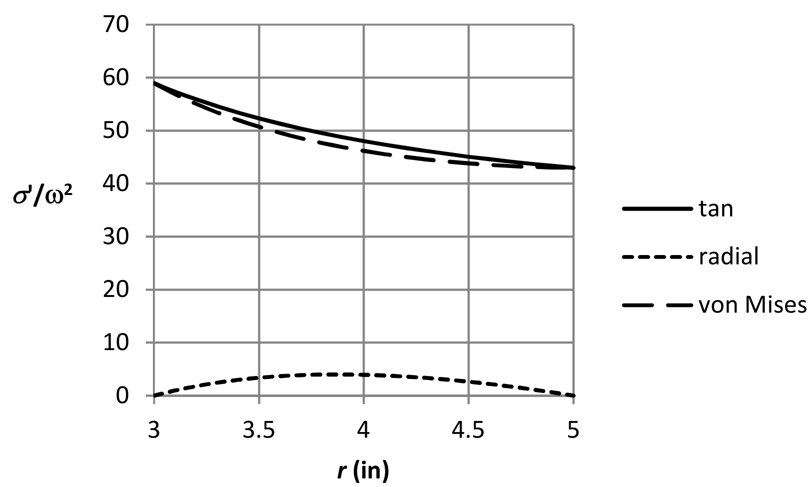
$$= 3.006\left(10^{-4}\right)\omega^2\left(34+\frac{225}{r^2}-0.5699r^2\right) = F(r)\,\omega^2 \qquad (1)$$

$$\sigma_r = 3.006\left(10^{-4}\right)\omega^2\left(34-\frac{225}{r^2}-r^2\right) = G(r)\,\omega^2 \qquad (2)$$

For the distortion-energy theory, the von Mises stress will be

$$\sigma' = \left(\sigma_t^2 - \sigma_t\sigma_r + \sigma_2^2\right)^{1/2} = \omega^2\left[F^2(r) - F(r)G(r) + G^2(r)\right]^{1/2} \qquad (3)$$

Although it was noted that the maximum radial stress occurs at $r = (r_o r_i)^{1/2}$ we are more interested as to where the von Mises stress is a maximum. One could take the derivative of Eq. (3) and set it to zero to find where the maximum occurs. However, it is much easier to plot σ'/ω^2 for $3 \le r \le 5$ in. Plotting Eqs. (1) through (3) results in



It can be seen that there is no maxima, and the greatest value of the von Mises stress is the tangential stress at $r = r_i$. Substituting $r = 3$ in into Eq. (1) and setting $\sigma' = S_y$ gives

$$\omega = \left[\frac{30\left(10^3\right)}{3.006\left(10^{-4}\right)\left(34+\frac{225}{3^2}-0.5699\left(3^2\right)\right)}\right]^{1/2} = 1361 \text{ rad/s}$$

$$n = \frac{60\omega}{2\pi} = \frac{60(1361)}{2\pi} = 13\,000 \text{ rev/min} \qquad \textit{Ans.}$$

5-73 Since $r/t = 1.75/0.065 = 26.9 > 10$, we can use thin-walled equations. From Eqs. (3-53) and (3-54),

$$d_i = 3.5 - 2(0.065) = 3.37 \text{ in}$$

$$\sigma_t = \frac{p(d_i + t)}{2t}$$

$$\sigma_t = \frac{500(3.37 + 0.065)}{2(0.065)} = 13\,212 \text{ psi} = 13.2 \text{ kpsi}$$

$$\sigma_l = \frac{pd_i}{4t} = \frac{500(3.37)}{4(0.065)} = 6481 \text{ psi} = 6.48 \text{ kpsi}$$

$$\sigma_r = -p_i = -500 \text{ psi} = -0.5 \text{ kpsi}$$

These are all principal stresses, thus, from Eq. (5-12),

$$\sigma' = \frac{1}{\sqrt{2}}\left\{(13.2 - 6.48)^2 + [6.48 - (-0.5)]^2 + (-0.5 - 13.2)^2\right\}^{1/2}$$

$$= 11.87 \text{ kpsi}$$

$$n = \frac{S_y}{\sigma'} = \frac{46}{11.87}$$

$$n = 3.88 \quad \textit{Ans.}$$

5-74 From Table A-20, $S_y = 320$ MPa

With $p_i = 0$, Eqs. (3-49) are

$$\sigma_t = -\frac{r_o^2 p_o}{r_o^2 - r_i^2}\left(1 + \frac{r_i^2}{r^2}\right) = c\left(1 + \frac{b^2}{r^2}\right) \qquad (1)$$

$$\sigma_r = -\frac{r_o^2 p_o}{r_o^2 - r_i^2}\left(1 - \frac{r_i^2}{r^2}\right) = c\left(1 - \frac{b^2}{r^2}\right)$$

For the distortion-energy theory, the von Mises stress is

$$\sigma' = \left(\sigma_t^2 - \sigma_t\sigma_r + \sigma_r^2\right)^{1/2} = c\left[\left(1 + \frac{b^2}{r^2}\right)^2 - \left(1 + \frac{b^2}{r^2}\right)\left(1 - \frac{b^2}{r^2}\right) + \left(1 - \frac{b^2}{r^2}\right)^2\right]^{1/2}$$

$$= c\left(1 + 3\frac{b^4}{r^4}\right)^{1/2}$$

We see that the maximum von Mises stress occurs where r is a minimum at $r = r_i$. Here, $\sigma_r = 0$ and thus $\sigma' = -\sigma_t$. Setting $-\sigma_t = S_y = 320$ MPa at $r = 0.1$ m in Eq. (1) results in

$$-\sigma_t\big|_{r=r_i} = \frac{2r_o^2 p_o}{r_o^2 - r_i^2} = \frac{2\left(0.15^2\right)p_o}{0.15^2 - 0.1^2} = 3.6p_o = 320 \;\Rightarrow\; p_o = 88.9 \text{ MPa} \qquad \textit{Ans.}$$

