Chapter 3

3-1

$\Sigma M_O = 0$

$18R_B - 6(100) = 0$

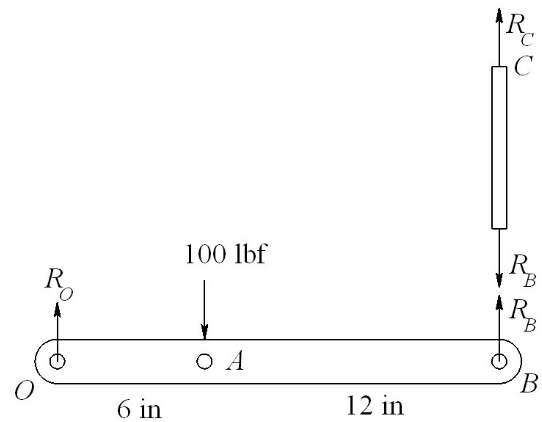
$R_B = 33.3$ lbf *Ans.*

$\Sigma F_y = 0$

$R_O + R_B - 100 = 0$

$R_O = 66.7$ lbf *Ans.*

$R_C = R_B = 33.3$ lbf *Ans.*



3-2

Body *AB*:

$\Sigma F_x = 0 \qquad R_{Ax} = R_{Bx}$

$\Sigma F_y = 0 \qquad R_{Ay} = R_{By}$

$\Sigma M_B = 0 \qquad R_{Ay}(10) - R_{Ax}(10) = 0$

$R_{Ax} = R_{Ay}$

Body *OAC*:

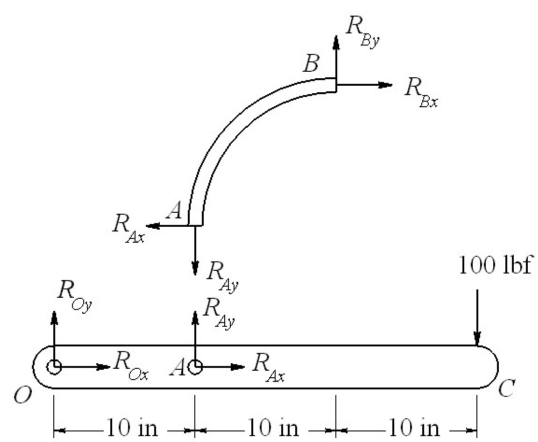
$\Sigma M_O = 0 \qquad R_{Ay}(10) - 100(30) = 0$

$R_{Ay} = 300$ lbf *Ans.*

$\Sigma F_x = 0 \qquad R_{Ox} = -R_{Ax} = -300$ lbf *Ans.*

$\Sigma F_y = 0 \qquad R_{Oy} + R_{Ay} - 100 = 0$

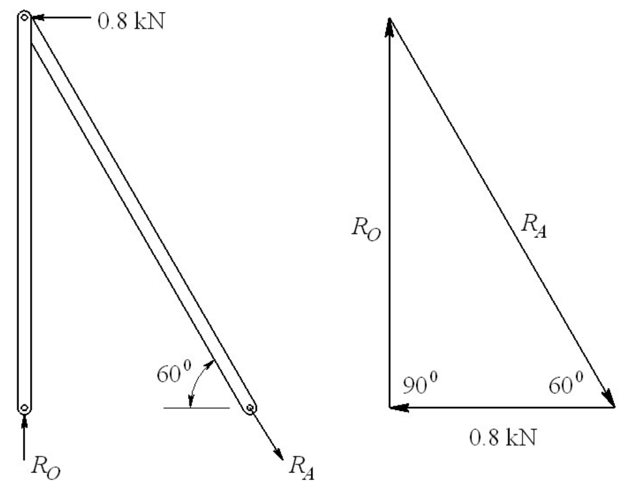
$R_{Oy} = -200$ lbf *Ans.*



3-3

$$R_O = \frac{0.8}{\tan 30^\circ} = 1.39 \text{ kN} \qquad Ans.$$

$$R_A = \frac{0.8}{\sin 30^\circ} = 1.6 \text{ kN} \qquad Ans.$$



3-4

Step 1: Find R_A & R_E

$$h = \frac{4.5}{\tan 30^\circ} = 7.794 \text{ m}$$

$$\Sigma M_A = 0$$

$$9R_E - 7.794(400\cos 30^\circ)$$

$$-4.5(400\sin 30^\circ) = 0$$

$$R_E = 400 \text{ N} \qquad Ans.$$

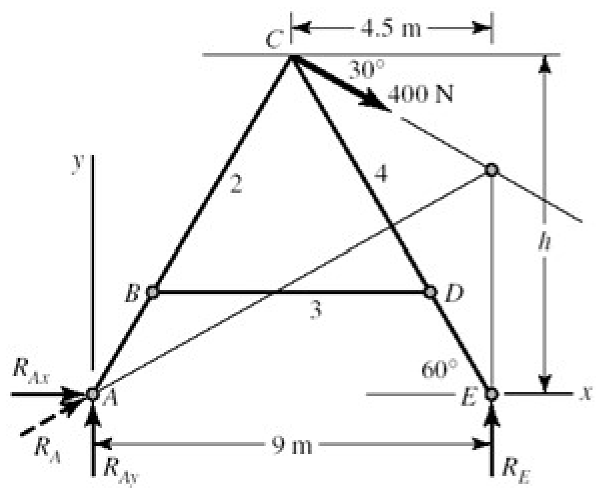
$$\sum F_x = 0 \qquad R_{Ax} + 400\cos 30^\circ = 0$$

$$R_{Ax} = -346.4 \text{ N}$$

$$\sum F_y = 0 \qquad R_{Ay} + 400 - 400\sin 30^\circ = 0$$

$$R_{Ay} = -200 \text{ N}$$

$$R_A = \sqrt{346.4^2 + 200^2} = 400 \text{ N} \qquad Ans.$$



Step 2: Find components of R_C and R_D on link 4

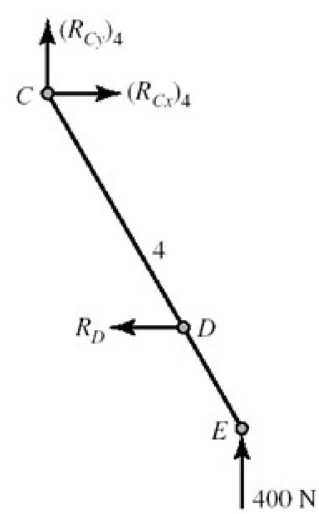
$$\sum M_C = 0$$

$$400(4.5) - (7.794 - 1.9)R_D = 0$$

$$R_D = 305.4 \text{ N} \qquad Ans.$$

$$\sum F_x = 0 \quad \Rightarrow \quad (R_{Cx})_4 = 305.4 \text{ N}$$

$$\sum F_y = 0 \quad \Rightarrow \quad (R_{Cy})_4 = -400 \text{ N}$$



Step 3: Find components of R_C on link 2

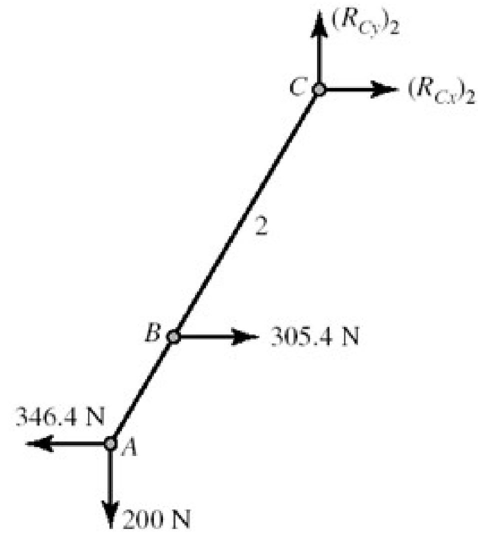
$$\sum F_x = 0$$

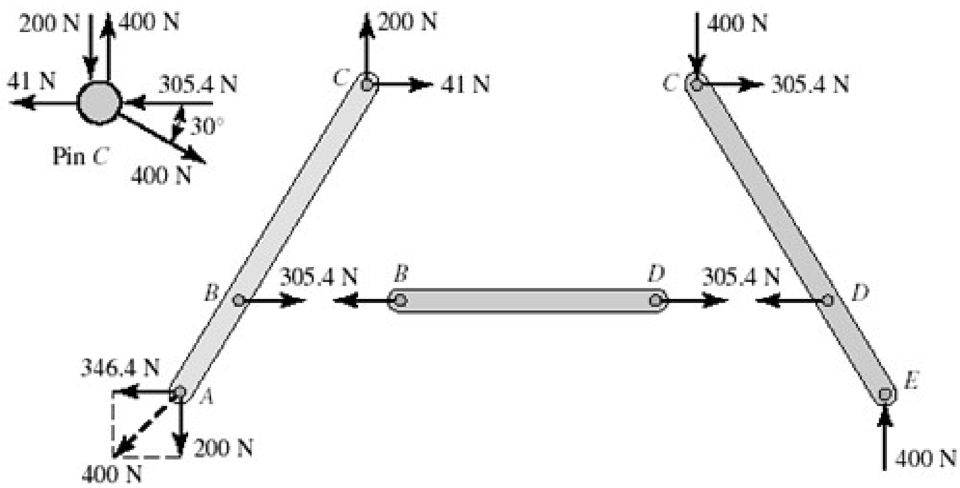
$$\left(R_{Cx}\right)_2 + 305.4 - 346.4 = 0$$

$$\left(R_{Cx}\right)_2 = 41 \text{ N}$$

$$\sum F_y = 0$$

$$\left(R_{Cy}\right)_2 = 200 \text{ N}$$





Ans.

3-5

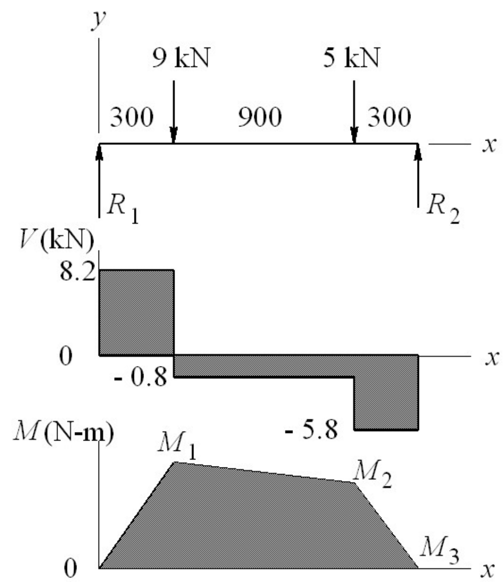
$\Sigma M_C = 0$

$-1500R_1 + 300(5) + 1200(9) = 0$

$R_1 = 8.2 \text{ kN}$ *Ans.*

$\Sigma F_y = 0$

$8.2 - 9 - 5 + R_2 = 0 \qquad R_2 = 5.8 \text{ kN}$ *Ans.*



$M_1 = 8.2(300) = 2460 \text{ N}\cdot\text{m}$ *Ans.*

$M_2 = 2460 - 0.8(900) = 1740 \text{ N}\cdot\text{m}$ *Ans.*

$M_3 = 1740 - 5.8(300) = 0$ checks!

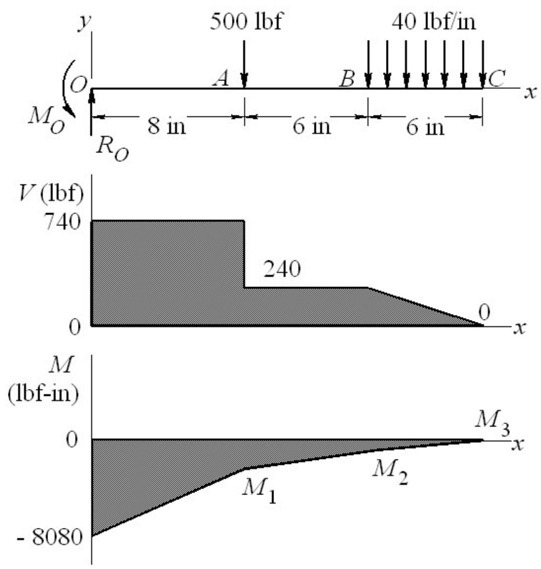
3-6

$\Sigma F_y = 0$

$R_O = 500 + 40(6) = 740 \text{ lbf}$ *Ans.*

$\Sigma M_O = 0$

$M_O = 500(8) + 40(6)(17) = 8080 \text{ lbf}\cdot\text{in}$ *Ans.*



$M_1 = -8080 + 740(8) = -2160 \text{ lbf}\cdot\text{in}$ *Ans.*

$M_2 = -2160 + 240(6) = -720 \text{ lbf}\cdot\text{in}$ *Ans.*

$M_3 = -720 + \frac{1}{2}(240)(6) = 0$ checks!

3-7

$\Sigma M_B = 0$

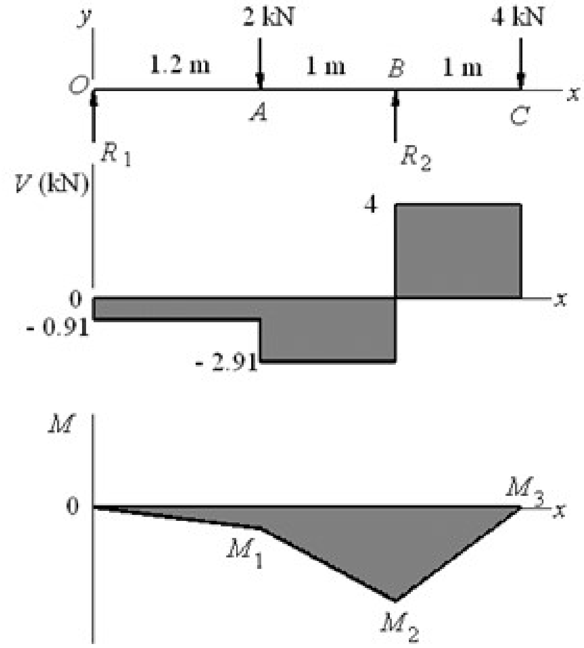
$-2.2R_1 + 1(2) - 1(4) = 0$

$R_1 = -0.91\text{ kN}$ *Ans.*

$\Sigma F_y = 0$

$-0.91 - 2 + R_2 - 4 = 0$

$R_2 = 6.91\text{ kN}$ *Ans.*



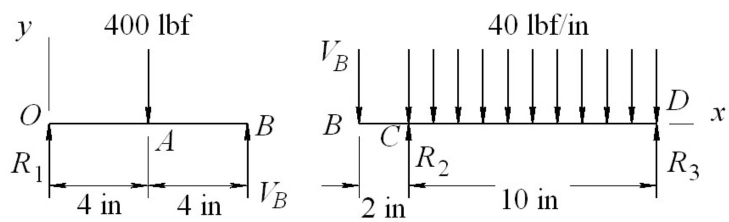
$M_1 = -0.91(1.2) = -1.09\text{ kN}\cdot\text{m}$ *Ans.*

$M_2 = -1.09 - 2.91(1) = -4\text{ kN}\cdot\text{m}$ *Ans.*

$M_3 = -4 + 4(1) = 0$ checks!

3-8

Break at the hinge at *B*



Beam *OB*:

From symmetry,

$R_1 = V_B = 200\text{ lbf}$ *Ans.*

Beam *BD*:

$\Sigma M_D = 0$

$200(12) - R_2(10) + 40(10)(5) = 0$

$R_2 = 440\text{ lbf}$ *Ans.*

$\Sigma F_y = 0$

$-200 + 440 - 40(10) + R_3 = 0$

$R_3 = 160\text{ lbf}$ *Ans.*

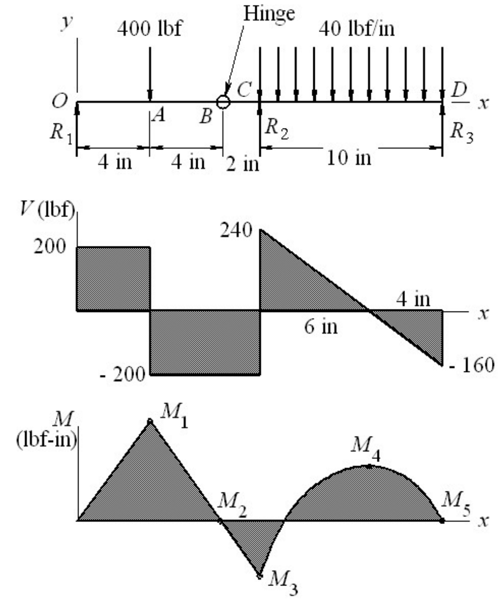
$M_1 = 200(4) = 800\ \text{lbf}\cdot\text{in}$ *Ans.*

$M_2 = 800 - 200(4) = 0$ checks at hinge

$M_3 = 800 - 200(6) = -400\ \text{lbf}\cdot\text{in}$ *Ans.*

$M_4 = -400 + \frac{1}{2}(240)(6) = 320\ \text{lbf}\cdot\text{in}$ *Ans.*

$M_5 = 320 - \frac{1}{2}(160)(4) = 0$ checks!



3-9

$$q = R_1\langle x\rangle^{-1} - 9\langle x-300\rangle^{-1} - 5\langle x-1200\rangle^{-1} + R_2\langle x-1500\rangle^{-1}$$

$$V = R_1 - 9\langle x-300\rangle^{0} - 5\langle x-1200\rangle^{0} + R_2\langle x-1500\rangle^{0} \qquad (1)$$

$$M = R_1 x - 9\langle x-300\rangle^{1} - 5\langle x-1200\rangle^{1} + R_2\langle x-1500\rangle^{1} \qquad (2)$$

At $x = 1500^+$ $V = M = 0$. Applying Eqs. (1) and (2),

$R_1 - 9 - 5 + R_2 = 0 \quad \Rightarrow \quad R_1 + R_2 = 14$

$1500R_1 - 9(1500-300) - 5(1500-1200) = 0 \quad \Rightarrow \quad R_1 = 8.2\ \text{kN}$ *Ans.*

$R_2 = 14 - 8.2 = 5.8\ \text{kN}$ *Ans.*

$0 \le x \le 300$: $V = 8.2\ \text{kN}$, $M = 8.2x\ \text{N}\cdot\text{m}$

$300 \le x \le 1200$: $V = 8.2 - 9 = -0.8\ \text{kN}$

$M = 8.2x - 9(x-300) = -0.8x + 2700\ \text{N}\cdot\text{m}$

$1200 \le x \le 1500$: $V = 8.2 - 9 - 5 = -5.8\ \text{kN}$

$M = 8.2x - 9(x-300) - 5(x-1200) = -5.8x + 8700\ \text{N}\cdot\text{m}$

Plots of V and M are the same as in Prob. 3-5.

3-10

$$q = R_O\langle x\rangle^{-1} - M_O\langle x\rangle^{-2} - 500\langle x-8\rangle^{-1} - 40\langle x-14\rangle^{0} + 40\langle x-20\rangle^{0}$$

$$V = R_O - M_O\langle x\rangle^{-1} - 500\langle x-8\rangle^{0} - 40\langle x-14\rangle^{1} + 40\langle x-20\rangle^{1} \qquad (1)$$

$$M = R_O x - M_O - 500\langle x-8\rangle^{1} - 20\langle x-14\rangle^{2} + 20\langle x-20\rangle^{2} \qquad (2)$$

at $x = 20^{+}$ in, $V = M = 0$, Eqs. (1) and (2) give

$$R_O - 500 - 40(20-14) = 0 \quad \Rightarrow \quad R_O = 740 \text{ lbf} \qquad \textit{Ans.}$$

$$R_O(20) - M_O - 500(20-8) - 20(20-14)^2 = 0 \quad \Rightarrow \quad M_O = 8080 \text{ lbf}\cdot\text{in} \qquad \textit{Ans.}$$

$0 \le x \le 8$: $V = 740$ lbf, $M = 740x - 8080$ lbf·in

$8 \le x \le 14$: $V = 740 - 500 = 240$ lbf

$M = 740x - 8080 - 500(x-8) = 240x - 4080$ lbf·in

$14 \le x \le 20$: $V = 740 - 500 - 40(x-14) = -40x + 800$ lbf

$M = 740x - 8080 - 500(x-8) - 20(x-14)^2 = -20x^2 + 800x - 8000$ lbf·in

Plots of *V* and *M* are the same as in Prob. 3-6.

3-11

$$q = R_1\langle x\rangle^{-1} - 2\langle x-1.2\rangle^{-1} + R_2\langle x-2.2\rangle^{-1} - 4\langle x-3.2\rangle^{-1}$$

$$V = R_1 - 2\langle x-1.2\rangle^{0} + R_2\langle x-2.2\rangle^{0} - 4\langle x-3.2\rangle^{0} \qquad (1)$$

$$M = R_1 x - 2\langle x-1.2\rangle^{1} + R_2\langle x-2.2\rangle^{1} - 4\langle x-3.2\rangle^{1} \qquad (2)$$

at $x = 3.2^{+}$, $V = M = 0$. Applying Eqs. (1) and (2),

$$R_1 - 2 + R_2 - 4 = 0 \quad \Rightarrow \quad R_1 + R_2 = 6 \qquad (3)$$

$$3.2R_1 - 2(2) + R_2(1) = 0 \quad \Rightarrow \quad 3.2R_1 + R_2 = 4 \qquad (4)$$

Solving Eqs. (3) and (4) simultaneously,
R_1 = -0.91 kN, R_2 = 6.91 kN *Ans.*

$0 \le x \le 1.2$: $V = -0.91$ kN, $M = -0.91x$ kN·m

$1.2 \le x \le 2.2$: $V = -0.91 - 2 = -2.91$ kN

$M = -0.91x - 2(x-1.2) = -2.91x + 2.4$ kN·m

$2.2 \le x \le 3.2$: $V = -0.91 - 2 + 6.91 = 4$ kN

$M = -0.91x - 2(x-1.2) + 6.91(x-2.2) = 4x - 12.8$ kN·m

Plots of *V* and *M* are the same as in Prob. 3-7.

3-12

$$q = R_1\langle x\rangle^{-1} - 400\langle x-4\rangle^{-1} + R_2\langle x-10\rangle^{-1} - 40\langle x-10\rangle^{0} + 40\langle x-20\rangle^{0} + R_3\langle x-20\rangle^{-1}$$

$$V = R_1 - 400\langle x-4\rangle^{0} + R_2\langle x-10\rangle^{0} - 40\langle x-10\rangle^{1} + 40\langle x-20\rangle^{1} + R_3\langle x-20\rangle^{0} \qquad (1)$$

$$M = R_1 x - 400\langle x-4\rangle^{1} + R_2\langle x-10\rangle^{1} - 20\langle x-10\rangle^{2} + 20\langle x-20\rangle^{2} + R_3\langle x-20\rangle^{1} \qquad (2)$$

$M = 0$ at $x = 8$ in $\therefore 8R_1 - 400(8-4) = 0 \quad \Rightarrow \quad R_1 = 200$ lbf *Ans.*

at $x = 20^+$, $V = M = 0$. Applying Eqs. (1) and (2),

$200 - 400 + R_2 - 40(10) + R_3 = 0 \quad \Rightarrow \quad R_2 + R_3 = 600$

$200(20) - 400(16) + R_2(10) - 20(10)^2 = 0 \quad \Rightarrow \quad R_2 = 440$ lbf *Ans.*

$R_3 = 600 - 440 = 160$ lbf *Ans.*

$0 \le x \le 4$: $V = 200$ lbf, $M = 200x$ lbf · in

$4 \le x \le 10$: $V = 200 - 400 = -200$ lbf,

$M = 200x - 400(x-4) = -200x + 1600$ lbf · in

$10 \le x \le 20$: $V = 200 - 400 + 440 - 40(x-10) = 640 - 40x$ lbf

$M = 200x - 400(x-4) + 440(x-10) - 20(x-10)^2 = -20x^2 + 640x - 4800$ lbf · in

Plots of V and M are the same as in Prob. 3-8.

3-13 Solution depends upon the beam selected.

3-14 **(a)** Moment at center,

$$x_c = \frac{(l-2a)}{2}$$

$$M_c = \frac{w}{2}\left[\frac{l}{2}(l-2a) - \left(\frac{l}{2}\right)^2\right] = \frac{wl}{2}\left(\frac{l}{4} - a\right)$$

At reaction, $|M_r| = wa^2/2$

$a = 2.25$, $l = 10$ in, $w = 100$ lbf/in

$$M_c = \frac{100(10)}{2}\left(\frac{10}{4} - 2.25\right) = 125 \text{ lbf} \cdot \text{in}$$

$$|M_r| = \frac{100(2.25^2)}{2} = 253 \text{ lbf} \cdot \text{in} \qquad \textit{Ans.}$$

(b) Optimal occurs when $M_c = |M_r|$

$$\frac{wl}{2}\left(\frac{l}{4}-a\right)=\frac{wa^2}{2}\Rightarrow a^2+al-0.25l^2=0$$

Taking the positive root

$$a=\frac{1}{2}\left[-l+\sqrt{l^2+4\left(0.25l^2\right)}\right]=\frac{l}{2}\left(\sqrt{2}-1\right)=0.207\ l \qquad \textit{Ans.}$$

for l = 10 in, w = 100 lbf, a = 0.207(10) = 2.07 in

$$M_{\min}=\left(100/2\right)2.07^2=214\ \text{lbf}\cdot\text{in}$$

3-15

(a)

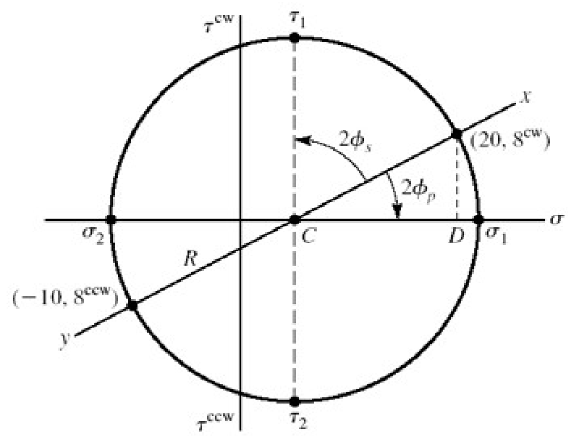
$$C=\frac{20-10}{2}=5\ \text{kpsi}$$

$$CD=\frac{20+10}{2}=15\ \text{kpsi}$$

$$R=\sqrt{15^2+8^2}=17\ \text{kpsi}$$

$$\sigma_1=5+17=22\ \text{kpsi}$$

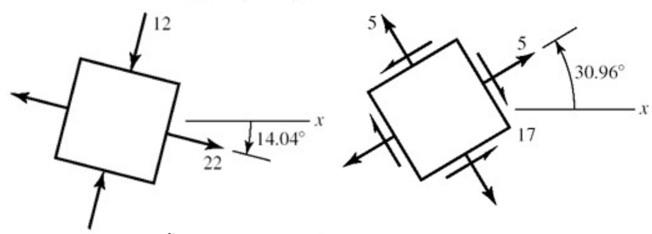
$$\sigma_2=5-17=-12\ \text{kpsi}$$



$$\phi_p=\frac{1}{2}\tan^{-1}\left(\frac{8}{15}\right)=14.04^\circ\ \text{cw}$$

$$\tau_1=R=17\ \text{kpsi}$$

$$\phi_s=45^\circ-14.04^\circ=30.96^\circ\ \text{ccw}$$



(b)

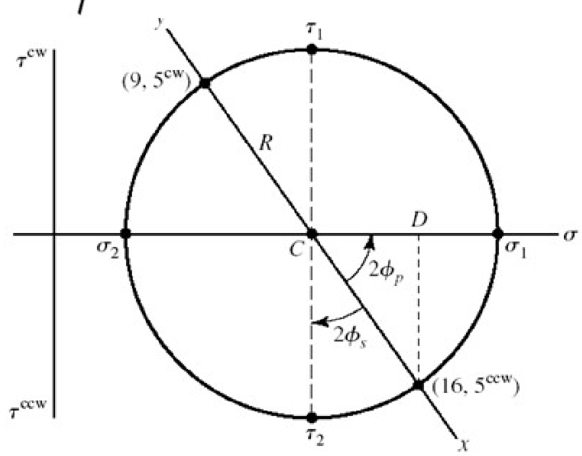
$$C=\frac{9+16}{2}=12.5\ \text{kpsi}$$

$$CD=\frac{16-9}{2}=3.5\ \text{kpsi}$$

$$R=\sqrt{5^2+3.5^2}=6.10\ \text{kpsi}$$

$$\sigma_1=12.5+6.1=18.6\ \text{kpsi}$$

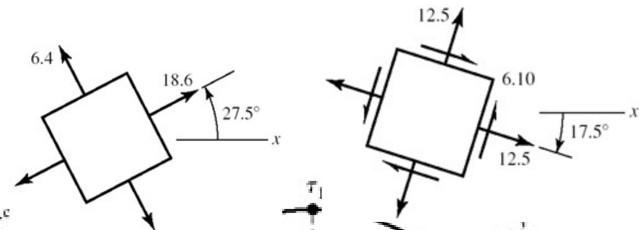
$$\sigma_2=12.5-6.1=6.4\ \text{kpsi}$$



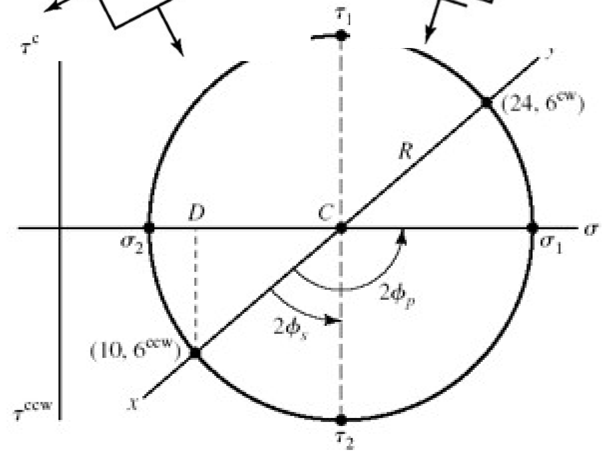
$$\phi_p = \frac{1}{2}\tan^{-1}\left(\frac{5}{3.5}\right) = 27.5^\circ \text{ ccw}$$

$$\tau_1 = R = 6.10 \text{ kpsi}$$

$$\phi_s = 45^\circ - 27.5^\circ = 17.5^\circ \text{ cw}$$



(c)



$$C = \frac{24+10}{2} = 17 \text{ kpsi}$$

$$CD = \frac{24-10}{2} = 7 \text{ kpsi}$$

$$R = \sqrt{7^2 + 6^2} = 9.22 \text{ kpsi}$$

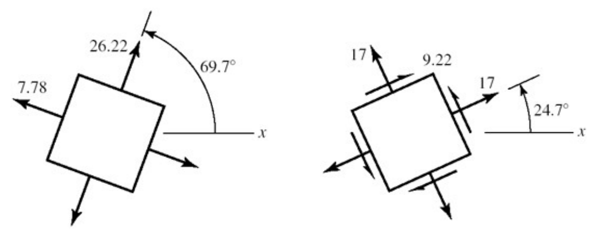
$$\sigma_1 = 17 + 9.22 = 26.22 \text{ kpsi}$$

$$\sigma_2 = 17 - 9.22 = 7.78 \text{ kpsi}$$

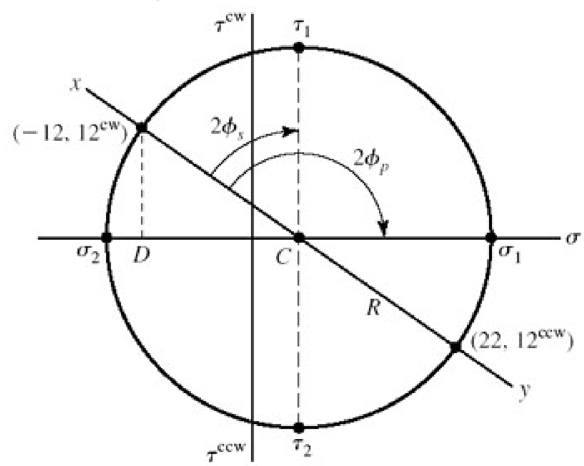
$$\phi_p = \frac{1}{2}\left[90^\circ + \tan^{-1}\left(\frac{7}{6}\right)\right] = 69.7^\circ \text{ ccw}$$

$$\tau_1 = R = 9.22 \text{ kpsi}$$

$$\phi_s = 69.7^\circ - 45^\circ = 24.7^\circ \text{ ccw}$$



(d)



$$C = \frac{-12+22}{2} = 5 \text{ kpsi}$$

$$CD = \frac{12+22}{2} = 17 \text{ kpsi}$$

$$R = \sqrt{17^2 + 12^2} = 20.81 \text{ kpsi}$$

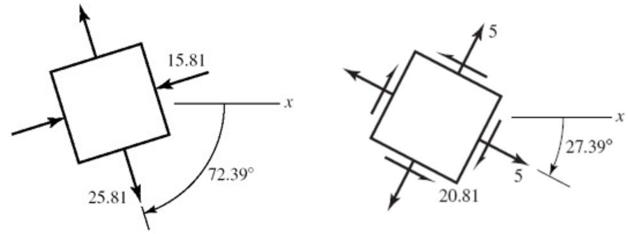
$$\sigma_1 = 5 + 20.81 = 25.81 \text{ kpsi}$$

$$\sigma_2 = 5 - 20.81 = -15.81 \text{ kpsi}$$

$$\phi_p = \frac{1}{2}\left[90^\circ + \tan^{-1}\left(\frac{17}{12}\right)\right] = 72.39^\circ \text{ cw}$$

$$\tau_1 = R = 20.81 \text{ kpsi}$$

$$\phi_s = 72.39 - 45 = 27.39^\circ \text{ cw}$$



3-16

(a)

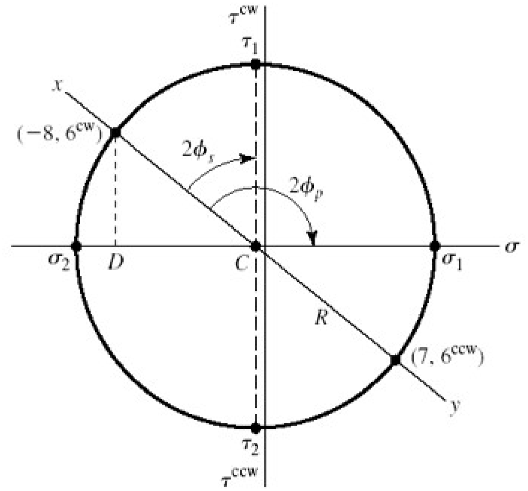
$$C = \frac{-8+7}{2} = -0.5 \text{ MPa}$$

$$CD = \frac{8+7}{2} = 7.5 \text{ MPa}$$

$$R = \sqrt{7.5^2 + 6^2} = 9.60 \text{ MPa}$$

$$\sigma_1 = 9.60 - 0.5 = 9.10 \text{ MPa}$$

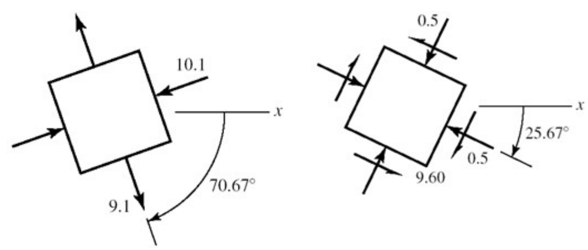
$$\sigma_2 = -0.5 - 9.6 = -10.1 \text{ Mpa}$$



$$\phi_p = \frac{1}{2}\left[90^\circ + \tan^{-1}\left(\frac{7.5}{6}\right)\right] = 70.67^\circ \text{ cw}$$

$$\tau_1 = R = 9.60 \text{ MPa}$$

$$\phi_s = 70.67^\circ - 45^\circ = 25.67^\circ \text{ cw}$$



(b)

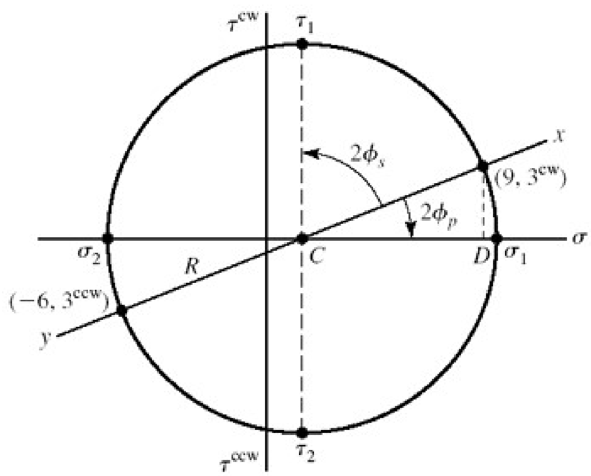
$$C = \frac{9-6}{2} = 1.5 \text{ MPa}$$

$$CD = \frac{9+6}{2} = 7.5 \text{ MPa}$$

$$R = \sqrt{7.5^2 + 3^2} = 8.078 \text{ MPa}$$

$$\sigma_1 = 1.5 + 8.078 = 9.58 \text{ MPa}$$

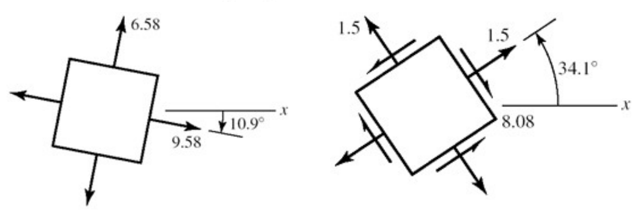
$$\sigma_2 = 1.5 - 8.078 = -6.58 \text{ MPa}$$



$$\phi_p = \frac{1}{2}\tan^{-1}\left(\frac{3}{7.5}\right) = 10.9^\circ \text{ cw}$$

$$\tau_1 = R = 8.078 \text{ MPa}$$

$$\phi_s = 45^\circ - 10.9^\circ = 34.1^\circ \text{ ccw}$$



(c)

$$C=\frac{12-4}{2}=4\text{ MPa}$$

$$CD=\frac{12+4}{2}=8\text{ MPa}$$

$$R=\sqrt{8^2+7^2}=10.63\text{ MPa}$$

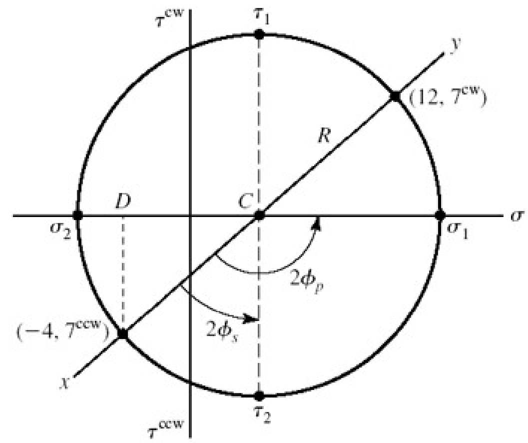
$$\sigma_1=4+10.63=14.63\text{ MPa}$$

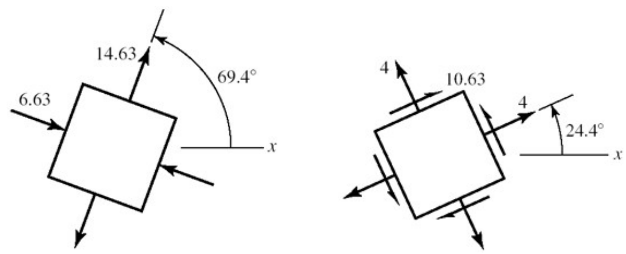
$$\sigma_2=4-10.63=-6.63\text{ MPa}$$

$$\phi_p=\frac{1}{2}\left[90^\circ+\tan^{-1}\left(\frac{8}{7}\right)\right]=69.4^\circ\text{ ccw}$$

$$\tau_1=R=10.63\text{ MPa}$$

$$\phi_s=69.4^\circ-45^\circ=24.4^\circ\text{ ccw}$$





(d)

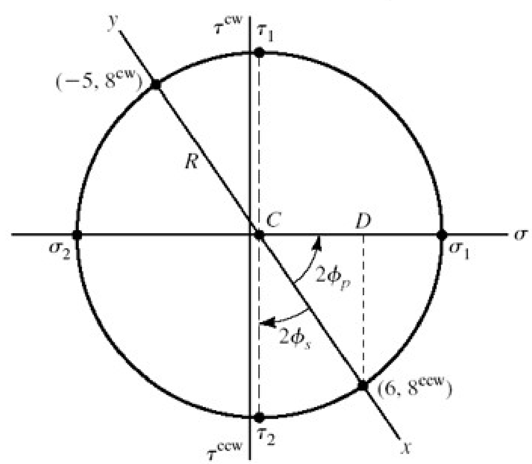
$$C=\frac{6-5}{2}=0.5\text{ MPa}$$

$$CD=\frac{6+5}{2}=5.5\text{ MPa}$$

$$R=\sqrt{5.5^2+8^2}=9.71\text{ MPa}$$

$$\sigma_1=0.5+9.71=10.21\text{ MPa}$$

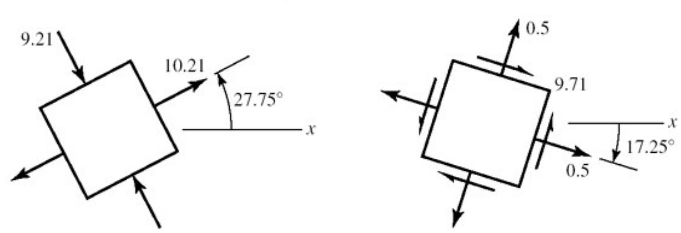
$$\sigma_2=0.5-9.71=-9.21\text{ MPa}$$



$$\phi_p=\frac{1}{2}\tan^{-1}\left(\frac{8}{5.5}\right)=27.75^\circ\text{ ccw}$$

$$\tau_1=R=9.71\text{ MPa}$$

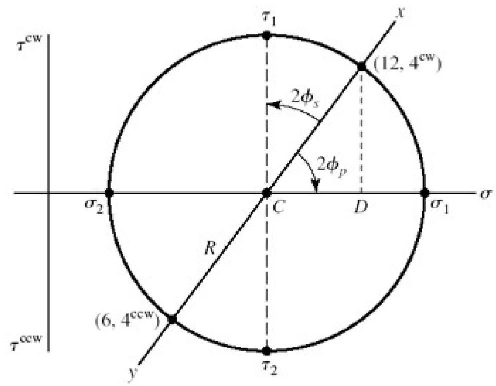
$$\phi_s=45^\circ-27.75^\circ=17.25^\circ\text{ cw}$$



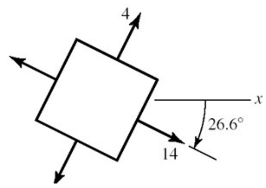
3-17

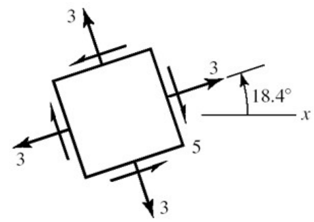
(a)

$$C = \frac{12+6}{2} = 9 \text{ kpsi}$$
$$CD = \frac{12-6}{2} = 3 \text{ kpsi}$$
$$R = \sqrt{3^2 + 4^2} = 5 \text{ kpsi}$$
$$\sigma_1 = 5 + 9 = 14 \text{ kpsi}$$
$$\sigma_2 = 9 - 5 = 4 \text{ kpsi}$$



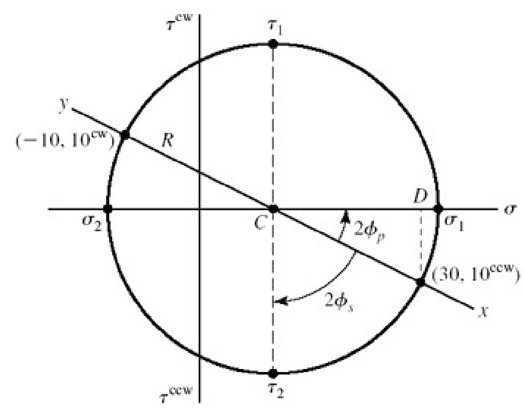
$$\phi_p = \frac{1}{2}\tan^{-1}\left(\frac{4}{3}\right) = 26.6^\circ \text{ ccw}$$
$$\tau_1 = R = 5 \text{ kpsi}$$
$$\phi_s = 45^\circ - 26.6^\circ = 18.4^\circ \text{ ccw}$$



