Chapter 13

13-1 $d_P = 17/8 = 2.125$ in

$$d_G = \frac{N_2}{N_3} d_P = \frac{1120}{544}(2.125) = 4.375 \text{ in}$$

$N_G = Pd_G = 8(4.375) = 35$ teeth *Ans.*

$C = (2.125 + 4.375)/2 = 3.25$ in *Ans.*

13-2 $n_G = 1600(15/60) = 400$ rev/min *Ans.*

$p = \pi m = 3\pi$ mm *Ans.*

$C = [3(15+60)]/2 = 112.5$ mm *Ans.*

13-3 $N_G = 16(4) = 64$ teeth *Ans.*

$d_G = N_G m = 64(6) = 384$ mm *Ans.*

$d_P = N_P m = 16(6) = 96$ mm *Ans.*

$C = (384 + 96)/2 = 240$ mm *Ans.*

13-4 *Mesh:* $a = 1/P = 1/3 = 0.3333$ in *Ans.*

$b = 1.25/P = 1.25/3 = 0.4167$ in *Ans.*

$c = b - a = 0.0834$ in *Ans.*

$p = \pi/P = \pi/3 = 1.047$ in *Ans.*

$t = p/2 = 1.047/2 = 0.523$ in *Ans.*

Pinion Base-Circle: $d_1 = N_1/P = 21/3 = 7$ in

$d_{1b} = 7\cos 20^\circ = 6.578$ in *Ans.*

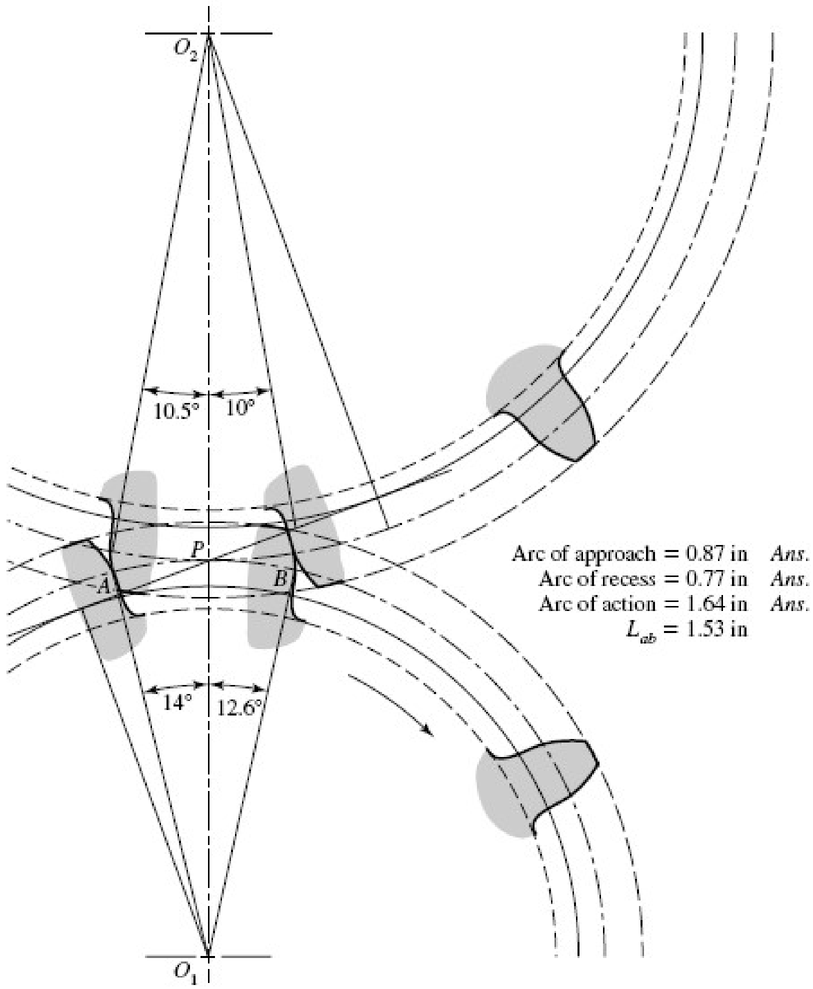
Gear Base-Circle: $d_2 = N_2/P = 28/3 = 9.333$ in

$d_{2b} = 9.333\cos 20^\circ = 8.770$ in *Ans.*

Base pitch: $p_b = p_c \cos\phi = (\pi/3)\cos 20^\circ = 0.984$ in *Ans.*

Contact Ratio: $m_c = L_{ab}/p_b = 1.53/0.984 = 1.55$ *Ans.*

See the following figure for a drawing of the gears and the arc lengths.



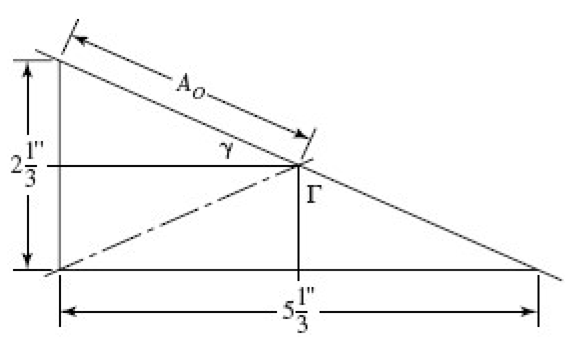
13-5

(a) $A_O = \left[\left(\frac{14/6}{2}\right)^2 + \left(\frac{32/6}{2}\right)^2\right]^{1/2} = 2.910 \text{ in}$ *Ans.*

(b)

$\gamma = \tan^{-1}(14/32) = 23.63^\circ$ *Ans.*

$\Gamma = \tan^{-1}(32/14) = 66.37^\circ$ *Ans.*



(c)

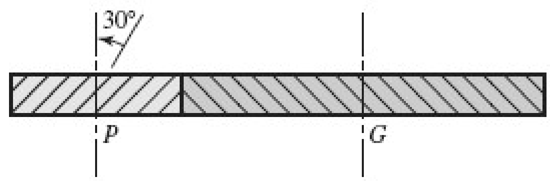
$d_P = 14/6 = 2.333$ in *Ans.*

$d_G = 32/6 = 5.333$ in *Ans.*

(d) From Table 13-3, $0.3A_O = 0.3(2.910)$ $= 0.873$ in and $10/P = 10/6 = 1.67$

$0.873 < 1.67 \quad \therefore \; F = 0.873$ in *Ans.*

13-6



(a) $p_n = \pi / P_n = \pi / 4 = 0.7854$ in

$p_t = p_n / \cos\psi = 0.7854 / \cos 30^\circ = 0.9069$ in

$p_x = p_t / \tan\psi = 0.9069 / \tan 30^\circ = 1.571$ in

(b) Eq. (13-7): $p_{nb} = p_n \cos\phi_n = 0.7854 \cos 25^\circ = 0.7380$ in *Ans.*

(c) $P_t = P_n \cos\psi = 4 \cos 30^\circ = 3.464$ teeth/in

$\phi_t = \tan^{-1}\left(\tan\phi_n / \cos\psi\right) = \tan^{-1}(\tan 25^\circ / \cos 30^\circ) = 28.3^\circ$ *Ans.*

(d) Table 13-4:

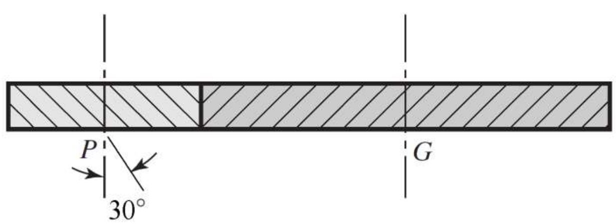
$a = 1/4 = 0.250$ in *Ans.*

$b = 1.25/4 = 0.3125$ in *Ans.*

$$d_P = \frac{20}{4\cos 30^\circ} = 5.774 \text{ in} \quad \textit{Ans.}$$

$$d_G = \frac{36}{4\cos 30^\circ} = 10.39 \text{ in} \quad \textit{Ans.}$$

13-7



$N_P = 19$ teeth, $N_G = 57$ teeth, $\phi_n = 20^\circ, m_n = 2.5$ mm

(a) $p_n = \pi m_n = \pi(2.5) = 7.854$ mm *Ans.*

$$p_t = \frac{p_n}{\cos\psi} = \frac{7.854}{\cos 30^\circ} = 9.069 \text{ mm} \quad \textit{Ans.}$$

$$p_x = \frac{p_t}{\tan\psi} = \frac{9.069}{\tan 30^\circ} = 15.71 \text{ mm} \quad \textit{Ans.}$$

(b) $$m_t = \frac{m_n}{\cos\psi} = \frac{2.5}{\cos 30^\circ} = 2.887 \text{ mm} \quad \textit{Ans.}$$

$$\phi_t = \tan^{-1}\left(\frac{\tan 20^\circ}{\cos 30^\circ}\right) = 22.80^\circ \quad Ans.$$

(c) $a = m_n = 2.5\text{ mm} \quad Ans.$

$$b = 1.25m_n = 1.25(2.5) = 3.125\text{ mm} \quad Ans.$$

$$d_P = \frac{N}{P_t} = Nm_t = 19(2.887) = 54.85\text{ mm} \quad Ans.$$

$$d_G = 57(2.887) = 164.6\text{ mm} \quad Ans.$$

13-8 **(a)** Using Eq. (13-11) with $k = 1$, $\phi = 20^\circ$, and $m = 2$,

$$N_P = \frac{2k}{(1+2m)\sin^2\phi}\left(m + \sqrt{m^2 + (1+2m)\sin^2\phi}\right)$$

$$= \frac{2(1)}{\left[1+2(2)\right]\sin^2(20^\circ)}\left\{(2) + \sqrt{(2)^2 + \left[1+2(2)\right]\sin^2(20^\circ)}\right\} = 14.16\text{ teeth}$$

Round up for the minimum integer number of teeth.