5-75 From Table A-24, S_{ut} = 31 kpsi for grade 30 cast iron. From Table A-5, ν = 0.211 and w = 0.260 lbf/in^3. In Prob. 5-72, it was determined that the maximum stress was the tangential stress at the inner radius, where the radial stress is zero. Thus at the inner radius, Eq. (3-55) gives

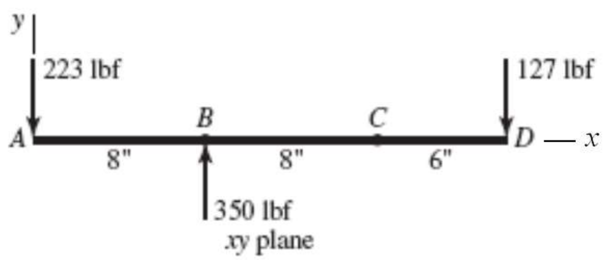
$$\sigma_t = \rho\omega^2\left(\frac{3+\nu}{8}\right)\left(2r_o^2 + r_i^2 - \frac{1+3\nu}{3+\nu}r_i^2\right) = \frac{0.260}{386}\omega^2\left(\frac{3.211}{8}\right)\left[2\left(5^2\right)+3^2 - \frac{1+3(0.211)}{3.211}3^2\right]$$

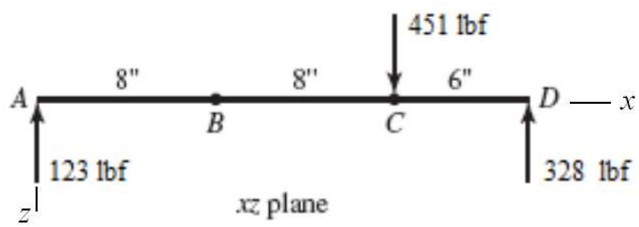
$$= 0.01471\omega^2 = 31\left(10^3\right) \qquad \Rightarrow \qquad 1452 \text{ rad/sec}$$

$$n = 60(1452)/(2\pi) = 13\,870 \text{ rev/min} \qquad \textit{Ans.}$$

5-76 From Table A-20, for AISI 1035 CD, S_y = 67 kpsi.

From force and bending-moment equations, the ground reaction forces are found in two planes as shown.





The maximum bending moment will be at B or C. Check which is larger. In the xy plane,

$M_B = 223(8) = 1784$ lbf · in and $M_C = 127(6) = 762$ lbf · in.

In the xz plane, $M_B = 123(8) = 984$ lbf · in and $M_C = 328(6) = 1968$ lbf · in.

$$M_B = [(1784)^2 + (984)^2]^{\frac{1}{2}} = 2037 \text{ lbf} \cdot \text{in}$$

$$M_C = [(762)^2 + (1968)^2]^{\frac{1}{2}} = 2110 \text{ lbf} \cdot \text{in}$$

So point C governs. The torque transmitted between B and C is T = (300 − 50)(4) = 1000 lbf·in. The stresses are

$$\tau_{xz} = \frac{16T}{\pi d^3} = \frac{16(1000)}{\pi d^3} = \frac{5093}{d^3} \text{ psi}$$

$$\sigma_x = \frac{32M_C}{\pi d^3} = \frac{32(2110)}{\pi d^3} = \frac{21\,492}{d^3} \text{ psi}$$

For combined bending and torsion, the maximum shear stress is found from

$$\tau_{\max} = \left[\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xz}^2\right]^{1/2} = \left[\left(\frac{21.49}{2d^3}\right)^2 + \left(\frac{5.09}{d^3}\right)^2\right]^{1/2} = \frac{11.89}{d^3} \text{ kpsi}$$

Max Shear Stress theory is chosen as a conservative failure theory. From Eq. (5-3)

$$\tau_{\max} = \frac{S_y}{2n} = \frac{11.89}{d^3} = \frac{67}{2(2)} \quad \Rightarrow \quad d = 0.892 \text{ in} \qquad \textit{Ans.}$$

5-77 As in Prob. 5-76, we will assume this to be a statics problem. Since the proportions are unchanged, the bearing reactions will be the same as in Prob. 5-76 and the bending moment will still be a maximum at point *C*. Thus

xy plane: $M_C = 127(3) = 381 \text{ lbf}\cdot\text{in}$
xz plane: $M_C = 328(3) = 984 \text{ lbf}\cdot\text{in}$

So

$$M_{\max} = \left[(381)^2 + (984)^2\right]^{1/2} = 1055 \text{ lbf}\cdot\text{in}$$

$$\sigma_x = \frac{32M_C}{\pi d^3} = \frac{32(1055)}{\pi d^3} = \frac{10\,746}{d^3} \text{ psi} = \frac{10.75}{d^3} \text{ kpsi}$$

Since the torsional stress is unchanged,

$$\tau_{xz} = \frac{5.09}{d^3} \text{ kpsi}$$

For combined bending and torsion, the maximum shear stress is found from

$$\tau_{\max} = \left[\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xz}^2\right]^{1/2} = \left[\left(\frac{10.75}{2d^3}\right)^2 + \left(\frac{5.09}{d^3}\right)^2\right]^{1/2} = \frac{7.40}{d^3} \text{ kpsi}$$

Using the MSS theory, as was used in Prob. 5-76, gives

$$\tau_{\max} = \frac{S_y}{2n} = \frac{7.40}{d^3} = \frac{67}{2(2)} \quad \Rightarrow \quad d = 0.762 \text{ in} \qquad \textit{Ans.}$$

5-78 For AISI 1018 HR, Table A-20 gives S_y = 32 kpsi. Transverse shear stress is a maximum at the neutral axis, and zero at the outer radius. Bending stress is a maximum at the outer radius, and zero at the neutral axis.