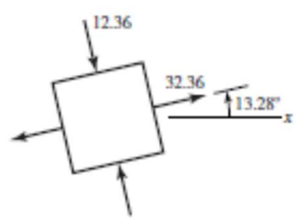


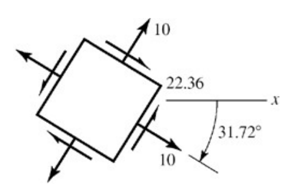
(b)

$$C = \frac{30-10}{2} = 10 \text{ kpsi}$$
$$CD = \frac{30+10}{2} = 20 \text{ kpsi}$$
$$R = \sqrt{20^2 + 10^2} = 22.36 \text{ kpsi}$$
$$\sigma_1 = 10 + 22.36 = 32.36 \text{ kpsi}$$
$$\sigma_2 = 10 - 22.36 = -12.36 \text{ kpsi}$$



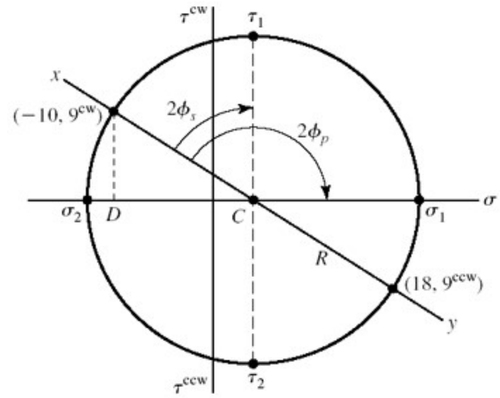
$$\phi_p = \frac{1}{2}\tan^{-1}\left(\frac{10}{20}\right) = 13.28^\circ \text{ ccw}$$
$$\tau_1 = R = 22.36 \text{ kpsi}$$
$$\phi_s = 45^\circ - 13.28^\circ = 31.72^\circ \text{ cw}$$



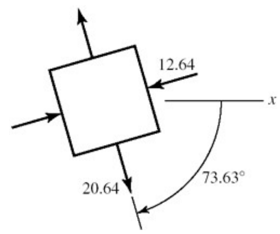


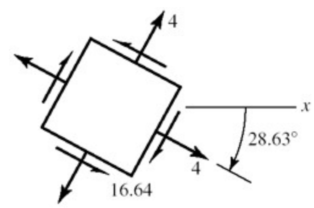
(c)

$$C = \frac{-10+18}{2} = 4 \text{ kpsi}$$
$$CD = \frac{10+18}{2} = 14 \text{ kpsi}$$
$$R = \sqrt{14^2 + 9^2} = 16.64 \text{ kpsi}$$
$$\sigma_1 = 4 + 16.64 = 20.64 \text{ kpsi}$$
$$\sigma_2 = 4 - 16.64 = -12.64 \text{ kpsi}$$



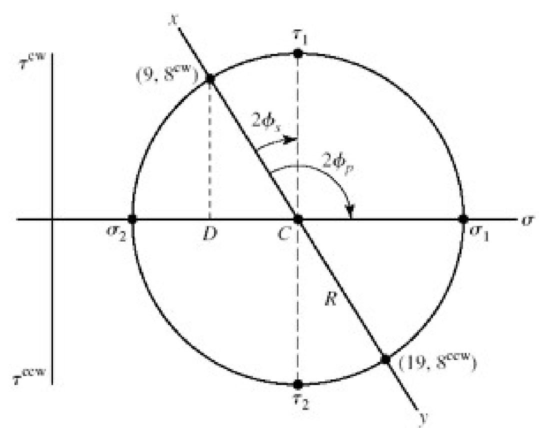
$$\phi_p = \frac{1}{2}\left[90^\circ + \tan^{-1}\left(\frac{14}{9}\right)\right] = 73.63^\circ \text{ cw}$$
$$\tau_1 = R = 16.64 \text{ kpsi}$$
$$\phi_s = 73.63 - 45 = 28.63^\circ \text{ cw}$$

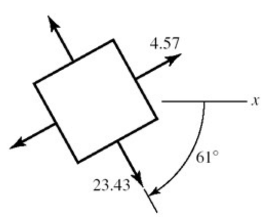


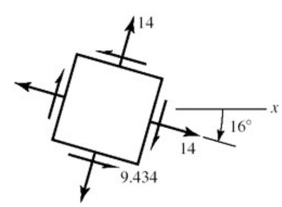


(d)

$$C = \frac{9+19}{2} = 14 \text{ kpsi}$$
$$CD = \frac{19-9}{2} = 5 \text{ kpsi}$$
$$R = \sqrt{5^2 + 8^2} = 9.434 \text{ kpsi}$$
$$\sigma_1 = 14 + 9.43 = 23.43 \text{ kpsi}$$
$$\sigma_2 = 14 - 9.43 = 4.57 \text{ kpsi}$$
$$\phi_p = \frac{1}{2}\left[90^\circ + \tan^{-1}\left(\frac{5}{8}\right)\right] = 61.0^\circ \text{ cw}$$
$$\tau_1 = R = 9.34 \text{ kpsi}$$
$$\phi_s = 61^\circ - 45^\circ = 16^\circ \text{ cw}$$







3-18

(a)

$$C = \frac{-80-30}{2} = -55 \text{ MPa}$$

$$CD = \frac{80-30}{2} = 25 \text{ MPa}$$

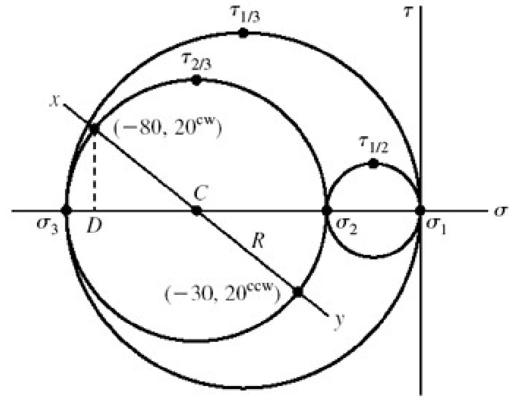
$$R = \sqrt{25^2 + 20^2} = 32.02 \text{ MPa}$$

$$\sigma_1 = 0 \text{ MPa}$$

$$\sigma_2 = -55 + 32.02 = -22.98 = -23.0 \text{ MPa}$$

$$\sigma_3 = -55 - 32.0 = -87.0 \text{ MPa}$$

$$\tau_{1/2} = \frac{23}{2} = 11.5 \text{ MPa}, \quad \tau_{2/3} = 32.0 \text{ MPa}, \quad \tau_{1/3} = \frac{87}{2} = 43.5 \text{ MPa}$$



(b)

$$C = \frac{30-60}{2} = -15 \text{ MPa}$$

$$CD = \frac{60+30}{2} = 45 \text{ MPa}$$

$$R = \sqrt{45^2 + 30^2} = 54.1 \text{ MPa}$$

$$\sigma_1 = -15 + 54.1 = 39.1 \text{ MPa}$$

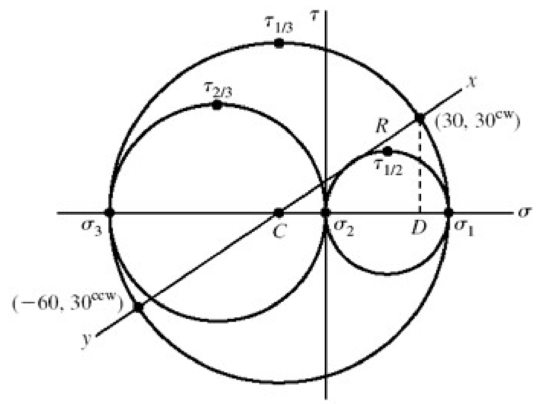
$$\sigma_2 = 0 \text{ MPa}$$

$$\sigma_3 = -15 - 54.1 = -69.1 \text{ MPa}$$

$$\tau_{1/3} = \frac{39.1 + 69.1}{2} = 54.1 \text{ MPa}$$

$$\tau_{1/2} = \frac{39.1}{2} = 19.6 \text{ MPa}$$

$$\tau_{2/3} = \frac{69.1}{2} = 34.6 \text{ MPa}$$



(c)

$$C = \frac{40+0}{2} = 20 \text{ MPa}$$

$$CD = \frac{40-0}{2} = 20 \text{ MPa}$$

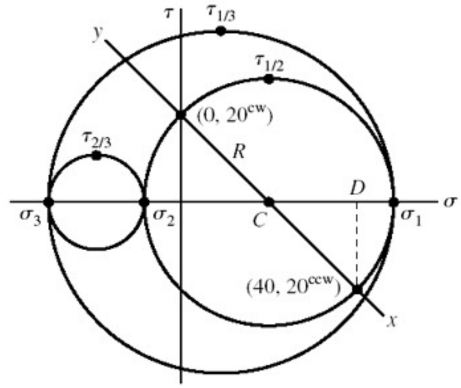
$$R = \sqrt{20^2 + 20^2} = 28.3 \text{ MPa}$$

$$\sigma_1 = 20 + 28.3 = 48.3 \text{ MPa}$$

$$\sigma_2 = 20 - 28.3 = -8.3 \text{ MPa}$$

$$\sigma_3 = \sigma_z = -30 \text{ MPa}$$

$$\tau_{1/3} = \frac{48.3+30}{2} = 39.1 \text{ MPa}, \qquad \tau_{1/2} = 28.3 \text{ MPa}, \qquad \tau_{2/3} = \frac{30-8.3}{2} = 10.9 \text{ MPa}$$



(d)

$$C = \frac{50}{2} = 25 \text{ MPa}$$

$$CD = \frac{50}{2} = 25 \text{ MPa}$$

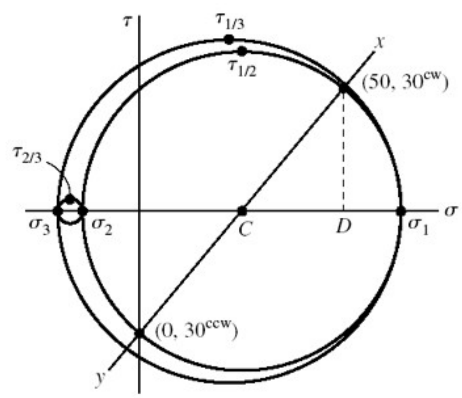
$$R = \sqrt{25^2 + 30^2} = 39.1 \text{ MPa}$$

$$\sigma_1 = 25 + 39.1 = 64.1 \text{ MPa}$$

$$\sigma_2 = 25 - 39.1 = -14.1 \text{ MPa}$$

$$\sigma_3 = \sigma_z = -20 \text{ MPa}$$

$$\tau_{1/3} = \frac{64.1+20}{2} = 42.1 \text{ MPa}, \qquad \tau_{1/2} = 39.1 \text{ MPa}, \qquad \tau_{2/3} = \frac{20-14.1}{2} = 2.95 \text{ MPa}$$



3-19

(a)

Since there are no shear stresses on the stress element, the stress element already represents principal stresses.

$$\sigma_1 = \sigma_x = 10 \text{ kpsi}$$

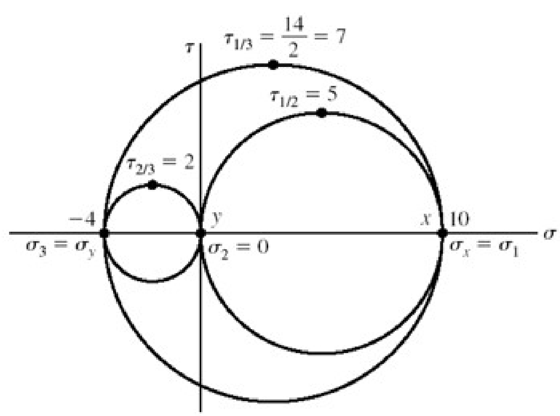
$$\sigma_2 = 0 \text{ kpsi}$$

$$\sigma_3 = \sigma_y = -4 \text{ kpsi}$$

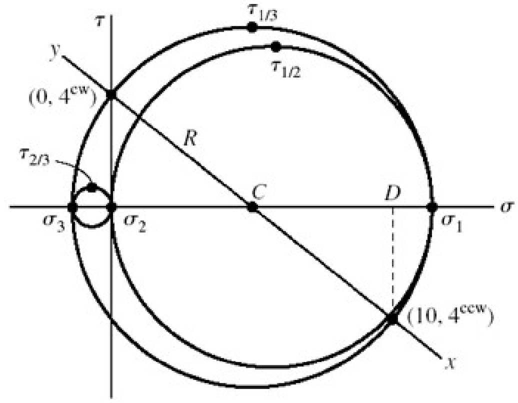
$$\tau_{1/3} = \frac{10-(-4)}{2} = 7 \text{ kpsi}$$

$$\tau_{1/2} = \frac{10}{2} = 5 \text{ kpsi}$$

$$\tau_{2/3} = \frac{0-(-4)}{2} = 2 \text{ kpsi}$$



(b)



$$C = \frac{0+10}{2} = 5 \text{ kpsi}$$

$$CD = \frac{10-0}{2} = 5 \text{ kpsi}$$

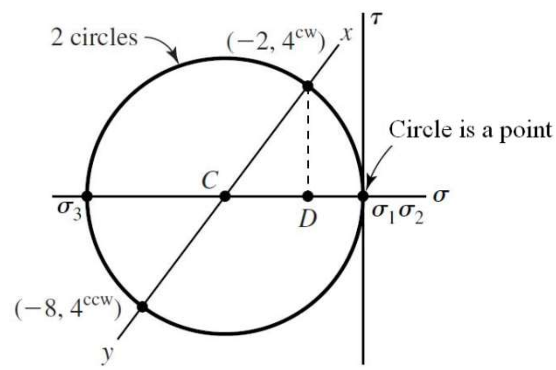
$$R = \sqrt{5^2 + 4^2} = 6.40 \text{ kpsi}$$

$$\sigma_1 = 5 + 6.40 = 11.40 \text{ kpsi}$$

$$\sigma_2 = 0 \text{ kpsi}, \quad \sigma_3 = 5 - 6.40 = -1.40 \text{ kpsi}$$

$$\tau_{1/3} = R = 6.40 \text{ kpsi}, \qquad \tau_{1/2} = \frac{11.40}{2} = 5.70 \text{ kpsi}, \qquad \tau_3 = \frac{1.40}{2} = 0.70 \text{ kpsi}$$

(c)



$$C = \frac{-2-8}{2} = -5 \text{ kpsi}$$

$$CD = \frac{8-2}{2} = 3 \text{ kpsi}$$

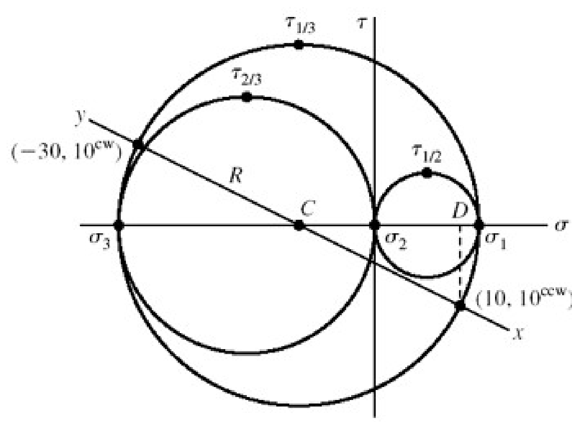
$$R = \sqrt{3^2 + 4^2} = 5 \text{ kpsi}$$

$$\sigma_1 = -5 + 5 = 0 \text{ kpsi}, \quad \sigma_2 = 0 \text{ kpsi}$$

$$\sigma_3 = -5 - 5 = -10 \text{ kpsi}$$

$$\tau_{1/3} = \frac{10}{2} = 5 \text{ kpsi}, \qquad \tau_{1/2} = 0 \text{ kpsi}, \qquad \tau_{2/3} = 5 \text{ kpsi}$$

(d)



$$C = \frac{10-30}{2} = -10 \text{ kpsi}$$

$$CD = \frac{10+30}{2} = 20 \text{ kpsi}$$

$$R = \sqrt{20^2 + 10^2} = 22.36 \text{ kpsi}$$

$$\sigma_1 = -10 + 22.36 = 12.36 \text{ kpsi}$$

$$\sigma_2 = 0 \text{ kpsi}$$

$$\sigma_3 = -10 - 22.36 = -32.36 \text{ kpsi}$$

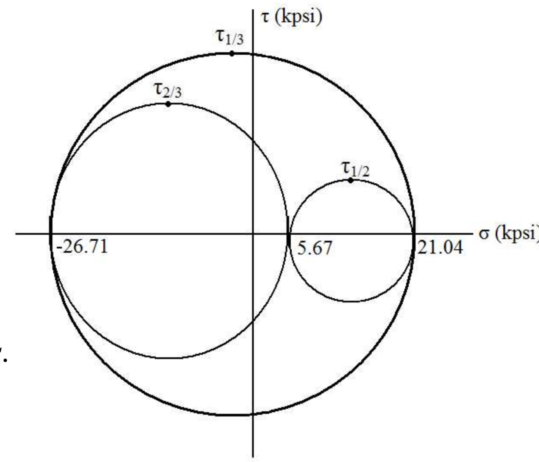
$$\tau_{1/3} = 22.36 \text{ kpsi}, \qquad \tau_{1/2} = \frac{12.36}{2} = 6.18 \text{ kpsi}, \qquad \tau_{2/3} = \frac{32.36}{2} = 16.18 \text{ kpsi}$$

3-20 From Eq. (3-15),

$$\sigma^3 - (-6+18-12)\sigma^2 + \left[-6(18)+(-6)(-12)+18(-12)-9^2-6^2-(-15)^2\right]\sigma$$
$$-\left[-6(18)(-12)+2(9)(6)(-15)-(-6)(6)^2-18(-15)^2-(-12)(9)^2\right]=0$$
$$\sigma^3 - 594\sigma + 3186 = 0$$

Roots are: 21.04, 5.67, –26.71 kpsi *Ans.*

$$\tau_{1/2} = \frac{21.04-5.67}{2} = 7.69 \text{ kpsi}$$
$$\tau_{2/3} = \frac{5.67+26.71}{2} = 16.19 \text{ kpsi}$$
$$\tau_{max} = \tau_{1/3} = \frac{21.04+26.71}{2} = 23.88 \text{ kpsi} \quad \textit{Ans.}$$



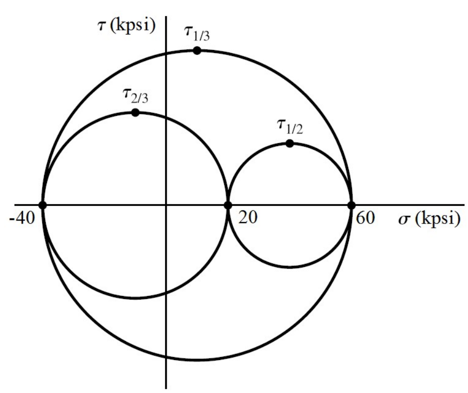
3-21

From Eq. (3-15)

$$\sigma^3 - (20+0+20)\sigma^2 + \left[20(0)+20(20)+0(20)-40^2-\left(-20\sqrt{2}\right)^2-0^2\right]\sigma$$
$$-\left[20(0)(20)+2(40)\left(-20\sqrt{2}\right)(0)-20\left(-20\sqrt{2}\right)^2-0(0)^2-20(40)^2\right]=0$$
$$\sigma^3 - 40\sigma^2 - 2\ 000\sigma + 48\ 000 = 0$$

Roots are: 60, 20, –40 kpsi *Ans.*

$$\tau_{1/2} = \frac{60-20}{2} = 20 \text{ kpsi}$$
$$\tau_{2/3} = \frac{20+40}{2} = 30 \text{ kpsi}$$
$$\tau_{max} = \tau_{1/3} = \frac{60+40}{2} = 50 \text{ kpsi} \quad \textit{Ans.}$$



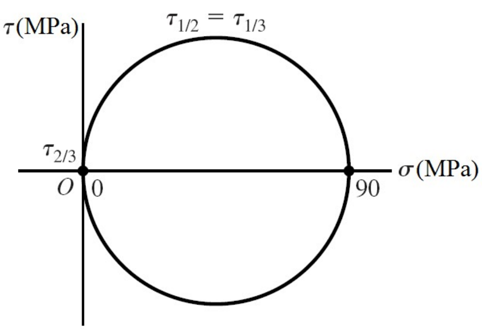
3-22 From Eq. (3-15)

$$\sigma^3 - (10+40+40)\sigma^2 + \left[10(40)+10(40)+40(40)-20^2-\left(-40\right)^2-\left(-20\right)^2\right]\sigma$$
$$-\left[10(40)(40)+2(20)(-40)(-20)-10(-40)^2-40(-20)^2-40(20)^2\right]=0$$
$$\sigma^3 - 90\sigma^2 = 0$$

Roots are: 90, 0, 0 MPa *Ans.*

$\tau_{2/3} = 0$

$\tau_{1/2} = \tau_{1/3} = \tau_{\max} = \dfrac{90}{2} = 45$ MPa *Ans.*



3-23

$$\sigma = \frac{F}{A} = \frac{15000}{(\pi/4)\left(0.75^2\right)} = 33\ 950 \text{ psi} = 34.0 \text{ kpsi} \quad \textit{Ans.}$$

$$\delta = \frac{FL}{AE} = \sigma\frac{L}{E} = 33\ 950\frac{60}{30\left(10^6\right)} = 0.0679 \text{ in} \quad \textit{Ans.}$$

$$\varepsilon_1 = \frac{\delta}{L} = \frac{0.0679}{60} = 1130\left(10^{-6}\right) = 1130\mu \quad \textit{Ans.}$$

From Table A-5, $v = 0.292$

$$\varepsilon_2 = -v\varepsilon_1 = -0.292(1130) = -330\mu \quad \textit{Ans.}$$

$$\Delta d = \varepsilon_2 d = -330\left(10^{-6}\right)(0.75) = -248\left(10^{-6}\right) \text{ in} \quad \textit{Ans.}$$

3-24

$$\sigma = \frac{F}{A} = \frac{3000}{(\pi/4)\left(0.75^2\right)} = 6790 \text{ psi} = 6.79 \text{ kpsi} \quad \textit{Ans.}$$

$$\delta = \frac{FL}{AE} = \sigma\frac{L}{E} = 6790\frac{60}{10.4\left(10^6\right)} = 0.0392 \text{ in} \quad \textit{Ans.}$$

$$\varepsilon_1 = \frac{\delta}{L} = \frac{0.0392}{60} = 653\left(10^{-6}\right) = 653\mu \quad \textit{Ans.}$$

From Table A-5, $v = 0.333$

$$\varepsilon_2 = -v\varepsilon_1 = -0.333(653) = -217\mu \quad \textit{Ans.}$$

$$\Delta d = \varepsilon_2 d = -217\left(10^{-6}\right)(0.75) = -163\left(10^{-6}\right) \text{ in} \quad \textit{Ans.}$$

3-25

$$\varepsilon_2 = \frac{\Delta d}{d} = \frac{-0.0001d}{d} = -0.0001$$

From Table A-5, $v = 0.326$, $E = 119$ GPa

$$\varepsilon_1 = \frac{-\varepsilon_2}{v} = \frac{-0.0001}{0.326} = 306.7\left(10^{-6}\right)$$

$$\delta = \frac{FL}{AE} \text{ and } \sigma = \frac{F}{A}, \text{ so}$$

$$\sigma = \frac{\delta E}{L} = \varepsilon_1 E = 306.7\left(10^{-6}\right)(119)\left(10^9\right) = 36.5 \text{ MPa}$$

$$F = \sigma A = 36.5\left(10^6\right)\frac{\pi\left(0.03\right)^2}{4} = 25\ 800 \text{ N} = 25.8 \text{ kN} \qquad Ans.$$

$S_y = 70$ MPa $> \sigma$, so elastic deformation assumption is valid.

3-26

$$\delta = \frac{FL}{AE} = \sigma\frac{L}{E} = 20\ 000\frac{8(12)}{10.4\left(10^6\right)} = 0.185 \text{ in} \qquad Ans.$$

3-27

$$\delta = \frac{FL}{AE} = \sigma\frac{L}{E} = 140\left(10^6\right)\frac{3}{71.7\left(10^9\right)} = 0.00586 \text{ m} = 5.86 \text{ mm} \qquad Ans.$$

3-28

$$\delta = \frac{FL}{AE} = \sigma\frac{L}{E} = 15\ 000\frac{10(12)}{10.4\left(10^6\right)} = 0.173 \text{ in} \qquad Ans.$$

3-29

With $\sigma_z = 0$, solve the first two equations of Eq. (3-19) simulatenously. Place E on the left-hand side of both equations, and using Cramer's rule,

$$\sigma_x = \frac{\begin{vmatrix} E\varepsilon_x & -v \\ E\varepsilon_y & 1 \end{vmatrix}}{\begin{vmatrix} 1 & -v \\ -v & 1 \end{vmatrix}} = \frac{E\varepsilon_x + vE\varepsilon_y}{1 - v^2} = \frac{E\left(\varepsilon_x + v\varepsilon_y\right)}{1 - v^2}$$

Likewise,

$$\sigma_y = \frac{E\left(\varepsilon_y + \nu\varepsilon_x\right)}{1-\nu^2}$$

From Table A-5, E = 207 GPa and ν = 0.292. Thus,

$$\sigma_x = \frac{E\left(\varepsilon_x + \nu\varepsilon_y\right)}{1-\nu^2} = \frac{207\left(10^9\right)\left[0.0019 + 0.292\left(-0.000\ 72\right)\right]}{1-0.292^2}\left(10^{-6}\right) = 382 \text{ MPa} \quad Ans.$$

$$\sigma_y = \frac{207\left(10^9\right)\left[-0.000\ 72 + 0.292\left(0.0019\right)\right]}{1-0.292^2}\left(10^{-6}\right) = -37.4 \text{ MPa} \quad Ans.$$

3-30

With $\sigma_z = 0$, solve the first two equations of Eq. (3-19) simulatenously. Place E on the left-hand side of both equations, and using Cramer's rule,

$$\sigma_x = \frac{\begin{vmatrix} E\varepsilon_x & -\nu \\ E\varepsilon_y & 1 \end{vmatrix}}{\begin{vmatrix} 1 & -\nu \\ -\nu & 1 \end{vmatrix}} = \frac{E\varepsilon_x + \nu E\varepsilon_y}{1-\nu^2} = \frac{E\left(\varepsilon_x + \nu\varepsilon_y\right)}{1-\nu^2}$$

Likewise,

$$\sigma_y = \frac{E\left(\varepsilon_y + \nu\varepsilon_x\right)}{1-\nu^2}$$

From Table A-5, E = 71.7 GPa and ν = 0.333. Thus,

$$\sigma_x = \frac{E\left(\varepsilon_x + \nu\varepsilon_y\right)}{1-\nu^2} = \frac{71.7\left(10^9\right)\left[0.0019 + 0.333\left(-0.000\ 72\right)\right]}{1-0.333^2}\left(10^{-6}\right) = 134 \text{ MPa} \quad Ans.$$

$$\sigma_y = \frac{71.7\left(10^9\right)\left[-0.000\ 72 + 0.333\left(0.0019\right)\right]}{1-0.333^2}\left(10^{-6}\right) = -7.04 \text{ MPa} \quad Ans.$$

3-31 For plane strain, $\varepsilon_z = 0$. From the third equation of Eq. (3-19),

$$\sigma_z = \nu\left(\sigma_x + \sigma_y\right) \qquad Ans.$$

First of Eq. (3-19),

$$\varepsilon_x = \frac{1}{E}\left\{\sigma_x - \nu\left[\sigma_y + \nu\left(\sigma_x + \sigma_y\right)\right]\right\}$$

$$= \frac{1}{E}\left[\left(1-\nu^2\right)\sigma_x - \nu\left(1+\nu\right)\sigma_y\right]$$

$$= \frac{1+\nu}{E}\left[\left(1-\nu\right)\sigma_x - \nu\sigma_y\right] \qquad Ans.$$

Similarly,

$$\varepsilon_y = \frac{1+\nu}{E}\left[(1-\nu)\sigma_y - \nu\sigma_x\right] \qquad Ans.$$

3-32

(a) $R_1 = \frac{c}{l}F \qquad M_{\max} = R_1 a = \frac{ac}{l}F$

$$\sigma = \frac{6M}{bh^2} = \frac{6}{bh^2}\frac{ac}{l}F \Rightarrow F = \frac{\sigma bh^2 l}{6ac} \qquad Ans.$$

(b) $\frac{F_m}{F} = \frac{(\sigma_m/\sigma)(b_m/b)(h_m/h)^2(l_m/l_1)}{(a_m/a)(c_m/c)} = \frac{1(s)(s)^2(s)}{(s)(s)} = s^2 \qquad Ans.$

For equal stress, the model load varies by the square of the scale factor.

3-33

(a) $R_1 = \frac{wl}{2}, \qquad M_{\max}\big|_{x=l/2} = \frac{w}{2}\frac{l}{2}\left(l - \frac{l}{2}\right) = \frac{wl^2}{8}$

$$\sigma = \frac{6M}{bh^2} = \frac{6}{bh^2}\frac{wl^2}{8} = \frac{3Wl}{4bh^2} \quad \Rightarrow \quad W = \frac{4}{3}\frac{\sigma bh^2}{l} \qquad Ans.$$

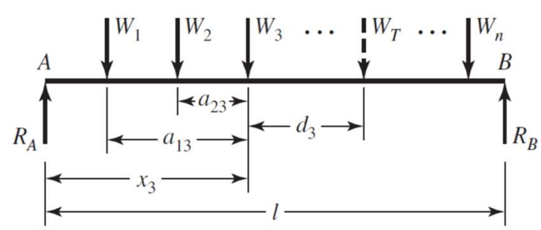
(b) $\frac{W_m}{W} = \frac{(\sigma_m/\sigma)(b_m/b)(h_m/h)^2}{l_m/l} = \frac{1(s)(s)^2}{s} = s^2 \qquad Ans.$

$$\frac{w_m l_m}{wl} = s^2 \quad \Rightarrow \quad \frac{w_m}{w} = \frac{s^2}{s} = s \qquad Ans.$$

For equal stress, the model load w varies linearly with the scale factor.

3-34

(a) Can solve by iteration *or* derive equations for the general case. Find maximum moment under wheel W_3.



$W_T = \Sigma W$ at centroid of W's

$$R_A = \frac{l - x_3 - d_3}{l}W_T$$

Under wheel 3,

$$M_3 = R_A x_3 - W_1 a_{13} - W_2 a_{23} = \frac{(l - x_3 - d_3)}{l}W_T x_3 - W_1 a_{13} - W_2 a_{23}$$

For maximum, $\frac{dM_3}{dx_3} = 0 = (l - d_3 - 2x_3)\frac{W_T}{l} \quad \Rightarrow \quad x_3 = \frac{l - d_3}{2}$

Substitute into $M \Rightarrow M_3 = \frac{(l - d_3)^2}{4l}W_T - W_1 a_{13} - W_2 a_{23}$

This means the midpoint of d_3 intersects the midpoint of the beam.

For wheel *i*, $x_i = \dfrac{l-d_i}{2}$, $M_i = \dfrac{(l-d_i)^2}{4l}W_T - \sum_{j=1}^{i-1} W_j a_{ji}$

Note for wheel 1: $\Sigma W_j a_{ji} = 0$

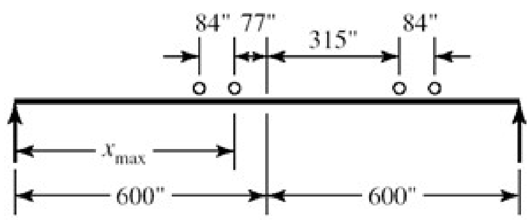
$$W_T = 104.4, \quad W_1 = W_2 = W_3 = W_4 = \frac{104.4}{4} = 26.1 \text{ kips}$$

Wheel 1: $d_1 = \dfrac{476}{2} = 238$ in, $M_1 = \dfrac{(1200-238)^2}{4(1200)}(104.4) = 20\,128 \text{ kip}\cdot\text{in}$

Wheel 2: $d_2 = 238 - 84 = 154$ in

$$M_2 = \frac{(1200-154)^2}{4(1200)}(104.4) - 26.1(84) = 21\,605 \text{ kip}\cdot\text{in} = M_{\max} \quad \textit{Ans.}$$

Check if all of the wheels are on the rail.



(b) $x_{\max} = 600 - 77 = 523$ in *Ans.*

(c) See above sketch.

(d) Inner axles

3-35

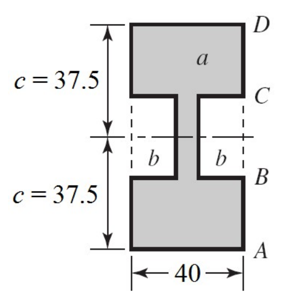
(a) Let *a* = total area of entire envelope

Let *b* = area of side notch

$$A = a - 2b = 40(2)(37.5) - 25(34) = 2150 \text{ mm}^2$$

$$I = I_a - 2I_b = \frac{1}{12}(40)(75)^3 - \frac{1}{12}(34)(25)^3$$

$$I = 1.36(10^6) \text{ mm}^4 \quad \textit{Ans.}$$



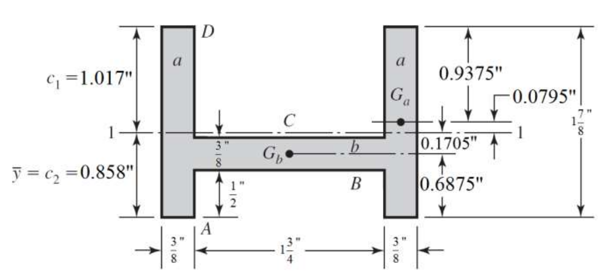
Dimensions in mm.

(b)

$$A_a = 0.375(1.875) = 0.703\,125 \text{ in}^2$$

$$A_b = 0.375(1.75) = 0.656\,25 \text{ in}^2$$

$$A = 2(0.703125) + 0.656\,25 = 2.0625 \text{ in}^2$$



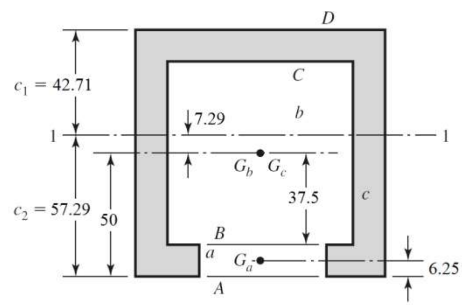
$$\bar{y} = \frac{2(0.703\ 125)(0.9375) + 0.656\ 25(0.6875)}{2.0625} = 0.858 \text{ in} \quad Ans.$$

$$I_a = \frac{0.375(1.875)^3}{12} = 0.206 \text{ in}^4$$

$$I_b = \frac{1.75(0.375)^3}{12} = 0.007\ 69 \text{ in}^4$$

$$I_1 = 2\left[0.206 + 0.703\ 125(0.0795)^2\right] + \left[0.00769 + 0.656\,25(0.1705)^2\right] = 0.448 \text{ in}^4 \quad Ans.$$

(c)
Use two negative areas.



$$A_a = 625 \text{ mm}^2,\ A_b = 5625 \text{ mm}^2,\ A_c = 10\,000 \text{ mm}^2$$

$$A = 10\,000 - 5625 - 625 = 3750 \text{ mm}^2;$$

$$\bar{y}_a = 6.25 \text{ mm}, \bar{y}_b = 50 \text{ mm}, \bar{y}_c = 50 \text{ mm}$$

$$\bar{y} = \frac{10\ 000(50) - 5625(50) - 625(6.25)}{3750} = 57.29 \text{ mm} \quad Ans.$$

$$c_1 = 100 - 57.29 = 42.71 \text{ mm} \quad Ans.$$

$$I_a = \frac{50(12.5)^3}{12} = 8138 \text{ mm}^4$$

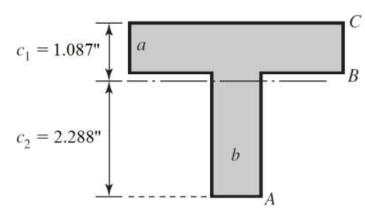
$$I_b = \frac{75(75)^3}{12} = 2.637\left(10^6\right) \text{ mm}^4$$

$$I_c = \frac{100(100)^3}{12} = 8.333\left(10^6\right) \text{in}^4$$

$$I_1 = \left[8.333\left(10^6\right) + 10\,000(7.29)^2\right] - \left[2.637\left(10^6\right) + 5625\left(7.29\right)^2\right] - \left[8138 + 625\left(57.29 - 6.25\right)^2\right]$$

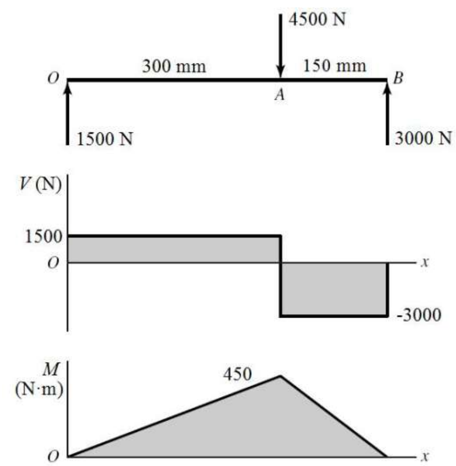
$$I_1 = 4.29\left(10^6\right) \text{ in}^4 \quad Ans.$$

(d)



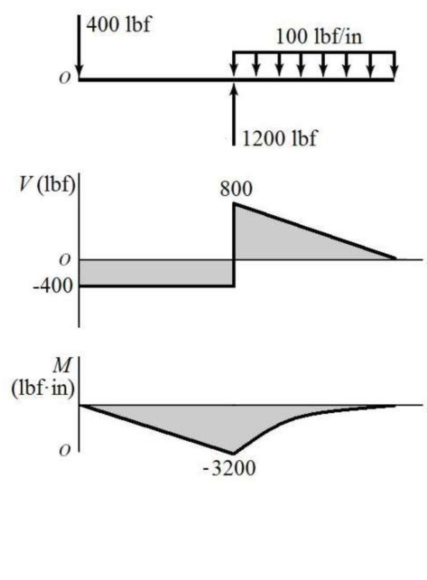
$$A_a = 4(0.875) = 3.5 \text{ in}^2$$
$$A_b = 2.5(0.875) = 2.1875 \text{ in}^2$$
$$A = A_a + A_b = 5.6875 \text{ in}^2$$
$$\bar{y} = \frac{2.9375(3.5) + 1.25(2.1875)}{5.6875} = 2.288 \text{ in} \quad \textit{Ans.}$$
$$I = \frac{1}{12}(4)(0.875)^3 + 3.5(2.9375 - 2.288)^2 + \frac{1}{12}(0.875)(2.5)^3 + 2.1875(2.288 - 1.25)^2$$
$$I = 5.20 \text{ in}^4 \quad \textit{Ans.}$$

3-36



$$I = \frac{1}{12}(20)(40)^3 = 1.067\left(10^5\right) \text{ mm}^4$$
$$A = 20(40) = 800 \text{ mm}^2$$
M_{max} is at A. At the bottom of the section,
$$\sigma_{max} = \frac{Mc}{I} = \frac{450\,000(20)}{1.067\left(10^5\right)} = 84.3 \text{ MPa} \quad \textit{Ans.}$$
Due to V, τ_{max} is between A and B at $y = 0$.
$$\tau_{max} = \frac{3}{2}\frac{V}{A} = \frac{3}{2}\left(\frac{3000}{800}\right) = 5.63 \text{ MPa} \quad \textit{Ans.}$$

3-37 $$I = \frac{1}{12}(1)(2)^3 = 0.6667 \text{ in}^4$$

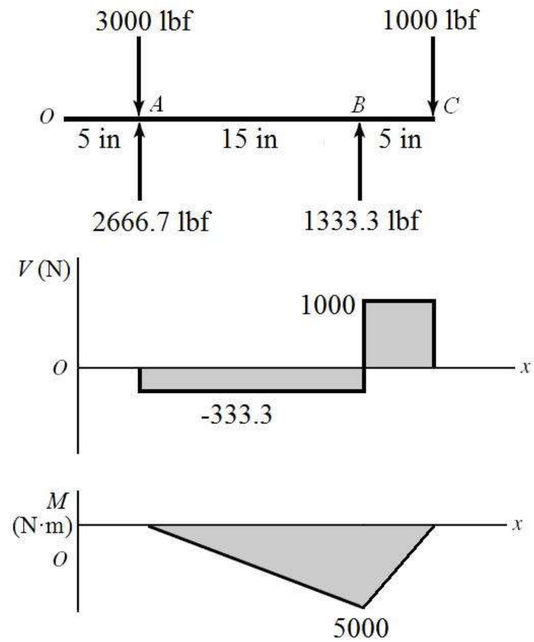


$$A = 1(2) = 2 \text{ in}^2$$

$$\Sigma M_O = 0$$
$$8R_A - 100(8)(12) = 0$$
$$R_A = 1200 \text{ lbf}$$
$$R_O = 1200 - 100(8) = 400 \text{ lbf}$$
M_{max} is at A. At the top of the beam,
$$\sigma_{max} = \frac{Mc}{I} = \frac{3200(0.5)}{0.6667} = 2400 \text{ psi} \quad \textit{Ans.}$$
Due to V, τ_{max} is at A, at $y = 0$.
$$\tau_{max} = \frac{3}{2}\frac{V}{A} = \frac{3}{2}\left(\frac{800}{2}\right) = 600 \text{ psi} \quad \textit{Ans.}$$

3-38 $I = \frac{1}{12}(0.75)(2)^3 = 0.5 \text{ in}^4$

$A = (0.75)(2) = 1.5 \text{ in}^2$



$\Sigma M_A = 0$

$15R_B - 1000(20) = 0$

$R_B = 1333.3 \text{ lbf}$

$R_A = 3000 - 1333.3 + 1000 = 2666.7 \text{ lbf}$

$M_{\max}$ is at B. At the top of the beam,

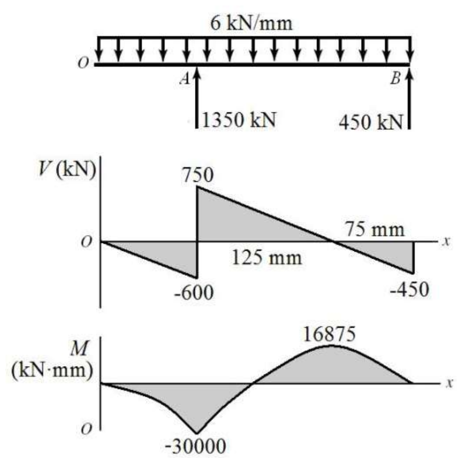
$$\sigma_{\max} = \frac{Mc}{I} = \frac{5000(1)}{0.5} = 10\,000 \text{ psi} \quad \textit{Ans.}$$

Due to V, $\tau_{\max}$ is between B and C at $y = 0$.

$$\tau_{\max} = \frac{3}{2}\frac{V}{A} = \frac{3}{2}\left(\frac{1000}{1.5}\right) = 1000 \text{ psi} \quad \textit{Ans.}$$

3-39 $I = \frac{\pi d^4}{64} = \frac{\pi (50)^4}{64} = 306.796\left(10^3\right) \text{ mm}^4$

$A = \frac{\pi d^2}{4} = \frac{\pi (50)^2}{4} = 1963 \text{ mm}^2$



$\Sigma M_B = 0$

$6(300)(150) - 200R_A = 0$

$R_A = 1350 \text{ kN}$

$R_B = 6(300) - 1350 = 450 \text{ kN}$

$M_{\max}$ is at A. At the top,

$$\sigma_{\max} = \frac{Mc}{I} = \frac{30\,000(25)}{30\,6796} = 2.44 \text{ kN/mm}^2 = 2.44 \text{ GPa} \quad \textit{Ans.}$$

Due to V, $\tau_{\max}$ is at A, at $y = 0$.

$$\tau_{\max} = \frac{4}{3}\frac{V}{A} = \frac{4}{3}\left(\frac{750}{1963}\right) = 0.509 \text{ kN/mm}^2 = 509 \text{ MPa} \quad \textit{Ans.}$$

3-40

$$M_{\max} = \frac{wl^2}{8} \Rightarrow \sigma_{\max} = \frac{wl^2 c}{8I} \Rightarrow \quad w = \frac{8\sigma_{\max} I}{cl^2}$$

(a) $l = 48$ in; Table A-8, $I = 0.537 \text{ in}^4$

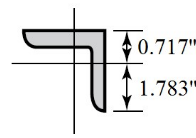
$$w = \frac{8(12)(10^3)(0.537)}{1(48^2)} = 22.38 \text{ lbf/in} \quad \textit{Ans.}$$