$N_P = 15$ teeth *Ans.*

(b) Repeating (a) with $m = 3$, $N_P = 14.98$ teeth. Rounding up, $N_P = 15$ teeth. *Ans.*
(c) Repeating (a) with $m = 4$, $N_P = 15.44$ teeth. Rounding up, $N_P = 16$ teeth. *Ans.*
(d) Repeating (a) with $m = 5$, $N_P = 15.74$ teeth. Rounding up, $N_P = 16$ teeth. *Ans.*

Alternatively, a useful table can be generated to determine the largest gear that can mesh with a specified pinion, and thus also the maximum gear ratio with a specified pinion. The Max N_G column was generated using Eq. (13-12) with $k = 1$, $\phi = 20^\circ$, and rounding up to the next integer.

Min N_P	**Max N_G**	**Max m = Max N_G / Min N_P**
13	16	1.23
14	26	1.86
15	45	3.00
16	101	6.31
17	1309	77.00
18	unlimited	unlimited

With this table, we can readily see that gear ratios up to 3 can be obtained with a minimum N_P of 15 teeth, and gear ratios up to 6.31 can be obtained with a minimum N_P of 16 teeth. This is consistent with the results previously obtained.

13-9 Repeating the process shown in the solution to Prob. 13-8, except with $\phi = 25°$, we obtain the following results.

(a) For $m = 2$, $N_P = 9.43$ teeth. Rounding up, $N_P = 10$ teeth. *Ans.*
(b) For $m = 3$, $N_P = 9.92$ teeth. Rounding up, $N_P = 10$ teeth. *Ans.*
(c) For $m = 4$, $N_P = 10.20$ teeth. Rounding up, $N_P = 11$ teeth. *Ans.*
(d) For $m = 5$, $N_P = 10.38$ teeth. Rounding up, $N_P = 11$ teeth. *Ans.*

For convenient reference, we will also generate the table from Eq. (13-12) for $\phi = 25°$.

Min N_P	**Max N_G**	**Max m = Max N_G / Min N_P**
9	13	1.44
10	32	3.20
11	249	22.64
12	unlimited	unlimited

13-10 **(a)** The smallest pinion tooth count that will run with itself is found from Eq. (13-10).

$$N_P \geq \frac{2k}{3\sin^2\phi}\left(1+\sqrt{1+3\sin^2\phi}\right)$$
$$\geq \frac{2(1)}{3\sin^2 20^\circ}\left(1+\sqrt{1+3\sin^2 20^\circ}\right)$$
$$\geq 12.32 \quad \rightarrow \quad 13 \text{ teeth} \quad \textit{Ans.}$$

(b) The smallest pinion that will mesh with a gear ratio of $m_G = 2.5$, from Eq. (13-11) is

$$N_P \geq \frac{2k}{(1+2m)\sin^2\phi}\left(m+\sqrt{m^2+(1+2m)\sin^2\phi}\right)$$
$$\geq \frac{2(1)}{\left[1+2(2.5)\right]\sin^2 20^\circ}\left\{2.5+\sqrt{2.5^2+\left[1+2(2.5)\right]\sin^2 20^\circ}\right\}$$
$$\geq 14.64 \quad \rightarrow \quad 15 \text{ teeth} \quad \textit{Ans.}$$

The largest gear-tooth count possible to mesh with this pinion, from Eq. (13-12) is

$$N_G \leq \frac{N_P^2\sin^2\phi - 4k^2}{4k - 2N_P\sin^2\phi}$$
$$\leq \frac{15^2\sin^2 20^\circ - 4(1)^2}{4(1)-2(15)\sin^2 20^\circ}$$
$$\leq 45.49 \quad \rightarrow \quad 45 \text{ teeth} \quad \textit{Ans.}$$

(c) The smallest pinion that will mesh with a rack, from Eq. (13-13),

$$N_P \geq \frac{2k}{\sin^2 \phi} = \frac{2(1)}{\sin^2 20^\circ}$$
$$\geq 17.097 \quad \rightarrow \quad 18 \text{ teeth} \quad \textit{Ans.}$$

13-11 $\phi_n = 20^\circ, \psi = 30^\circ$

From Eq. (13-19), $\phi_t = \tan^{-1}\left(\tan 20^\circ / \cos 30^\circ\right) = 22.80^\circ$

(a) The smallest pinion tooth count that will run with itself, from Eq. (13-21) is

$$N_P \geq \frac{2k\cos\psi}{3\sin^2\phi_t}\left(1+\sqrt{1+3\sin^2\phi_t}\right)$$
$$\geq \frac{2(1)\cos 30^\circ}{3\sin^2 22.80^\circ}\left(1+\sqrt{1+3\sin^2 22.80^\circ}\right)$$
$$\geq 8.48 \quad \rightarrow \quad 9 \text{ teeth} \quad \textit{Ans.}$$

(b) The smallest pinion that will mesh with a gear ratio of $m = 2.5$, from Eq. (13-22) is

$$N_P \geq \frac{2(1)\cos 30^\circ}{\left[1+2(2.5)\right]\sin^2 22.80^\circ}\left\{2.5+\sqrt{2.5^2+\left[1+2(2.5)\right]\sin^2 22.80^\circ}\right\}$$
$$\geq 9.95 \quad \rightarrow \quad 10 \text{ teeth} \quad \textit{Ans.}$$

The largest gear-tooth count possible to mesh with this pinion, from Eq. (13-23) is

$$N_G \leq \frac{N_P^2\sin^2\phi_t - 4k^2\cos^2\psi}{4k\cos\psi - 2N_P\sin^2\phi_t}$$
$$\leq \frac{10^2\sin^2 22.80^\circ - 4(1)\cos^2 30^\circ}{4(1)\cos^2 30^\circ - 2(20)\sin^2 22.80^\circ}$$
$$\leq 26.08 \quad \rightarrow \quad 26 \text{ teeth} \quad \textit{Ans.}$$

(c) The smallest pinion that will mesh with a rack, from Eq. (13-24) is

$$N_P \geq \frac{2k\cos\psi}{\sin^2\phi_t} = \frac{2(1)\cos 30^\circ}{\sin^2 22.80^\circ}$$
$$\geq 11.53 \quad \rightarrow \quad 12 \text{ teeth} \quad \textit{Ans.}$$

13-12 From Eq. (13-19), $\displaystyle \phi_t = \tan^{-1}\left(\frac{\tan\phi_n}{\cos\psi}\right) = \tan^{-1}\left(\frac{\tan 20^\circ}{\cos 30^\circ}\right) = 22.796^\circ$

Program Eq. (13-23) on a computer using a spreadsheet or code, and increment N_P. The first value of N_P that can be doubled is $N_P = 10$ teeth, where $N_G \le 26.01$ teeth. So $N_G = 20$ teeth will work. Higher tooth counts will work also, for example 11:22, 12:24, etc.

Use $N_P = 10$ teeth, $N_G = 20$ teeth *Ans.*

Note that the given diametral pitch (tooth size) is not relevant to the interference problem.

13-13 From Eq. (13-19), $\phi_t = \tan^{-1}\left(\dfrac{\tan\phi_n}{\cos\psi}\right) = \tan^{-1}\left(\dfrac{\tan 20^\circ}{\cos 45^\circ}\right) = 27.236^\circ$

Program Eq. (13-23) on a computer using a spreadsheet or code, and increment N_P. The first value of N_P that can be doubled is $N_P = 6$ teeth, where $N_G \le 17.6$ teeth. So $N_G = 12$ teeth will work. Higher tooth counts will work also, for example 7:14, 8:16, etc.

Use $N_P = 6$ teeth, $N_G = 12$ teeth *Ans.*

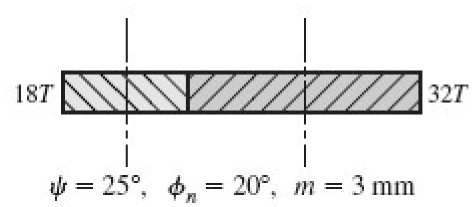
13-14 The smallest pinion that will operate with a rack without interference is given by Eq. (13-13).

$$N_P = \frac{2k}{\sin^2\phi}$$

Setting $k = 1$ for full depth teeth, $N_P = 9$ teeth, and solving for ϕ,

$$\phi = \sin^{-1}\sqrt{\frac{2k}{N_P}} = \sin^{-1}\sqrt{\frac{2(1)}{9}} = 28.126^\circ \qquad \textit{Ans.}$$

13-15



(a) Eq. (13-3): $p_n = \pi m_n = 3\pi$ mm *Ans.*

Eq. (13-16): $p_t = p_n / \cos\psi = 3\pi / \cos 25^\circ = 10.40$ mm *Ans.*

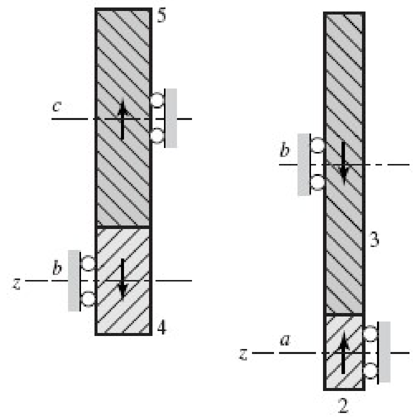
Eq. (13-17): $p_x = p_t / \tan\psi = 10.40 / \tan 25^\circ = 22.30$ mm *Ans.*

(b) Eq. (13-3): $m_t = p_t / \pi = 10.40 / \pi = 3.310$ mm *Ans.*

Eq. (13-19): $\phi_t = \tan^{-1}\dfrac{\tan\phi_n}{\cos\psi} = \tan^{-1}\dfrac{\tan 20^\circ}{\cos 25^\circ} = 21.88^\circ$ *Ans.*

(c) Eq. (13-2): $d_P = m_t N_P = 3.310\,(18) = 59.58$ mm *Ans.*
Eq. (13-2): $d_G = m_t N_G = 3.310\,(32) = 105.92$ mm *Ans.*

13-16 **(a)** Sketches of the figures are shown to determine the axial forces by inspection.