Model (*c*): From Prob. 3-41, at outer radius,

$$\sigma' = \sigma = 17.8 \text{ kpsi}$$

$$n = \frac{S_y}{\sigma'} = \frac{32}{17.8} = 1.80$$

At neutral axis,

$$\sigma' = \sqrt{3\tau^2} = \sqrt{3(3.4)^2} = 5.89 \text{ kpsi}$$

$$n = \frac{S_y}{\sigma'} = \frac{32}{5.89} = 5.43$$

The bending stress at the outer radius dominates. n = 1.80 *Ans.*

Model (*d*): Assume the bending stress at the outer radius will dominate, as in model (*c*). From Prob. 3-41,

$$\sigma' = \sigma = 25.5 \text{ kpsi}$$

$$n = \frac{S_y}{\sigma'} = \frac{32}{25.5} = 1.25 \qquad \textit{Ans.}$$

Model (*e*): From Prob. 3-41,

$$\sigma' = \sigma = 17.8 \text{ kpsi}$$

$$n = \frac{S_y}{\sigma'} = \frac{32}{17.8} = 1.80 \qquad \textit{Ans.}$$

Model (*d*) is the most conservative, thus safest, and requires the least modeling time. Model (*c*) is probably the most accurate, but model (*e*) yields the same results with less modeling effort.

5-79 For AISI 1018 HR, from Table A-20, S_y = 32 kpsi. Model (*d*) yields the largest bending moment, so designing to it is the most conservative approach. The bending moment is M = 312.5 lbf·in. For this case, the principal stresses are

$$\sigma_1 = \frac{32M}{\pi d^3}, \quad \sigma_2 = \sigma_3 = 0$$

Using a conservative yielding failure theory use the MSS theory and Eq. (5-3)

$$\sigma_1 - \sigma_3 = \frac{S_y}{n} \quad \Rightarrow \quad \frac{32M}{\pi d^3} = \frac{S_y}{n} \quad \Rightarrow \quad d = \left(\frac{32Mn}{\pi S_y}\right)^{1/3}$$

Thus, $d = \left[\dfrac{32(312.5)2.5}{\pi(32)10^3}\right]^{1/3} = 0.629 \text{ in} \quad \therefore \quad \text{Use } d = \dfrac{11}{16} \text{ in}$ *Ans.*

5-80 When the ring is set, the hoop tension in the ring is equal to the screw tension.

$$\sigma_t = \frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(1 + \frac{r_o^2}{r^2}\right)$$

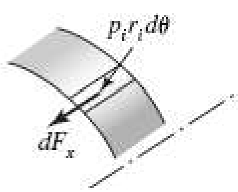
The differential hoop tension dF at r for the ring of width w, is $dF = w\sigma_t dr$. Integration yields

$$F = \int_{r_i}^{r_o} w\sigma_t dr = \frac{wr_i^2 p_i}{r_o^2 - r_i^2}\int_{r_i}^{r_o}\left(1 + \frac{r_o^2}{r^2}\right) dr = \frac{wr_i^2 p_i}{r_o^2 - r_i^2}\left(r - \frac{r_o^2}{r}\right)\Bigg|_{r_i}^{r_o} = wr_i p_i \quad (1)$$

The screw equation is

$$F_i = \frac{T}{0.2d} \qquad (2)$$

From Eqs. (1) and (2)



$$p_i = \frac{F}{wr_i} = \frac{T}{0.2dwr_i}$$

$$dF_x = fp_i r_i d\theta$$

$$F_x = \int_0^{2\pi} fp_i wr_i d\theta = \frac{f\,Tw}{0.2dwr_i} r_i \int_0^{2\pi} d\theta$$

$$= \frac{2\pi f\,T}{0.2d} \qquad Ans.$$

5-81 T = 20 N·m, S_y = 450 MPa

(a) From Prob. 5-80, $T = 0.2\, F_i\, d$

$$F_i = \frac{T}{0.2d} = \frac{20}{0.2\left[6\left(10^{-3}\right)\right]} = 16.7\left(10^3\right)\text{ N} = 16.7\text{ kN} \qquad Ans.$$

(b) From Prob. 5-80, $F = wr_i\, p_i$

$$p_i = \frac{F}{wr_i} = \frac{F_i}{wr_i} = \frac{16.7\left(10^3\right)}{\left[12\left(10^{-3}\right)\right]\left[(25/2)\left(10^{-3}\right)\right]} = 111.3\left(10^6\right)\text{ Pa} = 111.3\text{ MPa} \qquad Ans.$$

(c)
$$\sigma_t = \frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(1 + \frac{r_o^2}{r}\right)_{r=r_i} = \frac{p_i\left(r_i^2 + r_o^2\right)}{r_o^2 - r_i^2}$$

$$= \frac{111.3\left(0.0125^2 + 0.025^2\right)}{0.025^2 - 0.0125^2} = 185.5 \text{ MPa} \qquad Ans.$$

$$\sigma_r = -p_i = -111.3 \text{ MPa}$$

(d)
$$\tau_{\max} = \frac{\sigma_1 - \sigma_3}{2} = \frac{\sigma_t - \sigma_r}{2}$$
$$= \frac{185.5 - (-111.3)}{2} = 148.4 \text{ MPa} \qquad Ans.$$
$$\sigma' = (\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2)^{1/2}$$

$$= \left[185.5^2 - (185.5)(-111.3) + (-111.3)^2\right]^{1/2}$$

$$= 259.7 \text{ MPa} \qquad Ans.$$

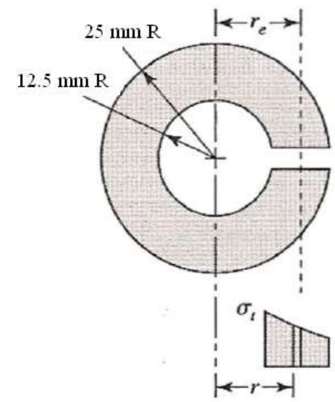
(e) Maximum Shear Stress Theory

$$n = \frac{S_y}{2\tau_{\max}} = \frac{450}{2(148.4)} = 1.52 \qquad Ans.$$

Distortion Energy theory

$$n = \frac{S_y}{\sigma'} = \frac{450}{259.7} = 1.73 \qquad Ans.$$

5-82 The moment about the center caused by the force F is $F\,r_e$ where r_e is the effective radius. This is balanced by the moment about the center caused by the tangential (hoop) stress. For the ring of width w



$$Fr_e = \int_{r_i}^{r_o} r\sigma_t w\,dr$$
$$= \frac{wp_i r_i^2}{r_o^2 - r_i^2}\int_{r_i}^{r_o}\left(r + \frac{r_o^2}{r}\right)dr$$
$$r_e = \frac{wp_i r_i^2}{F\left(r_o^2 - r_i^2\right)}\left(\frac{r_o^2 - r_i^2}{2} + r_o^2 \ln\frac{r_o}{r_i}\right)$$