(b) $l = 60$ in, $I \approx (1/12)(2)(3^3) - (1/12)(1.625)(2.625^3) = 2.051 \text{ in}^4$

$$w = \frac{8(12)(10^3)(2.051)}{(1.5)(60^2)} = 36.5 \text{ lbf/in} \quad \textit{Ans.}$$

(c) $l = 60$ in; Table A-6, $I = 2(0.703) = 1.406 \text{ in}^4$

$y = 0.717$ in, $c_{\max} = 1.783$ in



$$w = \frac{8(12)(10^3)(1.406)}{1.783(60^2)} = 21.0 \text{ lbf/in} \quad \textit{Ans.}$$

(d) $l = 60$ in, Table A-7, $I = 2.07 \text{ in}^4$

$$w = \frac{8(12)(10^3)(2.07)}{1.5(60^2)} = 36.8 \text{ lbf/in} \quad \textit{Ans.}$$

3-41

$$I = \frac{\pi}{64}(0.5^4) = 3.068(10^{-3}) \text{ in}^4, \quad A = \frac{\pi}{4}(0.5^2) = 0.1963 \text{ in}^2$$

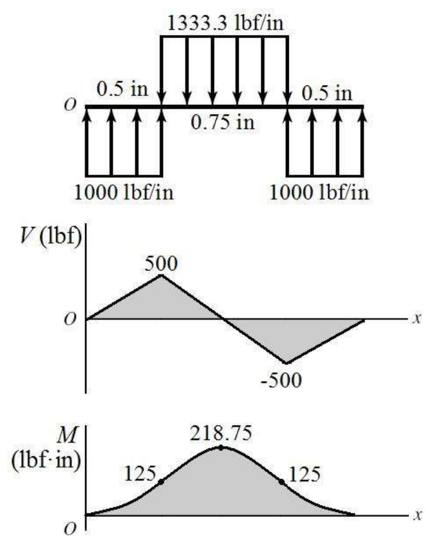
Model (*c*)

$$M = \frac{500(0.5)}{2} + \frac{500(0.75/2)}{2} = 218.75 \text{ lbf} \cdot \text{in}$$

$$\sigma = \frac{Mc}{I} = \frac{218.75(0.25)}{3.068(10^{-3})}$$

$$\sigma = 17\ 825 \text{ psi} = 17.8 \text{ kpsi} \quad \textit{Ans.}$$

$$\tau_{\max} = \frac{4}{3}\frac{V}{A} = \frac{4}{3}\frac{500}{0.1963} = 3400 \text{ psi} = 3.4 \text{ kpsi} \quad \textit{Ans.}$$



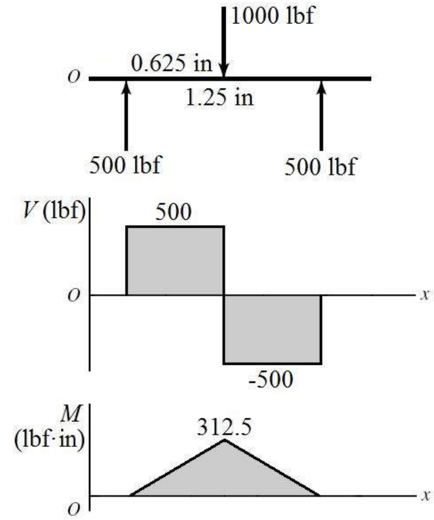
Model (*d*)

$$M = 500(0.625) = 312.5 \text{ lbf} \cdot \text{in}$$

$$\sigma = \frac{Mc}{I} = \frac{312.5(0.25)}{3.068\left(10^{-3}\right)}$$

$$\sigma = 25\ 464 \text{ psi} = 25.5 \text{ kpsi} \qquad Ans.$$

$$\tau_{\max} = \frac{4}{3}\frac{V}{A} = \frac{4}{3}\frac{500}{0.1963} = 3400 \text{ psi} = 3.4 \text{ kpsi} \qquad Ans.$$



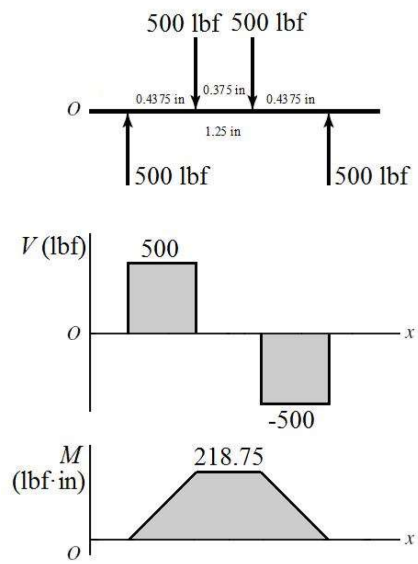
Model (*e*)

$$M = 500(0.4375) = 218.75 \text{ lbf} \cdot \text{in}$$

$$\sigma = \frac{Mc}{I} = \frac{218.75(0.25)}{3.068\left(10^{-3}\right)}$$

$$\sigma = 17\ 825 \text{ psi} = 17.8 \text{ kpsi} \qquad Ans.$$

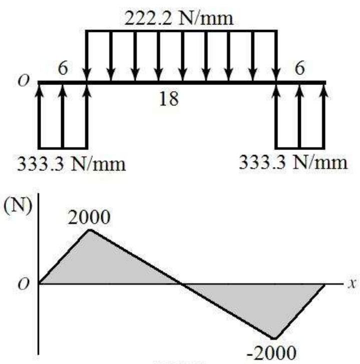
$$\tau_{\max} = \frac{4}{3}\frac{V}{A} = \frac{4}{3}\frac{500}{0.1963} = 3400 \text{ psi} = 3.4 \text{ kpsi} \qquad Ans.$$



3-42

$$I=\frac{\pi}{64}\left(12^4\right)=1018\ \text{mm}^4,\ A=\frac{\pi}{4}\left(12^2\right)=113.1\ \text{mm}^2$$

Model (c)



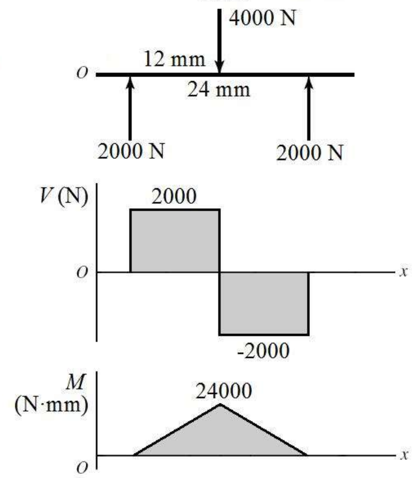
$$M=\frac{2000(6)}{2}+\frac{2000(9)}{2}=15\ 000\ \text{N}\cdot\text{mm}$$

$$\sigma=\frac{Mc}{I}=\frac{15\ 000(6)}{1018}$$

$\sigma=88.4\ \text{N/mm}^2=88.4\ \text{MPa}$ *Ans.*

$\tau_{\max}=\frac{4}{3}\frac{V}{A}=\frac{4}{3}\left(\frac{2000}{113.1}\right)=23.6\ \text{N/mm}^2=23.6\ \text{MPa}$ *Ans.*

Model (d)



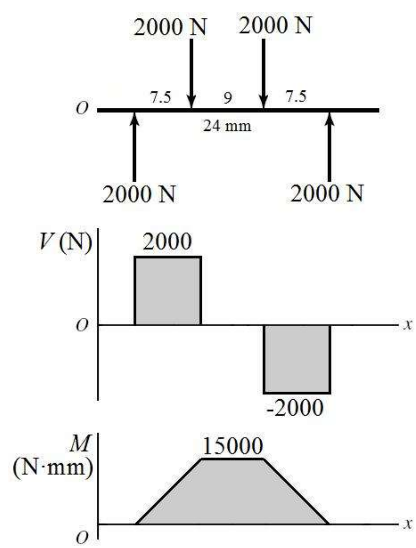
$$M=2000(12)=24\ 000\ \text{N}\cdot\text{mm}$$

$$\sigma=\frac{Mc}{I}=\frac{24\ 000(6)}{1018}$$

$\sigma=141.5\ \text{N/mm}^2=141.5\ \text{MPa}$ *Ans.*

$\tau_{\max}=\frac{4}{3}\frac{V}{A}=\frac{4}{3}\left(\frac{2000}{113.1}\right)=23.6\ \text{N/mm}^2=23.6\ \text{MPa}$ *Ans.*

Model (e)



$$M=2000(7.5)=15\,000\ \text{N}\cdot\text{mm}$$

$$\sigma=\frac{Mc}{I}=\frac{15\,000(6)}{1018}$$

$\sigma=88.4\ \text{N/mm}^2=88.4\ \text{MPa}$ *Ans.*

$\tau_{\max}=\frac{4}{3}\frac{V}{A}=\frac{4}{3}\left(\frac{2000}{113.1}\right)=23.6\ \text{N/mm}^2=23.6\ \text{MPa}$ *Ans.*

3-43 **(a)** $\sigma = \dfrac{Mc}{I} = \dfrac{M(d/2)}{\pi d^4/64} = \dfrac{32M}{\pi d^3}$

$$d = \sqrt[3]{\frac{32M}{\pi\sigma}} = \sqrt[3]{\frac{32(218.75)}{\pi(30\ 000)}} = 0.420 \text{ in} \quad \textit{Ans.}$$

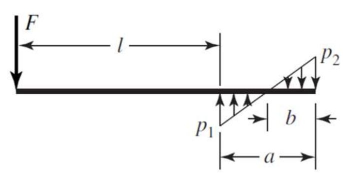
(b) $\tau = \dfrac{V}{A} = \dfrac{V}{\pi d^2/4}$

$$d = \sqrt{\frac{4V}{\pi\tau}} = \sqrt{\frac{4(500)}{\pi(15\,000)}} = 0.206 \text{ in} \quad \textit{Ans.}$$

(c) $\tau = \dfrac{4}{3}\dfrac{V}{A} = \dfrac{4}{3}\dfrac{V}{\left(\pi d^2/4\right)}$

$$d = \sqrt{\frac{4}{3}\frac{4V}{\pi\tau}} = \sqrt{\frac{4}{3}\frac{4(500)}{\pi(15\,000)}} = 0.238 \text{ in} \quad \textit{Ans.}$$

3-44



$$q = -F\langle x\rangle^{-1} + p_1\langle x-l\rangle^0 - \frac{p_1+p_2}{a}\langle x-l\rangle^1 + \text{terms for } x > l+a$$

$$V = -F + p_1\langle x-l\rangle^1 - \frac{p_1+p_2}{2a}\langle x-l\rangle^2 + \text{ terms for } x > l+a$$

$$M = -Fx + \frac{p_1}{2}\langle x-l\rangle^2 - \frac{p_1+p_2}{6a}\langle x-l\rangle^3 + \text{ terms for } x > l+a$$

At $x = (l+a)^+$, $V = M = 0$, terms for $x > l+a = 0$

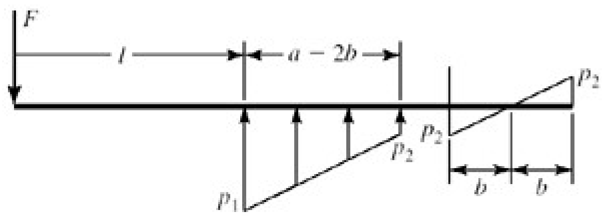
$$-F + p_1 a - \frac{p_1+p_2}{2a}a^2 = 0 \quad \Rightarrow \quad p_1 - p_2 = \frac{2F}{a} \qquad (1)$$

$$-F(l+a) + \frac{p_1 a^2}{2} - \frac{p_1+p_2}{6a}a^3 = 0 \quad \Rightarrow \quad 2p_1 - p_2 = \frac{6F(l+a)}{a^2} \qquad (2)$$

From (1) and (2) $\qquad p_1 = \dfrac{2F}{a^2}(3l+2a), \qquad p_2 = \dfrac{2F}{a^2}(3l+a) \qquad (3)$

From similar triangles $\quad \dfrac{b}{p_2} = \dfrac{a}{p_1 + p_2} \quad \Rightarrow \quad b = \dfrac{ap_2}{p_1 + p_2} \qquad (4)$

$M_{\max}$ occurs where $V = 0$



$$x_{\max} = l + a - 2b$$

$$M_{\max} = -F(l + a - 2b) + \frac{p_1}{2}(a-2b)^2 - \frac{p_1 + p_2}{6a}(a-2b)^3$$
$$= -Fl - F(a-2b) + \frac{p_1}{2}(a-2b)^2 - \frac{p_1 + p_2}{6a}(a-2b)^3$$

Normally $M_{\max} = -Fl$

The fractional increase in the magnitude is

$$\Delta = \frac{F(a-2b) - \left(p_1/2\right)(a-2b)^2 + \left[\left(p_1 + p_2\right)/6a\right](a-2b)^3}{Fl} \qquad (5)$$

For example, consider F = 1500 lbf, a = 1.2 in, l = 1.5 in

From (3) $\quad p_1 = \dfrac{2(1500)}{1.2^2}\left[3\left(1.5\right) + 2(1.2)\right] = 14\ 375$ lbf/in

$$p_2 = \frac{2(1500)}{1.2^2}\left[3\left(1.5\right) + 1.2\right] = 11\ 875 \text{ lbf/in}$$

From (4) $\quad b$ = 1.2(11 875)/(14 375 + 11 875) = 0.5429 in

Substituting into (5) yields

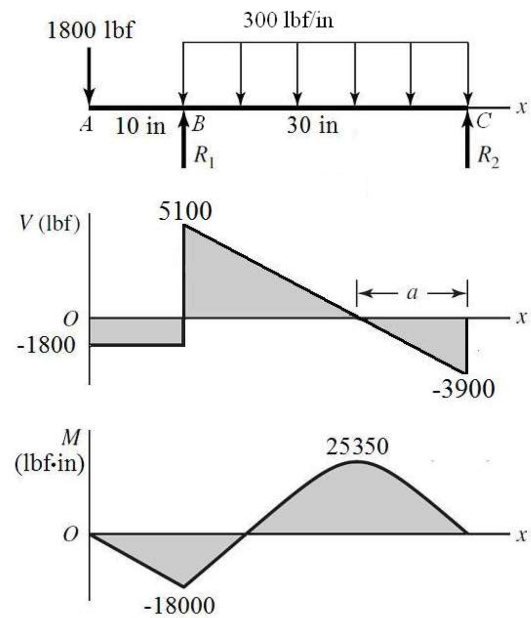
Δ = 0.036 89 or 3.7% higher than $-Fl$

3-45

$$R_1 = \frac{300(30)}{2} + \frac{40}{30}1800 = 6900 \text{ lbf}$$

$$R_2 = \frac{300(30)}{2} - \frac{10}{30}1800 = 3900 \text{ lbf}$$

$$a = \frac{3900}{300} = 13 \text{ in}$$



$M_B = -1800(10) = -18\ 000$ lbf·in

$M_{x = 27 \text{ in}} = (1/2)3900(13) = 25\ 350$ lbf·in

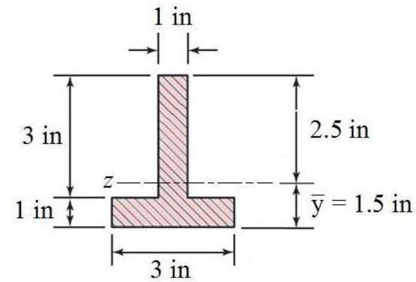
$$\bar{y} = \frac{0.5(3) + 2.5(3)}{6} = 1.5 \text{ in}$$

$$I_1 = \frac{1}{12}(3)(1^3) = 0.25 \text{ in}^4$$

$$I_2 = \frac{1}{12}(1)(3^3) = 2.25 \text{ in}^4$$

Applying the parallel-axis theorem,

$$I_z = \left[0.25 + 3(1.5-0.5)^2\right] + \left[2.25 + 3(2.5-1.5)^2\right] = 8.5 \text{ in}^4$$



(a)

At $x = 10$ in, $y = -1.5$ in, $\sigma_x = -\dfrac{-18000(-1.5)}{8.5} = -3176$ psi

At $x = 10$ in, $y = 2.5$ in, $\sigma_x = -\dfrac{-18000(2.5)}{8.5} = 5294$ psi

At $x = 27$ in, $y = -1.5$ in, $\sigma_x = -\dfrac{25350(-1.5)}{8.5} = 4474$ psi

At $x = 27$ in, $y = 2.5$ in, $\sigma_x = -\dfrac{25350(2.5)}{8.5} = -7456$ psi

Max tension $= 5294$ psi *Ans.*

Max compression $= -7456$ psi *Ans.*

(b) The maximum shear stress due to V is at B, at the neutral axis.

$V_{\max} = 5100$ lbf

$Q = \bar{y}'A' = 1.25(2.5)(1) = 3.125 \text{ in}^3$

$$(\tau_{\max})_V = \frac{VQ}{Ib} = \frac{5100(3.125)}{8.5(1)} = 1875 \text{ psi} \qquad \textit{Ans.}$$

(c) There are three potentially critical locations for the maximum shear stress, all at

x = 27 in: (i) at the top where the bending stress is maximum, (ii) at the neutral axis where the transverse shear is maximum, or (iii) in the web just above the flange where bending stress and shear stress are in their largest combination.

For (i):

The maximum bending stress was previously found to be -7456 psi, and the shear stress is zero. From Mohr's circle,

$$\tau_{\max} = \frac{|\sigma_{\max}|}{2} = \frac{7456}{2} = 3728 \text{ psi}$$

For (ii):

The bending stress is zero, and the transverse shear stress was found previously to be 1875 psi. Thus, $\tau_{\max}$ = 1875 psi.

For (iii):

The bending stress, at $y = -0.5$ in, is

$$\sigma_x = -\frac{-18000(-0.5)}{8.5} = -1059 \text{ psi}$$

The transverse shear stress is

$$Q = \bar{y}'A' = (1)(3)(1) = 3.0 \text{ in}^3$$

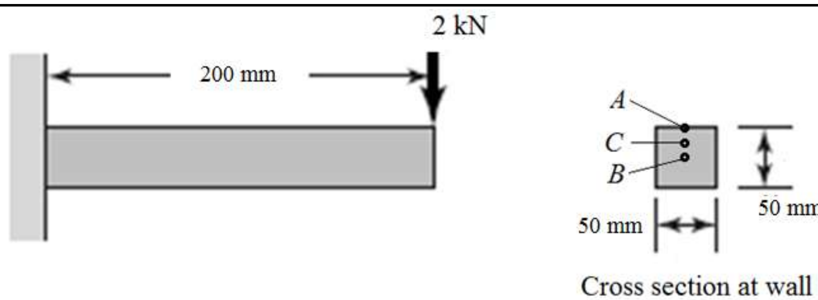
$$\tau = \frac{VQ}{Ib} = \frac{5100(3.0)}{8.5(1)} = 1800 \text{ psi}$$

From Mohr's circle,

$$\tau_{\max} = \sqrt{\left(\frac{-1059}{2}\right)^2 + 1800^2} = 1876 \text{ psi}$$

The critical location is at x = 27 in, at the top surface, where $\tau_{\max}$ = 3728 psi. *Ans.*

3-46



(a) $I = bh^3/12 = 50(50^3)/(12) = 520.83\ (10^3)$ mm^4

Element A:

$$\sigma_A = -\frac{My_A}{I} = -\frac{-2\,000(200)(50/2)}{520.83(10^3)} = 19.2 \text{ MPa}$$

Ans.

$$\tau_A = \frac{VQ_A}{Ib},\quad Q_A = 0 \;\Rightarrow\; \tau_A = 0$$

Element B:

$$\sigma_B = -\frac{My_B}{I}, \quad y_B = 0 \;\Rightarrow\; \sigma_B = 0$$

$$Q_B = \bar{y}'_B A' = (25/2)25(50) = 15.625\left(10^3\right) \text{ mm}^3$$

$$\tau_B = \frac{VQ_B}{Ib} = \frac{2000(15.625)10^3}{520.83\left(10^3\right)50} = 1.20 \text{ MPa} \qquad \textit{Ans.}$$

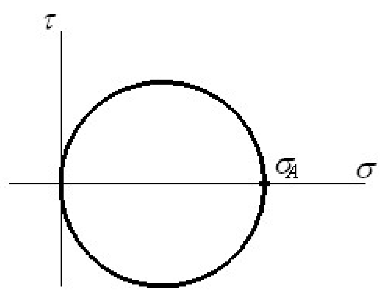
Element C:

$$\sigma_C = -\frac{-2\,000(200)(25/2)}{520.83\left(10^3\right)} = 9.60 \text{ MPa}$$

$$Q_C = (25/2 + 25/4)(25/2)50 = 11.719\left(10^3\right) \text{ mm}^3$$

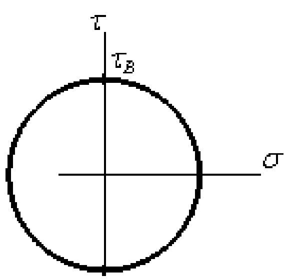
$$\tau_C = \frac{2\,000(11.719)10^3}{520.83\left(10^3\right)50} = 0.90 \text{ MPa} \qquad \textit{Ans.}$$

(b) Point A:



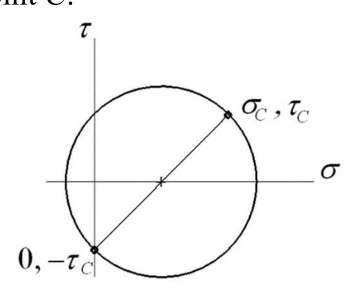
$$\tau_{\max} = \frac{\sigma_A}{2} = \frac{19.2}{2} = 9.6 \text{ MPa} \qquad \textit{Ans.}$$

Point B:



$$\tau_{\max} = \tau_B = 1.20 \text{ MPa} \qquad \textit{Ans.}$$

Point C:



$$\tau_{\max} = \sqrt{\left(\frac{\sigma_C}{2}\right)^2 + \tau_C^2}$$

$$= \sqrt{\left(\frac{9.60}{2}\right)^2 + 0.9^2} = 4.88 \text{ MPa} \qquad \textit{Ans.}$$

(c) Point A is critical. *Ans.*

(d) Transverse shear does not change with respect to the length of the beam.

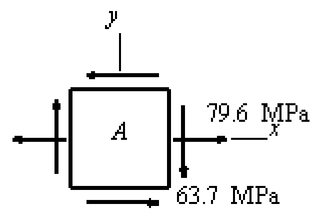
$$\left(\tau_A\right)_{\max} = \frac{\sigma_A}{2} = \frac{2\,000L\left(50/2\right)}{2\left(520.83\right)10^3} = 1.20 \quad \Rightarrow \quad L = 25.0 \text{ mm} \qquad Ans.$$

3-47 $I = (\pi/64)40^4 = 125.66(10^3)$ mm^4, $J = 2I$

(a) Point *A*:

$$\sigma_A = \frac{M_A c_A}{I} = \frac{50\left(10^3\right)10\left(40/2\right)}{125.66} = 79.6 \text{ MPa}$$

$$\tau_A = \frac{T_A r_A}{J} = \frac{800\left(10^3\right)\left(40/2\right)}{2\left(125.66\right)10^3} = 63.7 \text{ MPa}$$

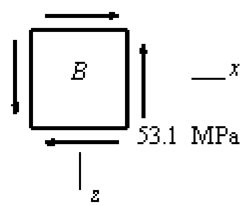


Ans.

Point *B*:

$$\sigma_B = 0$$

$$\tau_B = \frac{4}{3}\frac{V}{A} = \frac{4}{3}\frac{50\left(10^3\right)}{\pi\left(40/2\right)^2} = 53.1 \text{ MPa}$$

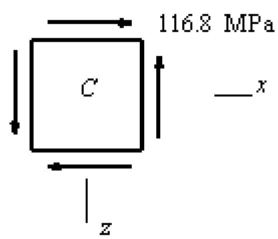


Ans.

Point *C*:

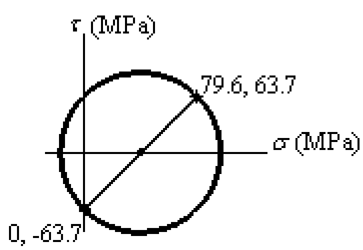
$$\sigma_C = 0$$

$$\tau_C = \tau_B + \frac{Tr}{J} = 53.1 + 63.7 = 116.8 \text{ MPa}$$



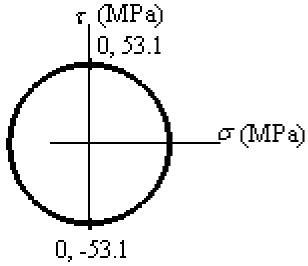
Ans.

(b) Point *A*:



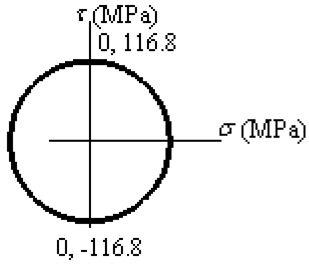
$$\left(\tau_{\max}\right)_A = \sqrt{\left(\frac{79.6}{2}\right)^2 + 63.7^2} = 75.1 \text{ MPa} \qquad Ans.$$

Point *B*:



$(\tau_{\max})_B = 53.1$ MPa *Ans.*

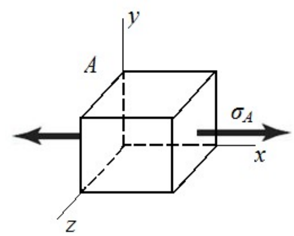
Point C:



$(\tau_{max})_C = 116.8$ MPa *Ans.*

(c) $\tau_{max} = 116.8$ MPa at point C *Ans.*

3-48 **(a)** $L = 10$ in. Element A:

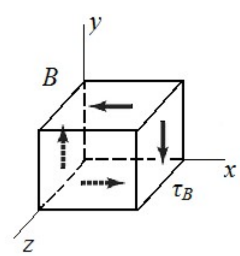


$$\sigma_A = -\frac{My}{I} = -\frac{-(1000)(10)(0.5)}{(\pi/64)(1)^4}\left(10^{-3}\right) = 101.9 \text{ kpsi}$$

$$\tau_A = \frac{VQ}{Ib}, \quad Q = 0 \quad \Rightarrow \quad \tau_A = 0$$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_A}{2}\right)^2 + \tau_A^2} = \sqrt{\left(\frac{101.9}{2}\right)^2 + (0)^2} = 50.9 \text{ kpsi} \quad \textit{Ans.}$$

Element B:



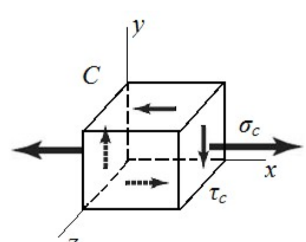
$$\sigma_B = -\frac{My}{I}, \quad y = 0 \quad \Rightarrow \quad \sigma_B = 0$$

$$Q = \bar{y}'A' = \left(\frac{4r}{3\pi}\right)\left(\frac{\pi r^2}{2}\right) = \frac{4r^3}{6} = \frac{4\left(0.5\right)^3}{6} = 1/12 \text{ in}^3$$

$$\tau_B = \frac{VQ}{Ib} = \frac{(1000)(1/12)}{(\pi/64)(1)^4(1)}\left(10^{-3}\right) = 1.698 \text{ kpsi}$$

$$\tau_{max} = \sqrt{\left(\frac{0}{2}\right)^2 + 1.698^2} = 1.698 \text{ kpsi} \quad \textit{Ans.}$$

Element C:

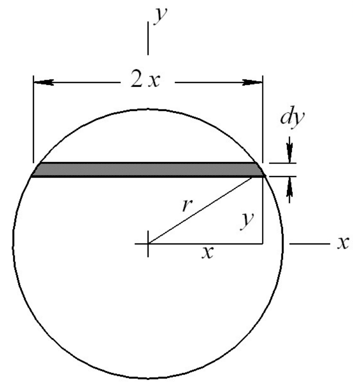


$$\sigma_C = -\frac{My}{I} = -\frac{-(1000)(10)(0.25)}{(\pi/64)(1)^4}\left(10^{-3}\right) = 50.93 \text{ kpsi}$$

$$Q = \int_{y_1}^{r} y\,dA = \int_{y_1}^{r} y(2x)dy = \int_{y_1}^{r} y\left(2\sqrt{r^2 - y^2}\right)dy$$

$$= -\frac{2}{3}\left(r^2 - y^2\right)^{3/2}\Big|_{y_1}^{r} = -\frac{2}{3}\left[\left(r^2 - r^2\right)^{3/2} - \left(r^2 - y_1^2\right)^{3/2}\right]$$

$$= \frac{2}{3}\left(r^2 - y_1^2\right)^{3/2}$$



For C, $y_1 = r/2 = 0.25$ in

$$Q = \frac{2}{3}\left(0.5^2 - 0.25^2\right)^{3/2} = 0.05413 \text{ in}^3$$

$$b = 2x = 2\sqrt{r^2 - y_1^2} = 2\sqrt{0.5^2 - 0.25^2} = 0.866 \text{ in}$$

$$\tau_C = \frac{VQ}{Ib} = \frac{(1000)(0.05413)}{(\pi/64)(1)^4(0.866)}\left(10^{-3}\right) = 1.273 \text{ kpsi}$$

$$\tau_{\max} = \sqrt{\left(\frac{50.93}{2}\right)^2 + (1.273)^2} = 25.50 \text{ kpsi} \quad \textit{Ans.}$$

(b) Neglecting transverse shear stress:

Element *A*: Since the transverse shear stress at point *A* is zero, there is no change.

$\tau_{\max} = 50.9$ kpsi *Ans.*

% error = 0% *Ans.*

Element *B*: Since the only stress at point *B* is transverse shear stress, neglecting the transverse shear stress ignores the entire stress.

$$\tau_{\max} = \sqrt{\left(\frac{0}{2}\right)^2} = 0 \text{ psi} \quad \textit{Ans.}$$

$$\% \text{ error} = \left(\frac{1.698 - 0}{1.698}\right) * (100) = 100\% \quad \textit{Ans.}$$

Element *C*:

$$\tau_{\max} = \sqrt{\left(\frac{50.93}{2}\right)^2} = 25.47 \text{ kpsi} \quad \textit{Ans.}$$

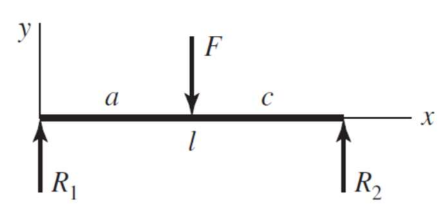
$$\% \text{ error} = \left(\frac{25.50 - 25.47}{25.50}\right) * (100) = 0.12\% \quad \textit{Ans.}$$

(c) Repeating the process with different beam lengths produces the results in the table.

	Bending stress, σ (kpsi)	Transverse shear stress, τ (kpsi)	Max shear stress, τ_{max} (kpsi)	Max shear stress, neglecting τ, τ_{max} (kpsi)	% error
***L* = 10 in**					
A	102	0	50.9	50.9	0
B	0	1.70	1.70	0	100
C	50.9	1.27	25.50	25.47	0.12
***L* = 4 in**					
A	40.7	0	20.4	20.4	0
B	0	1.70	1.70	0	100
C	20.4	1.27	10.26	10.19	0.77
***L* = 1 in**					
A	10.2	0	5.09	5.09	0
B	0	1.70	1.70	0	100
C	5.09	1.27	2.85	2.55	10.6
***L* = 0.1in**					
A	1.02	0	0.509	0.509	0
B	0	1.70	1.70	0	100
C	0.509	1.27	1.30	0.255	80.4

Discussion:
The transverse shear stress is only significant in determining the critical stress element as the length of the cantilever beam becomes smaller. As this length decreases, bending stress reduces greatly and transverse shear stress stays the same. This causes the critical element location to go from being at point *A*, on the surface, to point *B*, in the center. The maximum shear stress is on the outer surface at point *A* for all cases except *L* = 0.1 in, where it is at point *B* at the center. When the critical stress element is at point *A*, there is no error from neglecting transverse shear stress, since it is zero at that location. Neglecting the transverse shear stress has extreme significance at the stress element at the center at point *B*, but that location is probably only of practical significance for very short beam lengths.

3-49



$$R_1 = \frac{c}{l}F$$

$$M = \frac{c}{l}Fx \quad 0 \le x \le a$$

$$\sigma = \frac{6M}{bh^2} = \frac{6(c/l)Fx}{bh^2}$$

$$h = \sqrt{\frac{6Fcx}{lb\sigma_{max}}} \quad 0 \le x \le a \quad Ans.$$

3-50 From Problem 3-49, $R_1 = \dfrac{c}{l}F = V, \; 0 \le x \le a$

$$\tau_{\max} = \frac{3}{2}\frac{V}{bh} = \frac{3}{2}\frac{(c/l)F}{bh} \quad \Rightarrow \quad h = \frac{3}{2}\frac{Fc}{lb\tau_{\max}} \quad \textit{Ans.}$$

From Problem 3-49, $h(x) = \sqrt{\dfrac{6Fcx}{lb\sigma_{\max}}}$.

Sub in $x = e$ and equate to h above.

$$\frac{3}{2}\frac{Fc}{lb\tau_{\max}} = \sqrt{\frac{6Fce}{lb\sigma_{\max}}}$$

$$e = \frac{3}{8}\frac{Fc\sigma_{\max}}{lb\tau_{\max}^2} \quad \textit{Ans.}$$

3-51 **(a)**

<u>*x-z*</u> plane

$$\Sigma M_O = 0 = 1.5(0.5) + 2(1.5)\sin(30^\circ)(2.25) - R_{2z}(3)$$

$$R_{2z} = 1.375 \text{ kN} \quad \textit{Ans.}$$

$$\Sigma F_z = 0 = R_{1z} - 1.5 - 2(1.5)\sin(30^\circ) + 1.375$$

$$R_{1z} = 1.625 \text{ kN} \quad \textit{Ans.}$$

x-y plane

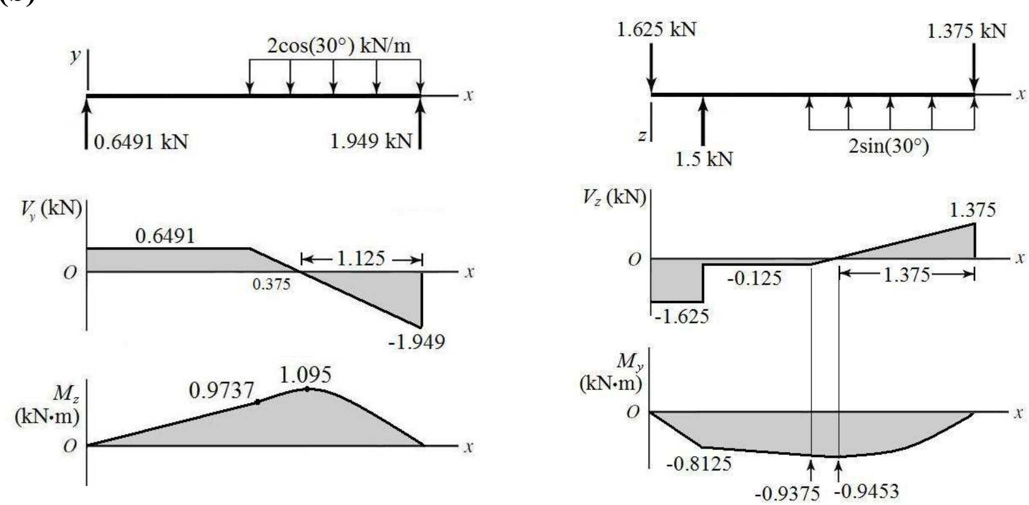
$$\Sigma M_O = 0 = -2(1.5)\cos(30^\circ)(2.25) + R_{2y}(3)$$

$$R_{2y} = 1.949 \text{ kN} \quad \textit{Ans.}$$

$$\Sigma F_y = 0 = R_{1y} - 2(1.5)\cos(30^\circ) + 1.949$$

$$R_{1y} = 0.6491 \text{ kN} \quad \textit{Ans.}$$

(b)



(c) The transverse shear and bending moments for most points of interest can readily be taken straight from the diagrams. For 1.5 < x < 3, the bending moment equations are parabolic, and are obtained by integrating the linear expressions for shear. For convenience, use a coordinate shift of $x' = x - 1.5$. Then, for $0 < x' < 1.5$,

$$V_z = x' - 0.125$$

$$M_y = \int V_z dx' = \frac{(x')^2}{2} - 0.125x' + C$$

$$\text{At } x' = 0, M_y = C = -0.9375 \;\Rightarrow\; M_y = 0.5(x')^2 - 0.125x' + 0.9375$$

$$V_y = -\frac{1.949}{1.125}x' + 0.6491 = -1.732x' + 0.6491$$

$$M_z = \frac{-1.732}{2}(x')^2 + 0.6491x' + C$$

$$\text{At } x' = 0, M_z = C = 0.9737 \;\Rightarrow\; M_z = -0.8662(x')^2 - 0.125x' - 0.9375$$

By programming these bending moment equations, we can find M_y, M_z, and their vector combination at any point along the beam. The maximum combined bending moment is found to be at x = 1.79 m, where M = 1.433 kN·m. The table below shows values at key locations on the shear and bending moment diagrams.

x (m)	V_z (kN)	V_y (kN)	V (kN)	M_y (kN·m)	M_z (kN·m)	M (kN·m)
0	–1.625	0.6491	1.750	0	0	0
0.5^-	–1.625	0.6491	1.750	–0.8125	0.3246	0.8749
1.5	–0.1250	0.6491	0.6610	0.9375	0.9737	1.352
1.625	0	0.4327	0.4327	–0.9453	1.041	1.406
1.875	0.2500	0	0.2500	–0.9141	1.095	1.427
3^-	1.375	–1.949	2.385	0	0	0

(d) The bending stress is obtained from Eq. (3-27),

$$\sigma_x = \frac{-M_z y_A}{I_z} + \frac{M_y z_A}{I_y}$$

The maximum tensile bending stress will be at point A in the cross section of Prob. 3-35 (a), where distances from the neutral axes for both bending moments will be maximum. At A, for M_z, y_A = –37.5 mm, and for M_y, z_A = –20 mm.

$$I_z = \frac{40(75)^3}{12} - \frac{34(25)^3}{12} = 1.36(10^6)\text{ mm}^4 = 1.36(10^{-6})\text{ m}^4$$

$$I_y = 2\left[\frac{25(40)^3}{12}\right] + \frac{25(6)^3}{12} = 2.67(10^5)\text{ mm}^4 = 2.67(10^{-7})\text{ m}^4$$

It is apparent the maximum bending moment, and thus the maximum stress, will be in the parabolic section of the bending moment diagrams. Programming Eq. (3-27) with the bending moment equations previously derived, the maximum tensile bending stress is found at x = 1.77 m, where M_y = – 0.9408 kN·m, M_z = 1.075 kN·m, and σ_x = 100.1 MPa. *Ans.*

3-52

(a) x-z plane

$$\Sigma M_O = 0 = \frac{3}{5}(1000)(4) - \frac{600}{\sqrt{2}}(10) + M_{Oy}$$

$$M_{Oy} = 1842.6 \text{ lbf}\cdot\text{in} \quad \textit{Ans.}$$

$$\Sigma F_z = 0 = R_{Oz} - \frac{3}{5}(1000) + \frac{600}{\sqrt{2}}$$

$$R_{Oz} = 175.7 \text{ lbf} \quad \textit{Ans.}$$

x-y plane

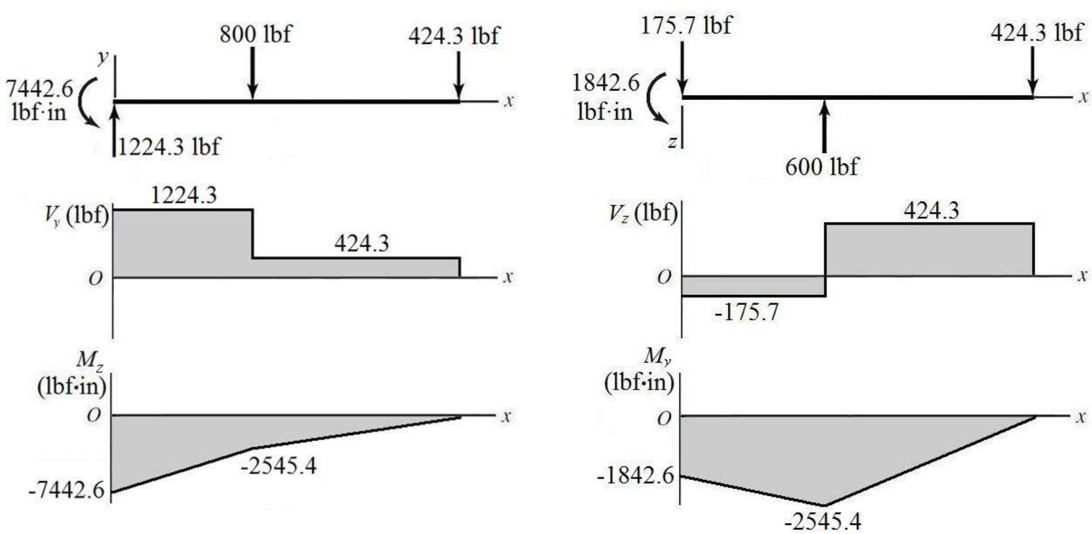
$$\Sigma M_O = 0 = -\frac{4}{5}(1000)(4) - \frac{600}{\sqrt{2}}(10) + M_{Oz}$$

$$M_{Oz} = 7442.5 \text{ lbf}\cdot\text{in} \quad \textit{Ans.}$$

$$\Sigma F_y = 0 = R_{Oy} - \frac{4}{5}(1000) - \frac{600}{\sqrt{2}}$$

$$R_{Oy} = 1224.3 \text{ lbf} \quad \textit{Ans.}$$

(b)



(c)

$$V(x) = \left[V_y(x)^2 + V_z(x)^2\right]^{1/2}$$

$$M(x) = \left[M_y(x)^2 + M_z(x)^2\right]^{1/2}$$

x (m)	V_z (kN)	V_y (kN)	V (kN)	M_y (kN·m)	M_z (kN·m)	M (kN·m)
0	–175.7	1224.3	1237	–1842.6	–7442.6	7667
4^-	–175.7	1224.3	1237	–2545.4	–2545.4	3600
10^-	424.3	424.3	600	0	0	0

(d) The maximum tensile bending stress will be at the outer corner of the cross section in the positive *y*, negative *z* quadrant, where $y = 1.5$ in and $z = -1$ in.

$$I_z = \frac{2(3)^3}{12} - \frac{(1.625)(2.625)^3}{12} = 2.051 \text{ in}^4$$

$$I_y = \frac{3(2)^3}{12} - \frac{(2.625)(1.625)^3}{12} = 1.601 \text{ in}^4$$

At $x = 0$, using Eq. (3-27),

$$\sigma_x = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$\sigma_x = -\frac{(-7442.6)(1.5)}{2.051} + \frac{(-1842.6)(-1)}{1.601} = 6594 \text{ psi}$$

Check at $x = 4$ in,

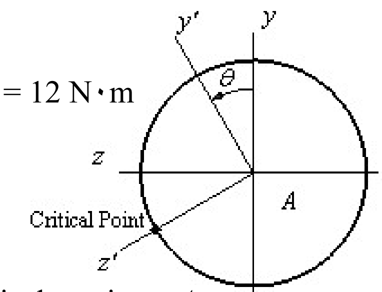
$$\sigma_x = -\frac{(-2545.4)(1.5)}{2.051} + \frac{(-2545.4)(-1)}{1.601} = 2706 \text{ psi}$$

The critical location is at $x = 0$, where $\sigma_x = 6594$ psi. *Ans.*

3-53 (a) Moments at *A*: $M_y = 300(0.050) = 15$ N·m, $M_z = 200(0.055) = 11$ N·m, Torque at *A*: $T_x = 200(0.060) = 12$ N·m

$$|M| = \sqrt{M_y^2 + M_z^2} = \sqrt{15^2 + 11^2} = 18.601 \text{ N}\cdot\text{m}$$

$$\theta = \tan^{-1}\left(\frac{M_z}{M_y}\right) = \tan^{-1}\left(\frac{11}{15}\right) = 36.25^\circ$$



Critical point will be 90° from θ, i.e. 126.25° from the vertical *y* axis. *Ans.*

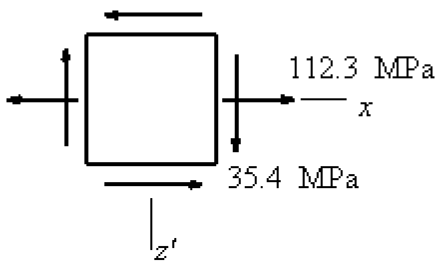
(b)

$$\sigma_{\text{bend}} = \frac{|M|c}{I} = \frac{32|M|}{\pi d^3} = \frac{32(18.601)}{\pi(0.012^3)}10^{-6} = 109.6 \text{ MPa}$$

$$\sigma_{\text{axial}} = \frac{F}{A} = \frac{4F}{\pi d^2} = \frac{4(300)}{\pi(0.012^2)}10^{-6} = 2.65 \text{ MPa}$$

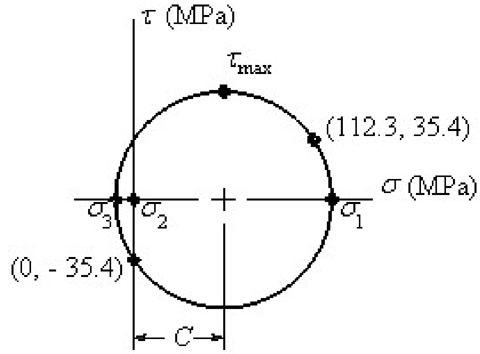
$$\sigma_{\text{total}} = \sigma_{\text{bend}} + \sigma_{\text{axial}} = 109.6 + 2.65 = 112.3 \text{ MPa}$$

$$\tau = \frac{T_x c}{J} = \frac{16T_x}{\pi d^3} = \frac{16(12)}{\pi(0.012^3)}10^{-6} = 35.4 \text{ MPa}$$



Ans.