The axial force of gear 2 on shaft *a* is in the negative *z*-direction. The axial force of gear 3 on shaft *b* is in the positive *z*-direction. *Ans.*

The axial force of gear 4 on shaft *b* is in the positive *z*-direction. The axial force of gear 5 on shaft *c* is in the negative *z*-direction. *Ans.*

(b) $$n_c = n_5 = \frac{12}{48}\left(\frac{16}{36}\right)(700) = +77.78 \text{ rev/min ccw} \quad Ans.$$

(c) $$d_{P2} = 12/\left(12\cos 30^\circ\right) = 1.155 \text{ in}$$
$$d_{G3} = 48/\left(12\cos 30^\circ\right) = 4.619 \text{ in}$$
$$C_{ab} = \frac{1.155 + 4.619}{2} = 2.887 \text{ in} \quad Ans.$$
$$d_{P4} = 16/\left(8\cos 25^\circ\right) = 2.207 \text{ in}$$
$$d_{G5} = 36/\left(8\cos 25^\circ\right) = 4.965 \text{ in}$$
$$C_{bc} = 3.586 \text{ in} \quad Ans.$$

13-17

$$e = \frac{20}{40}\left(\frac{8}{17}\right)\left(\frac{20}{60}\right) = \frac{4}{51}$$
$$n_d = \frac{4}{51}(600) = 47.06 \text{ rev/min cw} \quad Ans.$$

13-18

$$e = \frac{6}{10}\left(\frac{18}{38}\right)\left(\frac{20}{48}\right)\left(\frac{3}{36}\right) = \frac{3}{304}$$
$$n_9 = \frac{3}{304}(1200) = 11.84 \text{ rev/min cw} \quad Ans.$$

13-19 **(a)**

$$\left.\begin{array}{l} n_c = \dfrac{12}{40}\cdot\dfrac{1}{1}(540) = 162\text{ rev/min} \\ \\ \text{cw about } x,\ \text{as viewed from the positive } x \text{ axis} \end{array}\right\} \quad Ans.$$

(b)

$$d_P = 12/\left(8\cos 23^\circ\right) = 1.630\text{ in}$$

$$d_G = 40/\left(8\cos 23^\circ\right) = 5.432\text{ in}$$

$$\frac{d_P + d_G}{2} = 3.531\text{ in} \qquad Ans.$$

(c)

$$d = \frac{N}{P} = \frac{32}{4} = 8\text{ in} \qquad Ans.$$

13-20 Applying Eq. (13-30), $e = (N_2 / N_3)\ (N_4 / N_5) = 45$. For an exact ratio, we will choose to factor the train value into integers, such that

$N_2 / N_3 = 9$ (1)
$N_4 / N_5 = 5$ (2)

Assuming a constant diametral pitch in both stages, the geometry condition to satisfy the in-line requirement of the compound reverted configuration is

$N_2 + N_3 = N_4 + N_5$ (3)

With three equations and four unknowns, one free choice is available. It is necessary that all of the unknowns be integers. We will use a normalized approach to find the minimum free choice to guarantee integers; that is, set the smallest gear of the largest stage to unity, thus $N_3 = 1$. From (1), $N_2 = 9$. From (3),

$N_2 + N_3 = 9 + 1 = 10 = N_4 + N_5$

Substituting $N_4 = 5\ N_5$ from (2) gives

$10 = 5\ N_5 + N_5 = 6\ N_5$
$N_5 = 10 / 6 = 5 / 3$

To eliminate this fraction, we need to multiply the original free choice by a multiple of 3. In addition, the smallest gear needs to have sufficient teeth to avoid interference. From Eq. (13-11) with $k = 1$, $\phi = 20°$, and $m = 9$, the minimum number of teeth on the pinion to avoid interference is 17. Therefore, the smallest multiple of 3 greater than 17 is 18. Setting $N_3 = 18$ and repeating the solution of equations (1), (2), and (3) yields

$N_2 = 162$ teeth
$N_3 = 18$ teeth
$N_4 = 150$ teeth
$N_5 = 30$ teeth *Ans.*

13-21 The solution to Prob. 13-20 applies up to the point of determining the minimum number of teeth to avoid interference. From Eq. (13-11), with $k = 1$, $\phi = 25°$, and $m = 9$, the minimum number of teeth on the pinion to avoid interference is 11. Therefore, the smallest multiple of 3 greater than 11 is 12. Setting $N_3 = 12$ and repeating the solution of equations (1), (2), and (3) of Prob. 13-20 yields

$N_2 = 108$ teeth
$N_3 = 12$ teeth
$N_4 = 100$ teeth
$N_5 = 20$ teeth *Ans.*

13-22 Applying Eq. (13-30), $e = (N_2 / N_3)(N_4 / N_5) = 30$. For an exact ratio, we will choose to factor the train value into integers, such that

$$N_2 / N_3 = 6 \qquad (1)$$
$$N_4 / N_5 = 5 \qquad (2)$$

Assuming a constant diametral pitch in both stages, the geometry condition to satisfy the in-line requirement of the compound reverted configuration is

$$N_2 + N_3 = N_4 + N_5 \qquad (3)$$

With three equations and four unknowns, one free choice is available. It is necessary that all of the unknowns be integers. We will use a normalized approach to find the minimum free choice to guarantee integers; that is, set the smallest gear of the largest stage to unity, thus $N_3 = 1$. From (1), $N_2 = 6$. From (3),

$$N_2 + N_3 = 6 + 1 = 7 = N_4 + N_5$$

Substituting $N_4 = 5\,N_5$ from (2) gives

$$7 = 5\,N_5 + N_5 = 6\,N_5$$
$$N_5 = 7 / 6$$

To eliminate this fraction, we need to multiply the original free choice by a multiple of 6. In addition, the smallest gear needs to have sufficient teeth to avoid interference. From Eq. (13-11) with $k = 1$, $\phi = 20°$, and $m = 6$, the minimum number of teeth on the pinion to avoid interference is 16. Therefore, the smallest multiple of 6 greater than 16 is 18. Setting $N_3 = 18$ and repeating the solution of equations (1), (2), and (3) yields

N_2 = 108 teeth
N_3 = 18 teeth
N_4 = 105 teeth
N_5 = 21 teeth *Ans.*

13-23 Applying Eq. (13-30), $e = (N_2 / N_3)(N_4 / N_5) = 45$. For an approximate ratio, we will choose to factor the train value into two equal stages, such that

$$N_2 / N_3 = N_4 / N_5 = \sqrt{45}$$

If we choose identical pinions such that interference is avoided, both stages will be identical and the in-line geometry condition will automatically be satisfied. From Eq. (13-11) with $k = 1$, $\phi = 20°$, and $m = \sqrt{45}$, the minimum number of teeth on the pinions to avoid interference is 17. Setting $N_3 = N_5 = 17$, we get

$$N_2 = N_4 = 17\sqrt{45} = 114.04 \text{ teeth}$$

Rounding to the nearest integer, we obtain

$N_2 = N_4$ = 114 teeth
$N_3 = N_5$ = 17 teeth *Ans.*

Checking, the overall train value is $e = (114 / 17)(114 / 17) = 44.97$.

13-24 H = 25 hp, ω_i = 2500 rev/min
Let ω_o = 300 rev/min for minimal gear ratio to minimize gear size.

$$\frac{\omega_o}{\omega_i} = \frac{300}{2500} = \frac{1}{8.333} = \frac{N_2}{N_3}\frac{N_4}{N_5}$$

Let $$\frac{N_2}{N_3} = \frac{N_4}{N_5} = \sqrt{\frac{1}{8.333}} = \frac{1}{2.887}$$

From Eq. (13-11) with $k = 1$, $\phi = 20°$, and $m = 2.887$, the minimum number of teeth on the pinions to avoid interference is 15.

Let $N_2 = N_4$ = 15 teeth
$N_3 = N_5 = 2.887(15) = 43.31$ teeth

Try $N_3 = N_5$ = 43 teeth.

$$\omega_o = \left(\frac{15}{43}\right)\left(\frac{15}{43}\right)(2500) = 304.2$$

Too big. Try $N_3 = N_5 = 44$.

$$\omega_o = \left(\frac{15}{44}\right)\left(\frac{15}{44}\right)(2500) = 290.55 \text{ rev/min}$$

$N_2 = N_4 = 15$ teeth, $N_3 = N_5 = 44$ teeth *Ans.*

13-25 **(a)** The planet gears act as keys and the wheel speeds are the same as that of the ring gear. Thus,

$$n_A = n_3 = 900(16/48) = 300 \text{ rev/min} \quad \textit{Ans.}$$

(b)

$$n_F = n_5 = 0, \quad n_L = n_6, \quad e = -1$$

$$-1 = \frac{n_6 - 300}{0 - 300}$$

$$300 = n_6 - 300$$

$$n_6 = 600 \text{ rev/min} \quad \textit{Ans.}$$

(c) The wheel spins freely on icy surfaces, leaving no traction for the other wheel. The car is stalled. *Ans.*

13-26 **(a)** The motive power is divided equally among four wheels instead of two.

(b) Locking the center differential causes 50 percent of the power to be applied to the rear wheels and 50 percent to the front wheels. If one of the rear wheels rests on a slippery surface such as ice, the other rear wheel has no traction. But the front wheels still provide traction, and so you have two-wheel drive. However, if the rear differential is locked, you have 3-wheel drive because the rear-wheel power is now distributed 50-50.