From Prob. 5-80, $F = wr_i\,p_i$. Therefore,

$$r_e = \frac{r_i}{r_o^2 - r_i^2}\left(\frac{r_o^2 - r_i^2}{2} + r_o^2 \ln\frac{r_o}{r_i}\right)$$

For the conditions of Prob. 5-80, r_i = 12.5 mm and r_o = 25 mm

$$r_e = \frac{12.5}{25^2 - 12.5^2}\left(\frac{25^2 - 12.5^2}{2} + 25^2 \ln\frac{25}{12.5}\right) = 17.8 \text{ mm} \qquad \textit{Ans.}$$

5-83 (**a**) The nominal radial interference is δ_{nom} = (2.002 – 2.001) /2 = 0.0005 in.

From Eq. (3-57),

$$p = \frac{E\delta}{2R^3}\left[\frac{\left(r_o^2 - R^2\right)\left(R^2 - r_i^2\right)}{r_o^2 - r_i^2}\right]$$
$$= \frac{30\left(10^6\right)0.0005}{2\left(1^3\right)}\left[\frac{\left(1.5^2 - 1^2\right)\left(1^2 - 0.625^2\right)}{1.5^2 - 0.625^2}\right] = 3072 \text{ psi} \quad \textit{Ans.}$$

Inner member: $p_i = 0$, $p_o = p$ = 3072 psi. At fit surface $r = R$ = 1 in,

Eq. (3-49): $$\sigma_t = -p\frac{R^2 + r_i^2}{R^2 - r_i^2} = -3072\left(\frac{1^2 + 0.625^2}{1^2 - 0.625^2}\right) = -7\,010 \text{ psi}$$

$$\sigma_r = -p = -3072 \text{ psi}$$

Eq. (5-13):

$$\sigma' = \left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_A^2\right)^{1/2}$$

$$= \left[(-7010)^2 - (-7010)(-3072) + (-3072)\right]^{1/2} = 6086 \text{ psi} \quad \textit{Ans.}$$

Outer member: $p_i = p = 3072$ psi, $p_o = 0$. At fit surface $r = R = 1$ in,

Eq. (3-49):

$$\sigma_t = p\frac{r_o^2 + R^2}{r_o^2 - R^2} = 3072\left(\frac{1.5^2 + 1^2}{1.5^2 - 1^2}\right) = 7987 \text{ psi}$$

$$\sigma_r = -p = -3072 \text{ psi}$$

Eq. (5-13):

$$\sigma' = \left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_A^2\right)^{1/2}$$

$$= \left[7987^2 - 7987(-3072) + (-3072)\right]^{1/2} = 9888 \text{ psi} \quad \textit{Ans.}$$

(**b**) For a solid inner tube,

$$p = \frac{30(10^6)0.0005}{2(1^3)}\left[\frac{(1.5^2 - 1^2)(1^2)}{1.5^2}\right] = 4167 \text{ psi} \quad \textit{Ans.}$$

Inner member: $\sigma_t = \sigma_r = -p = -4167$ psi

$$\sigma' = \left[(-4167)^2 - (-4167)(-4167) + (-4167)^2\right]^{1/2} = 4167 \text{ psi} \quad \textit{Ans.}$$

Outer member: $p_i = p = 4167$ psi, $p_o = 0$. At fit surface $r = R = 1$ in,

Eq. (3-49):

$$\sigma_t = p\frac{r_o^2 + R^2}{r_o^2 - R^2} = 4167\left(\frac{1.5^2 + 1^2}{1.5^2 - 1^2}\right) = 10834 \text{ psi}$$

$$\sigma_r = -p = -4167 \text{ psi}$$

Eq. (5-13):

$$\sigma' = \left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_A^2\right)^{1/2}$$

$$= \left[10\,834^2 - 10\,834(-4167) + (-4167)\right]^{1/2} = 13\,410 \text{ psi} \quad \textit{Ans.}$$

5-84 From Table A-5, E = 207 (10^3) MPa. The nominal radial interference is δ_{nom} = (40 – 39.98) /2 = 0.01 mm.

From Eq. (3-57),

$$p = \frac{E\delta}{2R^3}\left[\frac{\left(r_o^2 - R^2\right)\left(R^2 - r_i^2\right)}{r_o^2 - r_i^2}\right]$$
$$= \frac{207\left(10^3\right)0.01}{2\left(20^3\right)}\left[\frac{\left(32.5^2 - 20^2\right)\left(20^2 - 10^2\right)}{32.5^2 - 10^2}\right] = 26.64 \text{ MPa} \quad \textit{Ans.}$$

Inner member: $p_i = 0$, $p_o = p$ = 26.64 MPa. At fit surface $r = R$ = 20 mm,

Eq. (3-49): $$\sigma_t = -p\frac{R^2 + r_i^2}{R^2 - r_i^2} = -26.64\left(\frac{20^2 + 10^2}{20^2 - 10^2}\right) = -44.40 \text{ MPa}$$

$$\sigma_r = -p = -26.64 \text{ MPa}$$

Eq. (5-13):
$$\sigma' = \left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_A^2\right)^{1/2}$$
$$= \left[\left(-44.40\right)^2 - \left(-44.40\right)\left(-26.64\right) + \left(-26.64\right)\right]^{1/2} = 38.71 \text{ MPa} \quad \textit{Ans.}$$

Outer member: $p_i = p$ = 26.64 MPa, p_o = 0. At fit surface $r = R$ = 20 mm,

Eq. (3-49): $$\sigma_t = p\frac{r_o^2 + R^2}{r_o^2 - R^2} = 26.64\left(\frac{32.5^2 + 20^2}{32.5^2 - 20^2}\right) = 59.12 \text{ MPa}$$

$$\sigma_r = -p = -26.64 \text{ MPa}$$

Eq. (5-13):
$$\sigma' = \left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_A^2\right)^{1/2}$$
$$= \left[59.12^2 - 59.12\left(-26.64\right) + \left(-26.64\right)\right]^{1/2} = 76.03 \text{ MPa} \quad \textit{Ans.}$$

5-85 From Table A-5, E = 207 (10^3) MPa. The nominal radial interference is δ_{nom} = (40.008 – 39.972) /2 = 0.018 mm.