(c)



Ans.

(d) $C = \dfrac{112.3}{2} = 56.2\ MPa, \quad R = \sqrt{\left(\dfrac{112.3}{2}\right)^2 + 35.4^2} = 66.4 \text{ MPa}$

$\sigma_1 = C + R = 56.2 + 66.4 = 122.6 \text{ MPa}, \ \sigma_2 = 0,$

$\sigma_3 = C - R = 56.2 - 66.4 = -10.2 \text{ MPa}$ *Ans.*

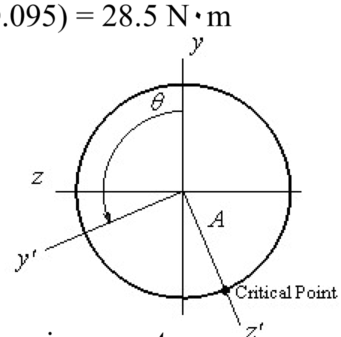
$\tau_{\max} = R = 66.4 \text{ MPa}$

3-54 **(a)** Moments at A: $M_y = -\,200(0.060) = -\,12$ N·m, $M_z = 300(0.095) = 28.5$ N·m

Torque at A: $T_x = -\,300(0.050) = -\,15$ N·m

$|M| = \sqrt{M_y^2 + M_z^2} = \sqrt{(-12)^2 + 28.5^2} = 30.92 \text{ N} \cdot \text{m}$

$\theta = \tan^{-1}\left(\dfrac{M_z}{M_y}\right) = \tan^{-1}\left(\dfrac{28.5}{-12}\right) = 112.8^{\circ}$



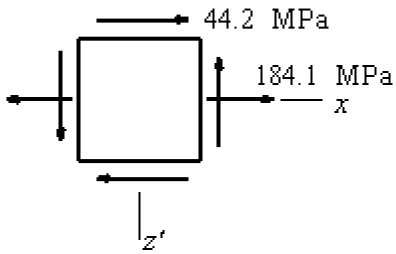
Critical point will be 90° from θ, i.e. 202.8° from the vertical y. axis. *Ans.*

$\sigma_{\text{bend}} = \dfrac{|M|c}{I} = \dfrac{32|M|}{\pi d^3} = \dfrac{32(30.92)}{\pi(0.012^3)} 10^{-6} = 182.3 \text{ MPa}$

$\sigma_{\text{axial}} = \dfrac{F}{A} = \dfrac{4F}{\pi d^2} = \dfrac{4(200)}{\pi(0.012^2)} 10^{-6} = 1.77 \text{ MPa}$

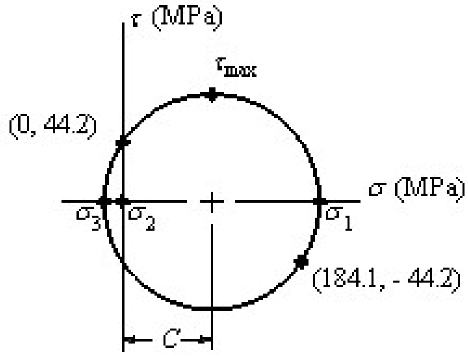
$\sigma_{\text{total}} = \sigma_{\text{bend}} + \sigma_{\text{axial}} = 182.3 + 1.77 = 184.1 \text{ MPa}$

$\tau = \dfrac{T_x c}{J} = \dfrac{16T_x}{\pi d^3} = \dfrac{16(15)}{\pi(0.012^3)} 10^{-6} = 44.2 \text{ MPa}$



Ans.

(c)



Ans.

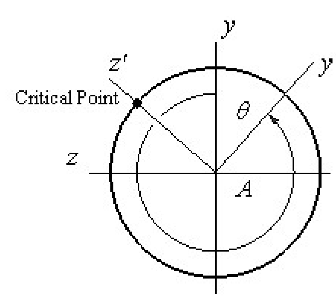
(d) $C = \dfrac{184.1}{2} = 92.1 \text{ MPa}, \quad R = \sqrt{\left(\dfrac{184.1}{2}\right)^2 + 44.2^2} = 102.1 \text{ MPa}$

$\sigma_1 = C + R = 92.1 + 102.1 = 194.2 \text{ MPa}, \ \sigma_2 = 0,$

$\sigma_3 = C - R = 92.1 - 102.1 = -10 \text{ MPa}$ *Ans.*

$\tau_{\max} = R = 102.1 \text{ MPa}$

3-55 **(a)** Moments at *A*: M_y = 60(5) + 200(5.5) = 1400 lbf·in,
M_z = – (300 – 75)5.5 = – 1238 lbf·in,
Torque at *A*: T_x = – 75(5) = – 375 lbf·in



$$|M| = \sqrt{M_y^2 + M_z^2} = \sqrt{1400^2 + (-1238)^2} = 1869 \text{ lbf} \cdot \text{in}$$

$$\theta = \tan^{-1}\left(\frac{M_z}{M_y}\right) = \tan^{-1}\left(\frac{-1238}{1400}\right) = 318.5 = -41.5^\circ$$

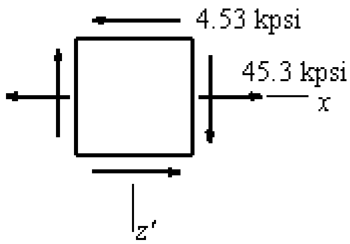
Critical point will be 90° from θ, i.e. 48.5° from the vertical y axis. *Ans.*

(b) $\sigma_{\text{bend}} = \dfrac{|M|c}{I} = \dfrac{32|M|}{\pi d^3} = \dfrac{32(1869)}{\pi(0.75^3)} 10^{-3} = 45.13 \text{ kpsi}$

$$\sigma_{\text{axial}} = \frac{F}{A} = \frac{4F}{\pi d^2} = \frac{4(60)}{\pi(0.75^2)} = 136 \text{ psi}$$

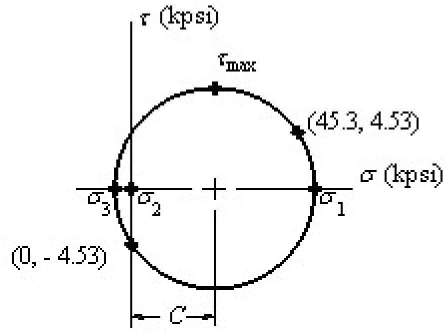
$$\sigma_{\text{total}} = \sigma_{\text{bend}} + \sigma_{\text{axial}} = 45.126 + 0.136 = 45.3 \text{ kpsi}$$

$$\tau = \frac{T_x c}{J} = \frac{16T_x}{\pi d^3} = \frac{16(375)}{\pi(0.75^3)} 10^{-3} = 4.53 \text{ kpsi}$$



Ans.

(c)



Ans.

(d) $C = \dfrac{45.3}{2} = 22.7\,\text{kpsi}, \quad R = \sqrt{\left(\dfrac{45.3}{2}\right)^2 + 4.53^2} = 23.1\text{ kpsi}$

$\sigma_1 = C + R = 22.7 + 23.1 = 45.8\,\text{kpsi}, \ \sigma_2 = 0,$

$\sigma_3 = C - R = 22.7 - 23.1 = -0.45\text{ kpsi}$ *Ans.*

$\tau_{\max} = R = 23.1\text{ kpsi}$

3-56 Given: b = 3.6 in, c = 2.5 in, and l = 40 in.
From Table A-5, G = 11.5 Mpsi. From Table A-20, S_y = 42 kpsi.
(a) For the table for Eq. (3-40), b/c = 3.6/2.5 = 1.44

α	b/c
0.208	1
α	1.44
0.231	1.5

$$\frac{0.231-\alpha}{0.231-0.208} = \frac{1.5-1.44}{1.5-1} = 0.12 \ \Rightarrow \ \alpha = 0.231 - 0.12(0.231-0.208) = 0.228$$

Equation (3-40):

$$\tau_{\max} = \frac{T}{\alpha bc^2} = \frac{30(10^3)}{0.228(3.6)2.5^2} = 5\,850\ \text{psi} = 5.85\text{ kpsi} \qquad \textit{Ans.}$$

(b) For the table for Eq. (3-41), b/c = 3.6/2.5 = 1.44

β	b/c
0.141	1
β	1.44
0.196	1.5

$$\frac{0.196-\beta}{0.196-0.141}=\frac{1.5-1.44}{1.5-1}=0.12 \;\Rightarrow\; \beta=0.196-0.12(0.196-0.141)=0.189$$

Equation (3-41):

$$\theta=\frac{Tl}{\beta bc^3G}=\frac{30(10^3)40}{0.189(3.6)2.5^3(11.5)10^6}=9.815(10^{-3})\;rad\;=0.562^\circ \qquad Ans.$$

(c) S_{sy} = 0.5(42) = 21 kpsi. Yield factor of safety,

$$n_y=\frac{S_{sy}}{\tau_{\max}}=\frac{21}{5.85}=3.59 \qquad Ans.$$

3-57 Given: 250 hp at 540 rev/min, τ_{allow} = 15 kpsi.

Eq. (3-42): $T=\frac{63\,025}{n}H=\frac{63\,025}{540}250=29.178(10^3)$ lbf · in

Eq. (3-41) with table where for square cross-section, b/c =1

$$\tau_{\max}=\frac{T}{0.208bc^2}=\frac{29.178(10^3)}{0.208b^3}=15(10^3) \;\Rightarrow\; b=2.107 \text{ in}$$

From Table A-17, use $b=2\frac{1}{4}$ in *Ans.*

3-58 Given: T = 50 kN-m. OD = 300 mm, t = 2 mm, and l = 2 m.

J = (π/32)(0.300^4 – 0.296^4) = 4.157(10^{-5}) m^4. From Table A-5, G = 79.3 GPa.

(a) Eq. (3-37):

$$\tau_{\max}=\frac{Tr}{J}=\frac{50(10^3)0.15}{4.157(10^{-5})}10^{-6}=180.4 \text{ MPa} \qquad Ans.$$

(b) Eq. (3-45):

$$\tau_{\max}=\frac{T}{2A_mt}=\frac{50(10^3)}{2(\pi/4)0.298^2(0.002)}10^{-6}=179.2\;MPa \qquad Ans.$$

Answer is 0.67 percent lower than part (a).

(c) Eq. (3-35):

$$\theta=\frac{TL}{JG}=\frac{50(10^3)2}{4.157(10^{-5})79.3(10^9)}=0.0303 \text{ rad}=1.74^\circ \qquad Ans.$$

(d) Eq. (3-46):

$$\theta=\theta_1 l=\frac{TL_ml}{4GA_m^2t}=\frac{50(10^3)\pi(0.298)2}{4(79.3)10^9\left[(\pi/4)0.298^2\right]^2 0.002}=0.303 \text{ rad}=1.74^\circ \qquad Ans.$$

Within the same accuracy, the answer is the same as part (c).

3-59 Given: Rectangular tube with inner dimensions 1.5 in × 2.0 in, $t = \frac{1}{8}$ in, 1035 CD steel, n = 540 rev/min, and S_{sy} =0.5 S_y.

(a) Table A-20, S_y = 67 kpsi. Factor of safety against yield, n_y = 2. For the table for Eq. (3-40), with b/c = 2/1.5 = 1.333,

α	b/c
0.208	1
α	1.333
0.231	1.5

$$\frac{0.231-\alpha}{0.231-0.208}=\frac{1.5-1.333}{1.5-1}=0.3333 \;\Rightarrow\; \alpha=0.231-0.3333(0.231-0.208)=0.2233$$

Eq. (3-40):

$$\tau_{\max}=\frac{T}{\alpha bc^2}=\frac{T}{0.2233(2)1.5^2}=\frac{0.5S_y}{n_y}=\frac{0.5(67)10^3}{2} \;\Rightarrow\; T=16.83\left(10^3\right)\ \text{lbf}\cdot\text{in}$$

Eq. (3-42):

$$H=\frac{Tn}{63\ 025}=\frac{16.83\left(10^3\right)540}{63\ 025}=144\ \text{hp} \qquad Ans.$$

(b) Eq. (3-45):

$$\tau_{\max}=\frac{0.5S_y}{n_y}=\frac{T}{2A_m t}$$

$$\frac{0.5(67)10^3}{2}=\frac{T}{2\left[\left(1.5+\frac{1}{8}\right)\left(2.0+\frac{1}{8}\right)\right]\frac{1}{8}} \;\Rightarrow\; T=14.46\left(10^3\right)\ \text{lbf}\cdot\text{in}$$

$$H=\frac{14.46\left(10^3\right)540}{63\ 025}=124\ \text{hp} \qquad Ans.$$

3-60 Outer dimensions 20 × 30 mm, t = 1 mm, l = 1 m, 1018 CD steel, S_{sy} = 0.5 S_y.
Table A-20, S_y = 370 MPa. S_{sy} = 0.5(370) = 185 MPa. Table A-5, G = 79.3 GPa.

(a) A_m = 19(29) = 551 mm^2.

Eq, (3-45):

$$\tau=\frac{T}{2A_m t}=185\left(10^6\right) \;\Rightarrow\; T=2(551)10^{-6}(0.001)185\left(10^6\right)=204\ \text{N}\cdot\text{m} \qquad Ans.$$

(b) Eq. (3-46):

$$\theta=\theta_1 l=\frac{TL_m l}{4GA_m^2 t} \;\Rightarrow\; 3\left(\frac{\pi}{180}\right)=\frac{T(2)(19+29)10^{-3}(1)}{4(79.3)10^9(551)^2 10^{-12}(0.001)} \;\Rightarrow\; T=52.5\ \text{N}\cdot\text{m} \quad Ans.$$

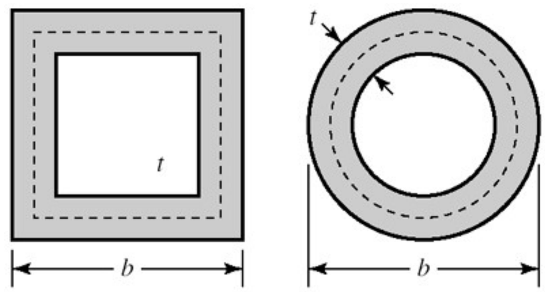
3-61 **(a)** The area within the wall median line, A_m, is

Square: $A_m = (b-t)^2$. From Eq. (3-45)

$$T_{\text{sq}} = 2A_m t\tau_{\text{all}} = 2(b-t)^2 t\tau_{\text{all}}$$

Round: $A_m = \pi(b-t)^2/4$

$$T_{\text{rd}} = 2\pi(b-t)^2 t\tau_{\text{all}}/4$$



Ratio of Torques

$$\frac{T_{\text{sq}}}{T_{\text{rd}}} = \frac{2(b-t)^2 t\tau_{\text{all}}}{\pi(b-t)^2 t\tau_{\text{all}}/2} = \frac{4}{\pi} = 1.27 \quad \textit{Ans.}$$

(b) Twist per unit length from Eq. (3-46) is

$$\theta_1 = \frac{TL_m}{4GA_m^2 t} = \frac{2A_m t\tau_{\text{all}} L_m}{4GA_m^2 t} = \frac{\tau_{\text{all}}}{2G}\frac{L_m}{A_m} = C\frac{L_m}{A_m}$$

Square:

$$\theta_{\text{sq}} = C\frac{4(b-t)}{(b-t)^2}$$

Round:

$$\theta_{\text{rd}} = C\frac{\pi(b-t)}{\pi(b-t)^2/4} = C\frac{4(b-t)}{(b-t)^2}$$

$\dfrac{\theta_{sq}}{\theta_{rd}} = 1$. Twists are the same. *Ans.*

3-62 **(a)** The area enclosed by the section median line is $A_m = (1 - 0.0625)^2 = 0.8789$ in^2 and the length of the section median line is $L_m = 4(1 - 0.0625) = 3.75$ in. From Eq. (3-45),

$$T = 2A_m t\tau = 2(0.8789)(0.0625)(12\ 000) = 1318 \text{ lbf}\cdot\text{in} \qquad \textit{Ans.}$$

From Eq. (3-46),

$$\theta = \theta_1 l = \frac{TL_m l}{4GA_m^2 t} = \frac{(1318)(3.75)(36)}{4(11.5)(10^6)(0.8789)^2(0.0625)} = 0.0801 \text{ rad} = 4.59^\circ \quad \textit{Ans.}$$

(b) The radius at the median line is $r_m = 0.125 + (0.5)(0.0625) = 0.15625$ in. The area enclosed by the section median line is $A_m = (1 - 0.0625)^2 - 4(0.15625)^2 + 4(\pi/4)(0.15625)^2 = 0.8579$ in^2. The length of the section median line is $L_m = 4[1 - 0.0625 - 2(0.15625)] + 2\pi(0.15625) = 3.482$ in.

From Eq. (3-45),

$$T = 2A_m t\tau = 2(0.8579)(0.0625)(12\ 000) = 1287 \text{ lbf}\cdot\text{in} \qquad Ans.$$

From Eq. (3-46),

$$\theta = \theta_1 l = \frac{TL_m l}{4GA_m^2 t} = \frac{(1287)(3.482)(36)}{4(11.5)(10^6)(0.8579)^2(0.0625)} = 0.0762 \text{ rad} = 4.37^\circ \quad Ans.$$

3-63

$$\theta_1 = \frac{3T_i}{GL_i c_i^3} \qquad \Rightarrow \qquad T_i = \frac{\theta_1 G L_i c_i^3}{3}$$

$$T = T_1 + T_2 + T_3 = \frac{\theta_1 G}{3}\sum_{i=1}^{3} L_i c_i^3 \qquad Ans.$$

From Eq. (3-47), $\tau = G\theta_1 c$

G and θ_1 are constant, therefore the largest shear stress occurs when c is a maximum.

$$\tau_{\max} = G\theta_1 c_{\max} \qquad Ans.$$

3-64

(b) Solve part (b) first since the angle of twist per unit length is needed for part (a).

$$\tau_{\max} = \tau_{\text{allow}} = 12(6.89) = 82.7 \text{ MPa}$$

$$\theta_1 = \frac{\tau_{\max}}{Gc_{\max}} = \frac{82.7(10^6)}{79.3(10^9)(0.003)} = 0.348 \text{ rad/m} \qquad Ans.$$

(a)

$$T_1 = \frac{\theta_1 G L_1 c_1^3}{3} = \frac{0.348(79.3)(10^9)(0.020)(0.002^3)}{3} = 1.47 \text{ N}\cdot\text{m} \qquad Ans.$$

$$T_2 = \frac{\theta_2 G L_2 c_2^3}{3} = \frac{0.348(79.3)(10^9)(0.030)(0.003^3)}{3} = 7.45 \text{ N}\cdot\text{m} \qquad Ans.$$

$$T_3 = \frac{\theta_3 G L_3 c_3^3}{3} = \frac{0.348(79.3)(10^9)(0)(0^3)}{3} = 0 \qquad Ans.$$

$$T = T_1 + T_2 + T_3 = 1.47 + 7.45 + 0 = 8.92 \text{ N}\cdot\text{m} \qquad Ans.$$

3-65

(b) Solve part (b) first since the angle of twist per unit length is needed for part (a).

$$\theta_1 = \frac{\tau_{\max}}{Gc_{\max}} = \frac{12\,000}{11.5\left(10^6\right)(0.125)} = 8.35\left(10^{-3}\right) \text{ rad/in} \quad \textit{Ans.}$$

(a)

$$T_1 = \frac{\theta_1 G L_1 c_1^3}{3} = \frac{(8.35)\left(10^{-3}\right)(11.5)\left(10^6\right)(0.75)\left(0.0625^3\right)}{3} = 5.86 \text{ lbf}\cdot\text{in} \quad \textit{Ans.}$$

$$T_2 = \frac{\theta_2 G L_2 c_2^3}{3} = \frac{(8.35)\left(10^{-3}\right)(11.5)\left(10^6\right)(1)\left(0.125^3\right)}{3} = 62.52 \text{ lbf}\cdot\text{in} \quad \textit{Ans.}$$

$$T_3 = \frac{\theta_3 G L_3 c_3^3}{3} = \frac{(8.35)\left(10^{-3}\right)(11.5)\left(10^6\right)(0.625)\left(0.0625^3\right)}{3} = 4.88 \text{ lbf}\cdot\text{in} \quad \textit{Ans.}$$

$$T = T_1 + T_2 + T_3 = 5.86 + 62.52 + 4.88 = 73.3 \text{ lbf}\cdot\text{in} \quad \textit{Ans.}$$

3-66

(b) Solve part (b) first since the angle of twist per unit length is needed for part (a).

$$\tau_{\max} = \tau_{\text{allow}} = 12(6.89) = 82.7 \text{ MPa}$$

$$\theta_1 = \frac{\tau_{\max}}{Gc_{\max}} = \frac{82.7\left(10^6\right)}{79.3\left(10^9\right)(0.003)} = 0.348 \text{ rad/m} \quad \textit{Ans.}$$

(a)

$$T_1 = \frac{\theta_1 G L_1 c_1^3}{3} = \frac{0.348(79.3)\left(10^9\right)(0.020)(0.002^3)}{3} = 1.47 \text{ N}\cdot\text{m} \quad \textit{Ans.}$$

$$T_2 = \frac{\theta_2 G L_2 c_2^3}{3} = \frac{0.348(79.3)\left(10^9\right)(0.030)(0.003^3)}{3} = 7.45 \text{ N}\cdot\text{m} \quad \textit{Ans.}$$

$$T_3 = \frac{\theta_3 G L_3 c_3^3}{3} = \frac{0.348(79.3)\left(10^9\right)(0.025)(0.002^3)}{3} = 1.84 \text{ N}\cdot\text{m} \quad \textit{Ans.}$$

$$T = T_1 + T_2 + T_3 = 1.47 + 7.45 + 1.84 = 10.8 \text{ N}\cdot\text{m} \quad \textit{Ans.}$$

3-67

(a) From Eq. (3-40), with two 2-mm strips,

$$T = \frac{\tau_{\max} b c^2}{3 + 1.8/(b/c)} = \frac{(80)\left(10^6\right)(0.030)\left(0.002^2\right)}{3 + 1.8/\left(0.030/0.002\right)} = 3.08 \text{ N}\cdot\text{m}$$

$$T_{\max} = 2(3.08) = 6.16 \text{ N}\cdot\text{m} \quad \textit{Ans.}$$

From the table for Eqs. (3-40) and (3-41), with *b/c* = 30/2 = 15, $\alpha = \beta$ and has a value between 0.313 and 0.333. From Eq. (3-40),

$$\alpha \approx \frac{1}{3+1.8/(30/2)} = 0.321$$

From Eq. (3-41),

$$\theta = \frac{Tl}{\beta bc^3 G} = \frac{3.08(0.3)}{0.321(0.030)\left(0.002^3\right)(79.3)\left(10^9\right)} = 0.151 \text{ rad} \quad \textit{Ans.}$$

$$k_t = \frac{T}{\theta} = \frac{6.16}{0.151} = 40.8 \text{ N}\cdot\text{m} \quad \textit{Ans.}$$

From Eq. (3-40), with a single 4-mm strip,

$$T_{\max} = \frac{\tau_{\max} bc^2}{3+1.8/(b/c)} = \frac{(80)\left(10^6\right)(0.030)\left(0.004^2\right)}{3+1.8/\left(0.030/0.004\right)} = 11.9 \text{ N}\cdot\text{m} \quad \textit{Ans.}$$

Interpolating from the table for Eqs. (3-40) and (3-41), with *b/c* = 30/4 = 7.5,

$$\beta = \frac{7.5-6}{8-6}(0.307-0.299)+0.299 = 0.305$$

From Eq. (3-41)

$$\theta = \frac{Tl}{\beta bc^3 G} = \frac{11.9(0.3)}{0.305(0.030)\left(0.004^3\right)(79.3)\left(10^9\right)} = 0.0769 \text{ rad} \quad \textit{Ans.}$$

$$k_t = \frac{T}{\theta} = \frac{11.9}{0.0769} = 155 \text{ N}\cdot\text{m} \quad \textit{Ans.}$$

(b) From Eq. (3-47), with two 2-mm strips,

$$T = \frac{Lc^2\tau}{3} = \frac{(0.030)\left(0.002^2\right)(80)\left(10^6\right)}{3} = 3.20 \text{ N}\cdot\text{m}$$

$$T_{\max} = 2(3.20) = 6.40 \text{ N}\cdot\text{m} \quad \textit{Ans.}$$

$$\theta = \frac{3Tl}{Lc^3 G} = \frac{3(3.20)(0.3)}{(0.030)\left(0.002^3\right)(79.3)\left(10^9\right)} = 0.151 \text{ rad} \quad \textit{Ans.}$$

$$k_t = T/\theta = 6.40/0.151 = 42.4 \text{ N}\cdot\text{m} \quad \textit{Ans.}$$

From Eq. (3-47), with a single 4-mm strip,

$$T_{\max} = \frac{Lc^2\tau}{3} = \frac{(0.030)\left(0.004^2\right)(80)\left(10^6\right)}{3} = 12.8 \text{ N}\cdot\text{m} \quad \textit{Ans.}$$

$$\theta = \frac{3Tl}{Lc^3G} = \frac{3(12.8)(0.3)}{(0.030)\left(0.004^3\right)(79.3)\left(10^9\right)} = 0.0757 \text{ rad} \qquad Ans.$$

$$k_t = T/\theta = 12.8/0.0757 = 169 \text{ N}\cdot\text{m} \qquad Ans.$$

The results for the spring constants when using Eq. (3-47) are slightly larger than when using Eq. (3-40) and Eq. (3-41) because the strips are not infinitesimally thin (i.e. b/c does not equal infinity). The spring constants when considering one solid strip are significantly larger (almost four times larger) than when considering two thin strips because two thin strips would be able to slip along the center plane.

3-68

(a) Obtain the torque from the given power and speed using Eq. (3-44).

$$T = 9.55\frac{H}{n} = 9.55\frac{(40\,000)}{2500} = 152.8 \text{ N}\cdot\text{m}$$

$$\tau_{\max} = \frac{Tr}{J} = \frac{16T}{\pi d^3}$$

$$d = \left(\frac{16T}{\pi\tau_{\max}}\right)^{1/3} = \left[\frac{16(152.8)}{\pi(70)\left(10^6\right)}\right]^{1/3} = 0.0223 \text{ m} = 22.3 \text{ mm} \qquad Ans.$$

(b) $T = 9.55\dfrac{H}{n} = 9.55\dfrac{(40\,000)}{250} = 1528 \text{ N}\cdot\text{m}$

$$d = \left[\frac{16(1528)}{\pi(70)\left(10^6\right)}\right]^{1/3} = 0.0481 \text{ m} = 48.1 \text{ mm} \qquad Ans.$$

3-69

(a) Obtain the torque from the given power and speed using Eq. (3-42).

$$T = \frac{63\,025H}{n} = \frac{63\,025(50)}{2500} = 1261 \text{ lbf}\cdot\text{in}$$

$$\tau_{\max} = \frac{Tr}{J} = \frac{16T}{\pi d^3}$$

$$d = \left(\frac{16T}{\pi\tau_{\max}}\right)^{1/3} = \left[\frac{16(1261)}{\pi(20\,000)}\right]^{1/3} = 0.685 \text{ in} \qquad Ans.$$

(b) $T = \dfrac{63\,025H}{n} = \dfrac{63\,025(50)}{250} = 12\,610 \text{ lbf}\cdot\text{in}$

$$d = \left[\frac{16(12\,610)}{\pi(20\,000)}\right]^{1/3} = 1.48 \text{ in} \qquad Ans.$$

3-70

$$\tau_{\max} = \frac{16T}{\pi d^3} \quad \Rightarrow \quad T = \frac{\tau_{\max} \pi d^3}{16} = \frac{(50)\left(10^6\right)\pi\left(0.03^3\right)}{16} = 265 \text{ N} \cdot \text{m}$$

Eq. (3-44), $H = \dfrac{Tn}{9.55} = \dfrac{265(2000)}{9.55} = 55.5\left(10^3\right) \text{ W} = 55.5 \text{ kW}$ *Ans.*

3-71

$$\tau = \frac{16T}{\pi d^3} \quad \Rightarrow \quad T = \frac{\pi}{16}\tau d^3 = \frac{\pi}{16}(110)\left(10^6\right)\left(0.020^3\right) = 173 \text{ N} \cdot \text{m}$$

$$\theta = \frac{Tl}{JG} \quad \Rightarrow \quad l = \frac{\pi d^4 G\theta}{32T} = \frac{\pi\left(0.020^4\right)(79.3)\left(10^9\right)\left(15\dfrac{\pi}{180}\right)}{32(173)}$$

$l = 1.89$ m *Ans.*

3-72

$$\tau = \frac{16T}{\pi d^3} \quad \Rightarrow \quad T = \frac{\pi}{16}\tau d^3 = \frac{\pi}{16}(30\,000)\left(0.75^3\right) = 2485 \text{ lbf} \cdot \text{in}$$

$$\theta = \frac{Tl}{JG} = \frac{32Tl}{\pi d^4 G} = \frac{32(2485)(24)}{\pi\left(0.75^4\right)(11.5)\left(10^6\right)} = 0.167 \text{ rad} = 9.57^\circ \quad \textit{Ans.}$$

3-73

(a) $T_{\text{solid}} = \dfrac{J\tau_{\max}}{r} = \dfrac{\pi d_o^4 \tau_{\max}}{16 d_o}$ $\qquad T_{\text{hollow}} = \dfrac{J\tau_{\max}}{r} = \dfrac{\pi(d_o^4 - d_i^4)\tau_{\max}}{16 d_o}$

$$\%\Delta T = \frac{T_{\text{solid}} - T_{\text{hollow}}}{T_{\text{solid}}}(100\%) = \frac{d_i^4}{d_o^4}(100\%) = \frac{\left(36^4\right)}{\left(40^4\right)}(100\%) = 65.6\% \quad \textit{Ans.}$$

(b) $W_{\text{solid}} = k d_o^2, \qquad W_{\text{hollow}} = k\left(d_o^{\,2} - d_i^{\,2}\right)$

$$\%\Delta W = \frac{W_{\text{solid}} - W_{\text{hollow}}}{W_{\text{solid}}}(100\%) = \frac{d_i^2}{d_o^2}(100\%) = \frac{\left(36^2\right)}{\left(40^2\right)}(100\%) = 81.0\% \quad \textit{Ans.}$$

3-74

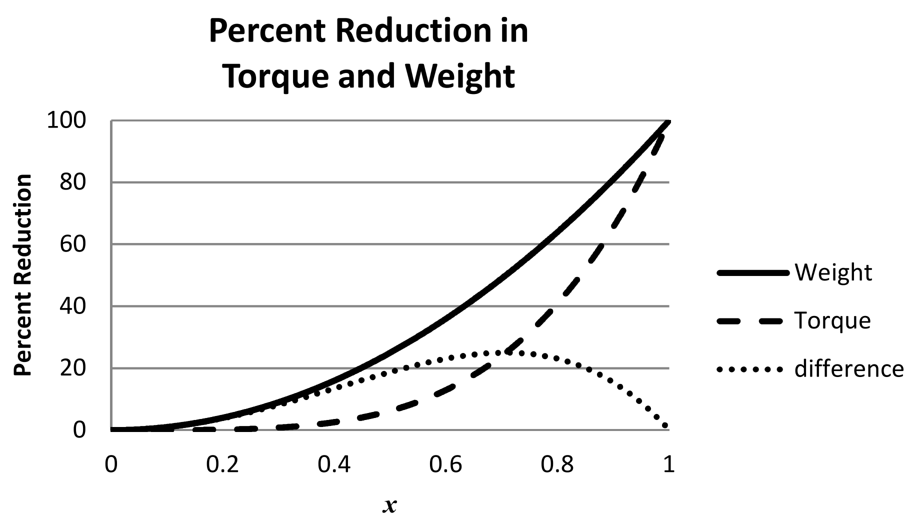
(a) $T_{\text{solid}} = \dfrac{J\tau_{\max}}{r} = \dfrac{\pi d^4 \tau_{\max}}{16 d}$ $\qquad T_{\text{hollow}} = \dfrac{J\tau_{\max}}{r} = \dfrac{\pi\left[d^4 - (xd)^4\right]\tau_{\max}}{16 d}$

$$\%\Delta T = \frac{T_{\text{solid}} - T_{\text{hollow}}}{T_{\text{solid}}}(100\%) = \frac{(xd)^4}{d^4}(100\%) = x^4(100\%) \quad \textit{Ans.}$$

(b) $W_{\text{solid}} = kd^2 \qquad W_{\text{hollow}} = k\left(d^2 - (xd)^2\right)$

$$\%\Delta W = \frac{W_{\text{solid}} - W_{\text{hollow}}}{W_{\text{solid}}}(100\%) = \frac{(xd)^2}{d^2}(100\%) = x^2(100\%) \qquad Ans.$$

Plot $\%\Delta T$ and $\%\Delta W$ versus x.



The value of greatest difference in percent reduction of weight and torque is 25% and occurs at $x = \sqrt{2}/2$.

3-75

(a) $\tau = \dfrac{Tc}{J} \quad \Rightarrow \quad 120\left(10^6\right) = \dfrac{4200(d/2)}{(\pi/32)\left[d^4 - (0.70d)^4\right]} = \dfrac{2.8149\left(10^4\right)}{d^3}$

$$d = \left(\frac{2.8149\left(10^4\right)}{120(10^6)}\right)^{1/3} = 6.17\left(10^{-2}\right)\text{ m} = 61.7\text{ mm}$$

From Table A-17, the next preferred size is d = 80 mm. *Ans.*

$d_i = 0.7d$ = 56 mm. The next preferred size smaller is d_i = 50 mm *Ans.*

(b)

$$\tau = \frac{Tc}{J} = \frac{4200(d_i/2)}{(\pi/32)\left[d^4 - (d_i)^4\right]} = \frac{4200(0.050/2)}{(\pi/32)\left[(0.080)^4 - (0.050)^4\right]} = 30.8\text{ MPa} \qquad Ans.$$

3-76

$$T = 9.55\frac{H}{n} = 9.55\frac{(1500)}{10} = 1433 \text{ N}\cdot\text{m}$$

$$\tau = \frac{16T}{\pi d_C^3} \quad \Rightarrow \quad d_C = \left(\frac{16T}{\pi\tau}\right)^{1/3} = \left[\frac{16(1433)}{\pi(80)(10^6)}\right]^{1/3} = 0.045 \text{ m} = 45 \text{ mm}$$

From Table A-17, select 50 mm. *Ans.*

(a) $\tau_{\text{start}} = \dfrac{16(2)(1433)}{\pi(0.050^3)} = 117(10^6) \text{ Pa} = 117 \text{ MPa}$ *Ans.*

(b) Design activity

3-77

$$T = \frac{63\ 025H}{n} = \frac{63\ 025(1)}{8} = 7880 \text{ lbf}\cdot\text{in}$$

$$\tau = \frac{16T}{\pi d_C^3} \quad \Rightarrow \quad d_C = \left(\frac{16T}{\pi\tau}\right)^{1/3} = \left[\frac{16(7880)}{\pi(15\ 000)}\right]^{1/3} = 1.39 \text{ in}$$

From Table A-17, select 1.40 in. *Ans.*

3-78 For a square cross section with side length b, and a circular section with diameter d,

$$A_{\text{square}} = A_{\text{circular}} \quad \Rightarrow \quad b^2 = \frac{\pi}{4}d^2 \quad \Rightarrow \quad b = \frac{\sqrt{\pi}}{2}d$$

From Eq. (3-40) with $b = c$,

$$(\tau_{\max})_{\text{square}} = \frac{T}{bc^2}\left(3 + \frac{1.8}{b/c}\right) = \frac{T}{b^3}\left(3 + \frac{1.8}{1}\right) = \frac{T}{d^3}\left(\frac{2}{\sqrt{\pi}}\right)^3 (4.8) = 6.896\frac{T}{d^3}$$

For the circular cross section,

$$(\tau_{\max})_{\text{circular}} = \frac{16T}{\pi d^3} = 5.093\frac{T}{d^3}$$

$$\frac{(\tau_{\max})_{\text{square}}}{(\tau_{\max})_{\text{circular}}} = \frac{6.896\dfrac{T}{d^3}}{5.093\dfrac{T}{d^3}} = 1.354$$

The shear stress in the square cross section is 35.4% greater. *Ans.*

(b) For the square cross section, from the table for Eq. (3-41), $\beta = 0.141$. From Eq. (3-41),

$$\theta_{\text{square}} = \frac{Tl}{\beta bc^3 G} = \frac{Tl}{\beta b^4 G} = \frac{Tl}{0.141\left(\dfrac{\sqrt{\pi}}{2}d\right)^4 G} = 11.50\frac{Tl}{d^4 G}$$

For the circular cross section,

$$\theta_{rd} = \frac{Tl}{GJ} = \frac{Tl}{G\left(\pi d^4/32\right)} = 10.19\frac{Tl}{d^4G}$$

$$\frac{\theta_{sq}}{\theta_{rd}} = \frac{11.50\dfrac{Tl}{d^4G}}{10.19\dfrac{Tl}{d^4G}} = 1.129$$

The angle of twist in the square cross section is 12.9% greater. *Ans.*

3-79 **(a)**

$$T_1 = 0.15T_2$$

$$\sum T = 0 = (500-75)(4) - (T_2 - T_1)(5) = 1700 - (T_2 - 0.15T_2)(5)$$

$$1700 - 4.25T_2 = 0 \qquad \Rightarrow \qquad T_2 = 400 \text{ lbf} \qquad \textit{Ans.}$$

$$T_1 = 0.15(400) = 60 \text{ lbf} \qquad \textit{Ans.}$$

(b)

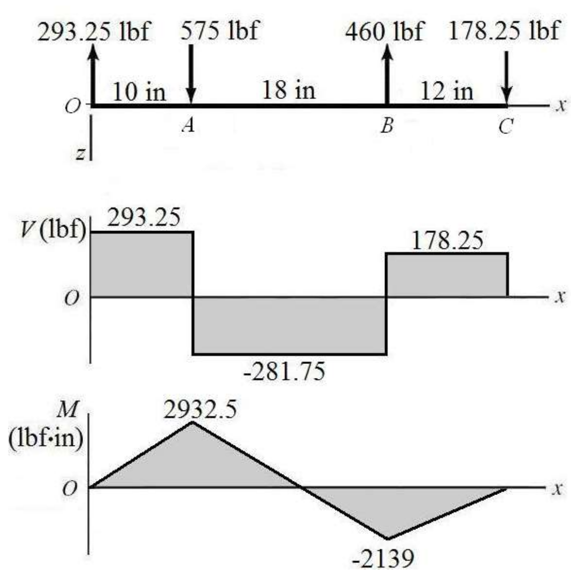
$$\sum M_O = 0 = -575(10) + 460(28) - R_C(40)$$

$$R_C = 178.25 \text{ lbf} \qquad \textit{Ans.}$$

$$\sum F = 0 = R_O + 575 - 460 + 178.25$$

$$R_O = -293.25 \text{ lbf} \qquad \textit{Ans.}$$

(c)



(d) The maximum bending moment is at $x = 10$ in, and is $M = 2932.5$ lbf·in. Since the shaft rotates, each stress element will experience both positive and negative bending stress as it moves from tension to compression. The torque transmitted through the shaft from A to B is $T = (500 - 75)(4) = 1700$ lbf·in. For a stress element on the outer surface where the bending stress and the torsional stress are both maximum,

$$\sigma = \frac{Mc}{I} = \frac{32M}{\pi d^3} = \frac{32(2932.5)}{\pi(1.25)^3} = 15\,294 \text{ psi} = 15.3 \text{ kpsi} \quad Ans.$$

$$\tau = \frac{Tr}{J} = \frac{16T}{\pi d^3} = \frac{16(1700)}{\pi(1.25)^3} = 4433 \text{ psi} = 4.43 \text{ kpsi} \quad Ans.$$

(e)

$$\sigma_1,\ \sigma_2 = \frac{\sigma_x}{2} \pm \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \left(\tau_{xy}\right)^2} = \frac{15.3}{2} \pm \sqrt{\left(\frac{15.3}{2}\right)^2 + \left(4.43\right)^2}$$

$\sigma_1 = 16.5$ kpsi *Ans.*

$\sigma_2 = -1.19$ kpsi *Ans.*

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \left(\tau_{xy}\right)^2} = \sqrt{\left(\frac{15.3}{2}\right)^2 + \left(4.43\right)^2} = 8.84 \text{ kpsi} \quad Ans.$$

3-80 **(a)**

$T_2 = 0.15T_1$

$$\sum T = 0 = (1800 - 270)(200) + (T_2 - T_1)(125) = 306(10^3) + 125(0.15T_1 - T_1)$$

$$306(10^3) - 106.25T_1 = 0 \quad \Rightarrow \quad T_1 = 2880 \text{ N} \quad Ans.$$

$T_2 = 0.15(2880) = 432$ N *Ans.*

(b)

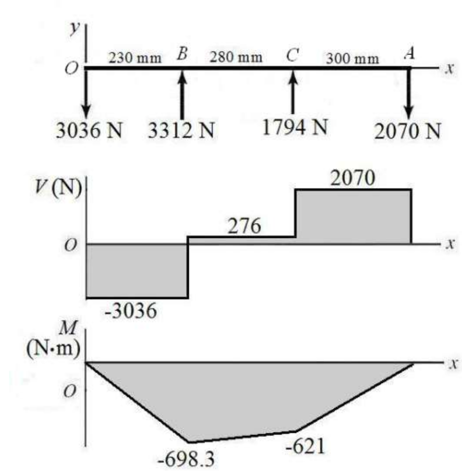
$$\sum M_O = 0 = 3312(230) + R_C(510) - 2070(810)$$

$R_C = 1794$ N *Ans.*

$$\sum F_y = 0 = R_O + 3312 + 1794 - 2070$$

$R_O = -3036$ N *Ans.*

(c)



(d) The maximum bending moment is at $x = 230$ mm, and is $M = -698.3$ N·m. Since the shaft rotates, each stress element will experience both positive and negative bending stress as it moves from tension to compression. The torque transmitted through the shaft from A to B is $T = (1800 - 270)(0.200) = 306$ N·m. For a stress element on the outer surface where the bending stress and the torsional stress are both maximum,

$$\sigma = \frac{Mc}{I} = \frac{32M}{\pi d^3} = \frac{32(698.3)}{\pi(0.030)^3} = 263\left(10^3\right)\text{ Pa} = 263\text{ MPa} \quad Ans.$$

$$\tau = \frac{Tr}{J} = \frac{16T}{\pi d^3} = \frac{16(306)}{\pi(0.030)^3} = 57.7\left(10^6\right)\text{ Pa} = 57.7\text{MPa} \quad Ans.$$