13-27 Let gear 2 be first, then $n_F = n_2 = 0$. Let gear 6 be last, then $n_L = n_6 = -12$ rev/min.

$$e = \frac{20}{30}\left(\frac{16}{34}\right) = \frac{16}{51} = \frac{n_L - n_A}{n_F - n_A}$$

$$(0 - n_A)\frac{16}{51} = -12 - n_A$$

$$n_A = \frac{-12}{35/51} = -17.49 \text{ rev/min}$$

(negative indicates cw as viewed from the bottom of the figure) } *Ans.*

13-28 Let gear 2 be first, then $n_F = n_2 = 0$ rev/min. Let gear 6 be last, then $n_L = n_6 = 85$ rev/min.

$$e = \frac{20}{30}\left(\frac{16}{34}\right) = \frac{16}{51} = \frac{n_L - n_A}{n_F - n_A}$$

$$(0 - n_A)\frac{16}{51} = (85 - n_A)$$

$$-n_A\left(\frac{16}{51}\right) + n_A = 85$$

$$n_A\left(1 - \frac{16}{51}\right) = 85$$

$$n_A = \frac{85}{1 - \frac{16}{51}} = 123.9 \text{ rev/min}$$

The positive sign indicates the same direction as n_6. $\therefore n_A = 123.9$ rev/min ccw *Ans.*

13-29 $\phi = 20^\circ$, $P = 6$ teeth/in. Since $N \alpha d$, then,

$$N_2 + N_3 = N_4 + N_5 \qquad (1)$$

Hour hand moves 1/12 of minute hand. Thus, $\omega_5 / \omega_2 = \frac{1}{12}$. Now,

$\omega_5 / \omega_4 = N_4 / N_5$, $\omega_3 / \omega_2 = N_2 / N_3$, and $\omega_4 = \omega_3$. Thus,

$$\frac{\omega_2}{\omega_2} = \frac{N_3}{N_2}\frac{N_5}{N_4} = 12 = 4 \times 3$$

So, try $N_3 = 4N_2$, and $N_5 = 3N_4$. Substituting $N_5 = 3N_4$ into Eq. (1) gives
$N_2 + N_3 = 4N_4 \Rightarrow N_4 = (N_2 + N_3)/4$. Let $N_2 = 1$. Then, $N_3 = 4N_2 = 4(1) = 4$,
$N_4 = (N_2 + N_3)/4 = (1 + 4)/4 = 5/4$, $N_5 = 3N_4 = 3(5/4) = 15/4$. Teeth must be a multiple of 4. To avoid interference for the smaller pinion use Eq. (13-11) with $m = N_3 / N_2 = 4$:

$$N_2 = \frac{2k}{(1+2m)\sin^2\phi}\left(m + \sqrt{m^2 + (1+2m)\sin^2\phi}\right)$$

$$= \frac{2(1)}{\left[1+2(4)\right]\sin^2 20^\circ}\left(4 + \sqrt{4^2 + \left[1+2(4)\right]\sin^2 20^\circ}\right) = 15.4 \text{ teeth}$$

Use $N_2 = 16$ teeth which is a multiple of 4. Then, $N_3 = 4(16) = 64$ teeth, $N_4 = (5/4)16 = 20$ teeth, and $N_5 = (15/4)\,16 = 60$ teeth. Thus,

$N_2 = 16$ teeth, $N_3 = 64$ teeth, $N_4 = 20$ teeth, and $N_5 = 60$ teeth *Ans.*

13-30 The geometry condition is $d_5 / 2 = d_2 / 2 + d_3 + d_4$. Since all the gears are meshed, they will all have the same diametral pitch. Applying $d = N / P$,

$$N_5 / (2P) = N_2 / (2P) + N_3 / P + N_4 / P$$
$$N_5 = N_2 + 2N_3 + 2N_4 = 12 + 2(16) + 2(12) = 68 \text{ teeth} \quad \textit{Ans.}$$

Let gear 2 be first, $n_F = n_2 = 320$ rev/min. Let gear 5 be last, $n_L = n_5 = 0$ rev/min.

$$e = \frac{12}{16}\left(\frac{16}{12}\right)\left(\frac{12}{68}\right) = \frac{3}{17} = \frac{n_L - n_A}{n_F - n_A}$$

$$320 - n_A = \frac{17}{3}(0 - n_A)$$
$$n_A = -\frac{3}{14}(320) = -68.57 \text{ rev/min}$$

The negative sign indicates opposite of n_2. $\therefore n_A = 68.57$ rev/min cw *Ans.*

13-31 Let $n_F = n_2$, then $n_L = n_7 = 0$.

$$e = -\frac{20}{16}\left(\frac{16}{30}\right)\left(\frac{36}{46}\right) = -0.5217 = \frac{n_L - n_5}{n_F - n_5}$$

$$\frac{0 - n_5}{10 - n_5} = -0.5217$$
$$-0.5217(10 - n_5) = -n_5$$
$$-5.217 + 0.5217 n_5 + n_5 = 0$$
$$n_5(1 + 0.5217) = 5.217$$
$$n_5 = \frac{5.217}{1.5217}$$
$$n_5 = n_b = 3.428 \text{ turns in same direction} \quad \textit{Ans.}$$

13-32 **(a)**

$$\omega = 2\pi n / 60$$
$$H = T\omega = 2\pi T n / 60 \quad (T \text{ in N}\cdot\text{m}, H \text{ in W})$$

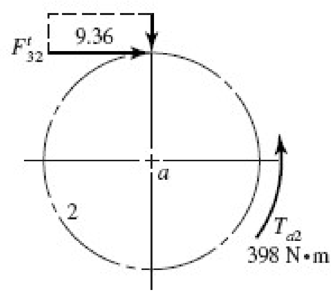
So $$T = \frac{60H\left(10^3\right)}{2\pi n}$$

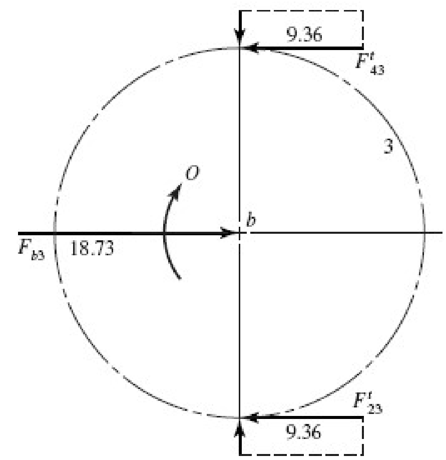
$$= 9550\ H/n \qquad (H \text{ in kW, } n \text{ in rev/min})$$

$$T_a = \frac{9550(75)}{1800} = 398 \text{ N}\cdot\text{m}$$

$$r_2 = \frac{mN_2}{2} = \frac{5(17)}{2} = 42.5 \text{ mm}$$

So $$F_{32}^t = \frac{T_a}{r_2} = \frac{398}{42.5} = 9.36 \text{ kN}$$



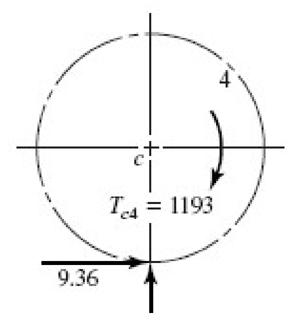


$$F_{3b} = -F_{b3} = 2(9.36) = 18.73 \text{ kN in the positive } x\text{-direction.} \quad \textit{Ans.}$$

(b) $$r_4 = \frac{mN_4}{2} = \frac{5(51)}{2} = 127.5 \text{ mm}$$

$$T_{c4} = 9.36(127.5) = 1193 \text{ N}\cdot\text{m ccw}$$

$$\therefore T_{4c} = 1193 \text{ N}\cdot\text{m cw} \quad \textit{Ans.}$$



Note: The solution is independent of the pressure angle.

13-33 $$d = \frac{N}{P} = \frac{N}{6}$$

$$d_2 = 4 \text{ in},\ \ d_4 = 4 \text{ in},\ \ d_5 = 6 \text{ in},\ \ d_6 = 24 \text{ in}$$

$$e = \left(-\frac{24}{24}\right)\left(-\frac{24}{36}\right)\left(+\frac{36}{144}\right) = 1/6$$

$$n_F = n_2 = 1000 \text{ rev/min}$$

$$n_L = n_6 = 0$$

$$e = \frac{n_L - n_A}{n_F - n_A} = \frac{0 - n_A}{1000 - n_A} = \frac{1}{6}$$

$$n_A = -200 \text{ rev/min}$$

Noting that power equals torque times angular velocity, the input torque is

$$T_2 = \frac{H}{n_2} = \frac{25 \text{ hp}}{1000 \text{ rev/min}}\left(\frac{550 \text{ lbf}\cdot\text{ft/s}}{\text{hp}}\right)\left(\frac{60 \text{ s}}{\text{min}}\right)\left(\frac{1 \text{ rev}}{2\pi \text{ rad}}\right)\left(\frac{12 \text{ in}}{\text{ft}}\right) = 1576 \text{ lbf}\cdot\text{in}$$

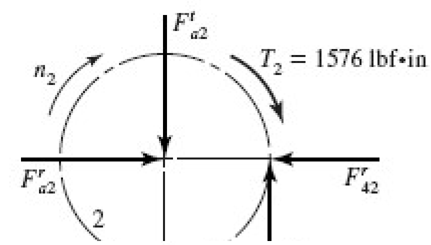
For 100 percent gear efficiency, the output power equals the input power, so

$$T_{arm} = \frac{H}{n_A} = \frac{25 \text{ hp}}{200 \text{ rev/min}}\left(\frac{550 \text{ lbf}\cdot\text{ft/s}}{\text{hp}}\right)\left(\frac{60 \text{ s}}{\text{min}}\right)\left(\frac{1 \text{ rev}}{2\pi \text{ rad}}\right)\left(\frac{12 \text{ in}}{\text{ft}}\right) = 7878 \text{ lbf}\cdot\text{in}$$

Next, we'll confirm the output torque as we work through the force analysis and complete the free body diagrams.