From Eq. (3-57),

$$p = \frac{E\delta}{2R^3}\left[\frac{\left(r_o^2 - R^2\right)\left(R^2 - r_i^2\right)}{r_o^2 - r_i^2}\right]$$

$$= \frac{207\left(10^3\right)0.018}{2\left(20^3\right)}\left[\frac{\left(32.5^2 - 20^2\right)\left(20^2 - 10^2\right)}{32.5^2 - 10^2}\right] = 47.94 \text{ MPa} \quad Ans.$$

Inner member: $p_i = 0$, $p_o = p = 47.94$ MPa. At fit surface $r = R = 20$ mm,

Eq. (3-49): $$\sigma_t = -p\frac{R^2 + r_i^2}{R^2 - r_i^2} = -47.94\left(\frac{20^2 + 10^2}{20^2 - 10^2}\right) = -79.90 \text{ MPa}$$

$$\sigma_r = -p = -47.94 \text{ MPa}$$

Eq. (5-13):

$$\sigma' = \left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_A^2\right)^{1/2}$$

$$= \left[\left(-79.90\right)^2 - \left(-79.90\right)\left(-47.94\right) + \left(-47.94\right)\right]^{1/2} = 69.66 \text{ MPa} \quad Ans.$$

Outer member: $p_i = p = 47.94$ MPa, $p_o = 0$. At fit surface $r = R = 20$ mm,

Eq. (3-49): $$\sigma_t = p\frac{r_o^2 + R^2}{r_o^2 - R^2} = 47.94\left(\frac{32.5^2 + 20^2}{32.5^2 - 20^2}\right) = 106.4 \text{ MPa}$$

$$\sigma_r = -p = -47.94 \text{ MPa}$$

Eq. (5-13):

$$\sigma' = \left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_A^2\right)^{1/2}$$

$$= \left[106.4^2 - 106.4\left(-47.94\right) + \left(-47.94\right)\right]^{1/2} = 136.8 \text{ MPa} \quad Ans.$$

5-86 From Table A-5, for carbon steel, $E_s = 30$ kpsi, and $\nu_s = 0.292$. While for $E_{ci} = 14.5$ Mpsi, and $\nu_{ci} = 0.211$. For ASTM grade 20 cast iron, from Table A-24, $S_{ut} = 22$ kpsi.

For midrange values, $\delta = (2.001 - 2.0002)/2 = 0.0004$ in.

Eq. (3-50):

$$p=\frac{\delta}{R\left[\frac{1}{E_o}\left(\frac{r_o^2+R^2}{r_o^2-R^2}+\nu_o\right)+\frac{1}{E_i}\left(\frac{R^2+r_i^2}{R^2-r_i^2}-\nu_i\right)\right]}$$

$$=\frac{0.0004}{1\left[\frac{1}{14.5\left(10^6\right)}\left(\frac{2^2+1^2}{2^2-1^2}+0.211\right)+\frac{1}{30\left(10^6\right)}\left(\frac{1^2}{1^2}-0.292\right)\right]}=2613 \text{ psi}$$

At fit surface, with $p_i = p$ =2613 psi, and p_o = 0, from Eq. (3-50)

$$\sigma_t = p\frac{r_o^2+R^2}{r_o^2-R^2}=2613\left(\frac{2^2+1^2}{2^2-1^2}\right)=4355 \text{ psi}$$

$$\sigma_r = -p = -2613 \text{ psi}$$

From Modified-Mohr theory, Eq. (5-32a), since $\sigma_A > 0 > \sigma_B$ and $|\sigma_B/\sigma_A| < 1$,

$$n=\frac{S_{ut}}{\sigma_A}=\frac{22}{4.355}=5.05 \qquad \textit{Ans.}$$

5-87 E = 207 GPa

Eq. (3-57) can be written in terms of diameters,

$$p=\frac{E\delta_d}{2D^3}\left[\frac{(d_o^2-D^2)(D^2-d_i^2)}{(d_o^2-d_i^2)}\right]=\frac{207\left(10^3\right)(0.062)}{2\left(45\right)^3}\left[\frac{(50^2-45^2)(45^2-40^2)}{(50^2-40^2)}\right]$$

$$p=15.80 \text{ MPa}$$

Outer member: From Eq. (3-50),

Outer radius: $$\left(\sigma_t\right)_o=\frac{45^2(15.80)}{50^2-45^2}(2)=134.7 \text{ MPa}$$

$$\left(\sigma_r\right)_o=0$$

Inner radius: $$\left(\sigma_t\right)_i=\frac{45^2(15.80)}{50^2-45^2}\left(1+\frac{50^2}{45^2}\right)=150.5 \text{ MPa}$$

$$\left(\sigma_r\right)_i=-15.80 \text{ MPa}$$

Bending (no slipping): $I = (\pi/64)(50^4 - 40^4) = 181.1\ (10^3)\ \text{mm}^4$

At r_o: $$(\sigma_x)_o = \pm\frac{Mc}{I} = \pm\frac{675(0.05/2)}{181.1\,(10^{-9})} = \pm 93.2(10^6)\text{ Pa} = \pm 93.2\text{ MPa}$$

At r_i: $$(\sigma_x)_i = \pm\frac{675(0.045/2)}{181.1\,(10^{-9})} = \pm 83.9(10^6)\text{ Pa} = \pm 83.9\text{ MPa}$$