(e)

$$\sigma_1,\ \sigma_2 = \frac{\sigma_x}{2} \pm \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \left(\tau_{xy}\right)^2} = \frac{263}{2} \pm \sqrt{\left(\frac{263}{2}\right)^2 + \left(57.7\right)^2}$$

$\sigma_1 = 275$ MPa $\quad$ *Ans.*

$\sigma_2 = -12.1$ MPa $\quad$ *Ans.*

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \left(\tau_{xy}\right)^2} = \sqrt{\left(\frac{263}{2}\right)^2 + \left(57.7\right)^2} = 144\text{ MPa} \quad Ans.$$

3-81

(a)

$T_2 = 0.15T_1$

$\sum T = 0 = (300-50)(4) + (T_2 - T_1)(3) = 1000 + (0.15T_1 - T_1)(3)$

$1000 - 2.55T_1 = 0 \quad \Rightarrow \quad T_1 = 392.16$ lbf *Ans.*

$T_2 = 0.15(392.16) = 58.82$ lbf *Ans.*

(b)

$\sum M_{Oy} = 0 = -450.98(16) - R_{Cz}(22)$

$R_{Cz} = -327.99$ lbf *Ans.*

$\sum F_z = 0 = R_{Oz} + 450.98 - 327.99$

$R_{Oz} = -122.99$ lbf *Ans.*

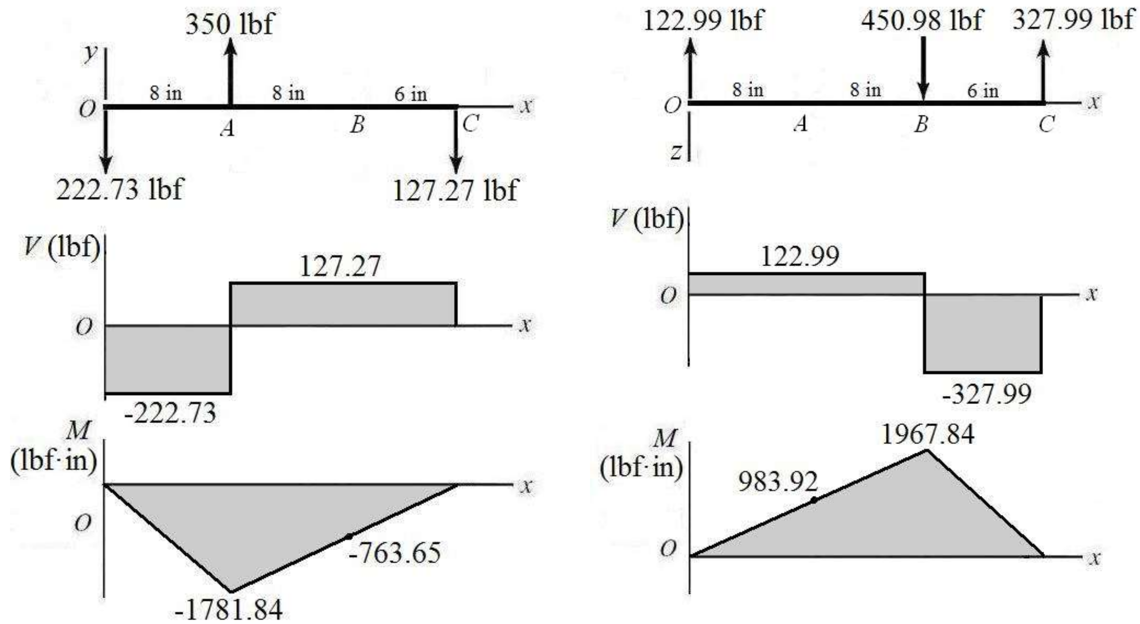
$\sum M_{Oz} = 0 = 350(8) + R_{Cy}(22)$

$R_{Cy} = -127.27$ lbf *Ans.*

$\sum F_y = 0 = R_{Oy} + 350 - 127.27$

$R_{Oy} = -222.73$ lbf *Ans.*

(c)



(d) Combine the bending moments from both planes at *A* and *B* to find the critical location.

$$M_A = \sqrt{(983.92)^2 + (-1781.84)^2} = 2035 \text{ lbf} \cdot \text{in}$$

$$M_B = \sqrt{(1967.84)^2 + (-763.65)^2} = 2111 \text{ lbf} \cdot \text{in}$$

The critical location is at *B*. The torque transmitted through the shaft from *A* to *B* is *T* = (300 − 50)(4) = 1000 lbf·in. For a stress element on the outer surface where the bending stress and the torsional stress are both maximum,

$$\sigma = \frac{Mc}{I} = \frac{32M}{\pi d^3} = \frac{32(2111)}{\pi(1)^3} = 21\,502 \text{ psi} = 21.5 \text{ kpsi} \quad \textit{Ans.}$$

$$\tau = \frac{Tr}{J} = \frac{16T}{\pi d^3} = \frac{16(1000)}{\pi(1)^3} = 5093 \text{ psi} = 5.09 \text{ kpsi} \qquad \textit{Ans.}$$

(e)

$$\sigma_1,\ \sigma_2 = \frac{\sigma_x}{2} \pm \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \left(\tau_{xy}\right)^2} = \frac{21.5}{2} \pm \sqrt{\left(\frac{21.5}{2}\right)^2 + \left(5.09\right)^2}$$

$$\sigma_1 = 22.6 \text{ kpsi} \qquad \textit{Ans.}$$

$$\sigma_2 = -1.14 \text{ kpsi} \qquad \textit{Ans.}$$

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \left(\tau_{xy}\right)^2} = \sqrt{\left(\frac{21.5}{2}\right)^2 + \left(5.09\right)^2} = 11.9 \text{ kpsi} \qquad \textit{Ans.}$$

3-82 **(a)**

$$T_2 = 0.15T_1$$

$$\sum T = 0 = (300-45)(125) + (T_2 - T_1)(150) = 31\,875 + (0.15T_1 - T_1)(150)$$

$$31\,875 - 127.5T_1 = 0 \qquad \Rightarrow \qquad T_1 = 250 \text{ N}\cdot\text{mm} \quad Ans.$$

$$T_2 = 0.15(250) = 37.5 \text{ N}\cdot\text{mm} \quad Ans.$$

(b)

$$\sum M_{Oy} = 0 = 345\sin 45^\circ(300) - 287.5(700) - R_{Cz}(850)$$

$$R_{Cz} = -150.7 \text{ N} \qquad Ans.$$

$$\sum F_z = 0 = R_{Oz} - 345\cos 45^\circ + 287.5 - 150.7$$

$$R_{Oz} = 107.2 \text{ N} \qquad Ans.$$

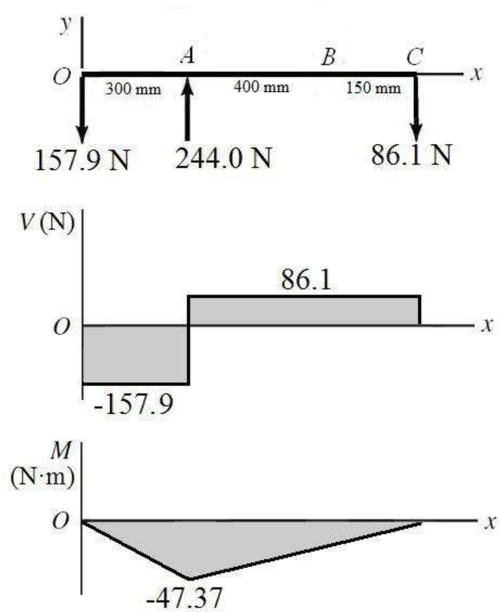
$$\sum M_{Oz} = 0 = 345\sin 45^\circ(300) + R_{Cy}(850)$$

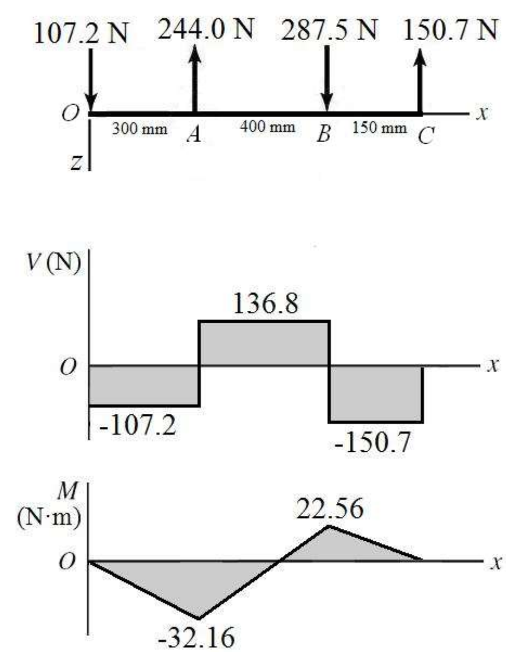
$$R_{Cy} = -86.10 \text{ N} \qquad Ans.$$

$$\sum F_y = 0 = R_{Oy} + 345\cos 45^\circ - 86.10$$

$$R_{Oy} = -157.9 \text{ N} \qquad Ans.$$

(c)





(d) From the bending moment diagrams, it is clear that the critical location is at *A* where both planes have the maximum bending moment. Combining the bending moments from the two planes,

$$M=\sqrt{(-47.37)^2+(-32.16)^2}=57.26\text{ N}\cdot\text{m}$$

The torque transmitted through the shaft from *A* to *B* is *T* = (300 − 45)(0.125) = 31.88 N·m. For a stress element on the outer surface where the bending stress and the torsional stress are both maximum,

$$\sigma=\frac{Mc}{I}=\frac{32M}{\pi d^3}=\frac{32(57.26)}{\pi(0.020)^3}=72.9\left(10^6\right)\text{ Pa}=72.9\text{ MPa}\qquad \textit{Ans.}$$

$$\tau=\frac{Tr}{J}=\frac{16T}{\pi d^3}=\frac{16(31.88)}{\pi(0.020)^3}=20.3\left(10^6\right)\text{ Pa}=20.3\text{ MPa}\qquad \textit{Ans.}$$

(e)

$$\sigma_1,\ \sigma_2=\frac{\sigma_x}{2}\pm\sqrt{\left(\frac{\sigma_x}{2}\right)^2+\left(\tau_{xy}\right)^2}=\frac{72.9}{2}\pm\sqrt{\left(\frac{72.9}{2}\right)^2+\left(20.3\right)^2}$$

$\sigma_1=78.2$ MPa *Ans.*

$\sigma_2=-5.27$ MPa *Ans.*

$$\tau_{\max}=\sqrt{\left(\frac{\sigma_x}{2}\right)^2+\left(\tau_{xy}\right)^2}=\sqrt{\left(\frac{72.9}{2}\right)^2+\left(20.3\right)^2}=41.7\text{ MPa}\qquad \textit{Ans.}$$

3-83

(a)

$$\sum T=0=-300(\cos 20^\circ)(10)+F_B(\cos 20^\circ)(4)$$

$F_B=750$ lbf *Ans.*

(b)

$$\sum M_{Oz}=0=300(\cos 20^\circ)(16)-750(\sin 20^\circ)(39)+R_{Cy}(30)$$

$R_{Cy}=183.1$ lbf *Ans.*

$$\sum F_y=0=R_{Oy}+300(\cos 20^\circ)+183.1-750(\sin 20^\circ)$$

$R_{Oy}=-208.5$ lbf *Ans.*

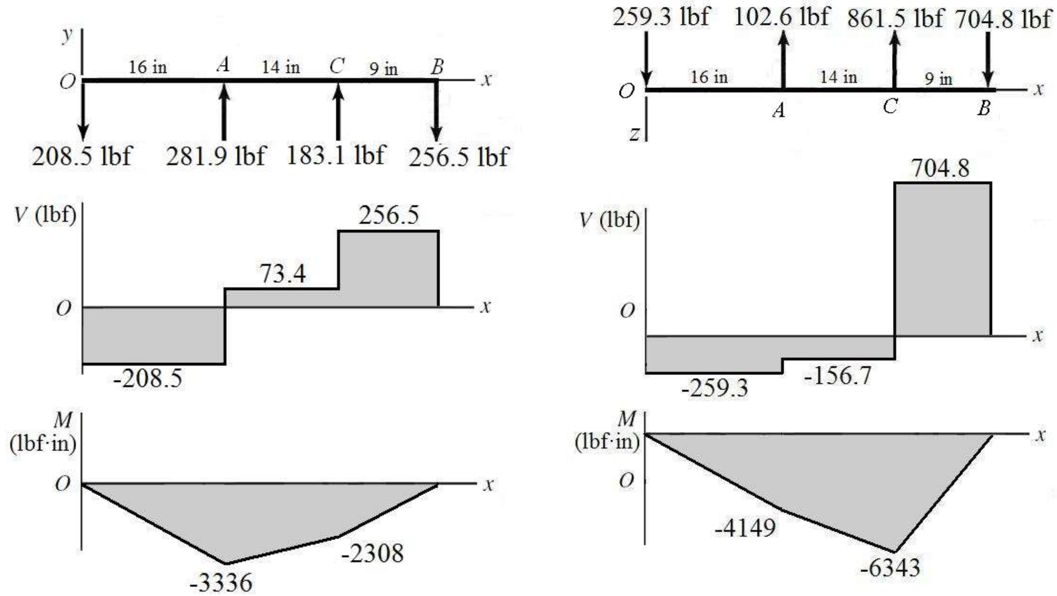
$$\sum M_{Oy}=0=300(\sin 20^\circ)(16)-R_{Cz}(30)-750(\cos 20^\circ)(39)$$

$R_{Cz}=-861.5$ lbf *Ans.*

$$\sum F_z=0=R_{Oz}-300(\sin 20^\circ)-861.5+750(\cos 20^\circ)$$

$R_{Oz}=259.3$ lbf *Ans.*

(c)



(d) Combine the bending moments from both planes at A and C to find the critical location.

$$M_A = \sqrt{(-3336)^2 + (-4149)^2} = 5324 \text{ lbf} \cdot \text{in}$$

$$M_C = \sqrt{(-2308)^2 + (-6343)^2} = 6750 \text{ lbf} \cdot \text{in}$$

The critical location is at C. The torque transmitted through the shaft from A to B is $T = 300\cos(20^\circ)(10) = 2819 \text{ lbf} \cdot \text{in}$. For a stress element on the outer surface where the bending stress and the torsional stress are both maximum,

$$\sigma = \frac{Mc}{I} = \frac{32M}{\pi d^3} = \frac{32(6750)}{\pi(1.25)^3} = 35\ 203 \text{ psi} = 35.2 \text{ kpsi} \qquad Ans.$$

$$\tau = \frac{Tr}{J} = \frac{16T}{\pi d^3} = \frac{16(2819)}{\pi(1.25)^3} = 7351 \text{ psi} = 7.35 \text{ kpsi} \qquad Ans.$$

(e)

$$\sigma_1,\ \sigma_2 = \frac{\sigma_x}{2} \pm \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \left(\tau_{xy}\right)^2} = \frac{35.2}{2} \pm \sqrt{\left(\frac{35.2}{2}\right)^2 + (7.35)^2}$$

$$\sigma_1 = 36.7 \text{ kpsi} \qquad Ans.$$

$$\sigma_2 = -1.47 \text{ kpsi} \qquad Ans.$$

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \left(\tau_{xy}\right)^2} = \sqrt{\left(\frac{35.2}{2}\right)^2 + (7.35)^2} = 19.1 \text{ kpsi} \qquad Ans.$$

3-84

(a)

$$\sum T = 0 = -11\,000(\cos 20^\circ)(300) + F_B(\cos 25^\circ)(150)$$

$$F_B = 22\,810 \text{ N} \qquad Ans.$$

(b)

$$\sum M_{Oz} = 0 = -11\,000(\sin 20^\circ)(400) - 22\,810(\sin 25^\circ)(750) + R_{Cy}(1050)$$

$$R_{Cy} = 8319 \text{ N} \qquad Ans.$$

$$\sum F_y = 0 = R_{Oy} - 11000(\sin 20^\circ) - 22\,810\sin(25^\circ) + 8319$$

$$R_{Oy} = 5083 \text{ N} \qquad Ans.$$

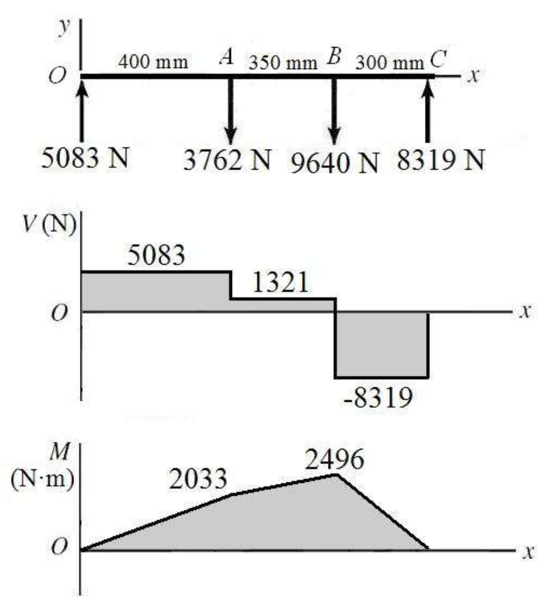
$$\sum M_{Oy} = 0 = 11\,000(\cos 20^\circ)(400) - 22\,810(\cos 25^\circ)(750) - R_{Cz}(1050)$$

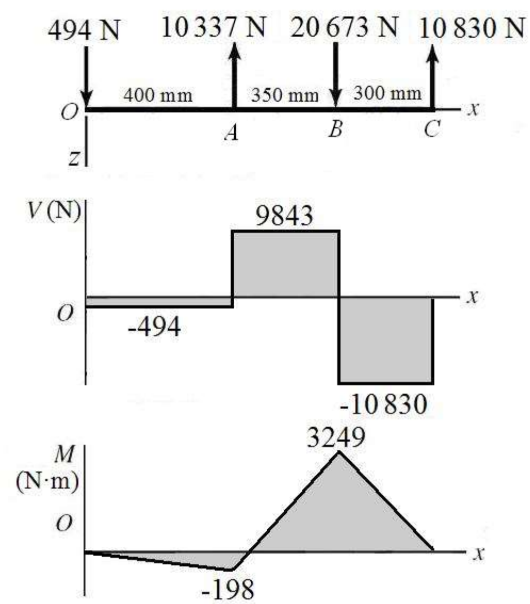
$$R_{Cz} = -10\,830 \text{ N} \quad Ans.$$

$$\sum F_z = 0 = R_{Oz} - 11\,000(\cos 20^\circ) + 22\,810(\cos 25^\circ) - 10\,830$$

$$R_{Oz} = 494 \text{ N} \quad Ans.$$

(c)





(d) From the bending moment diagrams, it is clear that the critical location is at *B* where both planes have the maximum bending moment. Combining the bending moments from the two planes,

$$M = \sqrt{(2496)^2 + (3249)^2} = 4097 \text{ N}\cdot\text{m}$$

The torque transmitted through the shaft from *A* to *B* is

$$T = 11\,000\cos(20^\circ)(0.3) = 3101 \text{ N}\cdot\text{m}.$$

For a stress element on the outer surface where the bending stress and the torsional stress are both maximum,

$$\sigma = \frac{Mc}{I} = \frac{32M}{\pi d^3} = \frac{32(4097)}{\pi(0.050)^3} = 333.9(10^6)\ \text{Pa} = 333.9\ \text{MPa} \qquad Ans.$$

$$\tau = \frac{Tr}{J} = \frac{16T}{\pi d^3} = \frac{16(3101)}{\pi(0.050)^3} = 126.3(10^6)\ \text{Pa} = 126.3\ \text{MPa} \qquad Ans.$$

(e)

$$\sigma_1,\ \sigma_2 = \frac{\sigma_x}{2} \pm \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + (\tau_{xy})^2} = \frac{333.9}{2} \pm \sqrt{\left(\frac{333.9}{2}\right)^2 + (126.3)^2}$$

$\sigma_1 = 376$ MPa *Ans.*

$\sigma_2 = -42.4$ MPa *Ans.*

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + (\tau_{xy})^2} = \sqrt{\left(\frac{333.9}{2}\right)^2 + (126.3)^2} = 209\ \text{MPa} \qquad Ans.$$

3-85

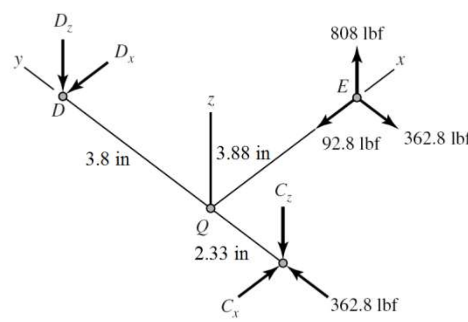
(a)

$$(\Sigma M_D)_z = 6.13C_x - 3.8(92.8) - 3.88(362.8) = 0$$

$C_x = 287.2$ lbf *Ans.*

$$(\Sigma M_C)_z = 6.13D_x + 2.33(92.8) - 3.88(362.8) = 0$$

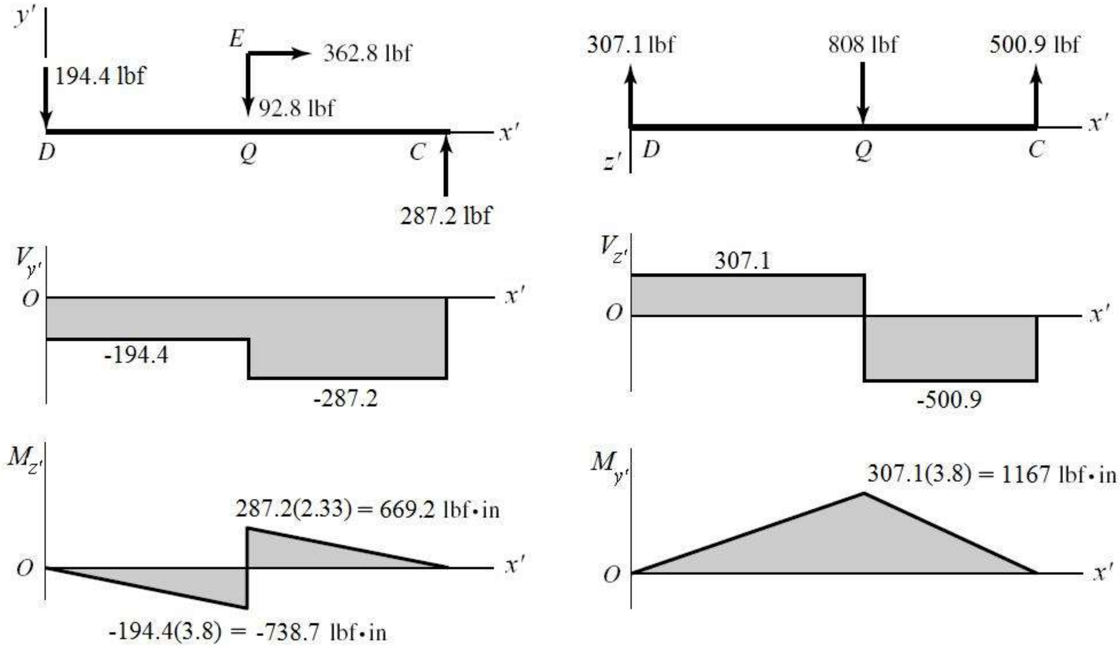
$D_x = 194.4$ lbf *Ans.*



$$(\Sigma M_D)_x = 0 \quad \Rightarrow \quad C_z = \frac{3.8}{6.13}(808) = 500.9\ \text{lbf} \qquad Ans.$$

$$(\Sigma M_C)_x = 0 \quad \Rightarrow \quad D_z = \frac{2.33}{6.13}(808) = 307.1\ \text{lbf} \qquad Ans.$$

(b) For *DQC*, let x', y', z' correspond to the original $-y$, x, z axes.



(c) The critical stress element is just to the right of *Q*, where the bending moment in both planes is a maximum, and where the torsional and axial loads exist.

$$T = 808(3.88) = 3135 \text{ lbf} \cdot \text{in}$$

$$M = \sqrt{669.2^2 + 1167^2} = 1345 \text{ lbf} \cdot \text{in}$$

$$\tau = \frac{16T}{\pi d^3} = \frac{16(3135)}{\pi\left(1.13^3\right)} = 11\,070 \text{ psi} \quad \textit{Ans.}$$

$$\sigma_b = \pm\frac{32M}{\pi d^3} = \pm\frac{32(1345)}{\pi\left(1.13^3\right)} = \pm 9495 \text{ psi} \quad \textit{Ans.}$$

$$\sigma_a = -\frac{F}{A} = -\frac{362.8}{(\pi/4)\left(1.13^2\right)} = -362 \text{ psi} \quad \textit{Ans.}$$

(d) The critical stress element will be where the bending stress and axial stress are both in compression.

$$\sigma_{\max} = -9495 - 362 = -9857 \text{ psi}$$

$$\tau_{\max} = \sqrt{\left(\frac{-9857}{2}\right)^2 + 11\,070^2} = 12\,118 \text{ psi} = 12.1 \text{ kpsi} \quad \textit{Ans.}$$

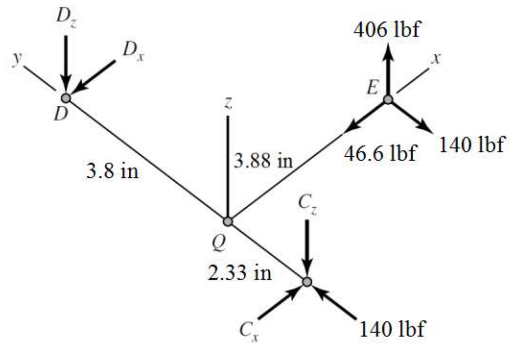
$$\sigma_1, \sigma_2 = \frac{-9857}{2} \pm \sqrt{\left(\frac{-9857}{2}\right)^2 + 11\,070^2}$$

$$\sigma_1 = 7189 \text{ psi} = 7.19 \text{ kpsi} \quad \textit{Ans.}$$

$$\sigma_2 = -17\,046 \text{ psi} = -17.0 \text{ kpsi} \quad \textit{Ans.}$$

3-86

(a)



$$\left(\Sigma M_D\right)_z = 0$$

$$6.13C_x - 3.8(46.6) - 3.88(140) = 0$$

$$C_x = 117.5 \text{ lbf} \quad \textit{Ans.}$$

$$\left(\Sigma M_C\right)_z = 0$$

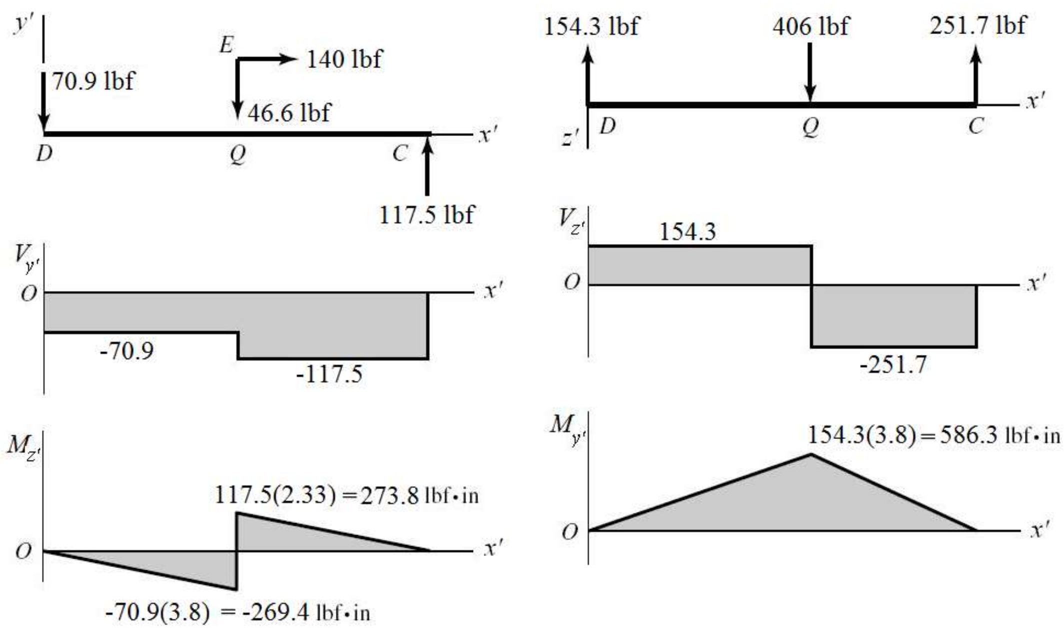
$$-6.13D_x - 2.33(46.6) + 3.88(140) = 0$$

$$D_x = 70.9 \text{ lbf} \quad \textit{Ans.}$$

$$\left(\Sigma M_D\right)_x = 0 \quad \Rightarrow \quad C_z = \frac{3.8}{6.13}(406) = 251.7 \text{ lbf} \quad \textit{Ans.}$$

$$\left(\Sigma M_C\right)_x = 0 \quad \Rightarrow \quad D_z = \frac{2.33}{6.13}(406) = 154.3 \text{ lbf} \quad \textit{Ans.}$$

(b) For *DQC*, let x', y', z' correspond to the original $-y$, x, z axes.



(c) The critical stress element is just to the right of *Q*, where the bending moment in both planes is a maximum, and where the torsional and axial loads exist.

$$T = 406(3.88) = 1575 \text{ lbf}\cdot\text{in}$$

$$M = \sqrt{273.8^2 + 586.3^2} = 647.1 \text{ lbf}\cdot\text{in}$$

$$\tau = \frac{16T}{\pi d^3} = \frac{16(1575)}{\pi\left(1^3\right)} = 8021 \text{ psi} \quad \textit{Ans.}$$

$$\sigma_b = \pm\frac{32M}{\pi d^3} = \pm\frac{32(647.1)}{\pi\left(1^3\right)} = \pm 6591 \text{ psi} \quad \textit{Ans.}$$

$$\sigma_a = -\frac{F}{A} = -\frac{140}{(\pi/4)\left(1^2\right)} = -178.3 \text{ psi} \quad \textit{Ans.}$$

(d) The critical stress element will be where the bending stress and axial stress are both in compression.

$$\sigma_{\max} = -6591 - 178.3 = -6769 \text{ psi}$$

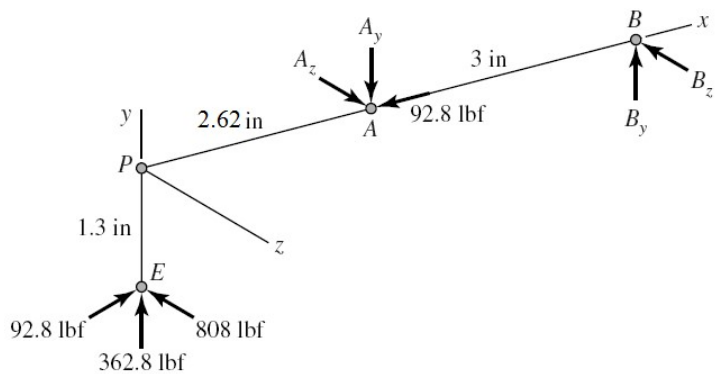
$$\tau_{\max} = \sqrt{\left(\frac{-6769}{2}\right)^2 + 8021^2} = 8706 \text{ psi} = 8.71 \text{ kpsi} \quad \textit{Ans.}$$

$$\sigma_1, \sigma_2 = \frac{-6769}{2} \pm \sqrt{\left(\frac{-6769}{2}\right)^2 + 8021^2}$$

$$\sigma_1 = 5321 \text{ psi} = 5.32 \text{ kpsi} \quad \textit{Ans.}$$

$$\sigma_2 = -12\,090 \text{ psi} = -12.1 \text{ kpsi} \quad \textit{Ans.}$$

3-87



$$\left(\Sigma M_B\right)_z = -5.62(362.8) + 1.3(92.8) + 3A_y = 0$$

$$A_y = 639.4 \text{ lbf} \quad \textit{Ans.}$$

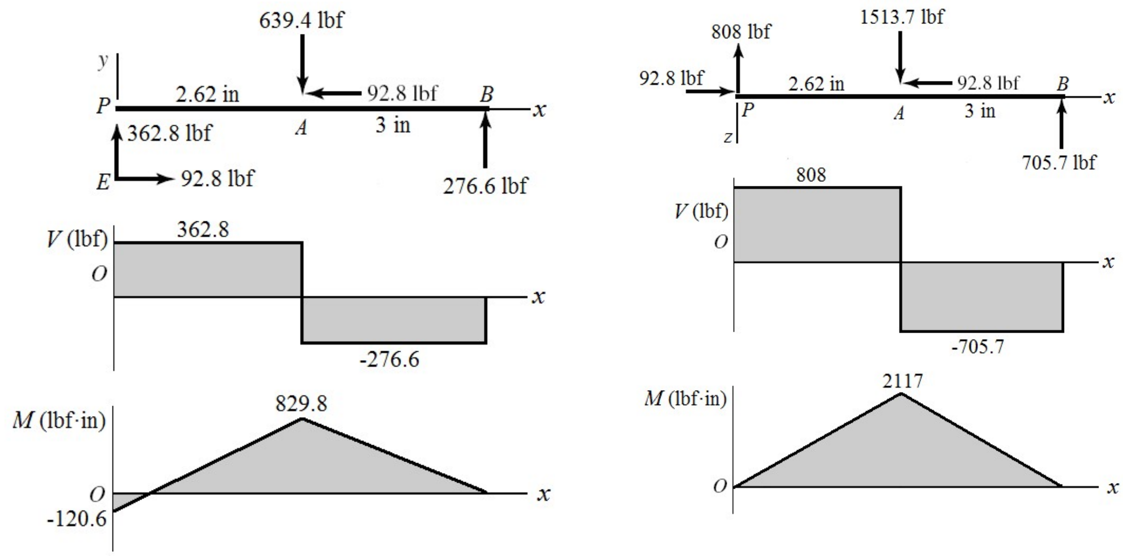
$$\left(\Sigma M_A\right)_z = -2.62(362.8) + 1.3(92.8) + 3B_y = 0$$

$$B_y = 276.6 \text{ lbf} \quad \textit{Ans.}$$

$$\left(\Sigma M_B\right)_y = 0 \quad \Rightarrow \quad A_z = \frac{5.62}{3}(808) = 1513.7 \text{ lbf} \quad \textit{Ans.}$$

$$\left(\Sigma M_A\right)_y = 0 \quad \Rightarrow \quad B_z = \frac{2.62}{3}(808) = 705.7 \text{ lbf} \quad \textit{Ans.}$$

(b)



(c) The critical stress element is just to the left of *A*, where the bending moment in both planes is a maximum, and where the torsional and axial loads exist.

$$T = 808(1.3) = 1050 \text{ lbf}\cdot\text{in}$$

$$\tau = \frac{16(1050)}{\pi\left(0.88^3\right)} = 7847 \text{ psi} \quad Ans.$$

$$M = \sqrt{(829.8)^2 + (2117)^2} = 2274 \text{ lbf}\cdot\text{in}$$

$$\sigma_b = \pm\frac{32M}{\pi d^3} = \pm\frac{32(2274)}{\pi\left(0.88^3\right)} = \pm 33\,990 \text{ psi} \quad Ans.$$

$$\sigma_a = -\frac{F}{A} = -\frac{92.8}{(\pi/4)\left(0.88^2\right)} = -153 \text{ psi} \quad Ans.$$

(d) The critical stress will occur when the bending stress and axial stress are both in compression.

$$\sigma_{\max} = -33\,990 - 153 = -34\,143 \text{ psi}$$

$$\tau_{\max} = \sqrt{\left(\frac{-34\,143}{2}\right)^2 + 7847^2} = 18\,789 \text{ psi} = 18.8 \text{ kpsi} \quad Ans.$$

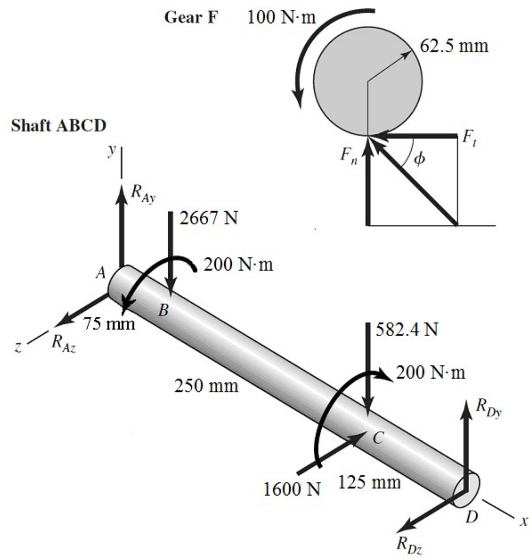
$$\sigma_1, \sigma_2 = \frac{-34143}{2} \pm \sqrt{\left(\frac{-34143}{2}\right)^2 + 7847^2}$$

$$\sigma_1 = 1717 \text{ psi} = 1.72 \text{ kpsi} \quad Ans.$$

$$\sigma_2 = -35\,860 \text{ psi} = -35.9 \text{ kpsi} \quad Ans.$$

3-88

$$F_t = \frac{T}{c/2} = \frac{100}{0.125/2} = 1600 \text{ N}$$



$$F_n = 1600 \tan 20 = 582.4 \text{ N}$$

$$T_C = F_t(b/2) = 1600(0.250/2) = 200 \text{ N}\cdot\text{m}$$

$$P = \frac{T_C}{(a/2)} = \frac{200}{(0.150/2)} = 2667 \text{ N}$$

$$\sum (M_A)_z = 0$$

$$450R_{Dy} - 582.4(325) - 2667(75) = 0$$

$$R_{Dy} = 865.1 \text{ N}$$

$$\sum (M_A)_y = 0 = -450R_{Dz} + 1600(325) \quad \Rightarrow \quad R_{Dz} = 1156 \text{ N}$$

$$\sum F_y = 0 = R_{Ay} + 865.1 - 582.4 - 2667 \quad \Rightarrow \quad R_{Ay} = 2384 \text{ N}$$

$$\sum F_z = 0 = R_{Az} + 1156 - 1600 \quad \Rightarrow \quad R_{Az} = 444 \text{ N}$$

AB The maximum bending moment will either be at *B* or *C*. If this is not obvious, sketch the shear and bending moment diagrams. We will directly obtain the combined moments from each plane.

$$M_B = \overline{AB}\sqrt{R_{A_y}^2 + R_{A_z}^2} = 0.075\sqrt{2384^2 + 444^2} = 181.9 \text{ N}\cdot\text{m}$$

$$M_C = \overline{CD}\sqrt{R_{D_y}^2 + R_{D_z}^2} = 0.125\sqrt{865.1^2 + 1156^2} = 180.5 \text{ N}\cdot\text{m}$$

The stresses at *B* and *C* are almost identical, but the maximum stresses occur at *B*. *Ans.*

$$\sigma_B = \frac{32M_B}{\pi d^3} = \frac{32(181.9)}{\pi(0.030^3)} = 68.6(10^6) \text{ Pa} = 68.6 \text{ MPa}$$

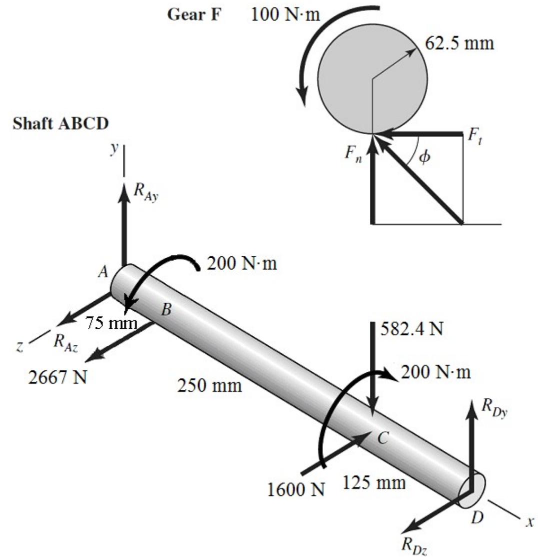
$$\tau_B = \frac{16T_B}{\pi d^3} = \frac{16(200)}{\pi(0.030^3)} = 37.7(10^6) \text{ Pa} = 37.7 \text{ MPa}$$

$$\sigma_{\max} = \frac{\sigma_B}{2} + \sqrt{\left(\frac{\sigma_B}{2}\right)^2 + \tau_B^2} = \frac{68.6}{2} + \sqrt{\left(\frac{68.6}{2}\right)^2 + 37.7^2} = 85.3 \text{ MPa} \quad \textit{Ans.}$$

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_B}{2}\right)^2 + \tau_B^2} = \sqrt{\left(\frac{68.6}{2}\right)^2 + 37.7^2} = 51.0 \text{ MPa} \quad \textit{Ans.}$$

3-89

$$F_t = \frac{T}{c/2} = \frac{100}{0.125/2} = 1600 \text{ N}$$



$$F_n = 1600 \tan 20 = 582.4 \text{ N}$$
$$T_C = F_t\left(b/2\right) = 1600\left(0.250/2\right) = 200 \text{ N}\cdot\text{m}$$
$$P = \frac{T_C}{\left(a/2\right)} = \frac{200}{\left(0.150/2\right)} = 2667 \text{ N}$$

$$\sum\left(M_A\right)_z = 0 = 450R_{Dy} - 582.4(325) \quad \Rightarrow \quad R_{Dy} = 420.6 \text{ N}$$
$$\sum\left(M_A\right)_y = 0 = -450R_{Dz} + 1600(325) - 2667(75) \quad \Rightarrow \quad R_{Dz} = 711.1 \text{ N}$$
$$\sum F_y = 0 = R_{Ay} + 420.6 - 582.4 \quad \Rightarrow \quad R_{Ay} = 161.8 \text{ N}$$
$$\sum F_z = 0 = R_{Az} + 711.1 - 1600 + 2667 \quad \Rightarrow \quad R_{Az} = -1778 \text{ N}$$

The maximum bending moment will either be at *B* or *C*. If this is not obvious, sketch shear and bending moment diagrams. We will directly obtain the combined moments from each plane.

$$M_B = \overline{AB}\sqrt{R_{A_y}^2 + R_{A_z}^2} = 0.075\sqrt{161.8^2 + \left(-1778\right)^2} = 133.9 \text{ N}\cdot\text{m}$$
$$M_C = \overline{CD}\sqrt{R_{D_y}^2 + R_{D_z}^2} = 0.125\sqrt{420.6^2 + 711.1^2} = 103.3 \text{ N}\cdot\text{m}$$

The maximum stresses occur at *B*. *Ans.*

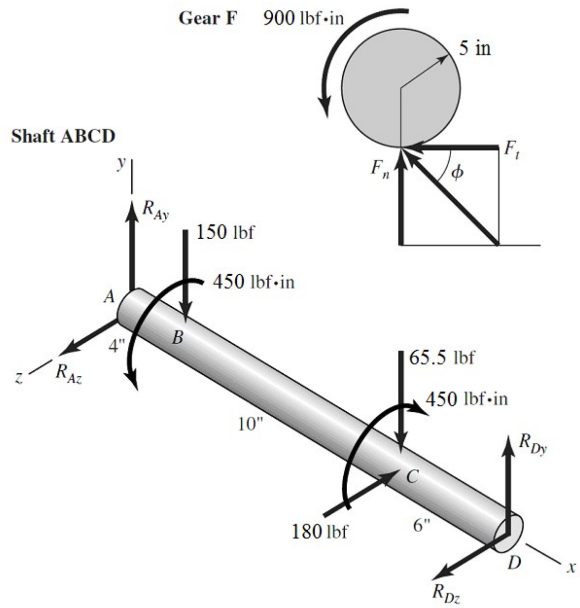
$$\sigma_B = \frac{32M_B}{\pi d^3} = \frac{32(133.9)}{\pi\left(0.030^3\right)} = 50.5\left(10^6\right) \text{ Pa} = 50.5 \text{ MPa}$$

$$\tau_B = \frac{16T_B}{\pi d^3} = \frac{16(200)}{\pi\left(0.030^3\right)} = 37.7\left(10^6\right) \text{ Pa} = 37.7 \text{ MPa}$$

$$\sigma_{max} = \frac{\sigma_B}{2} + \sqrt{\left(\frac{\sigma_B}{2}\right)^2 + \tau_B^2} = \frac{50.5}{2} + \sqrt{\left(\frac{50.5}{2}\right)^2 + 37.7^2} = 70.6 \text{ MPa} \quad \textit{Ans.}$$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_B}{2}\right)^2 + \tau_B^2} = \sqrt{\left(\frac{50.5}{2}\right)^2 + 37.7^2} = 45.4 \text{ MPa} \quad \textit{Ans.}$$

3-90



$$F_t = \frac{T}{c/2} = \frac{900}{10/2} = 180 \text{ lbf}$$

$$F_n = 180 \tan 20 = 65.5 \text{ lbf}$$

$$T_C = F_t\left(b/2\right) = 180\left(5/2\right) = 450 \text{ lbf}\cdot\text{in}$$

$$P = \frac{T_C}{\left(a/2\right)} = \frac{450}{\left(6/2\right)} = 150 \text{ lbf}$$

$$\sum\left(M_A\right)_z = 0 = 20R_{Dy} - 65.5(14) - 150(4) \quad \Rightarrow \quad R_{Dy} = 75.9 \text{ lbf}$$

$$\sum\left(M_A\right)_y = 0 = -20R_{Dz} + 180(14) \quad \Rightarrow \quad R_{Dz} = 126 \text{ lbf}$$

$$\sum F_y = 0 = R_{Ay} + 75.9 - 65.5 - 150 \quad \Rightarrow \quad R_{Ay} = 140 \text{ lbf}$$

$$\sum F_z = 0 = R_{Az} + 126 - 180 \quad \Rightarrow \quad R_{Az} = 54.0 \text{ lbf}$$

The maximum bending moment will either be at *B* or *C*. If this is not obvious, sketch shear and bending moment diagrams. We will directly obtain the combined moments from each plane.

$$M_B = \overline{AB}\sqrt{R_{A_y}^2 + R_{A_z}^2} = 4\sqrt{140^2 + 54^2} = 600 \text{ lbf}\cdot\text{in}$$

$$M_C = \overline{CD}\sqrt{R_{D_y}^2 + R_{D_z}^2} = 6\sqrt{75.9^2 + 126^2} = 883 \text{ lbf}\cdot\text{in}$$

The maximum stresses occur at *C*. *Ans.*

$$\sigma_C = \frac{32M_C}{\pi d^3} = \frac{32(883)}{\pi\left(1.375^3\right)} = 3460 \text{ psi}$$

$$\tau_C = \frac{16T_C}{\pi d^3} = \frac{16(450)}{\pi\left(1.375^3\right)} = 882 \text{ psi}$$

$$\sigma_{\max} = \frac{\sigma_C}{2} + \sqrt{\left(\frac{\sigma_C}{2}\right)^2 + \tau_C^2} = \frac{3460}{2} + \sqrt{\left(\frac{3460}{2}\right)^2 + 882^2} = 3670 \text{ psi} \quad \textit{Ans.}$$

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_C}{2}\right)^2 + \tau_C^2} = \sqrt{\left(\frac{3460}{2}\right)^2 + 882^2} = 1940 \text{ psi} \quad \textit{Ans.}$$

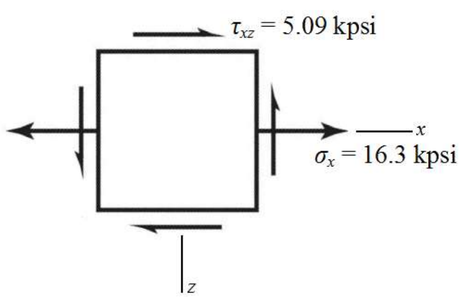
3-91

(a) Rod *AB* experiences constant torsion throughout its length, and maximum bending moment at the wall. Both torsional shear stress and bending stress will be a maximum on the outer surface. The transverse shear will be very small compared to bending and torsion, due to the reasonably high length to diameter ratio, so it will not dominate the determination of the critical location. The critical stress element will be at the wall, at either the top (compression) or the bottom (tension) on the *y* axis. We will select the bottom element for this analysis.