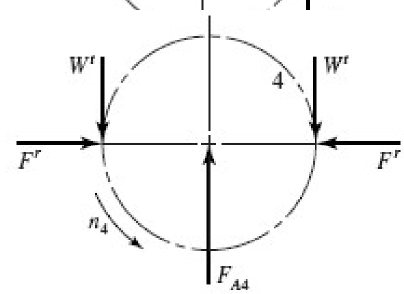
Gear 2

$$W^t = \frac{1576}{2} = 788 \text{ lbf}$$
$$F_{32}^r = 788 \tan 20^\circ = 287 \text{ lbf}$$

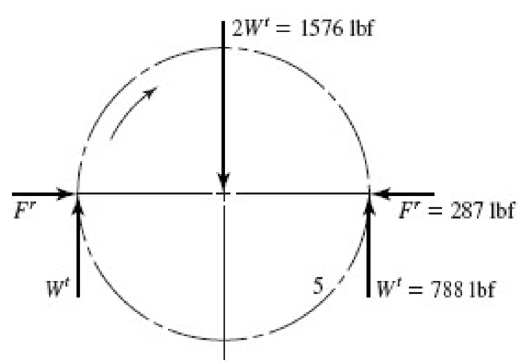


Gear 4

$$F_{A4} = 2W^t = 2(788) = 1576 \text{ lbf}$$

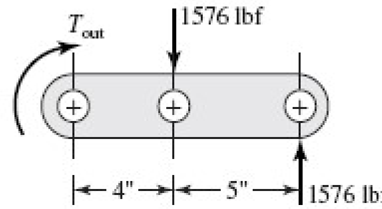


Gear 5



Arm

$$T_{\text{out}} = 1576(9) - 1576(4) = 7880 \text{ lbf} \cdot \text{in} \quad \textit{Ans.}$$



13-34 **(a)** $\phi = 20°$, $P = 6$ teeth/in. Since $N \propto d$, then,

$$N_2 + N_3 = N_4 + N_5 \qquad (1)$$

$e = 40 = 8 \times 5$. Then, let $N_2 / N_3 = 8$ and $N_4 / N_5 = 5$. Thus, $N_2 = 8\,N_3$ and $N_4 = 5\,N_5$. Substituting $N_4 = 5\,N_5$ into Eq. (1) gives $N_2 + N_3 = 6N_5 \;\Rightarrow\; N_5 = (N_2 + N_3)/6$.
To avoid interference use Eq. (13-11) with $m = N_2 / N_3 = 8$:

$$N_2 = \frac{2k}{(1+2m)\sin^2\phi}\left(m + \sqrt{m^2 + (1+2m)\sin^2\phi}\right)$$

$$= \frac{2(1)}{\left[1+2(8)\right]\sin^2 20°}\left(8 + \sqrt{8^2 + \left[1+2(8)\right]\sin^2 20°}\right) = 16.2 \text{ teeth}$$

Try $N_3 = 17$ teeth $\Rightarrow$ $N_2 = 8\,N_3 = 8(17) = 136$ teeth, $N_5 = (136 + 17)/6 = 25.5$ teeth which is unacceptable. Next, try $N_3 = 18$ teeth $\Rightarrow$ $N_2 = 8\,N_3 = 8(18) = 144$ teeth, $N_5 = (144 + 18)/6 = 27$ teeth, and $N_4 = 5\,N_5 = 5\,(27) = 135$ teeth. Thus,

$N_2 = 144$ teeth, $N_3 = 18$ teeth, $N_4 = 135$ teeth, and $N_5 = 27$ teeth *Ans.*

(b) Gear diameter is $d = N / P$ (with $P = 6$ teeth/in), 0.5 in wall clearance on two sides, addendum of each gear is $1/P = 1/6$ in, and each wall thickness is 0.75 in. Thus,

$$Y = \frac{N_2}{P} + \frac{N_3/2}{P} + \frac{N_4/2}{P} + 2(0.5) + 2\left(\frac{1}{P}\right) + 2(0.75)$$

$$= \frac{144}{6} + \frac{18/2}{6} + \frac{135/2}{6} + 2(0.5) + 2\left(\frac{1}{6}\right) + 2(0.75) = 39.6 \text{ in} \qquad \textit{Ans.}$$

13-35 **(a)** $\phi = 25°$, $P = 6$ teeth/in. Since $N \propto d$, then,

$$N_2 + N_3 = N_4 + N_5 \qquad (1)$$

$e = 40 = 8 \times 5$. Let $N_2 / N_3 = 8$ and $N_4 / N_5 = 5$. Thus, $N_2 = 8\,N_3$ and $N_4 = 5\,N_5$. Substituting $N_4 = 5\,N_5$ into Eq. (1) gives $N_2 + N_3 = 6N_5 \;\Rightarrow\; N_5 = (N_2 + N_3)/6$.
To avoid interference use Eq. (13-11) with $m = N_2 / N_3 = 8$:

$$N_2 = \frac{2k}{(1+2m)\sin^2\phi}\left(m + \sqrt{m^2 + (1+2m)\sin^2\phi}\right)$$

$$= \frac{2(1)}{\left[1+2(8)\right]\sin^2 25°}\left(8 + \sqrt{8^2 + \left[1+2(8)\right]\sin^2 25°}\right) = 10.7 \text{ teeth}$$

Try $N_3 = 11$ teeth $\Rightarrow$ $N_2 = 8\,N_3 = 8(11) = 88$ teeth, $N_5 = (88 + 11)/6 = 16.5$ teeth which is unacceptable. Next, try $N_3 = 12$ teeth $\Rightarrow$ $N_2 = 8\,N_3 = 8(12) = 96$ teeth, $N_5 = (96 + 12)/6 = 18$ teeth, and $N_4 = 5\,N_5 = 5\,(18) = 90$ teeth. Thus,

$N_2 = 96$ teeth, $N_3 = 12$ teeth, $N_4 = 90$ teeth, and $N_5 = 18$ teeth *Ans.*

(b) Gear diameter is $d = N / P$ (with P = 6 teeth/in), 0.5 in wall clearance on two sides, addendum of each gear is $1/P$ = 1/6 in, and each wall thickness is 0.75 in. Thus,

$$Y = \frac{N_2}{P} + \frac{N_3/2}{P} + \frac{N_4/2}{P} + 2(0.5) + 2\left(\frac{1}{P}\right) + 2(0.75)$$

$$= \frac{96}{6} + \frac{12/2}{6} + \frac{90/2}{6} + 2(0.5) + 2\left(\frac{1}{6}\right) + 2(0.75) = 27.3 \text{ in} \quad \textit{Ans.}$$

13-36 **(a)** $\phi_n = 20^\circ$, $\psi = 45^\circ$, P = 6 teeth/in. Since $N \,\alpha\, d$, then,

$$N_2 + N_3 = N_4 + N_5 \qquad (1)$$

$e = 40 = 8 \times 5$. Let $N_2 / N_3 = 8$ and $N_4 / N_5 = 5$. Thus $N_2 = 8\,N_3$ and $N_4 = 5\,N_5$. Substituting $N_4 = 5\,N_5$ into Eq. (1) gives $N_2 + N_3 = 6N_5 \;\Rightarrow\; N_5 = (N_2 + N_3)/6$.
From Eq. (13-19),

$$\phi_t = \tan^{-1}\left(\frac{\tan\phi_n}{\tan\psi}\right) = \tan^{-1}\left(\frac{\tan 20^\circ}{\tan 45^\circ}\right) = 20^\circ$$

Eq. (13-22):

$$N_p = \frac{2k\cos\psi}{(1+2m)\sin^2\phi_t}\left(m + \sqrt{m^2 + (1+2m)\sin^2\phi_t}\right)$$

$$= \frac{2(1)\cos 45^\circ}{\left[1+2(8)\right]\sin^2 20^\circ}\left(8 + \sqrt{8^2 + \left[1+2(8)\right]\sin^2 20^\circ}\right) = 11.5 \text{ teeth}$$

Try N_3 = 12 teeth $\Rightarrow$ $N_2 = 8\,N_3$ = 8(12) = 96 teeth, N_5 = (96 + 12)/6 = 18 teeth, and $N_4 = 5\,N_5$ = 5 (18) = 90 teeth. Thus,

N_2 = 96 teeth, N_3 = 12 teeth, N_4 = 90 teeth, and N_5 = 18 teeth *Ans.*

(b) Gear diameter is $d = N / P$ (with P = 6 teeth/in), 0.5 in wall clearance on two sides, addendum of each gear is $1/P$ = 1/6 in, and each wall thickness is 0.75 in. Thus,

$$Y = \frac{N_2}{P} + \frac{N_3/2}{P} + \frac{N_4/2}{P} + 2(0.5) + 2\left(\frac{1}{P}\right) + 2(0.75)$$

$$= \frac{96}{6} + \frac{12/2}{6} + \frac{90/2}{6} + 2(0.5) + 2\left(\frac{1}{6}\right) + 2(0.75) = 27.3 \text{ in} \quad \textit{Ans.}$$

13-37 $e \approx 40$, P = 6 teeth/in, $\phi = 20^\circ$.

(a) Minimum size is $N_2 / N_3 = N_4 / N_5 = \sqrt{40}$ = 6.325. To avoid interference use Eq. (13-11) with m = 6.325:

$$N_2 = \frac{2k}{(1+2m)\sin^2\phi}\left(m + \sqrt{m^2 + (1+2m)\sin^2\phi}\right)$$

$$= \frac{2(1)}{\left[1+2(6.325)\right]\sin^2 20^\circ}\left(6.325 + \sqrt{6.235^2 + \left[1+2(6.325)\right]\sin^2 20^\circ}\right) = 16.0 \text{ teeth}$$

Let N_3 = 16 teeth. $N_2 = 16\sqrt{40}$ = 101.2. Use N_2 = 101 teeth, $e = (101/16)^2 = 39.85$ which is ok since it is between 38 and 42. Thus,

$N_2 = N_4$ = 101 teeth, and $N_3 = N_5$ = 16 teeth *Ans.*

(b) Gear diameter is $d = N / P$ (with P = 6 teeth/in), 0.5 in wall clearance on two sides, addendum of each gear is $1/P$ = 1/6 in, and each wall thickness is 0.75 in. Thus,

$$Y = \frac{N_2}{P} + \frac{N_3/2}{P} + \frac{N_4/2}{P} + 2(0.5) + 2\left(\frac{1}{P}\right) + 2(0.75)$$

$$= \frac{101}{6} + \frac{116/2}{6} + \frac{101/2}{6} + 2(0.5) + 2\left(\frac{1}{6}\right) + 2(0.75) = 29.4 \text{ in} \qquad \textit{Ans.}$$

Comparing this to Prob. 13-34 where Y = 39.6 in, we see a large reduction in gearbox size.