Torsion: $J = 2I = 362.2\ (10^3)\ \text{mm}^4$

At r_o: $$(\tau_{xy})_o = \frac{Tc}{J} = \frac{900(0.05/2)}{362.2\,(10^{-9})} = 62.1(10^6)\text{ Pa} = 62.1\text{ MPa}$$

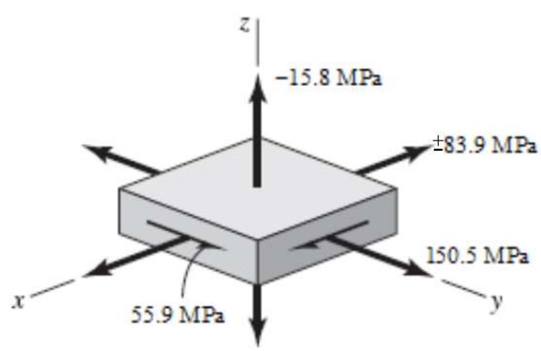
At r_i: $$(\tau_{xy})_i = \frac{900(0.045/2)}{362.2\,(10^{-9})} = 55.9(10^6)\text{ Pa} = 55.9\text{ MPa}$$

Outer radius, is plane stress. Since the tangential stress is positive the von Mises stress will be a maximum with a negative bending stress. That is,

$$\sigma_x = -93.2\text{ MPa},\ \sigma_y = 134.7\text{ MPa},\ \tau_{xy} = 62.1\text{ MPa}$$

Eq. (5-15)
$$\sigma' = \left(\sigma_x^2 - \sigma_x\sigma_y + \sigma_y^2 + 3\tau_{xy}^2\right)^{1/2}$$
$$= \left[(-93.2)^2 - (-93.2)(134.7) + 134.7^2 + 3(62.1)^2\right]^{1/2} = 226\text{ MPa}$$
$$n_o = \frac{S_y}{\sigma'} = \frac{415}{226} = 1.84 \qquad \textit{Ans.}$$

Inner radius, 3D state of stress



From Eq. (5-14) with $\tau_{yz} = \tau_{zx} = 0$ and $\sigma_x = +\ 83.9$ MPa
$$\sigma' = \frac{1}{\sqrt{2}}\left[(83.9-150.5)^2 + (150.5+15.8)^2 + (-15.8-83.9)^2 + 6(55.9)^2\right]^{1/2} = 174\text{ MPa}$$
With $\sigma_x = -\ 83.9$ MPa
$$\sigma' = \frac{1}{\sqrt{2}}\left[(-83.9-150.5)^2 + (150.5+15.8)^2 + (-15.8+83.9)^2 + 6(55.9)^2\right]^{1/2} = 230\text{ MPa}$$

$$n_i = \frac{S_y}{\sigma'} = \frac{415}{230} = 1.80 \qquad Ans.$$

5-88 From the solution of Prob. 5-87, p = 15.80 MPa

Inner member: From Eq. (3-50),

Outer radius: $(\sigma_t)_o = -\frac{r_o^2 + r_i^2}{r_o^2 - r_i^2} p_o = -\frac{45^2 + 40^2}{45^2 - 40^2}(15.80) = -134.8 \text{ MPa}$

$$(\sigma_r)_o = -p = -15.80 \text{ MPa}$$

Inner radius: $(\sigma_t)_i = -\frac{2r_o^2}{r_o^2 - r_i^2} p_o = -\frac{2\left(45^2\right)}{45^2 - 40^2}(15.80) = -150.6 \text{ MPa}$

$$(\sigma_r)_i = 0$$

Bending (no slipping): $I = (\pi/64)(50^4 - 40^4) = 181.1\ (10^3)\ \text{mm}^4$

At r_o: $(\sigma_x)_o = \pm\frac{Mc}{I} = \pm\frac{675(0.045/2)}{181.1\left(10^{-9}\right)} = \pm 83.9(10^6) \text{ Pa} = \pm 83.9 \text{ MPa}$

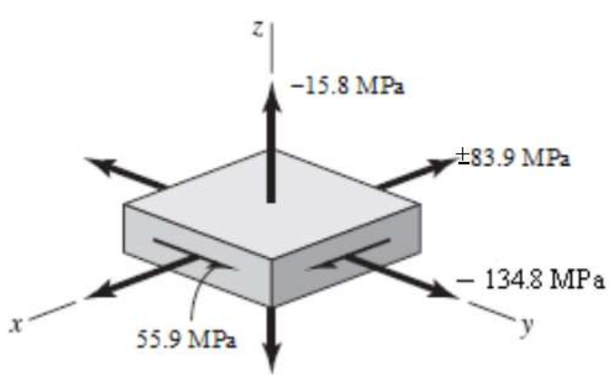
At r_i: $(\sigma_x)_i = \pm\frac{675(0.040/2)}{181.1\left(10^{-9}\right)} = \pm 74.5\left(10^6\right) \text{ Pa} = \pm 74.5 \text{ MPa}$

Torsion: $J = 2I = 362.2\ (10^3)\ \text{mm}^4$

At r_o: $(\tau_{xy})_o = \frac{Tc}{J} = \frac{900(0.045/2)}{362.2\left(10^{-9}\right)} = 55.9\left(10^6\right) \text{ Pa} = 55.9 \text{ MPa}$

At r_i: $(\tau_{xy})_i = \frac{900(0.040/2)}{362.2\left(10^{-9}\right)} = 49.7\left(10^6\right) \text{ Pa} = 49.7 \text{ MPa}$

Outer radius, 3D state of stress



From Eq. (5-14) with $\tau_{yz} = \tau_{zx} = 0$ and $\sigma_x = +\ 83.9$ MPa

$$\sigma' = \frac{1}{\sqrt{2}}\left[(83.9+134.8)^2+(-134.8+15.8)^2+(-15.8-83.9)^2+6(55.9)^2\right]^{1/2} = 213 \text{ MPa}$$