(b) Transverse shear is zero at the critical stress elements on the top and bottom surfaces.

$$\sigma_x = \frac{Mc}{I} = \frac{M(d/2)}{\pi d^4/64} = \frac{32M}{\pi d^3} = \frac{32(8)(200)}{\pi(1)^3} = 16\,297 \text{ psi} = 16.3 \text{ kpsi}$$

$$\tau_{xz} = \frac{Tr}{J} = \frac{T(d/2)}{\pi d^4/32} = \frac{16T}{\pi d^3} = \frac{16(5)(200)}{\pi(1)^3} = 5093 \text{ psi} = 5.09 \text{ kpsi}$$



(c)

$$\sigma_1,\ \sigma_2 = \frac{\sigma_x}{2} \pm \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + (\tau_{xz})^2} = \frac{16.3}{2} \pm \sqrt{\left(\frac{16.3}{2}\right)^2 + (5.09)^2}$$

$\sigma_1 = 17.8$ kpsi *Ans.*

$\sigma_2 = -1.46$ kpsi *Ans.*

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + (\tau_{xz})^2} = \sqrt{\left(\frac{16.3}{2}\right)^2 + (5.09)^2} = 9.61 \text{ kpsi} \quad \textit{Ans.}$$

3-92

(a) Rod *AB* experiences constant torsion throughout its length, and maximum bending moments at the wall in both planes of bending. Both torsional shear stress and bending stress will be a maximum on the outer surface. The transverse shear will be very small compared to bending and torsion, due to the reasonably high length to diameter ratio, so it will not dominate the determination of the critical location. The critical stress element will be on the outer surface at the wall, with its critical location determined by the plane of the combined bending moments.

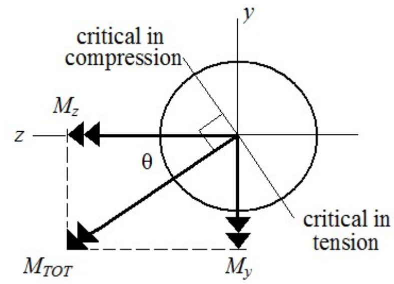
$M_y = -(100)(8) = -800$ lbf·in

$M_z = (175)(8) = 1400$ lbf·in

$$M_{\text{tot}} = \sqrt{M_y^2 + M_z^2}$$

$$= \sqrt{(-800)^2 + 1400^2} = 1612 \text{ lbf} \cdot \text{in}$$

$$\theta = \tan^{-1}\left(\left|\frac{M_y}{M_z}\right|\right) = \tan^{-1}\left(\frac{800}{1400}\right) = 29.7^\circ$$

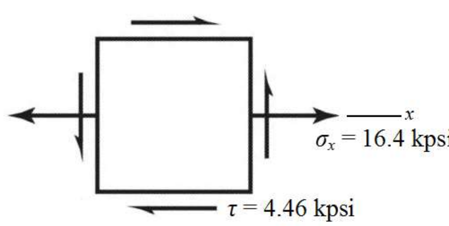


The combined bending moment vector is at an angle of 29.7º CCW from the *z* axis. The critical bending stress location, and thus the critical stress element, will be ±90º from this vector, as shown. There are two equally critical stress elements, one in tension (119.7º CCW from the *z* axis) and the other in compression (60.3º CW from the *z* axis). We'll continue the analysis with the element in tension.

(b) Transverse shear is zero at the critical stress elements on the outer surfaces.

$$\sigma_x = \frac{M_{\text{tot}}c}{I} = \frac{M_{\text{tot}}(d/2)}{\pi d^4/64} = \frac{32M_{\text{tot}}}{\pi d^3} = \frac{32(1612)}{\pi(1)^3} = 16\,420 \text{ psi} = 16.4 \text{ kpsi}$$

$$\tau = \frac{Tr}{J} = \frac{T(d/2)}{\pi d^4/32} = \frac{16T}{\pi d^3} = \frac{16(5)(175)}{\pi(1)^3} = 4456 \text{ psi} = 4.46 \text{ kpsi}$$



(c)

$$\sigma_1,\ \sigma_2 = \frac{\sigma_x}{2} \pm \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau^2} = \frac{16.4}{2} \pm \sqrt{\left(\frac{16.4}{2}\right)^2 + (4.46)^2}$$

$\sigma_1 = 17.5$ kpsi *Ans.*

$\sigma_2 = -1.13$ kpsi *Ans.*

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau^2} = \sqrt{\left(\frac{16.4}{2}\right)^2 + (4.46)^2} = 9.33 \text{ kpsi} \qquad \textit{Ans.}$$

3-93

(a) Rod *AB* experiences constant torsion and constant axial tension throughout its length, and maximum bending moments at the wall from both planes of bending. Both torsional shear stress and bending stress will be maximum on the outer surface. The transverse shear will be very small compared to bending and torsion, due to the reasonably high length to diameter ratio, so it will not dominate the determination of the critical location. The critical stress element will be on the outer surface at the wall, with its critical location determined by the plane of the combined bending moments.

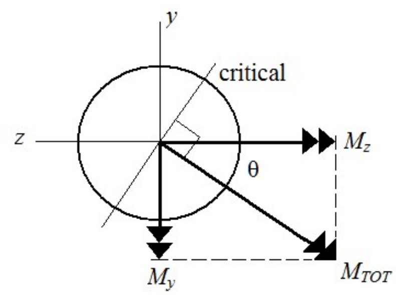
$M_y = -(100)(8) - (75)(5) = -1175$ lbf·in

$M_z = (-200)(8) = -1600$ lbf·in

$$M_{\text{tot}} = \sqrt{M_y^2 + M_z^2}$$

$$= \sqrt{(-1175)^2 + (-1600)^2} = 1985 \text{ lbf} \cdot \text{in}$$

$$\theta = \tan^{-1}\left(\left|\frac{M_y}{M_z}\right|\right) = \tan^{-1}\left(\frac{1175}{1600}\right) = 36.3^\circ$$



The combined bending moment vector is at an angle of 36.3º CW from the negative *z* axis. The critical bending stress location will be ±90º from this vector, as shown. Since there is an axial stress in tension, the critical stress element will be where the bending is also in tension. The critical stress element is therefore on the outer surface at the wall, at an angle of 36.3º CW from the *y* axis.

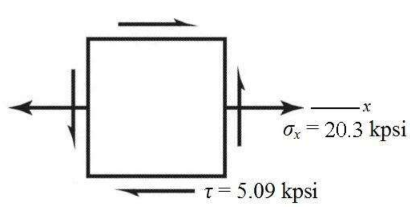
(b) Transverse shear is zero at the critical stress element on the outer surface.

$$\sigma_{x,\text{bend}} = \frac{M_{\text{tot}}c}{I} = \frac{M_{\text{tot}}(d/2)}{\pi d^4/64} = \frac{32M_{\text{tot}}}{\pi d^3} = \frac{32(1985)}{\pi(1)^3} = 20\,220 \text{ psi} = 20.2 \text{ kpsi}$$

$$\sigma_{x,\text{axial}} = \frac{F_x}{A} = \frac{F_x}{\pi d^2/4} = \frac{75}{\pi(1)^2/4} = 95.5 \text{ psi} = 0.1 \text{ kpsi}$$, which is essentially negligible

$$\sigma_x = \sigma_{x,\text{axial}} + \sigma_{x,\text{bend}} = 20\,220 + 95.5 = 20\,316 \text{ psi} = 20.3 \text{ kpsi}$$

$$\tau = \frac{Tr}{J} = \frac{16T}{\pi d^3} = \frac{16(5)(200)}{\pi(1)^3} = 5093 \text{ psi} = 5.09 \text{ kpsi}$$



(c)

$$\sigma_1,\ \sigma_2 = \frac{\sigma_x}{2} \pm \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau^2} = \frac{20.3}{2} \pm \sqrt{\left(\frac{20.3}{2}\right)^2 + (5.09)^2}$$

$\sigma_1 = 21.5$ kpsi *Ans.*

$\sigma_2 = -1.20$ kpsi *Ans.*

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau^2} = \sqrt{\left(\frac{20.3}{2}\right)^2 + (5.09)^2} = 11.4 \text{ kpsi} \qquad \textit{Ans.}$$

3-94

$T = (2)(200) = 400$ lbf·in

The maximum shear stress due to torsion occurs in the middle of the longest side of the rectangular cross section. From the table for Eq. (3-40), with $b/c = 1.5/0.25 = 6$, $\alpha = 0.299$. From Eq. (3-40),

$$\tau_{\max} = \frac{T}{\alpha b c^2} = \frac{400}{(0.299)(1.5)(0.25)^2} = 14\,270 \text{ psi} = 14.3 \text{ kpsi} \qquad \textit{Ans.}$$

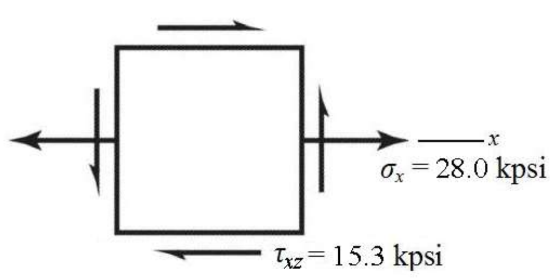
3-95

(a) The cross section at *A* will experience bending, torsion, and transverse shear. Both torsional shear stress and bending stress will be a maximum on the outer surface. The transverse shear will be very small compared to bending and torsion, due to the reasonably high length to diameter ratio, so it will not dominate the determination of the critical location. The critical stress element will be at either the top (compression) or the bottom (tension) on the *y* axis. We'll select the bottom element for this analysis.

(b) Transverse shear is zero at the critical stress elements on the top and bottom surfaces.

$$\sigma_x = \frac{Mc}{I} = \frac{M(d/2)}{\pi d^4/64} = \frac{32M}{\pi d^3} = \frac{32(11)(250)}{\pi (1)^3} = 28\,011 \text{ psi} = 28.0 \text{ kpsi}$$

$$\tau_{xz} = \frac{Tr}{J} = \frac{T(d/2)}{\pi d^4/32} = \frac{16T}{\pi d^3} = \frac{16(12)(250)}{\pi (1)^3} = 15\,279 \text{ psi} = 15.3 \text{ kpsi}$$



(c)

$$\sigma_1,\ \sigma_2 = \frac{\sigma_x}{2} \pm \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \left(\tau_{xz}\right)^2} = \frac{28.0}{2} \pm \sqrt{\left(\frac{28.0}{2}\right)^2 + \left(15.3\right)^2}$$

$\sigma_1 = 34.7$ kpsi *Ans.*

$\sigma_2 = -6.7$ kpsi *Ans.*

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \left(\tau_{xz}\right)^2} = \sqrt{\left(\frac{28.0}{2}\right)^2 + \left(15.3\right)^2} = 20.7 \text{ kpsi} \qquad \textit{Ans.}$$

3-96

(a) The cross section at *A* will experience bending, torsion, axial, and transverse shear. Both torsional shear stress and bending stress will be a maximum on the outer surface. The transverse shear will be very small compared to bending and torsion, due to the reasonably high length to diameter ratio, so it will not dominate the determination of the critical location. The critical stress element will be on the outer surface, with its critical location determined by the plane of the combined bending moments.

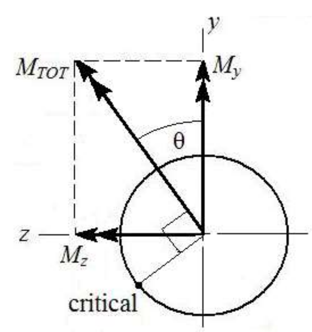
$M_y = (300)(12) = 3600$ lbf·in

$M_z = (250)(11) = 2750$ lbf·in

$$M_{\text{tot}} = \sqrt{M_y^2 + M_z^2}$$

$$= \sqrt{\left(3600\right)^2 + \left(2750\right)^2} = 4530 \text{ lbf} \cdot \text{in}$$

$$\theta = \tan^{-1}\left(\left|\frac{M_z}{M_y}\right|\right) = \tan^{-1}\left(\frac{2750}{3600}\right) = 37.4^{\circ}$$



The combined bending moment vector is at an angle of 37.4º CCW from the *y* axis. The critical bending stress location will be 90º CCW from this vector, where the tensile bending stress is additive with the tensile axial stress. The critical stress element is therefore on the outer surface, at an angle of 37.4º CCW from the *z* axis.

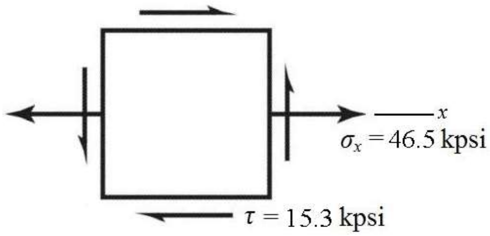
(b)

$$\sigma_{x,\text{bend}} = \frac{M_{\text{tot}}c}{I} = \frac{M_{\text{tot}}\left(d/2\right)}{\pi d^4/64} = \frac{32M_{\text{tot}}}{\pi d^3} = \frac{32\left(4530\right)}{\pi\left(1\right)^3} = 46\,142 \text{ psi} = 46.1 \text{ kpsi}$$

$$\sigma_{x,\text{axial}} = \frac{F_x}{A} = \frac{F_x}{\pi d^2/4} = \frac{300}{\pi\left(1\right)^2/4} = 382 \text{ psi} = 0.382 \text{ kpsi}$$

$$\sigma_x = \sigma_{x,\text{axial}} + \sigma_{x,\text{bend}} = 46\,142 + 382 = 46\,524 \text{ psi} = 46.5 \text{ kpsi}$$

$$\tau = \frac{Tr}{J} = \frac{16T}{\pi d^3} = \frac{16\left(12\right)\left(250\right)}{\pi\left(1\right)^3} = 15\,279 \text{ psi} = 15.3 \text{ kpsi}$$



(c)

$$\sigma_1,\ \sigma_2 = \frac{\sigma_x}{2} \pm \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau^2} = \frac{46.5}{2} \pm \sqrt{\left(\frac{46.5}{2}\right)^2 + (15.3)^2}$$

$\sigma_1 = 51.1$ kpsi *Ans.*

$\sigma_2 = -4.58$ kpsi *Ans.*

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau^2} = \sqrt{\left(\frac{46.5}{2}\right)^2 + (15.3)^2} = 27.8 \text{ kpsi} \qquad Ans.$$

3-97

(a) The cross section at *A* will experience bending, torsion, axial, and transverse shear. Both torsional shear stress and bending stress will be a maximum on the outer surface. The transverse shear will be very small compared to bending and torsion, due to the reasonably high length to diameter ratio, so it will not dominate the determination of the critical location. The critical stress element will be on the outer surface, with its critical location determined by the plane of the combined bending moments.

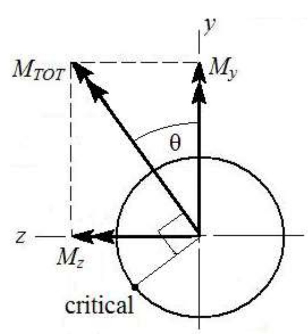
$M_y = (300)(12) - (-100)(11) = 4700$ lbf·in

$M_z = (250)(11) = 2750$ lbf·in

$$M_{\text{tot}} = \sqrt{M_y^2 + M_z^2}$$

$$= \sqrt{(4700)^2 + (2750)^2} = 5445 \text{ lbf} \cdot \text{in}$$

$$\theta = \tan^{-1}\left(\left|\frac{M_z}{M_y}\right|\right) = \tan^{-1}\left(\frac{2750}{4700}\right) = 30.3^\circ$$



The combined bending moment vector is at an angle of 30.3º CCW from the *y* axis. The critical bending stress location will be 90º CCW from this vector, where the tensile bending stress is additive with the tensile axial stress. The critical stress element is therefore on the outer surface, at an angle of 30.3º CCW from the *z* axis.

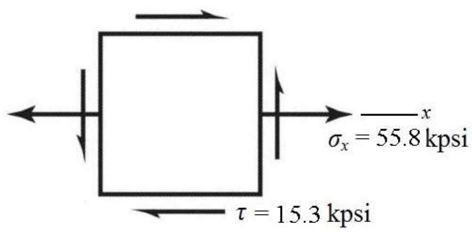
(b)

$$\sigma_{x,\text{bend}} = \frac{M_{\text{tot}}c}{I} = \frac{M_{\text{tot}}(d/2)}{\pi d^4/64} = \frac{32M_{\text{tot}}}{\pi d^3} = \frac{32(5445)}{\pi(1)^3} = 55\,462 \text{ psi} = 55.5 \text{ kpsi}$$

$$\sigma_{x,\text{axial}} = \frac{F_x}{A} = \frac{F_x}{\pi d^2/4} = \frac{300}{\pi(1)^2/4} = 382 \text{ psi} = 0.382 \text{ kpsi}$$

$$\sigma_x = \sigma_{x,\text{axial}} + \sigma_{x,\text{bend}} = 55\,462 + 382 = 55\,844 \text{ psi} = 55.8 \text{ kpsi}$$

$$\tau = \frac{Tr}{J} = \frac{16T}{\pi d^3} = \frac{16(12)(250)}{\pi(1)^3} = 15\,279 \text{ psi} = 15.3 \text{ kpsi}$$



(c)

$$\sigma_1,\ \sigma_2 = \frac{\sigma_x}{2} \pm \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau^2} = \frac{55.8}{2} \pm \sqrt{\left(\frac{55.8}{2}\right)^2 + (15.3)^2}$$

$\sigma_1 = 59.7$ kpsi *Ans.*

$\sigma_2 = -3.92$ kpsi *Ans.*

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau^2} = \sqrt{\left(\frac{55.8}{2}\right)^2 + (15.3)^2} = 31.8 \text{ kpsi} \qquad Ans.$$

3-98

(a) The cross section at *A* will experience bending, torsion, and transverse shear. Both torsional shear stress and bending stress will be a maximum on the outer surface, where the stress concentration will also be applicable. The transverse shear will be very small compared to bending and torsion, due to the reasonably high length to diameter ratio, so it will not dominate the determination of the critical location. The critical stress element will be at either the top (compression) or the bottom (tension) on the *y* axis. We'll select the bottom element for this analysis.

(b) Transverse shear is zero at the critical stress elements on the top and bottom surfaces.

$r/d = 0.125/1 = 0.125$

$D/d = 1.5/1 = 1.5$

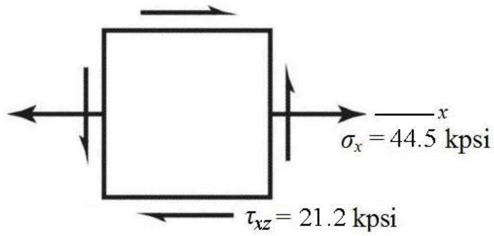
$K_{t,\text{torsion}} = 1.39$ Fig. A-15-8

$K_{t,\text{bend}} = 1.59$ Fig. A-15-9

$$\sigma_x = K_{t,\text{bend}} \frac{Mc}{I} = K_{t,\text{bend}} \frac{32M}{\pi d^3} = (1.59)\frac{32(11)(250)}{\pi(1)^3} = 44\,538 \text{ psi} = 44.5 \text{ kpsi}$$

$$\tau_{xz} = K_{t,\text{torsion}} \frac{Tr}{J} = K_{t,\text{torsion}} \frac{16T}{\pi d^3} = (1.39)\frac{16(12)(250)}{\pi(1)^3} = 21\,238 \text{ psi} = 21.2 \text{ kpsi}$$

(c)



$$\sigma_1,\ \sigma_2 = \frac{\sigma_x}{2} \pm \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \left(\tau_{xz}\right)^2} = \frac{44.5}{2} \pm \sqrt{\left(\frac{44.5}{2}\right)^2 + \left(21.2\right)^2}$$

$\sigma_1 = 53.0$ kpsi *Ans.*

$\sigma_2 = -8.48$ kpsi *Ans.*

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \left(\tau_{xz}\right)^2} = \sqrt{\left(\frac{44.5}{2}\right)^2 + \left(21.2\right)^2} = 30.7 \text{ kpsi} \qquad \textit{Ans.}$$

3-99

(a) The cross section at *A* will experience bending, torsion, axial, and transverse shear. Both torsional shear stress and bending stress will be a maximum on the outer surface, where the stress concentration will also be applicable. The transverse shear will be very small compared to bending and torsion, due to the reasonably high length to diameter ratio, so it will not dominate the determination of the critical location. The critical stress element will be on the outer surface, with its critical location determined by the plane of the combined bending moments.

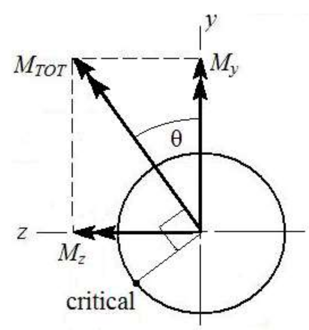
$M_y = (300)(12) = 3600$ lbf·in

$M_z = (250)(11) = 2750$ lbf·in

$$M_{\text{tot}} = \sqrt{M_y^2 + M_z^2}$$

$$= \sqrt{(3600)^2 + (2750)^2} = 4530 \text{ lbf}\cdot\text{in}$$

$$\theta = \tan^{-1}\left(\left|\frac{M_z}{M_y}\right|\right) = \tan^{-1}\left(\frac{2750}{3600}\right) = 37.4^\circ$$



The combined bending moment vector is at an angle of 37.4º CCW from the *y* axis. The critical bending stress location will be 90º CCW from this vector, where the tensile bending stress is additive with the tensile axial stress. The critical stress element is therefore on the outer surface, at an angle of 37.4º CCW from the *z* axis.

(b)

$r/d = 0.125/1 = 0.125$

$D/d = 1.5/1 = 1.5$

$K_{t,axial} = 1.75$ Fig. A-15-7

$K_{t,\text{torsion}} = 1.39$ Fig. A-15-8

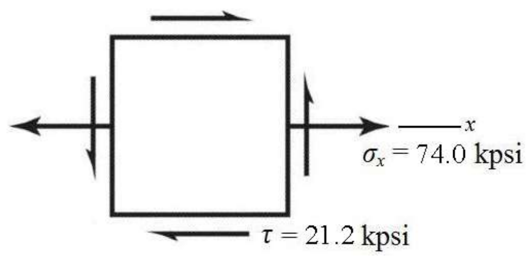
$K_{t,\text{bend}} = 1.59$ Fig. A-15-9

$$\sigma_{x,\text{bend}} = K_{t,\text{bend}} \frac{Mc}{I} = K_{t,\text{bend}} \frac{32M}{\pi d^3} = (1.59)\frac{32(4530)}{\pi(1)^3} = 73\,366 \text{ psi} = 73.4 \text{ kpsi}$$

$$\sigma_{x,\text{axial}} = K_{t,\text{axial}} \frac{F_x}{A} = (1.75)\frac{300}{\pi(1)^2/4} = 668 \text{ psi} = 0.668 \text{ kpsi}$$

$$\sigma_x = \sigma_{x,\text{axial}} + \sigma_{x,\text{bend}} = 73\,366 + 668 = 74\,034 \text{ psi} = 74.0 \text{ kpsi}$$

$$\tau = K_{t,\text{torsion}} \frac{Tr}{J} = K_{t,\text{torsion}} \frac{16T}{\pi d^3} = (1.39)\frac{16(12)(250)}{\pi(1)^3} = 21\,238 \text{ psi} = 21.2 \text{ kpsi}$$



(c)

$$\sigma_1,\ \sigma_2 = \frac{\sigma_x}{2} \pm \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau^2} = \frac{74.0}{2} \pm \sqrt{\left(\frac{74.0}{2}\right)^2 + (21.2)^2}$$

$\sigma_1 = 79.6$ kpsi *Ans.*

$\sigma_2 = -5.64$ kpsi *Ans.*

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau^2} = \sqrt{\left(\frac{74.0}{2}\right)^2 + (21.2)^2} = 42.6 \text{ kpsi} \qquad \textit{Ans.}$$

3-100

(a) The cross section at *A* will experience bending, torsion, axial, and transverse shear. Both torsional shear stress and bending stress will be a maximum on the outer surface, where the stress concentration is also applicable. The transverse shear will be very small compared to bending and torsion, due to the reasonably high length to diameter ratio, so it will not dominate the determination of the critical location. The critical stress element will be on the outer surface, with its critical location determined by the plane of the combined bending moments.

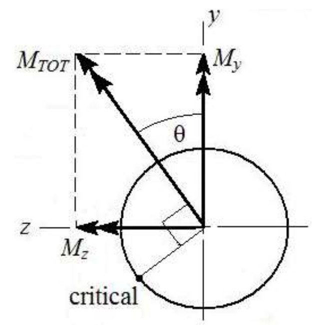
$M_y = (300)(12) - (-100)(11) = 4700$ lbf·in

$M_z = (250)(11) = 2750$ lbf·in

$$M_{\text{tot}} = \sqrt{M_y^2 + M_z^2}$$

$$= \sqrt{(4700)^2 + (2750)^2} = 5445 \text{ lbf} \cdot \text{in}$$

$$\theta = \tan^{-1}\left(\left|\frac{M_z}{M_y}\right|\right) = \tan^{-1}\left(\frac{2750}{4700}\right) = 30.3^{\circ}$$



The combined bending moment vector is at an angle of 30.3º CCW from the *y* axis. The critical bending stress location will be 90º CCW from this vector, where the tensile bending stress is additive with the tensile axial stress. The critical stress element is therefore on the outer surface, at an angle of 30.3º CCW from the *z* axis.

(b)

$$r/d = 0.125/1 = 0.125$$

$$D/d = 1.5/1 = 1.5$$

$K_{t,axial} = 1.75$ Fig. A-15-7

$K_{t,\text{torsion}} = 1.39$ Fig. A-15-8

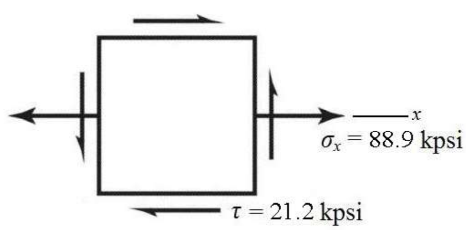
$K_{t,\text{bend}} = 1.59$ Fig. A-15-9

$$\sigma_{x,\text{bend}} = K_{t,\text{bend}} \frac{Mc}{I} = K_{t,\text{bend}} \frac{32M}{\pi d^3} = (1.59)\frac{32(5445)}{\pi(1)^3} = 88\,185 \text{ psi} = 88.2 \text{ kpsi}$$

$$\sigma_{x,\text{axial}} = K_{t,\text{axial}} \frac{F_x}{A} = (1.75)\frac{300}{\pi(1)^2/4} = 668 \text{ psi} = 0.668 \text{ kpsi}$$

$$\sigma_x = \sigma_{x,\text{axial}} + \sigma_{x,\text{bend}} = 88\,185 + 668 = 88\,853 \text{ psi} = 88.9 \text{ kpsi}$$

$$\tau = K_{t,\text{torsion}} \frac{Tr}{J} = K_{t,\text{torsion}} \frac{16T}{\pi d^3} = (1.39)\frac{16(12)(250)}{\pi(1)^3} = 21\,238 \text{ psi} = 21.2 \text{ kpsi}$$



(c)

$$\sigma_1,\ \sigma_2 = \frac{\sigma_x}{2} \pm \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau^2} = \frac{88.9}{2} \pm \sqrt{\left(\frac{88.9}{2}\right)^2 + (21.2)^2}$$

$\sigma_1 = 93.7$ kpsi *Ans.*

$\sigma_2 = -4.80$ kpsi *Ans.*

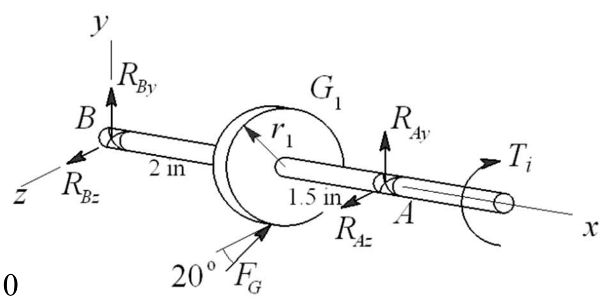
$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau^2} = \sqrt{\left(\frac{88.9}{2}\right)^2 + (21.2)^2} = 49.2 \text{ kpsi} \qquad \textit{Ans.}$$

3-101 **(a)** (Eq. 3-42):

$$P = \frac{Tn}{63\,025} = \frac{200(60)}{63\,025} = 0.1904 \text{ hp} \qquad \textit{Ans.}$$

(b) The output torque $T_o = (\omega_i / \omega_o)\, T_i$. But $\omega_i r_1 = \omega_o r_2$. Thus, $T_o = (r_2 / r_1)\, T_i$. So,
$T_o = (2.5/1)\, 200 = 500$ lbf·in *Ans.*

(c)

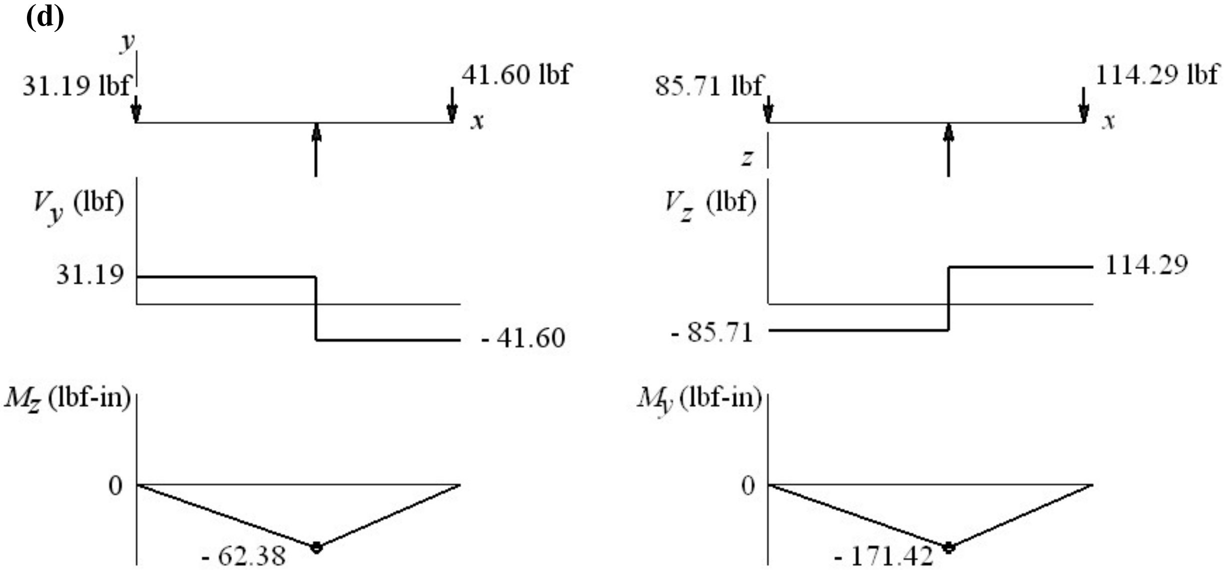


$\Sigma M_x = 0$, $(F_G \cos 20^\circ)(1) - 200 = 0$
$F_G = 212.84$ lbf
$\Sigma (M_B)_y = 0$, $2(F_G \cos 20^\circ) - 3.5R_{Az} = 0$
$R_{Az} = 2(212.84 \cos 20^\circ)/3.5 = 114.29$ lbf
$\Sigma F_z = 0$, $R_{Bz} + 114.29 - 212.84 \cos 20^\circ = 0$
$R_{Bz} = 85.71$ lbf
$\Sigma (M_B)_z = 0$, $2\,(212.84 \sin 20^\circ) + 3.5\,(F_A)_y = 0$
$R_{Ay} = -41.60$ lbf
$\Sigma F_y = 0$, $R_{By} - 41.60 + 212.84 \sin 20^\circ = 0 \ \Rightarrow\ R_{By} = -31.19$ lbf

$$R_A = \sqrt{R_{Ay}^2 + R_{Az}^2} = \sqrt{(-41.60)^2 + 114.29^2} = 121.6 \text{ lbf} \qquad \textit{Ans.}$$

$$R_B = \sqrt{R_{By}^2 + R_{Bz}^2} = \sqrt{(-31.19)^2 + 85.71^2} = 91.21 \text{ lbf} \qquad \textit{Ans.}$$

(d)



(e) $(M_y)_{max} = 2(-85.71) = -171.42$ lbf-in, $(M_z)_{max} = 2(-31.19) = -62.38$ lbf-in

$$M_{max} = \sqrt{(-171.42)^2 + (-62.38)^2} = 182.42 \text{ lbf-in}$$

$$\sigma = \frac{32M}{\pi d^3} = \frac{32(182.42)}{\pi(0.5)^3}10^{-3} = 14.87 \text{ kpsi} \qquad \textit{Ans.}$$

$$\tau = \frac{16T}{\pi d^3} = \frac{16(200)}{\pi(0.5)^3}10^{-3} = 8.15 \text{ kpsi} \qquad \textit{Ans.}$$

(f)

$$\sigma_1, \sigma_2 = \frac{\sigma}{2} \pm \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} = \frac{14.87}{2} \pm \sqrt{\left(\frac{14.87}{2}\right)^2 + 8.15^2}$$

$$= 18.47, -3.60 \quad \text{kpsi} \qquad \textit{Ans.}$$

$$\tau_{max} = \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} = \sqrt{\left(\frac{14.87}{2}\right)^2 + 8.15^2} = 11.03 \text{ kpsi} \qquad \textit{Ans.}$$

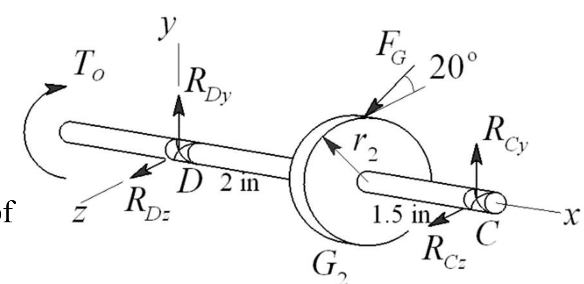
3-102 **(a)** (Eq. 3-42):

$$P = \frac{Tn}{63\,025} = \frac{200(60)}{63\,025} = 0.1904 \text{ hp} \qquad \textit{Ans.}$$

(b) The output torque $T_o = (\omega_i / \omega_o)\, T_i$. But $\omega_i r_1 = \omega_o r_2$. Thus, $T_o = (r_2 / r_1)\, T_i$. So,

$T_o = (2.5/1)\, 200 = 500$ lbf·in *Ans.*

(c)



$\Sigma M_x = 0 = (F_G \cos 20^o)(1) - 200$

$F_G = 212.84$ lbf

$\Sigma (M_B)_y = 0 = -2(F_G \cos 20^o) - 3.5R_{Cz}$

$R_{Cz} = -2(212.84 \cos 20^o)/3.5 = -114.29$ lbf

$\Sigma F_z = 0 = R_{Dz} - 114.29 + 212.84 \cos 20^o$

$R_{Dz} = -85.71$ lbf

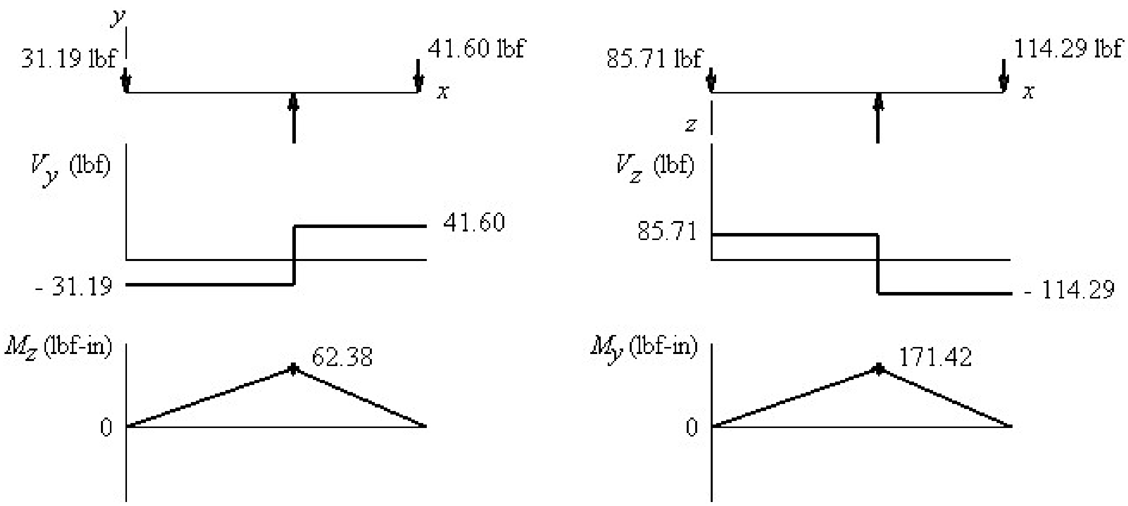
$\Sigma (M_B)_z = 0 = -2\,(212.84 \sin 20^o) + 3.5\, R_{Cy} \Rightarrow R_{Cy} = 41.60$ lbf

$\Sigma F_y = 0 = R_{Dy} + 41.60 - 212.84 \sin 20^o \Rightarrow R_{Dy} = 31.19$ lbf

$$R_C = \sqrt{R_{Cy}^2 + R_{Cz}^2} = \sqrt{41.60^2 + (-114.29)^2} = 121.6 \text{ lbf} \qquad \textit{Ans.}$$

$$R_D = \sqrt{R_{Dy}^2 + R_{Dz}^2} = \sqrt{31.19^2 + (-85.71)^2} = 91.21 \text{ lbf} \qquad \textit{Ans.}$$

(d)



(e) $(M_y)_{\max} = 2(85.71) = 171.42$ lbf·in, $(M_z)_{\max} = 2(31.19) = 62.38$ lbf·in

$$M_{\max} = \sqrt{171.42^2 + 62.38^2} = 182.42 \text{ lbf} \cdot \text{in}$$

$$\sigma = \frac{32M}{\pi d^3} = \frac{32(182.42)}{\pi(0.5)^3}10^{-3} = 14.87 \text{ kpsi} \qquad \textit{Ans.}$$

$$\tau = \frac{16T}{\pi d^3} = \frac{16(500)}{\pi(0.5)^3}10^{-3} = 20.37 \text{ kpsi} \qquad \textit{Ans.}$$

(f)

$$\sigma_1, \sigma_2 = \frac{\sigma}{2} \pm \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} = \frac{14.87}{2} \pm \sqrt{\left(\frac{14.87}{2}\right)^2 + 20.37^2}$$

$$= 29.1, -14.2 \text{ kpsi} \qquad \textit{Ans.}$$

$$\tau_{\max} = \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} = \sqrt{\left(\frac{14.87}{2}\right)^2 + 20.37^2} = 21.7 \text{ kpsi} \qquad \textit{Ans.}$$

3-103

(a) $M = F(p/4)$, $c = p/4$, $I = bh^3/12$, $b = \pi d_r n_t$, $h = p/2$

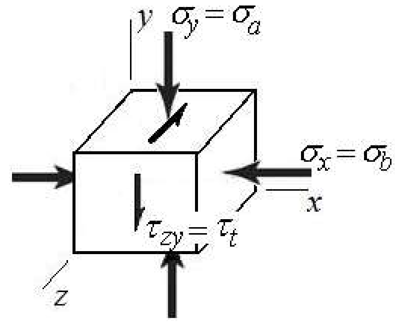
$$\sigma_b = \pm\frac{Mc}{I} = \pm\frac{[F(p/4)](p/4)}{bh^3/12} = \pm\frac{Fp^2}{16(\pi d_r n_t)(p/2)^3/12}$$

$$\sigma_b = \pm\frac{6F}{\pi d_r n_t p} \qquad \textit{Ans.}$$

(b) $$\sigma_a = -\frac{F}{A} = -\frac{F}{\pi d_r^2/4} = -\frac{4F}{\pi d_r^2} \qquad \textit{Ans.}$$

$$\tau_t = \frac{Tr}{J} = \frac{T(d_r/2)}{\pi d_r^4/32} = \frac{16T}{\pi d_r^3} \qquad \textit{Ans.}$$

(c) The bending stress causes compression in the *x* direction. The axial stress causes compression in the *y* direction. The torsional stress shears across the *y* face in the negative *z* direction.