13-38 **(a)** P = 6 teeth/in, $\phi = 20°$. Since $N \propto d$, then,

$$\frac{N_2}{2} + N_3 + \frac{N_4}{2} = \frac{N_5}{2} + N_6 + \frac{N_7}{2} \quad \Rightarrow \quad \frac{30}{2} + 20 + \frac{60}{2} = \frac{20}{2} + N_6 + \frac{80}{2}$$

Solving for N_6 yields N_6 = 15 teeth *Ans.*

(b) $$Y = \frac{N_4}{P} + \frac{N_3}{P} + \frac{N_2/2}{P} + \frac{N_7/2}{P} + \frac{2}{P} = \frac{60}{6} + \frac{20}{6} + \frac{30/2}{6} + \frac{80/2}{6} + \frac{2}{6} = 22.83 \text{ in} \qquad \textit{Ans.}$$

(c) Gear 2, n_2 = 300 rev/min,

Eq, (13-34): $$V_2 = \frac{\pi d_2 n_2}{12} = \frac{\pi (N_2/P) n_2}{12} = \frac{\pi (30/6)300}{12} = 392.7 \text{ ft/min} \qquad \textit{Ans.}$$

(d) Eq. (13-35): $$W_t = 33\,000\frac{H}{V} = 33\,000\frac{4}{392.7} = 336.1 \text{ lbf} \qquad \textit{Ans.}$$

(e) $F_r = W_t \tan\phi$ = 336.1 tan20° = 122.3 lbf *Ans*,

(f) $$T_i = \frac{63\,025H}{12n} = \frac{63\,025(4)}{12(300)} = 70.0 \text{ lbf}\cdot\text{in} \qquad \textit{Ans.}$$

(g) $$e = \frac{N_2}{\cancel{N_3}}\frac{\cancel{N_3}}{N_4}\frac{N_5}{\cancel{N_6}}\frac{\cancel{N_6}}{N_7} = \frac{N_2 N_5}{N_4 N_7} = \frac{30(20)}{60(80)} = 0.125$$

$$T_o = T_i\left(\frac{1}{e}\right) = 70.0\left(\frac{1}{0.125}\right) = 560 \text{ lbf}\cdot\text{ft} \qquad \textit{Ans.}$$

(h) $\omega_o = e\omega_i$ = 0.125 (300) = 37.5 rev/min *Ans.*

(i) Assuming no losses, $P_o = P_i$ = 4 hp *Ans.*

13-39 Given: m = 12 mm, n_P = 1800 rev/min cw,

$N_2 = 18T$, $N_3 = 32T$, $N_4 = 18T$, $N_5 = 48T$

Pitch Diameters: d_2 = 18(12) = 216 mm, d_3 = 32(12) = 384 mm,
d_4 = 18(12) = 216 mm, d_5 = 48(12) = 576 mm

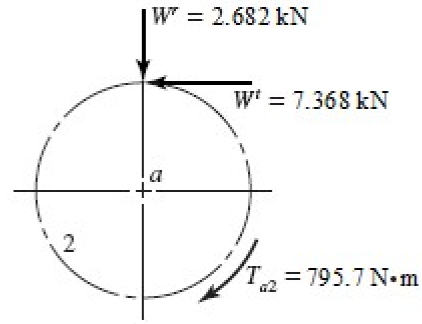
Gear 2

From Eq. (13-36),

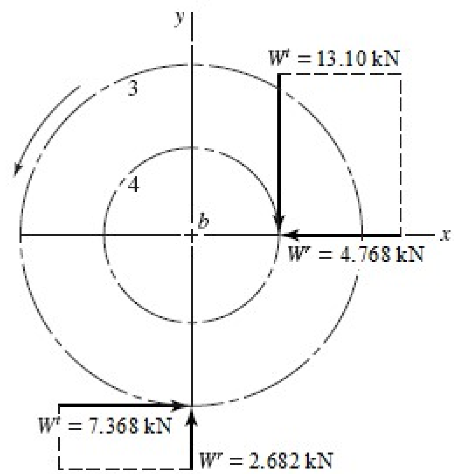
$$W_t = \frac{60\,000H}{\pi dn} = \frac{60\,000(150)}{\pi(216)(1800)} = 7.368 \text{ kN}$$

$$T_{a2} = W_t\left(\frac{d_2}{2}\right) = 7.368\left(\frac{216}{2}\right) = 795.7 \text{ N}\cdot\text{m}$$

$$W^r = 7.368 \tan 20^\circ = 2.682 \text{ kN}$$



Gears 3 *and* 4



$$W^t\left(\frac{216}{2}\right) = 7.368\frac{(384)}{2}$$

$$W^t = 13.10 \text{ kN}$$

$$W^r = 13.10 \tan 20^\circ = 4.768 \text{ kN}$$

Ans.

13-40 Given: $P = 5$ teeth/in, $N_2 = 18T$, $N_3 = 45T$,
$\phi_n = 20^\circ$, $H = 32$ hp, $n_2 = 1800$ rev/min

Gear 2

$$T_{\text{in}} = \frac{63025(32)}{1800} = 1120 \text{ lbf}\cdot\text{in}$$

$$d_P = \frac{18}{5} = 3.600 \text{ in}$$

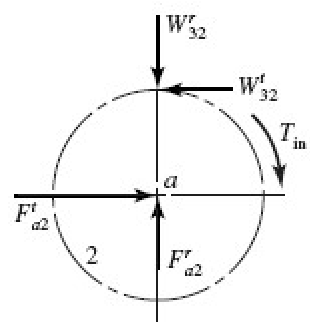
$$d_G = \frac{45}{5} = 9.000 \text{ in}$$

$$W_{32}^t = \frac{1120}{3.6/2} = 622 \text{ lbf}$$

$$W_{32}^r = 622 \tan 20^\circ = 226 \text{ lbf}$$

$$F_{a2}^t = W_{32}^t = 622 \text{ lbf}, \quad F_{a2}^r = W_{32}^r = 226 \text{ lbf}$$

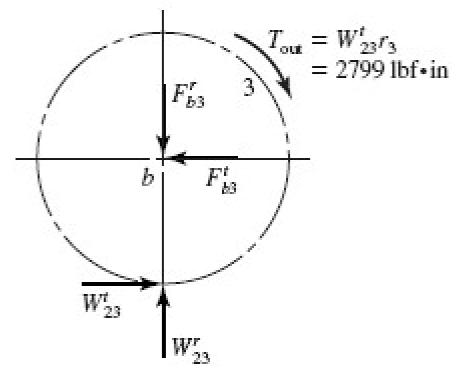
$$F_{a2} = \left(622^2 + 226^2\right)^{1/2} = 662 \text{ lbf}$$



Each bearing on shaft *a* has the same radial load of $R_A = R_B = 662/2 = 331$ lbf.

Gear 3

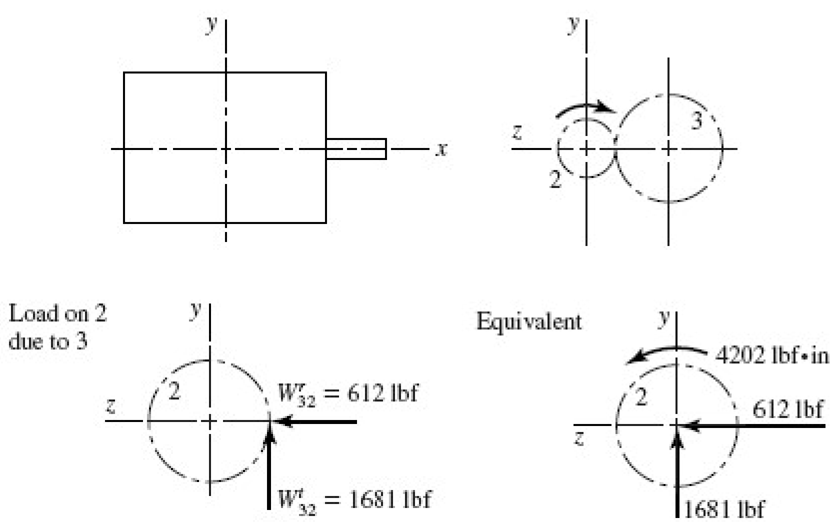
$$W_{23}^t = W_{32}^t = 622 \text{ lbf}$$
$$W_{23}^r = W_{32}^r = 226 \text{ lbf}$$
$$F_{b3} = F_{b2} = 662 \text{ lbf}$$
$$R_C = R_D = 662/2 = 331 \text{ lbf}$$



Each bearing on shaft *b* has the same radial load which is equal to the radial load of bearings *A* and *B*. Thus, all four bearings have the same radial load of 331 lbf. *Ans.*

13-41 Given: $P = 4$ teeth/in, $\phi_n = 20^\circ$, $N_P = 20T$, $n_2 = 900$ rev/min

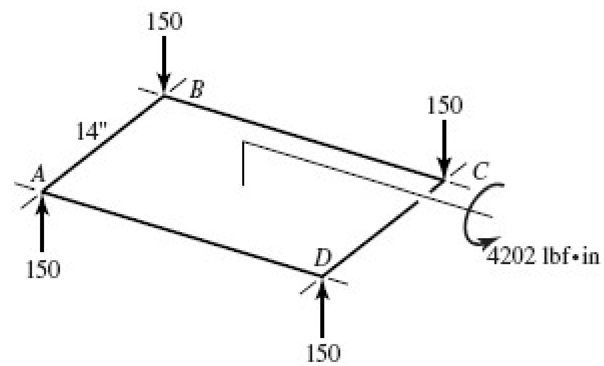
$$d_2 = \frac{N_P}{P} = \frac{20}{4} = 5.000 \text{ in}$$
$$T_{\text{in}} = \frac{63\,025(30)(2)}{900} = 4202 \text{ lbf} \cdot \text{in}$$
$$W_{32}^t = T_{\text{in}} / (d_2/2) = 4202/(5/2) = 1681 \text{ lbf}$$
$$W_{32}^r = 1681 \tan 20^\circ = 612 \text{ lbf}$$



The motor mount resists the equivalent forces and torque.
The radial force due to torque is

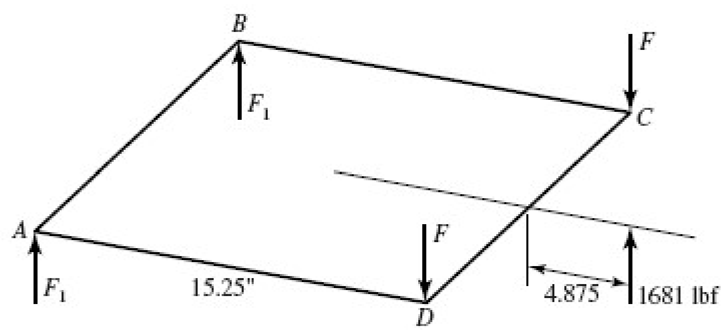
$$F^r = \frac{4202}{14(2)} = 150 \text{ lbf}$$

Forces reverse with rotational sense as torque reverses.