With $\sigma_x = -\ 83.9$ MPa

$$\sigma' = \frac{1}{\sqrt{2}}\left[(-83.9+134.8)^2+(-134.8+15.8)^2+(-15.8+83.9)^2+6(55.9)^2\right]^{1/2} = 142 \text{ MPa}$$

$$n_o = \frac{S_y}{\sigma'} = \frac{415}{213} = 1.95 \qquad Ans.$$

Inner radius, plane stress. Worst case is when σ_x is positive

$\sigma_x = 74.5$ MPa, $\sigma_y = -150.6$ MPa, $\tau_{xy} = 49.7$ MPa

Eq. (5-15)

$$\sigma' = \left(\sigma_x^2 - \sigma_x\sigma_y + \sigma_y^2 + 3\tau_{xy}^2\right)^{1/2}$$

$$= \left[74.5^2 - 74.5(-150.6) + (-150.6)^2 + 3(49.7)^2\right]^{1/2} = 216 \text{ MPa}$$

$$n_i = \frac{S_y}{\sigma'} = \frac{415}{216} = 1.92 \qquad Ans.$$

5-89 For AISI 1040 HR, from Table A-20, $S_y = 290$ MPa.

From Prob. 3-124, $p_{\max} = 65.2$ MPa. From Eq. (3-50) at the inner radius R of the outer member,

$$\sigma_t = p\frac{r_o^2 + R^2}{r_o^2 - R^2} = 65.2\frac{50^2+25^2}{50^2-25^2} = 108.7 \text{ MPa}$$

$$\sigma_r = -p = -65.2 \text{ MPa}$$

These are principal stresses. From Eq. (5-13)

$$\sigma_o' = \left(\sigma_t^2 - \sigma_t\sigma_r + \sigma_r^2\right)^{1/2} = \left[108.7^2 - 108.7(-65.2) + (-65.2)^2\right]^{1/2} = 152.2 \text{ MPa}$$

$$n = \frac{S_y}{\sigma_o'} = \frac{290}{152.2} = 1.91 \qquad Ans.$$

5-90 For AISI 1040 HR, from Table A-20, $S_y = 42$ kpsi.

From Prob. 3-125, p_{max} = 9 kpsi. From Eq. (3-50) at the inner radius R of the outer member,

$$\sigma_t = p\frac{r_o^2 + R^2}{r_o^2 - R^2} = 9\frac{2^2 + 1^2}{2^2 - 1^2} = 15 \text{ kpsi}$$

$$\sigma_r = -p = -9 \text{ kpsi}$$

These are principal stresses. From Eq. (5-13)

$$\sigma'_o = \left(\sigma_t^2 - \sigma_t\sigma_r + \sigma_r^2\right)^{1/2} = \left[15^2 - 15(-9) + (-9)^2\right]^{1/2} = 21 \text{ kpsi}$$

$$n = \frac{S_y}{\sigma'_o} = \frac{42}{21} = 2 \qquad \textit{Ans.}$$

5-91 For AISI 1040 HR, from Table A-20, S_y = 290 MPa.

From Prob. 3-126, p_{max} = 91.6 MPa. From Eq. (3-50) at the inner radius R of the outer member,

$$\sigma_t = p\frac{r_o^2 + R^2}{r_o^2 - R^2} = 91.6\frac{50^2 + 25^2}{50^2 - 25^2} = 152.7 \text{ MPa}$$

$$\sigma_r = -p = -91.6 \text{ MPa}$$

These are principal stresses. From Eq. (5-13)

$$\sigma'_o = \left(\sigma_t^2 - \sigma_t\sigma_r + \sigma_r^2\right)^{1/2} = \left[152.7^2 - 152.7(-91.6) + (-91.6)^2\right]^{1/2} = 213.8 \text{ MPa}$$

$$n = \frac{S_y}{\sigma'_o} = \frac{290}{213.8} = 1.36 \qquad \textit{Ans.}$$

5-92 For AISI 1040 HR, from Table A-20, S_y = 42 kpsi.

From Prob. 3-127, p_{max} = 12.94 kpsi. From Eq. (3-50) at the inner radius R of the outer member,

$$\sigma_t = p\frac{r_o^2 + R^2}{r_o^2 - R^2} = 12.94\frac{2^2 + 1^2}{2^2 - 1^2} = 21.57 \text{ kpsi}$$

$$\sigma_r = -p = -12.94 \text{ kpsi}$$

These are principal stresses. From Eq. (5-13)

$$\sigma'_o = \left(\sigma_t^2 - \sigma_t\sigma_r + \sigma_r^2\right)^{1/2} = \left[21.57^2 - 21.57(-12.94) + (-12.94)^2\right]^{1/2} = 30.20 \text{ kpsi}$$

$$n = \frac{S_y}{\sigma'_o} = \frac{42}{30.2} = 1.39 \qquad Ans.$$

5-93 For AISI 1040 HR, from Table A-20, S_y = 290 MPa.

From Prob. 3-128, $p_{\max}$ = 134 MPa. From Eq. (3-50) at the inner radius R of the outer member,

$$\sigma_t = p\frac{r_o^2 + R^2}{r_o^2 - R^2} = 134\frac{50^2 + 25^2}{50^2 - 25^2} = 223.3 \text{ MPa}$$

$$\sigma_r = -p = -134 \text{ MPa}$$

These are principal stresses. From Eq. (5-13)

$$\sigma'_o = \left(\sigma_t^2 - \sigma_t\sigma_r + \sigma_r^2\right)^{1/2} = \left[223.3^2 - 223.3(-134) + \left(-134\right)^2\right]^{1/2} = 312.6 \text{ MPa}$$

$$n = \frac{S_y}{\sigma'_o} = \frac{290}{312.6} = 0.93 \qquad Ans.$$

5-94 For AISI 1040 HR, from Table A-20, S_y = 42 kpsi.