(d) Analyze the stress element from part (c) using the equations developed in parts (a) and (b).

$$d_r = d - p = 1.5 - 0.25 = 1.25 \text{ in}$$

$$\sigma_x = \sigma_b = -\frac{6F}{\pi d_r n_t p} = -\frac{6(1500)}{\pi(1.25)(2)(0.25)} = -4584 \text{ psi} = -4.584 \text{ kpsi}$$

$$\sigma_y = \sigma_a = -\frac{4F}{\pi d_r^2} = -\frac{4(1500)}{\pi(1.25^2)} = -1222 \text{ psi} = -1.222 \text{ kpsi}$$

$$\tau_{yz} = -\tau_t = -\frac{16T}{\pi d_r^3} = -\frac{16(235)}{\pi(1.25^3)} = -612.8 \text{ psi} = -0.6128 \text{ kpsi}$$

Use Eq. (3-15) for the three-dimensional stress element.

$$\sigma^3 - (-4.584 - 1.222)\sigma^2 + \left[(-4.584)(-1.222) - (-0.6128)^2\right]\sigma - \left[-(-4.584)(-0.6128)^2\right] = 0$$

$$\sigma^3 + 5.806\sigma^2 + 5.226\sigma - 1.721 = 0$$

The roots are at 0.2543, – 4.584, and –1.476. Thus, the ordered principal stresses are

σ_1 = 0.2543 kpsi, σ_2 = –1.476 kpsi, and σ_3 = – 4.584 kpsi. *Ans.*

From Eq. (3-16), the principal shear stresses are

$$\tau_{1/2} = \frac{\sigma_1 - \sigma_2}{2} = \frac{0.2543 - (-1.476)}{2} = 0.8652 \text{ kpsi} \qquad \textit{Ans.}$$

$$\tau_{2/3} = \frac{\sigma_2 - \sigma_3}{2} = \frac{(-1.476) - (-4.584)}{2} = 1.554 \text{ kpsi} \qquad \textit{Ans.}$$

$$\tau_{1/3} = \frac{\sigma_1 - \sigma_3}{2} = \frac{0.2543 - (-4.584)}{2} = 2.419 \text{ kpsi} \qquad \textit{Ans.}$$

3-104 As shown in Fig. 3-32, the maximum stresses occur at the inside fiber where $r = r_i$. Therefore, from Eq. (3-50)

$$\sigma_{t,\max} = \frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(1 + \frac{r_o^2}{r_i^2}\right)$$

$$= p_i\left(\frac{r_o^2 + r_i^2}{r_o^2 - r_i^2}\right) \quad Ans.$$

$$\sigma_{r,\max} = \frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(1 - \frac{r_o^2}{r_i^2}\right) = -p_i \quad Ans.$$

3-105 If $p_i = 0$, Eq. (3-49) becomes

$$\sigma_t = \frac{-p_o r_o^2 - r_i^2 r_o^2 p_o / r^2}{r_o^2 - r_i^2}$$

$$= -\frac{p_o r_o^2}{r_o^2 - r_i^2}\left(1 + \frac{r_i^2}{r^2}\right)$$

The maximum tangential stress occurs at $r = r_i$. So

$$\sigma_{t,\max} = -\frac{2 p_o r_o^2}{r_o^2 - r_i^2} \quad Ans.$$

For σ_r, we have

$$\sigma_r = \frac{-p_o r_o^2 - r_i^2 r_o^2 p_o / r^2}{r_o^2 - r_i^2}$$

$$= \frac{p_o r_o^2}{r_o^2 - r_i^2}\left(\frac{r_i^2}{r^2} - 1\right)$$

So $\sigma_r = 0$ at $r = r_i$. Thus at $r = r_o$

$$\sigma_{r,\max} = \frac{p_o r_o^2}{r_o^2 - r_i^2}\left(\frac{r_i^2 - r_o^2}{r_o^2}\right) = -p_o \quad Ans.$$

3-106 The force due to the pressure on half of the sphere is resisted by the stress that is distributed around the center plane of the sphere. All planes are the same, so

$$(\sigma_t)_{\text{av}} = \sigma_1 = \sigma_2 = \frac{p(\pi/4)d_i^2}{\pi d_i t} = \frac{p d_i}{4t} \quad Ans.$$

The radial stress on the inner surface of the shell is, $\sigma_3 = -p$ *Ans.*

3-107 $\sigma_t > \sigma_l > \sigma_r$

$\tau_{max} = (\sigma_t - \sigma_r)/2$ at $r = r_i$

$$\tau_{max} = \frac{1}{2}\left[\frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(1 + \frac{r_o^2}{r_i^2}\right) - \frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(1 - \frac{r_o^2}{r_i^2}\right)\right] = \frac{r_o^2 p_i}{r_o^2 - r_i^2}$$

$$\Rightarrow \quad p_i = \frac{r_o^2 - r_i^2}{r_o^2}\tau_{max} = \frac{3^2 - 2.75^2}{3^2}(10\ 000) = 1597 \text{ psi} \quad \textit{Ans.}$$

3-108 $\sigma_t > \sigma_l > \sigma_r$

$\tau_{max} = (\sigma_t - \sigma_r)/2$ at $r = r_i$

$$\tau_{max} = \frac{1}{2}\left[\frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(1 + \frac{r_o^2}{r_i^2}\right) - \frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(1 - \frac{r_o^2}{r_i^2}\right)\right] = \frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(\frac{r_o^2}{r_i^2}\right) = \frac{r_o^2 p_i}{r_o^2 - r_i^2}$$

$$\Rightarrow \quad r_i = r_o\sqrt{\frac{(\tau_{max} - p_i)}{\tau_{max}}} = 100\sqrt{\frac{(25-4)10^6}{25\left(10^6\right)}} = 91.7 \text{ mm}$$

$$t = r_o - r_i = 100 - 91.7 = 8.3 \text{ mm} \quad \textit{Ans.}$$

3-109 $\sigma_t > \sigma_l > \sigma_r$

$\tau_{max} = (\sigma_t - \sigma_r)/2$ at $r = r_i$

$$\tau_{max} = \frac{1}{2}\left[\frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(1 + \frac{r_o^2}{r_i^2}\right) - \frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(1 - \frac{r_o^2}{r_i^2}\right)\right] = \frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(\frac{r_o^2}{r_i^2}\right) = \frac{r_o^2 p_i}{r_o^2 - r_i^2}$$

$$= \frac{4^2(500)}{4^2 - 3.75^2} = 4129 \text{ psi} \quad \textit{Ans.}$$

3-110 From Eq. (3-49) with $p_i = 0$,

$$\sigma_t = -\frac{r_o^2 p_o}{r_o^2 - r_i^2}\left(1 + \frac{r_i^2}{r^2}\right)$$

$$\sigma_r = -\frac{r_o^2 p_o}{r_o^2 - r_i^2}\left(1 - \frac{r_i^2}{r^2}\right)$$

$\sigma_t > \sigma_l > \sigma_r$, and since σ_t and σ_r are negative,

$\tau_{max} = (\sigma_r - \sigma_t)/2$ at $r = r_o$

$$\tau_{max} = \frac{1}{2}\left[-\frac{r_o^2 p_o}{r_o^2 - r_i^2}\left(1 - \frac{r_i^2}{r_o^2}\right) + \frac{r_o^2 p_o}{r_o^2 - r_i^2}\left(1 + \frac{r_i^2}{r_o^2}\right)\right] = \frac{r_o^2 p_o}{r_o^2 - r_i^2}\left(\frac{r_i^2}{r_o^2}\right) = \frac{r_i^2 p_o}{r_o^2 - r_i^2}$$

$$\Rightarrow \quad p_o = \frac{r_o^2 - r_i^2}{r_i^2}\tau_{max} = \frac{3^2 - 2.75^2}{2.75^2}(10\ 000) = 1900 \text{ psi} \quad \textit{Ans.}$$

3-111 From Eq. (3-49) with $p_i = 0$,

$$\sigma_t = -\frac{r_o^2 p_o}{r_o^2 - r_i^2}\left(1 + \frac{r_i^2}{r^2}\right)$$

$$\sigma_r = -\frac{r_o^2 p_o}{r_o^2 - r_i^2}\left(1 - \frac{r_i^2}{r^2}\right)$$

$\sigma_t > \sigma_l > \sigma_r$, and since σ_t and σ_r are negative,
$\tau_{max} = (\sigma_r - \sigma_t)/2$ at $r = r_o$

$$\tau_{\max} = \frac{1}{2}\left[-\frac{r_o^2 p_o}{r_o^2 - r_i^2}\left(1 - \frac{r_i^2}{r_o^2}\right) + \frac{r_o^2 p_o}{r_o^2 - r_i^2}\left(1 + \frac{r_i^2}{r_o^2}\right)\right] = \frac{r_o^2 p_o}{r_o^2 - r_i^2}\left(\frac{r_i^2}{r_o^2}\right) = \frac{r_i^2 p_o}{r_o^2 - r_i^2}$$

$$\Rightarrow \quad r_i = r_o\sqrt{\frac{\tau_{\max}}{(\tau_{\max} + p_o)}} = 100\sqrt{\frac{25\left(10^6\right)}{\left(25 + 4\right)10^6}} = 92.8 \text{ mm}$$

$$t = r_o - r_i = 100 - 92.8 = 7.2 \text{ mm} \qquad \textit{Ans.}$$

3-112 From Eq. (3-49) with $p_i = 0$,

$$\sigma_t = -\frac{r_o^2 p_o}{r_o^2 - r_i^2}\left(1 + \frac{r_i^2}{r^2}\right)$$

$$\sigma_r = -\frac{r_o^2 p_o}{r_o^2 - r_i^2}\left(1 - \frac{r_i^2}{r^2}\right)$$

$\sigma_t > \sigma_l > \sigma_r$, and since σ_t and σ_r are negative,
$\tau_{max} = (\sigma_r - \sigma_t)/2$ at $r = r_o$

$$\tau_{\max} = \frac{1}{2}\left[-\frac{r_o^2 p_o}{r_o^2 - r_i^2}\left(1 - \frac{r_i^2}{r_o^2}\right) + \frac{r_o^2 p_o}{r_o^2 - r_i^2}\left(1 + \frac{r_i^2}{r_o^2}\right)\right] = \frac{r_o^2 p_o}{r_o^2 - r_i^2}\left(\frac{r_i^2}{r_o^2}\right) = \frac{r_i^2 p_o}{r_o^2 - r_i^2}$$

$$= \frac{3.75^2(500)}{4^2 - 3.75^2} = 3629 \text{ psi} \quad \textit{Ans.}$$

3-113 From Table A-20, S_y=490 MPa
From Eq. (3-49) with $p_i = 0$,

$$\sigma_t = -\frac{r_o^2 p_o}{r_o^2 - r_i^2}\left(1 + \frac{r_i^2}{r^2}\right)$$

Maximum will occur at $r = r_i$

$$\sigma_{t,\max} = -\frac{2r_o^2 p_o}{r_o^2 - r_i^2} \Rightarrow p_o = -\frac{\sigma_{t,\max}(r_o^2 - r_i^2)}{2r_o^2} = -\frac{\left[0.8(-490)\right]\left(25^2 - 19^2\right)}{2(25^2)} = 82.8 \text{ MPa} \quad \textit{Ans.}$$

3-114 From Table A-20, $S_y = 71$ kpsi
From Eq. (3-49) with $p_i = 0$,

$$\sigma_t = -\frac{r_o^2 p_o}{r_o^2 - r_i^2}\left(1 + \frac{r_i^2}{r^2}\right)$$

Maximum will occur at $r = r_i$

$$\sigma_{t,\max} = -\frac{2r_o^2 p_o}{r_o^2 - r_i^2} \Rightarrow p_o = -\frac{\sigma_{t,\max}\left(r_o^2 - r_i^2\right)}{2r_o^2} = -\frac{\left[0.8(-71)\right]\left(1^2 - 0.75^2\right)}{2(1^2)} = 12.4 \text{ kpsi} \quad \textit{Ans.}$$

3-115 From Table A-20, S_y=490 MPa
From Eq. (3-50)

$$\sigma_t = \frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(1 + \frac{r_o^2}{r^2}\right)$$

Maximum will occur at $r = r_i$

$$\sigma_{t,\max} = \frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(1 + \frac{r_o^2}{r_i^2}\right) = \frac{p_i\left(r_o^2 + r_i^2\right)}{r_o^2 - r_i^2}$$

$$\Rightarrow \quad p_i = \frac{\sigma_{t,\max}(r_o^2 - r_i^2)}{r_o^2 + r_i^2} = \frac{\left[0.8(490)\right](25^2 - 19^2)}{(25^2 + 19^2)} = 105 \text{ MPa} \quad \textit{Ans.}$$

3-116 From Table A-20, S_y =71 MPa
From Eq. (3-50)

$$\sigma_t = \frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(1 + \frac{r_o^2}{r^2}\right)$$

Maximum will occur at $r = r_i$

$$\sigma_{t,\max} = \frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(1 + \frac{r_o^2}{r_i^2}\right) = \frac{p_i(r_o^2 + r_i^2)}{r_o^2 - r_i^2}$$

$$\Rightarrow \quad p_i = \frac{\sigma_{t,\max}(r_o^2 - r_i^2)}{r_o^2 + r_i^2} = \frac{\left[0.8(71)\right](1^2 - 0.75^2)}{(1^2 + 0.75^2)} = 15.9 \text{ ksi} \quad \textit{Ans.}$$

3-117 The longitudinal stress will be due to the weight of the vessel above the maximum stress point. From Table A-5, the unit weight of steel is $\gamma_s = 0.282$ lbf/in^3. The area of the wall is

$$A_{\text{wall}} = (\pi/4)(360^2 - 358.5^2) = 846.\ 5 \text{ in}^2$$

The volume of the wall and dome are

$$V_{\text{wall}} = A_{\text{wall}}\, h = 846.5\ (720) = 609.5\ (10^3) \text{ in}^3$$

$$V_{\text{dome}} = (2\pi/3)(180^3 - 179.25^3) = 152.0\ (10^3) \text{ in}^3$$

The weight of the structure on the wall area at the tank bottom is

$$W = \gamma_s\, V_{\text{total}} = 0.282(609.5 + 152.0)\ (10^3) = 214.7(10^3) \text{ lbf}$$

$$\sigma_l = -\frac{W}{A_{\text{wall}}} = -\frac{214.7\left(10^3\right)}{846.5} = -254 \text{ psi}$$

The maximum pressure will occur at the bottom of the tank, $p_i = \gamma_{\text{water}}\, h$. From Eq. (3-50) with $r = r_i$

$$\sigma_t = \frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(1 + \frac{r_o^2}{r_i^2}\right) = p_i\left(\frac{r_o^2 + r_i^2}{r_o^2 - r_i^2}\right)$$

$$= \left[62.4(55)\left(\frac{1 \text{ ft}^2}{144 \text{ in}^2}\right)\right]\left(\frac{180^2 + 179.25^2}{180^2 - 179.25^2}\right) = 5708 \approx 5710 \text{ psi} \quad \textit{Ans.}$$

$$\sigma_r = \frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(1 - \frac{r_o^2}{r_i^2}\right) = -p_i = -62.4(55)\left(\frac{1 \text{ ft}^2}{144 \text{ in}^2}\right) = -23.8 \text{ psi} \qquad \textit{Ans.}$$

Note: These stresses are very idealized as the floor of the tank will restrict the values calculated.

Since $\sigma_1 \geq \sigma_2 \geq \sigma_3$, $\sigma_1 = \sigma_t = 5708$ psi, $\sigma_2 = \sigma_r = -24$ psi and $\sigma_3 = \sigma_l = -254$ psi. From Eq. (3-16),

$$\tau_{1/3} = \frac{5708 + 254}{2} = 2981 \approx 2980 \text{ psi}$$

$$\tau_{1/2} = \frac{5708 + 24}{2} = 2866 \approx 2870 \text{ psi} \qquad \textit{Ans.}$$

$$\tau_{2/3} = \frac{-24 + 254}{2} = 115 \text{ psi}$$

3-118 Stresses from additional pressure are,
Eq. (3-52),

$$\left(\sigma_l\right)_{50\text{psi}} = \frac{50\left(179.25^2\right)}{180^2 - 179.25^2} = 5963 \text{ psi}$$

$$(\sigma_r)_{50\text{ psi}} = -\,50 \text{ psi}$$

Eq. (3-50)

$$\left(\sigma_t\right)_{50\text{psi}} = 50\frac{180^2 + 179.25^2}{180^2 - 179.25^2} = 11\,975 \text{ psi}$$

Adding these to the stresses found in Prob. 3-117 gives

$\sigma_t = 5708 + 11\,975 = 17683$ psi = 17.7 kpsi *Ans.*

$\sigma_r = -\,23.8 - 50 = -\,73.8$ psi *Ans.*

$\sigma_l = -\,254 + 5963 = 5709$ psi *Ans.*

Note: These stresses are very idealized as the floor of the tank will restrict the values calculated.

From Eq. (3-16)

$$\tau_{1/3} = \frac{17\,683 + 73.8}{2} = 8879 \text{ psi}$$

$$\tau_{1/2} = \frac{17\,683 - 5709}{2} = 5987 \text{ psi} \qquad \textit{Ans.}$$

$$\tau_{2/3} = \frac{5709 + 23.8}{2} = 2866 \text{ psi}$$

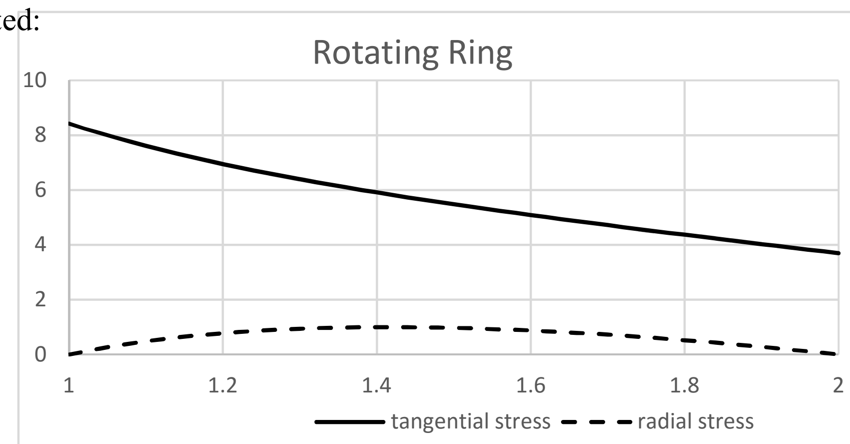
3-119 **(a)** For shapes let $\rho\omega\,(3 + \nu)/8 = 1$, $\nu = 0.3$, $r_o = 2$, and $r_i = 1$. Then,

$$\frac{1+3\nu}{3+\nu} = \frac{1+3(0.3)}{3+0.3} = 0.5758$$

Thus, Eqs. (3-55) are

$$\sigma_t = 1 + 4 + \frac{4}{r^2} - 0.5758r^2, \qquad \sigma_r = 1 + 4 - \frac{4}{r^2} - r^2$$

These equations are plotted:



(b) The tangential stress, σ_t is maximum at $r = r_i$ and is given by

$$\left(\sigma_t\right)_{\max} = \rho\omega^2\left(\frac{3+\nu}{8}\right)\left(r_i^2 + r_o^2 + \frac{r_i^2 r_o^2}{r_i^2} - \frac{1+3\nu}{3+\nu}r_i^2\right)$$

$$= \rho\omega^2\left(\frac{3+\nu}{8}\right)\left\{\left[\frac{3+\nu-(1+3\nu)}{3+\nu}\right]r_i^2 + 2r_o^2\right\}$$

$$= \frac{\rho\omega^2}{4}\left[(1-\nu)r_i^2 + (3+\nu)r_o^2\right] \qquad \textit{Ans.}$$

$\sigma_r = 0$ at $r = r_i$ and $r = r_o$. $(\sigma_r)_{\max}$ occurs where $d\sigma_r / dr = 0$.

$$\frac{d\sigma_r}{dr} = \rho\omega^2\left(\frac{3+\nu}{8}\right)\left(\frac{2r_i^2 r_o^2}{r^3} - 2r\right) = 0 \;\Rightarrow\; r^4 = r_i^2 r_o^2 \;\Rightarrow\; r = \sqrt{r_i r_o}$$

Thus,

$$\left(\sigma_r\right)_{\max} = \rho\omega^2\left(\frac{3+\nu}{8}\right)\left(r_i^2 + r_o^2 - \frac{r_i^2 r_o^2}{r_i r_o} - r_i r_o\right) = \rho\omega^2\left(\frac{3+\nu}{8}\right)\left(r_o - r_i\right)^2 \qquad \textit{Ans.}$$

3-120 Since σ_t and σ_r are both positive and $\sigma_t > \sigma_r$

$$\tau_{\max} = \left(\sigma_t\right)_{\max}/2$$

From Eq. (3-55), σ_t is maximum at $r = r_i = 0.3125$ in. The term

$$\rho\omega^2\left(\frac{3+\nu}{8}\right) = \frac{0.282}{386}\left[\frac{2\pi(5000)}{60}\right]^2\left(\frac{3+0.292}{8}\right) = 82.42 \text{ lbf/in}$$

$$\left(\sigma_t\right)_{\max} = 82.42\left[0.3125^2 + 2.75^2 + \frac{\left(0.3125^2\right)\left(2.75^2\right)}{0.3125^2} - \frac{1+3(0.292)}{3+0.292}\left(0.3125^2\right)\right]$$

$$= 1260 \text{ psi}$$

$$\tau_{\max} = \frac{1260}{2} = 630 \text{ psi} \qquad \textit{Ans.}$$

Radial stress:

$$\sigma_r = k\left(r_i^2 + r_o^2 - \frac{r_i^2 r_o^2}{r^2} - r^2\right)$$

Maxima:

$$\frac{d\sigma_r}{dr} = k\left(2\frac{r_i^2 r_o^2}{r^3} - 2r\right) = 0 \quad \Rightarrow \quad r = \sqrt{r_i r_o} = \sqrt{0.3125(2.75)} = 0.927 \text{ in}$$

$$(\sigma_r)_{\max} = 82.42\left[0.3125^2 + 2.75^2 - \frac{0.3125^2\left(2.75^2\right)}{0.927^2} - 0.927^2\right]$$

$$= 490 \text{ psi} \quad Ans.$$

3-121 $\omega = 2\pi(2000)/60 = 209.4$ rad/s, $\rho = 3320$ 20 kg/m^3, $\nu = 0.24$, $r_i = 0.01$ m, $r_o = 0.125$ m

Using Eq. (3-55)

$$\sigma_t = 3320(209.4)^2\left(\frac{3+0.24}{8}\right)\left[(0.01)^2 + (0.125)^2 + (0.125)^2 - \frac{1+3(0.24)}{3+0.24}(0.01)^2\right](10)^{-6}$$

$$= 1.85 \text{ MPa} \quad Ans.$$

3-122 Eq (3-55):

$$\sigma_t = K\left(r_i^2 + r_o^2 + \frac{r_i^2 r_o^2}{r^2} - \frac{r^2}{2}\right) \quad (1), \qquad \text{and} \qquad \sigma_r = K\left(r_i^2 + r_o^2 - \frac{r_i^2 r_o^2}{r^2} - r^2\right) \quad (2)$$

Where $K = \rho\omega^2 (3 + \nu)/8$ and $(1 + 3\nu)/(3 + \nu) = [1 + 3(0.2)] / (3 + 0.2) = 1/2$.
It can be seen that σ_t is always positive. Check for maxima.

$$\frac{d\sigma_t}{dr} = K\left(-2\frac{r_i^2 r_o^2}{r^3} - r\right) = 0 \;\Rightarrow\; r^4 = -2r_i^2 r_o^2 \quad \text{No roots, no maxima. Check at extremes.}$$

At $r = r_i$, $$(\sigma_t)_i = K\left(r_i^2 + r_o^2 + r_o^2 - \frac{r_i^2}{2}\right) = K\left(2r_o^2 + \frac{r_i^2}{2}\right) \quad (3)$$

At $r = r_o$, $$(\sigma_t)_i = K\left(r_i^2 + r_o^2 + r_i^2 - \frac{r_o^2}{2}\right) = K\left(2r_i^2 - \frac{r_o^2}{2}\right) \quad (4)$$

Eq. (3) > (4). Thus, $(\sigma_t)_{\max} = (\sigma_t)_i$.
For Eq. (2), $\sigma_r = 0$ at $r = r_i$ and $r = r_o$. Check for maxima.

$$\frac{d\sigma_r}{dr} = K\left(2\frac{r_i^2 r_o^2}{r^3} - 2r\right) = 0 \;\Rightarrow\; r = \sqrt{r_i r_o} \quad (5)$$

Substitute Eq. (5) into (2),

$$(\sigma_r)_{\max} = K\left(r_i^2 + r_o^2 - r_i r_o - r_i r_o\right) = K\left(r_o^2 - 2r_i r_o + r_i^2\right) = K\left(r_o - r_i\right)^2 \quad (6)$$

Comparing Eqs. (3) and (6) it is clear that $K\left(2r_o^2 + \frac{r_i^2}{2}\right) > K\left(r_o^2 + r_i^2 - 2r_i r_o\right)$. Thus, the maximum stress is given by Eq. (3) simplified as, $\sigma_{\max} = \rho\omega^2\left(\frac{3+\nu}{16}\right)\left(4r_o^2 + r_i^2\right)$ *Ans.*

(b) Vol = $(\pi/4)(5^2 - 0.75^2)(1/16) = 1.1996$ in^3. $\omega = 2\pi(12\,000)/60 = 1256.6$ rad/s.

$$\rho = \frac{5/16}{386(1.1996)} = 6.749\left(10^{-4}\right) \text{ lbf-s}^2/\text{in}^4$$

Eq. (3): $(\sigma_t)_{\max} = 6.749\left(10^{-4}\right)1256.6^2\left(\dfrac{3+0.2}{8}\right)\left[2\left(2.5\right)^2 + \dfrac{0.375^2}{2}\right] = 5\ 360\ \text{ psi}$ *Ans.*

The factor of safety corresponding to fracture is:

$$n = \frac{S_u}{\sigma_{\max}} = \frac{12}{5.36} = 2.24 \qquad \textit{Ans.}$$

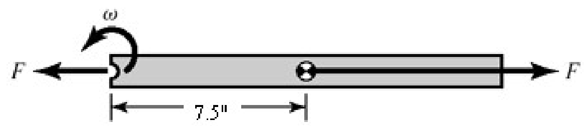
3-123 $\omega = 2\pi(3500)/60 = 366.5$ rad/s,

mass of blade = $m = \rho V = (0.282 / 386)\ [1.25(30)(0.125)] = 3.425(10^{-3})$ lbf·s²/in

$F = (m/2)\ \omega^2 r$

$= [3.425(10^{-3})/2](366.5^2)(7.5)$

$= 1725$ lbf



$A_{\text{nom}} = (1.25 - 0.5)(1/8) = 0.093\ 75\ \text{in}^2$

$\sigma_{\text{nom}} = F/ A_{\text{nom}} = 1725/0.093\ 75 = 18\ 400$ psi *Ans.*

Note: Stress concentration Fig. A-15-1 gives $K_t = 2.25$ which increases $\sigma_{\max}$ and fatigue.

3-124 $\nu = 0.292$, $E = 207$ GPa, $r_i = 0$, $R = 25$ mm, $r_o = 50$ mm

Eq. (3-57),

$$p = \frac{207(10^9)\delta}{2(0.025)^3}\left[\frac{(0.05^2 - 0.025^2)(0.025^2 - 0)}{(0.05^2 - 0)}\right]\left(10^{-9}\right) = 3.105(10^3)\delta \qquad (1)$$

where p is in MPa and δ is in mm.

Maximum interference,

$$\delta_{\max} = \frac{1}{2}[50.042 - 50.000] = 0.021\ \text{mm} \qquad \textit{Ans.}$$

Minimum interference,

$$\delta_{\min} = \frac{1}{2}[50.026 - 50.025] = 0.0005\ \text{mm} \qquad \textit{Ans.}$$

From Eq. (1)

$p_{\max} = 3.105(10^3)(0.021) = 65.2$ MPa *Ans.*

$p_{\min} = 3.105(10^3)(0.0005) = 1.55$ MPa *Ans.*

3-125 $\nu = 0.292$, $E = 30$ Mpsi, $r_i = 0$, $R = 1$ in, $r_o = 2$ in

Eq. (3-57),

$$p = \frac{30(10^6)\delta}{2(1^3)}\left[\frac{(2^2 - 1^2)(1^2 - 0)}{(2^2 - 0)}\right] = 1.125(10^7)\delta \qquad (1)$$

where p is in psi and δ is in inches.

Maximum interference,

$$\delta_{\max} = \frac{1}{2}[2.0016 - 2.0000] = 0.0008 \text{ in} \qquad Ans.$$

Minimum interference,

$$\delta_{\min} = \frac{1}{2}[2.0010 - 2.0010] = 0 \qquad Ans.$$

From Eq. (1),

$$p_{\max} = 1.125(10^7)(0.0008) = 9\,000 \text{ psi} \qquad Ans.$$

$$p_{\min} = 1.125(10^7)(0) = 0 \qquad Ans.$$

3-126 $\nu = 0.292$, $E = 207$ GPa, $r_i = 0$, $R = 25$ mm, $r_o = 50$ mm

Eq. (3-57),

$$p = \frac{207(10^9)\delta}{2(0.025)^3}\left[\frac{(0.05^2 - 0.025^2)(0.025^2 - 0)}{(0.05^2 - 0)}\right]\left(10^{-9}\right) = 3.105(10^3)\delta \qquad (1)$$

where p is in MPa and δ is in mm.

Maximum interference,

$$\delta_{\max} = \frac{1}{2}[50.059 - 50.000] = 0.0295 \text{ mm} \qquad Ans.$$

Minimum interference,

$$\delta_{\min} = \frac{1}{2}[50.043 - 50.025] = 0.009 \text{ mm} \qquad Ans.$$

From Eq. (1)

$$p_{\max} = 3.105(10^3)(0.0295) = 91.6 \text{ MPa} \qquad Ans.$$

$$p_{\min} = 3.105(10^3)(0.009) = 27.9 \text{ MPa} \qquad Ans.$$

3-127 $\nu = 0.292$, $E = 30$ Mpsi, $r_i = 0$, $R = 1$ in, $r_o = 2$ in

Eq. (3-57),

$$p = \frac{30(10^6)\delta}{2(1^3)}\left[\frac{(2^2 - 1^2)(1^2 - 0)}{(2^2 - 0)}\right] = 1.125(10^7)\delta \qquad (1)$$

where p is in psi and δ is in inches.

Maximum interference,

$$\delta_{\max} = \frac{1}{2}[2.0023 - 2.0000] = 0.00115 \text{ in} \qquad Ans.$$

Minimum interference,

$$\delta_{\min} = \frac{1}{2}[2.0017 - 2.0010] = 0.00035 \quad \textit{Ans.}$$

From Eq. (1),

$p_{\max} = 1.125(10^7)(0.00115) = 12\,940$ psi *Ans.*

$p_{\min} = 1.125(10^7)(0.00035) = 3\,938$ *Ans.*

3-128 $\nu = 0.292$, $E = 207$ GPa, $r_i = 0$, $R = 25$ mm, $r_o = 50$ mm

Eq. (3-57),

$$p = \frac{207(10^9)\delta}{2(0.025)^3}\left[\frac{(0.05^2 - 0.025^2)(0.025^2 - 0)}{(0.05^2 - 0)}\right]\left(10^{-9}\right) = 3.105(10^3)\delta \qquad (1)$$

where p is in MPa and δ is in mm.

Maximum interference,

$$\delta_{\max} = \frac{1}{2}[50.086 - 50.000] = 0.043 \text{ mm} \quad \textit{Ans.}$$

Minimum interference,

$$\delta_{\min} = \frac{1}{2}[50.070 - 50.025] = 0.0225 \text{ mm} \quad \textit{Ans.}$$

From Eq. (1)

$p_{\max} = 3.105(10^3)(0.043) = 134$ MPa *Ans.*

$p_{\min} = 3.105(10^3)(0.0225) = 69.9$ MPa *Ans.*

3-129 $\nu = 0.292$, $E = 30$ Mpsi, $r_i = 0$, $R = 1$ in, $r_o = 2$ in

Eq. (3-57),

$$p = \frac{30(10^6)\delta}{2(1^3)}\left[\frac{(2^2 - 1^2)(1^2 - 0)}{(2^2 - 0)}\right] = 1.125(10^7)\delta \qquad (1)$$

where p is in psi and δ is in inches.

Maximum interference,

$$\delta_{\max} = \frac{1}{2}[2.0034 - 2.0000] = 0.0017 \text{ in} \quad \textit{Ans.}$$

Minimum interference,

$$\delta_{\min} = \frac{1}{2}[2.0028 - 2.0010] = 0.0009 \quad \textit{Ans.}$$

From Eq. (1),

p_{max} = 1.125(10^7)(0.0017) = 19 130 psi *Ans.*

p_{min} = 1.125(10^7)(0.0009) = 10 130 *Ans.*

3-130 From Table A-5, $E_i = E_o$ = 30 Mpsi, $\nu_i = \nu_o$ = 0.292. r_i = 0, R = 1 in, r_o = 1.5 in

The radial interference is $\delta = \frac{1}{2}(2.002 - 2.000) = 0.001 \text{ in}$ *Ans.*

Eq. (3-57),

$$p = \frac{E\delta}{2R^3}\left[\frac{(r_o^2 - R^2)(R^2 - r_i^2)}{r_o^2 - r_i^2}\right] = \frac{30(10^6)0.001}{2(1^3)}\left[\frac{(1.5^2 - 1^2)(1^2 - 0)}{(1.5^2 - 0)}\right]$$

$$= 8333 \text{ psi} = 8.33 \text{ kpsi} \quad \textit{Ans.}$$

The tangential stresses at the interface for the inner and outer members are given by Eqs. (3-58) and (3-59), respectively.

$$(\sigma_t)_i\big|_{r=R} = -p\frac{R^2 + r_i^2}{R^2 - r_i^2} = -(8333)\frac{1^2 + 0^2}{1^2 - 0^2} = -8333 \text{ psi} = -8.33 \text{ kpsi} \quad \textit{Ans.}$$

$$(\sigma_t)_o\big|_{r=R} = p\frac{r_o^2 + R^2}{r_o^2 - R^2} = (8333)\frac{1.5^2 + 1^2}{1.5^2 - 1^2} = 21\,670 \text{ psi} = 21.7 \text{ kpsi} \quad \textit{Ans.}$$

3-131 From Table A-5, E_i = 30 Mpsi, E_o =14.5 Mpsi, ν_i = 0.292, ν_o = 0.211.
r_i = 0, R = 1 in, r_o = 1.5 in

The radial interference is $\delta = \frac{1}{2}(2.002 - 2.000) = 0.001 \text{ in}$ *Ans.*

Eq. (3-56),

$$p = \frac{\delta}{R\left[\frac{1}{E_o}\left(\frac{r_o^2 + R^2}{r_o^2 - R^2} + \nu_o\right) + \frac{1}{E_i}\left(\frac{R^2 + r_i^2}{R^2 - r_i^2} - \nu_i\right)\right]}$$

$$p = \frac{0.001}{1\left[\frac{1}{14.5(10^6)}\left(\frac{1.5^2 + 1^2}{1.5^2 - 1^2} + 0.211\right) + \frac{1}{30(10^6)}\left(\frac{1^2 + 0^2}{1^2 - 0^2} - 0.292\right)\right]} = 4599 \text{ psi} \quad \textit{Ans.}$$

The tangential stresses at the interface for the inner and outer members are given by Eqs. (3-58) and (3-59), respectively.

$$(\sigma_t)_i\big|_{r=R} = -p\frac{R^2 + r_i^2}{R^2 - r_i^2} = -(4599)\frac{1^2 + 0^2}{1^2 - 0^2} = -4599 \text{ psi} \quad \textit{Ans.}$$

$$(\sigma_t)_o\big|_{r=R} = p\frac{r_o^2+R^2}{r_o^2-R^2} = (4599)\frac{1.5^2+1^2}{1.5^2-1^2} = 11\,960 \text{ psi} \quad Ans.$$

3-132 From Table A-5, $E_i = E_o = 30$ Mpsi, $\nu_i = \nu_o = 0.292$. $r_i = 0$, $R = 0.5$ in, $r_o = 1$ in
The minimum and maximum radial interferences are

$$\delta_{\min} = \frac{1}{2}\left(1.002-1.002\right) = 0.000 \text{ in} \quad Ans.$$

$$\delta_{\max} = \frac{1}{2}\left(1.003-1.001\right) = 0.001 \text{ in} \quad Ans.$$

Since the minimum interference is zero, the minimum pressure and tangential stresses are zero. *Ans.*

The maximum pressure is obtained from Eq. (3-57).

$$p = \frac{E\delta}{2R^3}\left[\frac{\left(r_o^2-R^2\right)\left(R^2-r_i^2\right)}{r_o^2-r_i^2}\right]$$

$$p = \frac{30\left(10^6\right)0.001}{2\left(0.5^3\right)}\left[\frac{\left(1^2-0.5^2\right)\left(0.5^2-0\right)}{\left(1^2-0\right)}\right] = 22\,500 \text{ psi} \qquad Ans$$

The maximum tangential stresses at the interface for the inner and outer members are given by Eqs. (3-58) and (3-59), respectively.

$$(\sigma_t)_i\big|_{r=R} = -p\frac{R^2+r_i^2}{R^2-r_i^2} = -(22\,500)\frac{0.5^2+0^2}{0.5^2-0^2} = -22\,500 \text{ psi} \quad Ans.$$

$$(\sigma_t)_o\big|_{r=R} = p\frac{r_o^2+R^2}{r_o^2-R^2} = (22\,500)\frac{1^2+0.5^2}{1^2-0.5^2} = 37\,500 \text{ psi} \quad Ans.$$

3-133 From Table A-5, $E_i = 10.4$ Mpsi, $E_o = 30$ Mpsi, $\nu_i = 0.333$, $\nu_o = 0.292$.
$r_i = 0$, $R = 1$ in, $r_o = 1.5$ in

The minimum and maximum radial interferences are

$$\delta_{\min} = \frac{1}{2}[2.003-2.002] = 0.0005 \text{ in} \quad Ans.$$

$$\delta_{\max} = \frac{1}{2}[2.006-2.000] = 0.003 \text{ in} \quad Ans.$$

Eq. (3-56),

$$p=\frac{\delta}{R\left[\frac{1}{E_o}\left(\frac{r_o^2+R^2}{r_o^2-R^2}+\nu_o\right)+\frac{1}{E_i}\left(\frac{R^2+r_i^2}{R^2-r_i^2}-\nu_i\right)\right]}$$

$$p=\frac{\delta}{1\left[\frac{1}{30\left(10^6\right)}\left(\frac{1.5^2+1^2}{1.5^2-1^2}+0.292\right)+\frac{1}{10.4\left(10^6\right)}\left(\frac{1^2+0^2}{1^2-0^2}-0.333\right)\right]}$$

$$p=6.229\left(10^6\right)\delta\ \text{psi} \qquad Ans.$$

$$p_{\min}=6.229\left(10^6\right)\delta_{\min}=6.229\left(10^6\right)\left(0.0005\right)=3114.6\ \text{psi}=3.11\ \text{kpsi} \qquad Ans.$$

$$p_{\max}=6.229\left(10^6\right)\delta_{\max}=6.229\left(10^6\right)\left(0.003\right)=18\,687\ \text{psi}=18.7\ \text{kpsi} \qquad Ans.$$

The tangential stresses at the interface for the inner and outer members are given by Eqs. (3-58) and (3-59), respectively.

Minimum interference:

$$(\sigma_t)_i\big|_{\min}=-p_{\min}\frac{R^2+r_i^2}{R^2-r_i^2}=-(3.11)\frac{1^2+0^2}{1^2-0^2}=-3.11\ \text{kpsi} \quad Ans.$$

$$(\sigma_t)_o\big|_{\min}=p_{\min}\frac{r_o^2+R^2}{r_o^2-R^2}=(3.11)\frac{1.5^2+1^2}{1.5^2-1^2}=8.09\ \text{kpsi} \quad Ans.$$

Maximum interference:

$$(\sigma_t)_i\big|_{\max}=-p_{\max}\frac{R^2+r_i^2}{R^2-r_i^2}=-(18.7)\frac{1^2+0^2}{1^2-0^2}=-18.7\ \text{kpsi} \quad Ans.$$

$$(\sigma_t)_o\big|_{\max}=p_{\max}\frac{r_o^2+R^2}{r_o^2-R^2}=(18.7)\frac{1.5^2+1^2}{1.5^2-1^2}=48.6\ \text{kpsi} \quad Ans.$$

3-134 $d=20$ mm, $r_i=37.5$ mm, $r_o=57.5$ mm

From Table 3-4, for R = 10 mm,

$$r_c=37.5+10=47.5\ \text{mm}$$

$$r_n=\frac{10^2}{2\left(47.5-\sqrt{47.5^2-10^2}\right)}=46.96772\ \text{mm}$$

$$e=r_c-r_n=47.5-46.96772=0.53228\ \text{mm}$$

$$c_i=r_n-r_i=46.9677-37.5=9.4677\ \text{mm}$$

$$c_o=r_o-r_n=57.5-46.9677=10.5323\ \text{mm}$$

$$A=\pi d^2/4=\pi(20)^2/4=314.16\ \text{mm}^2$$

$$M=Fr_c=4000(47.5)=190\,000\ \text{N}\cdot\text{mm}$$

Using Eq. (3-65) for the bending stress, and combining with the axial stress,

$$\sigma_i = \frac{F}{A} + \frac{Mc_i}{Aer_i} = \frac{4000}{314.16} + \frac{190\ 000(9.4677)}{314.16(0.53228)(37.5)} = 300 \text{ MPa} \quad Ans.$$

$$\sigma_o = \frac{F}{A} - \frac{Mc_o}{Aer_o} = \frac{4000}{314.16} - \frac{190\ 000(10.5323)}{314.16(0.53228)(57.5)} = -195 \text{ MPa} \quad Ans.$$

3-135 $d = 0.75$ in, $r_i = 1.25$ in, $r_o = 2.0$ in

From Table 3-4, for R = 0.375 in,

$$r_c = 1.25 + 0.375 = 1.625 \text{ in}$$

$$r_n = \frac{0.375^2}{2\left(1.625 - \sqrt{1.625^2 - 0.375^2}\right)} = 1.60307 \text{ in}$$

$$e = r_c - r_n = 1.625 - 1.60307 = 0.02193 \text{ in}$$

$$c_i = r_n - r_i = 1.60307 - 1.25 = 0.35307 \text{ in}$$

$$c_o = r_o - r_n = 2.0 - 1.60307 = 0.39693 \text{ in}$$

$$A = \pi d^2 / 4 = \pi(0.75)^2 / 4 = 0.44179 \text{ in}^2$$

$$M = Fr_c = 750(1.625) = 1218.8 \text{ lbf} \cdot \text{in}$$

Using Eq. (3-65) for the bending stress, and combining with the axial stress,

$$\sigma_i = \frac{F}{A} + \frac{Mc_i}{Aer_i} = \frac{750}{0.44179} + \frac{1218.8(0.35307)}{0.44179(0.02193)(1.25)} = 37\ 230 \text{ psi} = 37.2 \text{ kpsi} \quad Ans.$$

$$\sigma_o = \frac{F}{A} - \frac{Mc_o}{Aer_o} = \frac{750}{0.44179} - \frac{1218.8(0.39693)}{0.44179(0.02193)(2.0)} = -23\ 269 \text{ psi} = -23.3 \text{ kpsi} \quad Ans.$$

3-136 $d = 6$ mm, $r_i = 10$ mm, $r_o = 16$ mm

From Table 3-4, for R = 3 mm,

$$r_c = 10 + 3 = 13 \text{ mm}$$

$$r_n = \frac{3^2}{2\left(13 - \sqrt{13^2 - 3^2}\right)} = 12.82456 \text{ mm}$$

$$e = r_c - r_n = 13 - 12.82456 = 0.17544 \text{ mm}$$

$$c_i = r_n - r_i = 12.82456 - 10 = 2.82456 \text{ mm}$$

$$c_o = r_o - r_n = 16 - 12.82456 = 3.17544 \text{ mm}$$

$$A = \pi d^2 / 4 = \pi(6)^2 / 4 = 28.2743 \text{ mm}^2$$

$$M = Fr_c = 300(13) = 3900 \text{ N} \cdot \text{mm}$$

Using Eq. (3-65) for the bending stress, and combining with the axial stress,

$$\sigma_i = \frac{F}{A} + \frac{Mc_i}{Aer_i} = \frac{300}{28.2743} + \frac{3900(2.82456)}{28.2743(0.17544)(10)} = 233 \text{ MPa} \quad Ans.$$

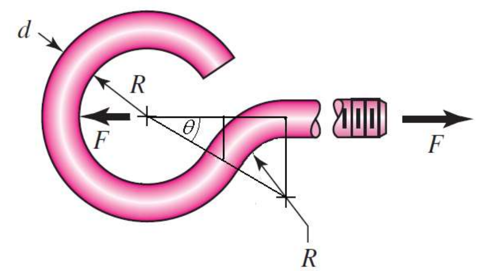
$$\sigma_o = \frac{F}{A} - \frac{Mc_o}{Aer_o} = \frac{300}{28.2743} - \frac{3900(3.17544)}{28.2743(0.17544)(16)} = -145 \text{ MPa} \quad Ans.$$

3-137 $d = 6$ mm, $r_i = 10$ mm, $r_o = 16$ mm

From Table 3-4, for R = 3 mm,

$$r_c = 10 + 3 = 13 \text{ mm}$$

$$r_n = \frac{3^2}{2\left(13 - \sqrt{13^2 - 3^2}\right)} = 12.82456 \text{ mm}$$



$$e = r_c - r_n = 13 - 12.82456 = 0.17544 \text{ mm}$$

$$c_i = r_n - r_i = 12.82456 - 10 = 2.82456 \text{ mm}$$

$$c_o = r_o - r_n = 16 - 12.82456 = 3.17544 \text{ mm}$$

$$A = \pi d^2/4 = \pi(6)^2/4 = 28.2743 \text{ mm}^2$$

The angle θ of the line of radius centers is

$$\theta = \sin^{-1}\left(\frac{R + d/2}{R + d + R}\right) = \sin^{-1}\left(\frac{10 + 6/2}{10 + 6 + 10}\right) = 30^\circ$$

$$M = F\left(R + d/2\right)\sin\theta = 300\left(10 + 6/2\right)\sin 30^\circ = 1950 \text{ N}\cdot\text{mm}$$

Using Eq. (3-65) for the bending stress, and combining with the axial stress,

$$\sigma_i = \frac{F\sin\theta}{A} + \frac{Mc_i}{Aer_i} = \frac{300\sin 30^\circ}{28.2743} + \frac{1950(2.82456)}{28.2743(0.17544)(10)} = 116 \text{ MPa} \quad Ans.$$

$$\sigma_o = \frac{F\sin\theta}{A} - \frac{Mc_o}{Aer_o} = \frac{300\sin 30^\circ}{28.2743} - \frac{1950(3.17544)}{28.2743(0.17544)(16)} = -72.7 \text{ MPa} \quad Ans.$$

Note that the shear stress due to the shear force is zero at the surface.