The compressive loads at *A* and *D* are absorbed by the base plate, not the bolts. For W_{32}^t, the tensions in *C* and *D* are

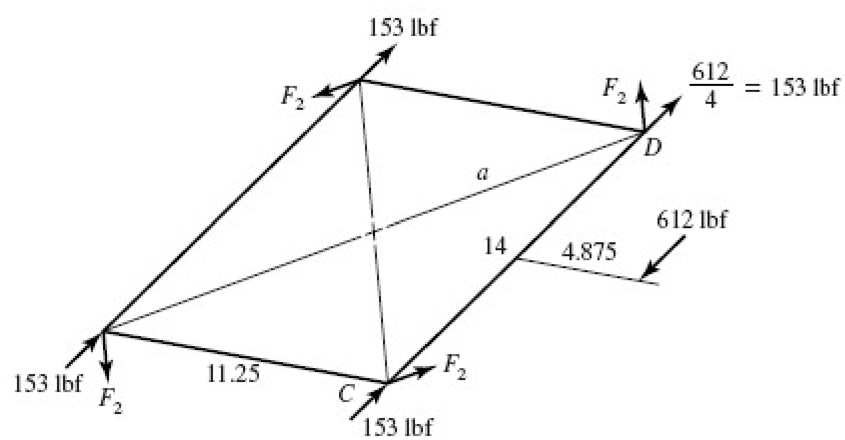
$$\Sigma M_{AB} = 0 \qquad 1681(4.875 + 15.25) - 2F(15.25) = 0 \qquad F = 1109 \text{ lbf}$$



If W_{32}^t reverses, 15.25 in changes to 13.25 in, 4.815 in changes to 2.875 in, and the forces change direction. For *A* and *B*,

$$1681(2.875) - 2F_1(13.25) = 0 \quad \Rightarrow \quad F_1 = 182.4 \text{ lbf}$$

For W_{32}^r,



$$M = 612(4.875 + 11.25/2) = 6426 \text{ lbf} \cdot \text{in}$$

$$a = \sqrt{(14/2)^2 + (11.25/2)^2} = 8.98 \text{ in}$$

$$F_2 = \frac{6426}{4(8.98)} = 179 \text{ lbf}$$

At C and D, the shear forces are:

$$F_{S1} = \sqrt{[153 + 179(5.625/8.98)]^2 + [179(7/8.98)]^2}$$

At A and B, the shear forces are:

$$F_{S2} = \sqrt{[153 - 179(5.625/8.98)]^2 + [179(7/8.98)]^2}$$
$$= 145 \text{ lbf}$$

The shear forces are independent of the rotational sense.
The bolt tensions and the shear forces for cw rotation are,

	Tension (lbf)	Shear (lbf)
A	0	145
B	0	145
C	1109	300
D	1109	300

For ccw rotation,

	Tension (lbf)	Shear (lbf)
A	182	145
B	182	145
C	0	300
D	0	300

13-42 **(a)** $N_2 = N_4 = 15$ teeth, $N_3 = N_5 = 44$ teeth

$$P = \frac{N}{d} \quad \Rightarrow \quad d = \frac{N}{P}$$

$$d_2 = d_4 = \frac{15}{6} = 2.5 \text{ in} \quad \textit{Ans.}$$

$$d_3 = d_5 = \frac{44}{6} = 7.33 \text{ in} \quad \textit{Ans.}$$

(b) $V_i = V_2 = V_3 = \dfrac{\pi d_2 n_2}{12} = \dfrac{\pi(2.5)(2500)}{12} = 1636 \text{ ft/min}$ *Ans.*

$V_o = V_4 = V_5 = \dfrac{\pi d_4 n_4}{12} = \dfrac{\pi(2.5)\left[(2500)(15/44)\right]}{12} = 558 \text{ ft/min}$ *Ans.*

(c) Input gears:

$W_{ti} = 33\,000\dfrac{H}{V_i} = \dfrac{33\,000(25)}{1636} = 504.3 \text{ lbf} = 504 \text{ lbf}$ *Ans.*

$W_{ri} = W_{ti}\tan\phi = 504.3\tan 20^\circ = 184 \text{ lbf}$ *Ans.*

$W_i = \dfrac{W_{ti}}{\cos\phi} = \dfrac{504.3}{\cos 20^\circ} = 537 \text{ lbf}$ *Ans.*

Output gears:

$W_{to} = 33\,000\dfrac{H}{V_o} = \dfrac{33\,000(25)}{558} = 1478 \text{ lbf}$ *Ans.*

$W_{ro} = W_{to}\tan\phi = 1478\tan 20^\circ = 538 \text{ lbf}$ *Ans.*

$W_o = \dfrac{W_{to}}{\cos 20^\circ} = \dfrac{1478}{\cos 20^\circ} = 1573 \text{ lbf}$ *Ans.*

(d) $T_i = W_{ti}\left(\dfrac{d_2}{2}\right) = 504.3\left(\dfrac{2.5}{2}\right) = 630 \text{ lbf}\cdot\text{in}$ *Ans.*

(e) $T_o = T_i\left(\dfrac{44}{15}\right)^2 = 630\left(\dfrac{44}{15}\right)^2 = 5420 \text{ lbf}\cdot\text{in}$ *Ans.*

13-43 $H = 35$ hp, $n_i = 1200$ rev/min, $\phi = 20^\circ$

$N_2 = N_4 = 16$ teeth, $N_3 = N_5 = 48$ teeth, $P = 10$ teeth/in

(a) $n_{\text{intermediate}} = n_3 = n_4 = \dfrac{N_2}{N_3}n_i = \dfrac{16}{48}(1200) = 400 \text{ rev/min}$ *Ans.*

$n_o = \dfrac{N_2}{N_3}\dfrac{N_4}{N_5}n_i = \dfrac{16}{48}\left(\dfrac{16}{48}\right)(1200) = 133.3 \text{ rev/min}$ *Ans.*

(b) $P = \dfrac{N}{d} \Rightarrow d = \dfrac{N}{P}$

$d_2 = d_4 = \dfrac{16}{10} = 1.6 \text{ in}$ *Ans.*

$d_3 = d_5 = \dfrac{48}{10} = 4.8 \text{ in}$ *Ans.*

$$V_i = V_2 = V_3 = \frac{\pi d_2 n_2}{12} = \frac{\pi(1.6)(1200)}{12} = 502.7 \text{ ft/min} \quad \textit{Ans.}$$

$$V_o = V_4 = V_5 = \frac{\pi d_4 n_4}{12} = \frac{\pi(1.6)(400)}{12} = 167.6 \text{ ft/min} \quad \textit{Ans.}$$

(c) $$W_{ti} = 33\,000\frac{H}{V_i} = \frac{33\,000(35)}{502.7} = 2298 \text{ lbf lbf} \quad \textit{Ans.}$$

$$W_{ri} = W_{ti}\tan\phi = 2298\tan 20^\circ = 836.4 \text{ lbf} \quad \textit{Ans.}$$

$$W_i = \frac{W_{ti}}{\cos\phi} = \frac{2298}{\cos 20^\circ} = 2445 \text{ lbf} \quad \textit{Ans.}$$

$$W_{to} = 33\,000\frac{H}{V_o} = \frac{33\,000(35)}{167.6} = 6891 \text{ lbf} \quad \textit{Ans.}$$

$$W_{ro} = W_{to}\tan\phi = 6891\tan 20^\circ = 2508 \text{ lbf} \quad \textit{Ans.}$$

$$W_o = \frac{W_{to}}{\cos 20^\circ} = \frac{6891}{\cos 20^\circ} = 7333 \text{ lbf} \quad \textit{Ans.}$$

(d) $$T_i = W_{ti}\left(\frac{d_2}{2}\right) = 2298\left(\frac{1.6}{2}\right) = 1838 \text{ lbf}\cdot\text{in} \quad \textit{Ans.}$$

(e) $$T_o = T_i\left(\frac{48}{16}\right)^2 = 1838\left(\frac{48}{16}\right)^2 = 16\,540 \text{ lbf}\cdot\text{in} \quad \textit{Ans.}$$

13-44 **(a)** For $\frac{\omega_o}{\omega_i} = \frac{2}{1}$, from Eq. (13-11), with $m = 2$, $k = 1$, $\phi = 20^\circ$

$$N_P = \frac{2(1)}{[1+2(2)]\sin^2 20^\circ}\left\{2 + \sqrt{2^2 + [1+2(2)]\sin^2 20^\circ}\right\} = 14.16$$

So $$N_{P\min} = 15 \quad \textit{Ans.}$$

(b) $$P = \frac{N}{d} = \frac{15}{8} = 1.875 \text{ teeth/in} \quad \textit{Ans.}$$

(c) To transmit the same power with no change in pitch diameters, the speed and transmitted force must remain the same.