From Prob. 3-129, $p_{\max}$ = 19.13 kpsi. From Eq. (3-50) at the inner radius R of the outer member,

$$\sigma_t = p\frac{r_o^2 + R^2}{r_o^2 - R^2} = 19.13\frac{2^2 + 1^2}{2^2 - 1^2} = 31.88 \text{ kpsi}$$

$$\sigma_r = -p = -19.13 \text{ kpsi}$$

These are principal stresses. From Eq. (5-13)

$$\sigma'_o = \left(\sigma_t^2 - \sigma_t\sigma_r + \sigma_r^2\right)^{1/2} = \left[31.88^2 - 31.88(-19.13) + \left(-19.13\right)^2\right]^{1/2} = 44.63 \text{ kpsi}$$

$$n = \frac{S_y}{\sigma'_o} = \frac{42}{44.63} = 0.94 \qquad Ans.$$

5-95

$$\sigma_p = \frac{1}{2}\left(2\frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\right) \pm \left[\left(\frac{K_I}{\sqrt{2\pi r}}\sin\frac{\theta}{2}\cos\frac{\theta}{2}\sin\frac{3\theta}{2}\right)^2 + \left(\frac{K_I}{\sqrt{2\pi r}}\sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2}\right)^2\right]^{1/2}$$

$$= \frac{K_I}{\sqrt{2\pi r}}\left[\cos\frac{\theta}{2} \pm \left(\sin^2\frac{\theta}{2}\cos^2\frac{\theta}{2}\sin^2\frac{3\theta}{2} + \sin^2\frac{\theta}{2}\cos^2\frac{\theta}{2}\cos^2\frac{3\theta}{2}\right)^{1/2}\right]$$

$$= \frac{K_I}{\sqrt{2\pi r}}\left(\cos\frac{\theta}{2} \pm \sin\frac{\theta}{2}\cos\frac{\theta}{2}\right) = \frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1 \pm \sin\frac{\theta}{2}\right)$$

Plane stress: The third principal stress is zero and

$$\sigma_1 = \frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1 + \sin\frac{\theta}{2}\right),\ \sigma_2 = \frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1 - \sin\frac{\theta}{2}\right),\ \sigma_3 = 0 \quad \textit{Ans.}$$

Plane strain: Equations for σ_1 and σ_2 are still valid,. However,

$$\sigma_3 = \nu\left(\sigma_1 + \sigma_2\right) = 2\nu\frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2} \qquad \textit{Ans.}$$

5-96 For $\theta = 0$ and plane strain, the principal stress equations of Prob. 5-95 give

$$\sigma_1 = \sigma_2 = \frac{K_I}{\sqrt{2\pi r}},\quad \sigma_3 = 2\nu\frac{K_I}{\sqrt{2\pi r}} = 2\nu\sigma_1$$

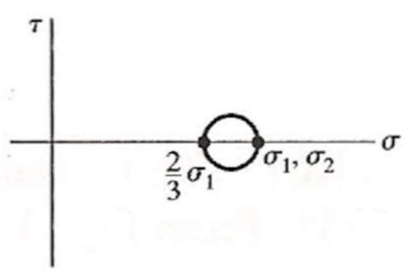
(**a**) DE: Eq. (5-18) $\frac{1}{\sqrt{2}}\left[\left(\sigma_1 - \sigma_1\right)^2 + \left(\sigma_1 - 2\nu\sigma_1\right)^2 + \left(2\nu\sigma_1 - \sigma_1\right)^2\right]^{1/2} = S_y$

or, $\sigma_1 - 2\nu\sigma_1 = S_y$

For $\nu = \frac{1}{3}$, $\left[1 - 2\left(\frac{1}{3}\right)\right]\sigma_1 = S_y \quad \Rightarrow \quad \sigma_1 = 3S_y \quad$ *Ans.*

(**a**) MSS: Eq. (5-3) , with n =1 $\qquad \sigma_1 - \sigma_3 = S_y \quad \Rightarrow \quad \sigma_1 - 2\nu\sigma_1 = S_y$

$$\nu = \frac{1}{3} \quad \Rightarrow \quad \sigma_1 = 3S_y \qquad \textit{Ans.}$$



$$\sigma_3 = \sigma_1 - S_y = 2S_y \quad \Rightarrow \quad \sigma_3 = \frac{2}{3}\sigma_1$$

Radius of largest circle

$$R = \frac{1}{2}\left(\sigma_1 - \frac{2}{3}\sigma_1\right) = \frac{\sigma_1}{6}$$

5-97 Given: a = 16 mm, K_{Ic} = 80 MPa$\cdot\sqrt{\text{m}}$ and $S_y = 950$ MPa

(a) Ignoring stress concentration

$F = S_yA$ = 950(100 − 16)(12) = 958(10^3) N = 958 kN *Ans.*

(b) From Fig. 5-26: h/b = 1, a/b = 16/100 = 0.16, β = 1.3

Eq. (5-37) $K_I = \beta\sigma\sqrt{\pi a}$

$$80 = 1.3\frac{F}{100(12)}\sqrt{\pi(16)10^{-3}}$$

F = 329.4(10^3) N = 329.4 kN *Ans.*

5-98 Given: a = 0.5 in, K_{Ic} = 72 kpsi$\cdot\sqrt{\text{in}}$ and S_y = 170 kpsi, S_{ut} = 192 kpsi

r_o = 14/2 = 7 in, r_i = (14 − 2)/2 = 6 in

$$\frac{a}{r_o - r_i} = \frac{0.5}{7-6} = 0.5, \qquad \frac{r_i}{r_o} = \frac{6}{7} = 0.857$$

Fig. 5-30: β B 2.4

Eq. (5-37): $K_{Ic} = \beta\sigma\sqrt{\pi a} \quad\Rightarrow\quad 72 = 2.4\sigma\sqrt{\pi(0.5)} \quad\Rightarrow\quad \sigma = 23.9$ kpsi

Eq. (3-50) at $r = r_o$ = 7 in:

$$\sigma_t = \frac{r_i^2 p_i}{r_o^2 - r_i^2}(2) \quad\Rightarrow\quad 23.9 = \frac{6^2 p_i}{7^2 - 6^2}(2) \quad\Rightarrow\quad p_i = 4.315 \text{ kpsi} \qquad \textit{Ans.}$$