3-138 $d = 0.25$ in, $r_i = 0.5$ in, $r_o = 0.75$ in

From Table 3-4, for R = 0.125 in,

$$r_c = 0.5 + 0.125 = 0.625 \text{ in}$$

$$r_n = \frac{0.125^2}{2\left(0.625 - \sqrt{0.625^2 - 0.125^2}\right)} = 0.618686 \text{ in}$$

$$e = r_c - r_n = 0.625 - 0.618686 = 0.006314 \text{ in}$$

$$c_i = r_n - r_i = 0.618686 - 0.5 = 0.118686 \text{ in}$$

$$c_o = r_o - r_n = 0.75 - 0.618686 = 0.131314 \text{ in}$$

$$A = \pi d^2 / 4 = \pi(0.25)^2 / 4 = 0.049087 \text{ in}^2$$
$$M = Fr_c = 75(0.625) = 46.875 \text{ lbf} \cdot \text{in}$$

Using Eq. (3-65) for the bending stress, and combining with the axial stress,

$$\sigma_i = \frac{F}{A} + \frac{Mc_i}{Aer_i} = \frac{75}{0.049087} + \frac{46.875(0.118686)}{0.049087(0.006314)(0.5)} = 37\,428 \text{ psi} = 37.4 \text{ kpsi} \quad \textit{Ans.}$$

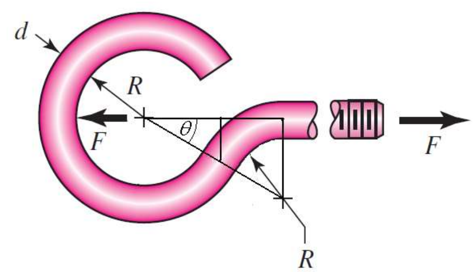
$$\sigma_o = \frac{F}{A} - \frac{Mc_o}{Aer_o} = \frac{75}{0.049087} - \frac{46.875(0.131314)}{0.049087(0.006314)(0.75)} = -24\,952 \text{ psi} = -25.0 \text{ kpsi} \quad \textit{Ans.}$$

3-139 $d = 0.25$ in, $r_i = 0.5$ in, $r_o = 0.75$ in

From Table 3-4, for R = 0.125 in,

$$r_c = 0.5 + 0.125 = 0.625 \text{ in}$$

$$r_n = \frac{0.125^2}{2\left(0.625 - \sqrt{0.625^2 - 0.125^2}\right)} = 0.618686 \text{ in}$$



$$e = r_c - r_n = 0.625 - 0.618686 = 0.006314 \text{ in}$$
$$c_i = r_n - r_i = 0.618686 - 0.5 = 0.118686 \text{ in}$$
$$c_o = r_o - r_n = 0.75 - 0.618686 = 0.131314 \text{ in}$$
$$A = \pi d^2 / 4 = \pi(0.25)^2 / 4 = 0.049087 \text{ in}^2$$

The angle θ of the line of radius centers is

$$\theta = \sin^{-1}\left(\frac{R + d/2}{R + d + R}\right) = \sin^{-1}\left(\frac{0.5 + 0.25/2}{0.5 + 0.25 + 0.5}\right) = 30^\circ$$

$$M = F\left(R + d/2\right)\sin\theta = 75\left(0.5 + 0.25/2\right)\sin 30^\circ = 23.44 \text{ lbf} \cdot \text{in}$$

Using Eq. (3-65) for the bending stress, and combining with the axial stress,

$$\sigma_i = \frac{F\sin\theta}{A} + \frac{Mc_i}{Aer_i} = \frac{75\sin 30^\circ}{0.049087} + \frac{23.44(0.118686)}{0.049087(0.006314)(0.5)} = 18\,716 \text{ psi} = 18.7 \text{ kpsi} \quad \textit{Ans.}$$

$$\sigma_o = \frac{F\sin\theta}{A} - \frac{Mc_o}{Aer_o} = \frac{75\sin 30^\circ}{0.049087} - \frac{23.44(0.131314)}{0.049087(0.006314)(0.75)} = -12\,478 \text{ psi} = -12.5 \text{ kpsi} \quad \textit{Ans.}$$

Note that the shear stress due to the shear force is zero at the surface.

3-140

(a) $\sigma = \pm\dfrac{Mc}{I} = \pm\dfrac{[3(4)][0.5(0.1094)]}{(0.75)\left(0.1094^3\right)/12} = \pm 8021 \text{ psi} = \pm 8.02 \text{ kpsi} \quad \textit{Ans.}$

(b) r_i = 0.125 in, $r_o = r_i + h$ = 0.125 + 0.1094 = 0.2344 in

From Table 3-4,

$$r_c = 0.125 + (0.5)(0.1094) = 0.1797 \text{ in}$$
$$r_n = \frac{0.1094}{\ln(0.2344/0.125)} = 0.174006 \text{ in}$$
$$e = r_c - r_n = 0.1797 - 0.174006 = 0.005694 \text{ in}$$
$$c_i = r_n - r_i = 0.174006 - 0.125 = 0.049006 \text{ in}$$
$$c_o = r_o - r_n = 0.2344 - 0.174006 = 0.060394 \text{ in}$$
$$A = bh = 0.75(0.1094) = 0.08205 \text{ in}^2$$
$$M = -3(4) = -12 \text{ lbf} \cdot \text{in}$$

The negative sign on the bending moment is due to the sign convention shown in Fig. 3-35. Using Eq. (3-65),

$$\sigma_i = \frac{Mc_i}{Aer_i} = \frac{-12(0.049006)}{0.08205(0.005694)(0.125)} = -10\ 070 \text{ psi} = -10.1 \text{ kpsi} \quad \textit{Ans.}$$
$$\sigma_o = -\frac{Mc_o}{Aer_o} = -\frac{-12(0.060394)}{0.08205(0.005694)(0.2344)} = 6618 \text{ psi} = 6.62 \text{ kpsi} \quad \textit{Ans.}$$

(c) $K_i = \dfrac{\sigma_i}{\sigma} = \dfrac{-10.1}{-8.02} = 1.26 \quad \textit{Ans.}$

$$K_o = \frac{\sigma_o}{\sigma} = \frac{6.62}{8.02} = 0.825 \quad \textit{Ans.}$$

3-141

(a) $\sigma = \pm\dfrac{Mc}{I} = \pm\dfrac{[3(4)][0.5(0.1406)]}{(0.75)(0.1406^3)/12} = \pm 4856 \text{ psi} = \pm 4.86 \text{ kpsi} \quad \textit{Ans.}$

(b) $r_i = 0.125$ in, $r_o = r_i + h = 0.125 + 0.1406 = 0.2656$ in

From Table 3-4,

$$r_c = 0.125 + (0.5)(0.1406) = 0.1953 \text{ in}$$
$$r_n = \frac{0.1406}{\ln(0.2656/0.125)} = 0.186552 \text{ in}$$
$$e = r_c - r_n = 0.1953 - 0.186552 = 0.008748 \text{ in}$$
$$c_i = r_n - r_i = 0.186552 - 0.125 = 0.061552 \text{ in}$$
$$c_o = r_o - r_n = 0.2656 - 0.186552 = 0.079048 \text{ in}$$
$$A = bh = 0.75(0.1406) = 0.10545 \text{ in}^2$$
$$M = -3(4) = -12 \text{ lbf} \cdot \text{in}$$

The negative sign on the bending moment is due to the sign convention shown in Fig. 3-35. Using Eq. (3-65),

$$\sigma_i = \frac{Mc_i}{Aer_i} = \frac{-12(0.061552)}{0.10545(0.008748)(0.125)} = -6406 \text{ psi} = -6.41 \text{ kpsi} \quad \textit{Ans.}$$

$$\sigma_o = -\frac{Mc_o}{Aer_o} = -\frac{-12(0.079048)}{0.10545(0.008748)(0.2656)} = 3872 \text{ psi} = 3.87 \text{ kpsi} \quad \textit{Ans.}$$

(c) $K_i = \dfrac{\sigma_i}{\sigma} = \dfrac{-6.41}{-4.86} = 1.32 \quad \textit{Ans.}$

$K_o = \dfrac{\sigma_o}{\sigma} = \dfrac{3.87}{4.86} = 0.80 \quad \textit{Ans.}$

3-142

(a) $\sigma = \pm\dfrac{Mc}{I} = \pm\dfrac{[3(4)][0.5(0.1094)]}{(0.75)\left(0.1094^3\right)/12} = \pm 8021 \text{ psi} = \pm 8.02 \text{ kpsi} \quad \textit{Ans.}$

(b) $r_i = 0.25$ in, $r_o = r_i + h = 0.25 + 0.1094 = 0.3594$ in

From Table 3-4,

$$r_c = 0.25 + (0.5)(0.1094) = 0.3047 \text{ in}$$

$$r_n = \frac{0.1094}{\ln(0.3594/0.25)} = 0.301398 \text{ in}$$

$$e = r_c - r_n = 0.3047 - 0.301398 = 0.003302 \text{ in}$$

$$c_i = r_n - r_i = 0.301398 - 0.25 = 0.051398 \text{ in}$$

$$c_o = r_o - r_n = 0.3594 - 0.301398 = 0.058002 \text{ in}$$

$$A = bh = 0.75(0.1094) = 0.08205 \text{ in}^2$$

$$M = -3(4) = -12 \text{ lbf}\cdot\text{in}$$

The negative sign on the bending moment is due to the sign convention shown in Fig. 3-35. Using Eq. (3-65),

$$\sigma_i = \frac{Mc_i}{Aer_i} = \frac{-12(0.051398)}{0.08205(0.003302)(0.25)} = -9106 \text{ psi} = -9.11 \text{ kpsi} \quad \textit{Ans.}$$

$$\sigma_o = -\frac{Mc_o}{Aer_o} = -\frac{-12(0.058002)}{0.08205(0.003302)(0.3594)} = 7148 \text{ psi} = 7.15 \text{ kpsi} \quad \textit{Ans.}$$

(c) $K_i = \dfrac{\sigma_i}{\sigma} = \dfrac{-9.11}{-8.02} = 1.14 \quad \textit{Ans.}$

$K_o = \dfrac{\sigma_o}{\sigma} = \dfrac{7.15}{8.02} = 0.89 \quad \textit{Ans.}$

3-143 $r_i = 25$ mm, $r_o = r_i + h = 25 + 87 = 112$ mm, $r_c = 25 + 87/2 = 68.5$ mm

The radius of the neutral axis is found from Eq. (3-63), given below.

$$r_n = \frac{A}{\int (dA/r)}$$

For a rectangular area with constant width b, the denominator is

$$\int_{r_i}^{r_o} \left(\frac{b\,dr}{r} \right) = b \ln \frac{r_o}{r_i}$$

Applying this equation over each of the four rectangular areas,

$$\int \frac{dA}{r} = 9\left(\ln \frac{45}{25} \right) + 31\left(\ln \frac{54.5}{45} \right) + 31\left(\ln \frac{92}{82.5} \right) + 9\left(\ln \frac{112}{92} \right) = 16.3769$$

$$A = 2\left[20(9) + 31(9.5) \right] = 949 \text{ mm}^2$$

$$r_n = \frac{A}{\int (dA/r)} = \frac{949}{16.3769} = 57.9475 \text{ mm}$$

$$e = r_c - r_n = 68.5 - 57.9475 = 10.5525 \text{ mm}$$

$$c_i = r_n - r_i = 57.9475 - 25 = 32.9475 \text{ mm}$$

$$c_o = r_o - r_n = 112 - 57.9475 = 54.0525 \text{ mm}$$

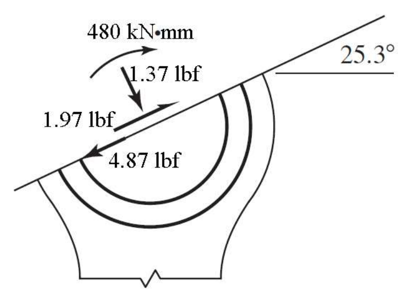
$$M = 150F_2 = 150(3.2) = 480 \text{ kN·mm}$$

We need to find the forces transmitted through the section in order to determine the axial stress. It is not immediately obvious which plane should be used for resolving the axial versus shear directions. It is convenient to use the plane containing the reaction force at the bushing, which assumes its contribution resolves entirely into shear force. To find the angle of this plane, find the resultant of F_1 and F_2.

$$F_x = F_{1x} + F_{2x} = 2.4\cos 60^\circ + 3.2\cos 0^\circ = 4.40 \text{ kN}$$

$$F_y = F_{1y} + F_{2y} = 2.4\sin 60^\circ + 3.2\sin 0^\circ = 2.08 \text{ kN}$$

$$F = \left(4.40^2 + 2.08^2 \right)^{1/2} = 4.87 \text{ kN}$$



This is the pin force on the lever which acts in a direction

$$\theta = \tan^{-1} \frac{F_y}{F_x} = \tan^{-1} \frac{2.08}{4.40} = 25.3^\circ$$

On the surface 25.3° from the horizontal, find the internal forces in the tangential and normal directions. Resolving F_1 into components,

$$F_t = 2.4\cos\left(60^\circ - 25.3^\circ \right) = 1.97 \text{ kN}$$

$$F_n = 2.4\sin\left(60^\circ - 25.3^\circ \right) = 1.37 \text{ kN}$$

The transverse shear stress is zero at the inner and outer surfaces. Using Eq. (3-65) for the bending stress, and combining with the axial stress due to F_n,

$$\sigma_i = \frac{F_n}{A} + \frac{Mc_i}{Aer_i} = \frac{1370}{949} + \frac{\left[(3200)(150)\right](32.9475)}{949(10.5525)(25)} = 64.6 \text{ MPa} \quad Ans.$$

$$\sigma_o = \frac{F_n}{A} - \frac{Mc_o}{Aer_o} = \frac{1370}{949} - \frac{\left[(3200)(150)\right](54.0525)}{949(10.5525)(112)} = -21.7 \text{ MPa} \quad Ans.$$

3-144 $r_i = 2$ in, $r_o = r_i + h = 2 + 4 = 6$ in, $r_c = 2 + 0.5(4) = 4$ in

$$A = (6 - 2 - 0.75)(0.75) = 2.4375 \text{ in}^2$$

Similar to Prob. 3-143,

$$\int \frac{dA}{r} = 0.75 \ln \frac{3.625}{2} + 0.75 \ln \frac{6}{4.375} = 0.682\,920 \text{ in}$$

$$r_n = \frac{A}{\int (dA/r)} = \frac{2.4375}{0.682\,920} = 3.56923 \text{ in}$$

$$e = r_c - r_n = 4 - 3.56923 = 0.43077 \text{ in}$$

$$c_i = r_n - r_i = 3.56923 - 2 = 1.56923 \text{ in}$$

$$c_o = r_o - r_n = 6 - 3.56923 = 2.43077 \text{ in}$$

$$M = Fr_c = 6000(4) = 24\,000 \text{ lbf} \cdot \text{in}$$

Using Eq. (3-65) for the bending stress, and combining with the axial stress,

$$\sigma_i = \frac{F}{A} + \frac{Mc_i}{Aer_i} = \frac{6000}{2.4375} + \frac{24\,000(1.56923)}{2.4375(0.43077)(2)} = 20\,396 \text{ psi} = 20.4 \text{ kpsi} \quad Ans.$$

$$\sigma_o = \frac{F}{A} - \frac{Mc_o}{Aer_o} = \frac{6000}{2.4375} - \frac{24\,000(2.43077)}{2.4375(0.43077)(6)} = -6\,799 \text{ psi} = -6.80 \text{ kpsi} \quad Ans.$$

3-145 $r_i = 12$ in, $r_o = r_i + h = 12 + 3 = 15$ in, $r_c = 12 + 3/2 = 13.5$ in

$$I = \frac{\pi}{4} a^3 b = \frac{\pi}{4}(1.5^3)(0.75) = 1.988 \text{ in}^4$$

$$A = \pi ab = \pi(1.5)(0.75) = 3.534$$

$$M = 20(3 + 1.5) = 90 \text{ kip} \cdot \text{in}$$

Since the radius is large compared to the cross section, assume Eq. 3-67 is applicable for the bending stress. Combining the bending stress and the axial stress,

$$\sigma_i = \frac{F}{A} + \frac{Mc_i r_c}{Ir_i} = \frac{20}{3.534} + \frac{90(1.5)(13.5)}{(1.988)(12)} = 82.1 \text{ kpsi} \quad Ans.$$

$$\sigma_o = \frac{F}{A} - \frac{Mc_o r_c}{Ir_o} = \frac{20}{3.534} - \frac{90(1.5)(13.5)}{1.988(15)} = -55.5 \text{ kpsi} \quad Ans.$$

3-146 $r_i = 1.25$ in, $r_o = r_i + h = 1.25 + 0.5 + 1 + 0.5 = 3.25$ in

$r_c = (r_i + r_o) / 2 =$ (1.25 + 3.25)/2 = 2.25 in *Ans.*

For outer rectangle, $\left(\int \dfrac{dA}{r}\right)_{\square} = b \ln \dfrac{r_o}{r_i}$

For circle, $\left[\dfrac{A}{\int (dA/r)}\right]_{\text{O}} = \left[\dfrac{r^2}{2\left(r_c - \sqrt{r_c^2 - r^2}\right)}\right]_{\text{O}}, \quad A_{\text{O}} = \pi r^2$

$$\therefore \left[\int \frac{dA}{r}\right]_{\text{O}} = 2\pi (r_c - \sqrt{r_c^2 - r^2})$$

Combine the integrals subtracting the circle from the rectangle

$$\Sigma \int \frac{dA}{r} = 1.25 \ln \frac{3.25}{1.25} - 2\pi\left(2.25 - \sqrt{2.25^2 - 0.5^2}\right) = 0.840\,904 \text{ in}$$

$$A = 1.25(2) - \pi(0.5^2) = 1.714\,60 \text{ in}^2 \qquad \textit{Ans.}$$

$$r_n = \frac{A}{\sum \int (dA/r)} = \frac{1.714\,60}{0.840\,904} = 2.0390 \text{ in} \qquad \textit{Ans.}$$

$$e = r_c - r_n = 2.25 - 2.0390 = 0.2110 \text{ in} \qquad \textit{Ans.}$$

$$c_i = r_n - r_i = 2.0390 - 1.25 = 0.7890 \text{ in}$$

$$c_o = r_o - r_n = 3.25 - 2.0390 = 1.2110 \text{ in}$$

$$M = 2000(4.5 + 1.25 + 0.5 + 0.5) = 13\,500 \text{ lbf} \cdot \text{in}$$

$$\sigma_i = \frac{F}{A} + \frac{Mc_i}{Aer_i} = \frac{2000}{1.7146} + \frac{13\,500(0.7890)}{1.7146(0.2110)(1.25)} = 20\,720 \text{ psi} = 20.7 \text{ kpsi} \qquad \textit{Ans.}$$

$$\sigma_o = \frac{F}{A} - \frac{Mc_o}{Aer_o} = \frac{2000}{1.7146} - \frac{13\,500(1.2110)}{1.7146(0.2110)(3.25)} = -12\,738 \text{ psi} = -12.7 \text{ kpsi} \qquad \textit{Ans.}$$

3-147 Table A-5: Glass: E_G = 46.2 GPa, ν_G = 0.245, Steel: E_S =207 GPa, ν_S = 0.292

Eq. (3-68):

$$a = \sqrt[3]{\frac{3F}{8} \frac{\left(1-\nu_1^2\right)/E_1 + \left(1-\nu_2^2\right)/E_2}{1/d_1 + 1/d_2}}$$

$$= \sqrt[3]{\frac{3(5)}{8} \frac{\left(1-0.245^2\right)/\left[46.2\left(10^9\right)\right] + \left(1-0.292^2\right)/\left[207\left(10^9\right)\right]}{1/0.030 + 1/\infty}}$$

$$= 1.1168\left(10^{-4}\right) \text{ m} = 0.11168 \text{ mm}$$

Eq. (3-69):

$$p_{\max} = \frac{3F}{2\pi a^2} = \frac{3(5)}{2\pi\left(0.11168\right)^2} = 191.4 \text{ MPa} \qquad \textit{Ans.}$$

(b) Eq. (3-70):

$$\sigma_1 = \sigma_2 = -p_{max}\left\{\left(1 - \left|\frac{z}{a}\right|\tan^{-1}\frac{1}{|z/a|}\right)(1+\nu_G) - \frac{1}{2\left[1+(z/a)^2\right]}\right\}$$

$$= -191.4\left\{\left[1 - \left(\frac{z}{0.11168}\right)\tan^{-1}\frac{1}{(z/0.11168)}\right](1+0.245) - \frac{1}{2\left[1+(z/0.11168)^2\right]}\right\} \qquad (1)$$

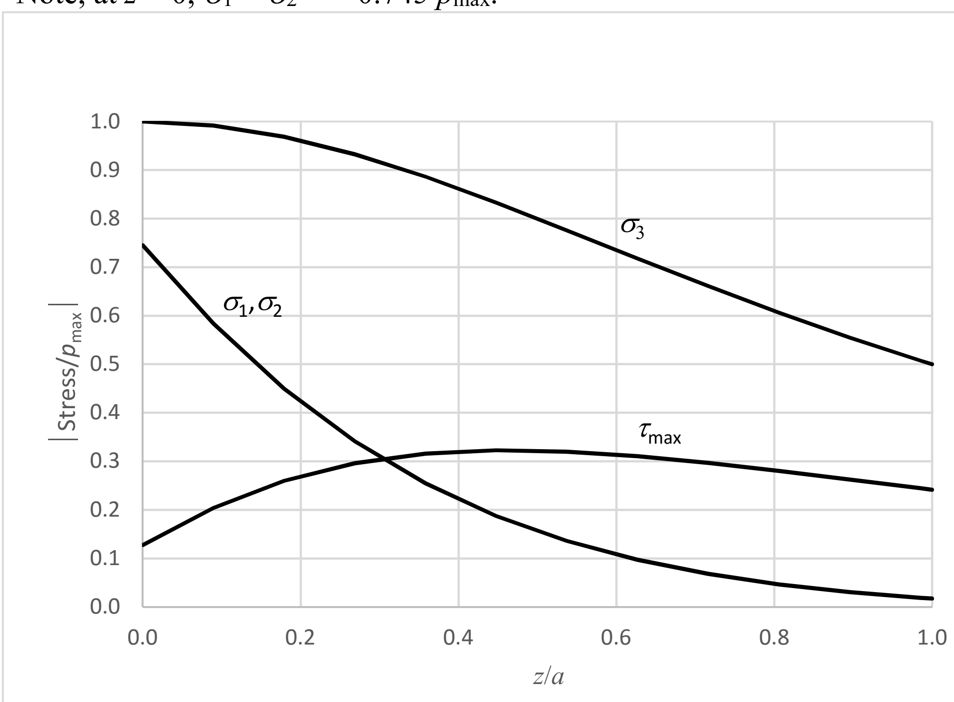
Eq. (3-71):

$$\sigma_3 = \frac{-p_{max}}{1+(z/a)^2} = \frac{-191.4}{1+(z/0.11168)^2} \qquad (2)$$

The maximum shear stress is given by

$$\tau_{max} = \left|\frac{\sigma_1 - \sigma_3}{2}\right| \qquad (3)$$

(c) Plots of Eqs. (1) – (3) as absolute dimensionless stresses, $|\text{stress} / p_{max}|$, are given. Note, at $z = 0$, $\sigma_1 = \sigma_2 = -0.745\, p_{max}$.



Ans.

(d) $\tau_{max} \approx 0.323\, p_{max} = 0.323(191.4) = 61.8$ MPa at $z = 0.05$ mm *Ans.*

(e) From Fig. 3-38, $\tau_{max} = 0.3(191.4) = 57.4$ MPa *Ans.*

3-148 From Eq. (3-68),

$$a = KF^{1/3} = F^{1/3}\left\{\left(\frac{3}{8}\right)\frac{2\left[\left(1-\nu^2\right)/E\right]}{2\left(1/d\right)}\right\}^{1/3}$$

Use ν = 0.292, F in newtons, E in N/mm^2 and d in mm, then

$$K = \left\{\left(\frac{3}{8}\right)\frac{[(1-0.292^2)/207\,000]}{1/30}\right\}^{1/3} = 0.03685$$

From Eq. (3-69),

$$p_{\max} = \frac{3F}{2\pi a^2} = \frac{3F}{2\pi(KF^{1/3})^2} = \frac{3F^{1/3}}{2\pi K^2} = \frac{3F^{1/3}}{2\pi(0.03685)^2} = 352F^{1/3}\ \text{MPa}$$

From Eq. (3-71), the maximum principal stress occurs on the surface where z = 0, and is equal to $-\,p_{\max}$.

$$\sigma_{\max} = \sigma_z = -p_{\max} = -352F^{1/3}\ \text{MPa} \qquad \textit{Ans.}$$

From Fig. 3-38,

$$\tau_{\max} = 0.3p_{\max} = 106F^{1/3}\ \text{MPa} \qquad \textit{Ans.}$$

3-149 From Eq. (3-68),

$$a = \sqrt[3]{\left(\frac{3F}{8}\right)\frac{\left(1-\nu_1^2\right)/E_1 + \left(1-\nu_2^2\right)/E_2}{1/d_1 + 1/d_2}}$$

$$a = \sqrt[3]{\left(\frac{3(10)}{8}\right)\frac{\left(1-0.292^2\right)/\left(207\,000\right) + \left(1-0.333^2\right)/\left(71\,700\right)}{1/25 + 1/40}} = 0.0990\ \text{mm}$$

From Eq. (3-69),

$$p_{\max} = \frac{3F}{2\pi a^2} = \frac{3(10)}{2\pi\left(0.0990^2\right)} = 487.2\ \text{MPa}$$

From Fig. 3-38, the maximum shear stress occurs at a depth of z = 0.48 a.

$$z = 0.48a = 0.48(0.0990) = 0.0475\ \text{mm} \qquad \textit{Ans.}$$

The principal stresses are obtained from Eqs. (3-70) and (3-71) at a depth of z/a = 0.48.

$$\sigma_1 = \sigma_2 = -487.2\left\{\left[1-0.48\tan^{-1}\left(1/0.48\right)\right](1+0.333) - \frac{1}{2\left(1+0.48^2\right)}\right\} = -101.3\ \text{MPa}$$

$$\sigma_3 = \frac{-487.2}{1+0.48^2} = -396.0\ \text{MPa}$$

From Eq. (3-72),

$$\tau_{\max} = \frac{\sigma_1 - \sigma_3}{2} = \frac{(-101.3) - (-396.0)}{2} = 147.4 \text{ MPa} \qquad \textit{Ans.}$$

Note that if a closer examination of the applicability of the depth assumption from Fig. 3-38 is desired, implementing Eqs. (3-70), (3-71), and (3-72) on a spreadsheet will allow for calculating and plotting the stresses versus the depth for specific values of ν. For ν = 0.333 for aluminum, the maximum shear stress occurs at a depth of $z = 0.492a$ with τ_{max} = 0.3025 p_{max}.

This gives τ_{max} = 0.3025 p_{max} = (0.3025)(487.2) = 147.38 MPa. Even though the depth assumption was a little off, it did not have significant effect on the the maximum shear stress.

3-150 From the solution to Prob. 3-149, a = 0.0990 mm and p_{max} = 487.2 MPa. Assuming applicability of Fig. 3-38, the maximum shear stress occurs at a depth of z = 0.48 a = 0.0475 mm. *Ans.*

The principal stresses are obtained from Eqs. (3-70) and (3-71) at a depth of z/a = 0.48.

$$\sigma_1 = \sigma_2 = -487.2\left\{\left[1 - 0.48\tan^{-1}(1/0.48)\right](1+0.292) - \frac{1}{2(1+0.48^2)}\right\} = -92.09 \text{ MPa}$$

$$\sigma_3 = \frac{-487.2}{1+0.48^2} = -396.0 \text{ MPa}$$

From Eq. (3-72),

$$\tau_{\max} = \frac{\sigma_1 - \sigma_3}{2} = \frac{(-92.09) - (-396.0)}{2} = 152.0 \text{ MPa} \qquad \textit{Ans.}$$

Note that if a closer examination of the applicability of the depth assumption from Fig. 3-38 is desired, implementing Eqs. (3-70), (3-71), and (3-72) on a spreadsheet will allow for calculating and plotting the stresses versus the depth for specific values of ν. For ν = 0.292 for steel, the maximum shear stress occurs at a depth of $z = 0.478a$ with τ_{max} = 0.3119 p_{max}.

3-151 From Eq. (3-68),

$$a = \sqrt[3]{\left(\frac{3F}{8}\right)\frac{2(1-\nu^2)/E}{1/d_1 + 1/d_2}}$$

$$a = \sqrt[3]{\left(\frac{3(20)}{8}\right)\frac{2(1-0.292^2)/(207\,000)}{1/30 + 1/\infty}} = 0.1258 \text{ mm}$$

From Eq. (3-69),

$$p_{\max} = \frac{3F}{2\pi a^2} = \frac{3(20)}{2\pi\left(0.1258^2\right)} = 603.4 \text{ MPa}$$

From Fig. 3-38, the maximum shear stress occurs at a depth of

$$z = 0.48a = 0.48(0.1258) = 0.0604 \text{ mm} \qquad \textit{Ans.}$$

Also from Fig. 3-38, the maximum shear stress is

$$\tau_{\max} = 0.3p_{\max} = 0.3(603.4) = 181 \text{ MPa} \qquad \textit{Ans.}$$

3-152 Aluminum **Plate-Ball interface:** From Eq. (3-68),

$$a = \sqrt[3]{\left(\frac{3F}{8}\right)\frac{\left(1-\nu_1^2\right)/E_1 + \left(1-\nu_2^2\right)/E_2}{1/d_1 + 1/d_2}}$$

$$a = \sqrt[3]{\left(\frac{3F}{8}\right)\frac{\left(1-0.292^2\right)/\left[(30)\left(10^6\right)\right] + \left(1-0.333^2\right)/\left[(10.4)\left(10^6\right)\right]}{1/1 + 1/\infty}} = 3.517\left(10^{-3}\right)F^{1/3} \text{ in}$$

From Eq. (3-69),

$$p_{\max} = \frac{3F}{2\pi a^2} = \frac{3F}{2\pi\left[3.517\left(10^{-3}\right)F^{1/3}\right]^2} = 3.860\left(10^4\right)F^{1/3} \text{ psi}$$

By examination of Eqs. (3-70), (3-71), and (3-72), it can be seen that the only difference in the maximum shear stress for the plate and the ball will be due to Poisson's ratio in Eq. (3-70). The larger Poisson's ratio will create the greater maximum shear stress, so the aluminum plate will be the critical element in this interface. Applying the equations for the aluminum plate,

$$\sigma_1 = -3.86\left(10^4\right)F^{1/3}\left\{\left[1 - 0.48\tan^{-1}(1/0.48)\right](1+0.333) - \frac{1}{2\left(1+0.48^2\right)}\right\} = -8025F^{1/3} \text{ psi}$$

$$\sigma_3 = \frac{-3.86\left(10^4\right)F^{1/3}}{1+0.48^2} = -3.137\left(10^4\right)F^{1/3} \text{ psi}$$

From Eq. (3-72),

$$\tau_{\max} = \frac{\sigma_1 - \sigma_3}{2} = \frac{\left(-8025F^{1/3}\right) - \left(-3.137\left(10^4\right)F^{1/3}\right)}{2} = 1.167\left(10^4\right)F^{1/3} \text{ psi}$$

Comparing this stress to the allowable stress, and solving for F,

$$F = \left[\frac{20\,000}{1.167\left(10^4\right)}\right]^3 = 5.03 \text{ lbf}$$

Table-Ball interface: From Eq. (3-68),

$$a = \sqrt[3]{\left(\frac{3F}{8}\right)\frac{\left(1-0.292^2\right)/\left[(30)\left(10^6\right)\right]+\left(1-0.211^2\right)/\left[(14.5)\left(10^6\right)\right]}{1/1+1/\infty}} = 3.306\left(10^{-3}\right)F^{1/3} \text{ in}$$

From Eq. (3-69),

$$p_{max} = \frac{3F}{2\pi a^2} = \frac{3F}{2\pi\left[3.306\left(10^{-3}\right)F^{1/3}\right]^2} = 4.369\left(10^4\right)F^{1/3} \text{ psi}$$

The steel ball has a higher Poisson's ratio than the cast iron table, so it will dominate.

$$\sigma_1 = -4.369\left(10^4\right)F^{1/3}\left\{\left[1-0.48\tan^{-1}(1/0.48)\right](1+0.292) - \frac{1}{2\left(1+0.48^2\right)}\right\} = -8258F^{1/3} \text{ psi}$$

$$\sigma_3 = \frac{-4.369\left(10^4\right)F^{1/3}}{1+0.48^2} = -3.551\left(10^4\right)F^{1/3} \text{ psi}$$

From Eq. (3-72),

$$\tau_{max} = \frac{\sigma_1-\sigma_3}{2} = \frac{\left(-8258F^{1/3}\right)-\left(-3.551\left(10^4\right)F^{1/3}\right)}{2} = 1.363\left(10^4\right)F^{1/3} \text{ psi}$$

Comparing this stress to the allowable stress, and solving for F,

$$F = \left[\frac{20\,000}{1.363\left(10^4\right)}\right]^3 = 3.16 \text{ lbf}$$

The steel ball is critical, with F = 3.16 lbf. *Ans.*

3-153 $\nu_1 = 0.333$, $E_1 = 10.4$ Mpsi, $l = 2$ in, $d_1 = 1.25$ in, $\nu_2 = 0.211$, $E_2 = 14.5$ Mpsi, $d_2 = -12$ in.

From Eq. 3-73, with $b = K_c F^{1/2}$

$$K_c = \left(\frac{2}{\pi(2)}\frac{\left(1-0.333^2\right)/\left[10.4\left(10^6\right)\right]+\left(1-0.211^2\right)/\left[14.5\left(10^6\right)\right]}{1/1.25+1/(-12)}\right)^{1/2}$$

$$= 2.593\left(10^{-4}\right)$$

By examination of Eqs. (3-75), (3-76), and (3-77), it can be seen that the only difference in the maximum shear stress for the two materials will be due to Poisson's ratio in Eq. (3-75). The larger Poisson's ratio will create the greater maximum shear stress, so the aluminum roller will be the critical element in this interface. Instead of applying these equations, we will assume the Poisson's ratio for aluminum of 0.333 is close enough to 0.3 to make Fig. 3-40 applicable.

$$\tau_{\max} = 0.3 p_{\max}$$

$$p_{\max} = \frac{4000}{0.3} = 13\ 300 \text{ psi}$$

From Eq. (3-74), $p_{\max} = 2F / (\pi bl)$, so we have

$$p_{\max} = \frac{2F}{\pi l K_c F^{1/2}} = \frac{2F^{1/2}}{\pi l K_c}$$

So,

$$F = \left(\frac{\pi l K_c p_{\max}}{2}\right)^2$$

$$= \left(\frac{\pi(2)(2.593)\left(10^{-4}\right)(13\ 300)}{2}\right)^2$$

$$= 117.4 \text{ lbf} \quad \textit{Ans.}$$

3-154

$v = 0.292$, $E = 30$ Mpsi, $l = 0.75$ in, $d_1 = 2(0.47) = 0.94$ in, $d_2 = 2(0.62) = 1.24$ in.
Eq. (3-73):

$$b = \left(\frac{2(40)}{\pi(0.75)} \frac{2\left(1-0.292^2\right)\Big/\left[30\left(10^6\right)\right]}{1/0.94 + 1/1.24}\right)^{1/2} = 1.052\left(10^{-3}\right) \text{ in}$$

Eq. (3-74):

$$p_{\max} = \frac{2F}{\pi bl} == \frac{2(40)}{\pi(1.052)\left(10^{-3}\right)(0.75)} = 32\ 275 \text{ psi} = 32.3 \text{ kpsi} \qquad \textit{Ans.}$$

From Fig. 3-40,

$$\tau_{\max} = 0.3 p_{\max} = 0.3(32\ 275) = 9682.5 \text{ psi} = 9.68 \text{ kpsi} \qquad \textit{Ans.}$$

3-155 Use Eqs. (3-73) through (3-77).

$$b = \left(\frac{2F}{\pi l} \frac{(1-\nu_1^2)/E_1 + (1-\nu_2^2)/E_2}{(1/d_1) + (1/d_2)}\right)^{1/2}$$

$$= \left(\frac{2(600)}{\pi(2)} \frac{(1-0.292^2)/(30(10^6)) + (1-0.292^2)/(30(10^6))}{1/5 + 1/\infty}\right)^{1/2}$$

$$b = 0.007\ 631 \text{ in}$$

$$p_{\max} = \frac{2F}{\pi bl} = \frac{2(600)}{\pi(0.007\ 631)(2)} = 25\ 028 \text{ psi}$$

$$\sigma_x = -2\nu p_{\max}\left(\sqrt{1+\frac{z^2}{b^2}} - \left|\frac{z}{b}\right|\right) = -2(0.292)(25\ 028)\left(\sqrt{1+0.786^2} - 0.786\right)$$

$$= -7102 \text{ psi} = -7.10 \text{ kpsi} \qquad \textit{Ans.}$$

$$\sigma_y = -p_{\max}\left(\frac{1+2\dfrac{z^2}{b^2}}{\sqrt{1+\dfrac{z^2}{b^2}}} - 2\left|\frac{z}{b}\right|\right) = -25\ 028\left(\frac{1+2\left(0.786^2\right)}{\sqrt{1+\left(0.786^2\right)}} - 2(0.786)\right)$$

$$= -4\ 646 \text{ psi} = -4.65 \text{ kpsi} \qquad \textit{Ans.}$$

$$\sigma_z = \frac{-p_{\max}}{\sqrt{1+\dfrac{z^2}{b^2}}} = \frac{-25\ 028}{\sqrt{1+0.786^2}} = -19\ 677 \text{ psi} = -19.7 \text{ kpsi} \qquad \textit{Ans.}$$

$$\tau_{\max} = \frac{\sigma_y - \sigma_z}{2} = \frac{-4\ 646 - (-19\ 677)}{2} = 7\ 516 \text{ psi} = 7.52 \text{ kpsi} \qquad \textit{Ans.}$$

3-156 Use Eqs. (3-73) through (3-77).

$$b = \left(\frac{2F}{\pi l}\frac{\left(1-\nu_1^2\right)/E_1 + \left(1-\nu_2^2\right)/E_2}{1/d_1 + 1/d_2}\right)^{1/2}$$

$$= \left(\frac{2(2000)}{\pi(40)}\frac{\left(1-0.292^2\right)/\left[207\left(10^3\right)\right] + \left(1-0.211^2\right)/\left[100\left(10^3\right)\right]}{1/150 + 1/\infty}\right)^{1/2}$$

$$b = 0.2583 \text{ mm}$$

$$p_{\max} = \frac{2F}{\pi bl} = \frac{2(2000)}{\pi(0.2583)(40)} = 123.2 \text{ MPa}$$

$$\sigma_x = -2\nu p_{\max}\left(\sqrt{1+\frac{z^2}{b^2}} - \left|\frac{z}{b}\right|\right) = -2(0.292)(123.2)\left(\sqrt{1+0.786^2} - 0.786\right)$$

$$= -35.0 \text{ MPa} \qquad \textit{Ans.}$$

$$\sigma_y = -p_{\max}\left(\frac{1+2\dfrac{z^2}{b^2}}{\sqrt{1+\dfrac{z^2}{b^2}}} - 2\left|\frac{z}{b}\right|\right) = -123.2\left(\frac{1+2\left(0.786^2\right)}{\sqrt{1+\left(0.786^2\right)}} - 2(0.786)\right)$$

$$= -22.9 \text{ MPa} \qquad \textit{Ans.}$$

$$\sigma_z = \frac{-p_{\max}}{\sqrt{1+\frac{z^2}{b^2}}} = \frac{-123.2}{\sqrt{1+0.786^2}} = -96.9 \text{ MPa} \qquad \textit{Ans.}$$

$$\tau_{\max} = \frac{\sigma_y - \sigma_z}{2} = \frac{-22.9-(-96.9)}{2} = 37.0 \text{ MPa} \qquad \textit{Ans.}$$

3-157

Use Eqs. (3-73) through (3-77).

$$b = \left(\frac{2F}{\pi l} \frac{\left(1-\nu_1^{\,2}\right)/E_1 + \left(1-\nu_2^{\,2}\right)/E_2}{1/d_1 + 1/d_2} \right)^{1/2}$$

$$= \left(\frac{2(250)}{\pi(1.25)} \frac{\left(1-0.211^2\right)/\left[14.5\left(10^6\right)\right] + \left(1-0.211^2\right)/\left[14.5\left(10^6\right)\right]}{1/3+1/\infty} \right)^{1/2}$$

$$b = 0.007\ 095 \text{ in}$$

$$p_{\max} = \frac{2F}{\pi b l} = \frac{2(250)}{\pi(0.007\ 095)(1.25)} = 17\ 946 \text{ psi}$$

$$\sigma_x = -2\nu p_{\max}\left(\sqrt{1+\frac{z^2}{b^2}} - \left|\frac{z}{b}\right|\right) = -2(0.211)(17\,946)\left(\sqrt{1+0.786^2} - 0.786\right)$$

$$= -3\,680 \text{ psi} = -3.68 \text{ kpsi} \qquad \textit{Ans.}$$

$$\sigma_y = -p_{\max}\left(\frac{1+2\frac{z^2}{b^2}}{\sqrt{1+\frac{z^2}{b^2}}} - 2\left|\frac{z}{b}\right| \right) = -17\,946\left(\frac{1+2\left(0.786^2\right)}{\sqrt{1+\left(0.786^2\right)}} - 2(0.786) \right)$$

$$= -3\,332 \text{ psi} = -3.33 \text{ kpsi} \qquad \textit{Ans.}$$

$$\sigma_z = \frac{-p_{\max}}{\sqrt{1+\frac{z^2}{b^2}}} = \frac{-17\,946}{\sqrt{1+0.786^2}} = -14\,109 \text{ psi} = -14.1 \text{ kpsi} \qquad \textit{Ans.}$$

$$\tau_{\max} = \frac{\sigma_y - \sigma_z}{2} = \frac{-3\,332-(-14\,109)}{2} = 5\,389 \text{ psi} = 5.39 \text{ kpsi} \qquad \textit{Ans.}$$