For A, with $\phi = 20°$,

$$W_{tA} = F_A \cos 20° = 300 \cos 20° = 281.9 \text{ lbf}$$

For A, with $\phi = 25°$, same transmitted load,

$F_A = W_{tA}/\cos 25° = 281.9/\cos 25° = 311.0$ lbf *Ans.*

Summing the torque about the shaft axis,

$$W_{tA}\left(\frac{d_A}{2}\right) = W_{tB}\left(\frac{d_B}{2}\right)$$

$$W_{tB} = W_{tA}\frac{(d_A/2)}{(d_B/2)} = W_{tA}\left(\frac{d_A}{d_B}\right) = (281.9)\left(\frac{20}{8}\right) = 704.75 \text{ lbf}$$

$$F_B = \frac{W_{tB}}{\cos 25^\circ} = \frac{704.75}{\cos 25^\circ} = 777.6 \text{ lbf} \quad \textit{Ans.}$$

13-45 **(a)** For $\frac{\omega_o}{\omega_i} = \frac{5}{1}$, from Eq. (13-11), with $m = 5$, $k = 1$, $\phi = 20^\circ$

$$N_P = \frac{2(1)}{[1+2(5)]\sin^2 25^\circ}\left\{5 + \sqrt{5^2 + [1+2(5)]\sin^2 25^\circ}\right\} = 10.4$$

So $N_{P\min} = 11$ *Ans.*

(b) $$m = \frac{d}{N} = \frac{300}{11} = 27.3 \text{ mm/tooth} \quad \textit{Ans.}$$

(c) To transmit the same power with no change in pitch diameters, the speed and transmitted force must remain the same.
For A, with $\phi = 20°$,

$W_{tA} = F_A \cos 20° = 11 \cos 20° = 10.33$ kN

For A, with $\phi = 25°$, same transmitted load,

$F_A = W_{tA}/\cos 25° = 10.33 / \cos 25° = 11.40$ kN *Ans.*

Summing the torque about the shaft axis,

$$W_{tA}\left(\frac{d_A}{2}\right) = W_{tB}\left(\frac{d_B}{2}\right)$$

$$W_{tB} = W_{tA}\frac{(d_A/2)}{(d_B/2)} = W_{tA}\left(\frac{d_A}{d_B}\right) = (11.40)\left(\frac{600}{300}\right) = 22.80 \text{ kN}$$

$$F_B = \frac{W_{tB}}{\cos 25^\circ} = \frac{22.80}{\cos 25^\circ} = 25.16 \text{ kN} \quad \textit{Ans.}$$

13-46 **(a)** Using Eq. (13-11) with $k = 1$, $\phi = 20°$, and $m = 2$,

$$N_P = \frac{2k}{(1+2m)\sin^2\phi}\left(m+\sqrt{m^2+(1+2m)\sin^2\phi}\right)$$
$$= \frac{2(1)}{[1+2(2)]\sin^2(20^\circ)}\left\{(2)+\sqrt{(2)^2+[1+2(2)]\sin^2(20^\circ)}\right\} = 14.16 \text{ teeth}$$

Round up for the minimum integer number of teeth.

N_F = 15 teeth, N_C = 30 teeth *Ans.*

(b) $$m = \frac{d}{N} = \frac{125}{15} = 8.33 \text{ mm/tooth} \quad Ans.$$

(c) $$T = \frac{H}{\omega} = \frac{2 \text{ kW}}{191 \text{ rev/min}}\left(\frac{1000 \text{ W}}{\text{kW}}\right)\left(\frac{\text{rev}}{2\pi \text{ rad}}\right)\left(\frac{60 \text{ s}}{\text{min}}\right) = 100 \text{ N}\cdot\text{m}$$

(d) From Eq. (13-36),

$$W_t = \frac{60\,000H}{\pi dn} = \frac{60\,000(2)}{\pi(125)(191)} = 1.60 \text{ kN} = 1600 \text{ N} \qquad Ans.$$

Or, we could have obtained W_t directly from the torque and radius,

$$W_t = \frac{T}{d/2} = \frac{100}{0.125/2} = 1600 \text{ N}$$

$$W_r = W_t \tan\phi = 1600 \tan 20^\circ = 583 \text{ N} \quad Ans.$$
$$W = \frac{W_t}{\cos\phi} = \frac{1600}{\cos 20^\circ} = 1700 \text{ N} \quad Ans.$$

13-47 **(a)** Using Eq. (13-11) with $k = 1$, $\phi = 20^\circ$, and $m = 2$,

$$N_P = \frac{2k}{(1+2m)\sin^2\phi}\left(m+\sqrt{m^2+(1+2m)\sin^2\phi}\right)$$
$$= \frac{2(1)}{[1+2(2)]\sin^2(20^\circ)}\left\{(2)+\sqrt{(2)^2+[1+2(2)]\sin^2(20^\circ)}\right\} = 14.16 \text{ teeth}$$

Round up for the minimum integer number of teeth.

N_C = 15 teeth, N_F = 30 teeth *Ans.*

(b) $$P = \frac{N}{d} = \frac{30}{10} = 3 \text{ teeth/in} \quad \textit{Ans.}$$

(c) $$T = \frac{H}{\omega} = \frac{1 \text{ hp}}{70 \text{ rev/min}}\left(\frac{550 \text{ lbf}\cdot\text{ft/s}}{\text{hp}}\right)\left(\frac{12 \text{ in}}{\text{ft}}\right)\left(\frac{\text{rev}}{2\pi \text{ rad}}\right)\left(\frac{60 \text{ s}}{\text{min}}\right)$$
$$T = 900 \text{ lbf}\cdot\text{in} \quad \textit{Ans.}$$

(d) From Eqs. (13-34) and (13-35),

$$V = \frac{\pi d n}{12} = \frac{\pi(10)(70)}{12} = 183.3 \text{ ft/min}$$
$$W_t = 33\,000\frac{H}{V} = \frac{33\,000(1)}{183.3} = 180 \text{ lbf} \quad \textit{Ans.}$$
$$W_r = W_t \tan\phi = 180\tan 20^\circ = 65.5 \text{ lbf} \quad \textit{Ans.}$$
$$W = \frac{W_t}{\cos\phi} = \frac{180}{\cos 20^\circ} = 192 \text{ lbf} \quad \textit{Ans.}$$

13-48 **(a)** Eq. (13-14): $\gamma = \tan^{-1}\left(\frac{N_P}{N_G}\right) = \tan^{-1}\left(\frac{d_P}{d_G}\right) = \tan^{-1}\left(\frac{1.30}{3.88}\right) = 18.5^\circ$ *Ans.*

(b) Eq. (13-34): $V = \frac{\pi d n}{12} = \frac{\pi(2)(1.30)(600)}{12} = 408.4 \text{ ft/min}$ *Ans.*

(c) Eq. (13-35): $W_t = 33\,000\frac{H}{V} = 33\,000\left(\frac{10}{408.4}\right) = 808 \text{ lbf}$ *Ans.*

Eq. (13-38): $W_r = W_t \tan\phi\cos\gamma = 808\tan 20^\circ\cos 18.5^\circ = 279 \text{ lbf}$ *Ans.*

Eq. (13-38): $W_a = W_t \tan\phi\sin\gamma = 808\tan 20^\circ\sin 18.5^\circ = 93.3 \text{ lbf}$ *Ans.*

The tangential and axial forces agree with Prob. 3-85, but the radial force given in Prob. 3-85 is shown here to be incorrect. *Ans.*

13-49

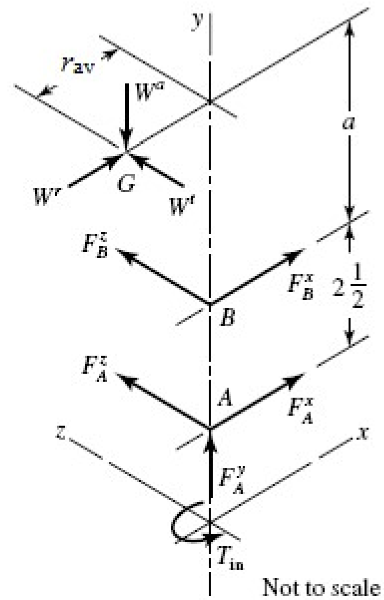
$$\gamma = \tan^{-1}(2/4) = 26.565^\circ$$
$$\Gamma = \tan^{-1}(4/2) = 63.435^\circ$$
$$r_{\text{av}} = 2 - (1.5\sin 26.565^0)/2 = 1.665 \text{ in}$$
$$T_{\text{in}} = 63\,025H/n = 63025(2.5)/240 = 656.5 \text{ lbf}\cdot\text{in}$$
$$W^t = T/r_{\text{av}} = 656.5/1.665 = 394.3 \text{ lbf}$$
$$a = 2 + (1.5\cos 26.565^\circ)/2 = 2.671 \text{ in}$$
$$W^r = 394.3\tan 20^\circ\cos 26.565^\circ = 128.4 \text{ lbf}$$

$W^a = 394.3\tan 20^\circ \sin 26.565^\circ = 64.2$ lbf

W = 128.4**i** – 64.2**j** + 394.3**k** lbf

$\mathbf{R}_{AG} = -1.665\mathbf{i} + 5.171\mathbf{j}, \quad \mathbf{R}_{AB} = 2.5\mathbf{j}$

$\Sigma\mathbf{M}_4 = \mathbf{R}_{AG}\times\mathbf{W} + \mathbf{R}_{AB}\times\mathbf{F}_B + \mathbf{T} = \mathbf{0}$



Solving gives

$\mathbf{R}_{AB}\times\mathbf{F}_B = 2.5F_B^z\mathbf{i} - 2.5F_B^x\mathbf{k}$

$\mathbf{R}_{AG}\times\mathbf{W} = 2039\mathbf{i} + 656.5\mathbf{j} - 557.1\mathbf{k}$

So,

$$\left(2039\mathbf{i} + 656.5\mathbf{j} - 557.1\mathbf{k}\right) + \left(2.5F_B^z\mathbf{i} - 2.5F_B^x\mathbf{k} + T\mathbf{j}\right) = \mathbf{0}$$

$F_B^z = -2039/2.5 = -815.6$ lbf

$T = -656.5$ lbf · in

$F_B^x = -557.1/2.5 = -222.8$ lbf

So, **F**$_B$ = –222.8**i** –815.6**k** lbf *Ans.*

$F_B = \left[\left(-222.88\right)^2 + \left(-815.6\right)^2\right]^{1/2} = 845.5$ lbf

$\mathbf{F}_A = -(\mathbf{F}_B + \mathbf{W})$
$= -(-222.8\mathbf{i} - 815.6\mathbf{k} + 128.4\mathbf{i} - 64.2\mathbf{j} + 394.3\mathbf{k})$
$= 94.4\mathbf{i} + 64.2\mathbf{j} + 421.3\mathbf{k}$ *Ans.*

$F_A(\text{radial}) = \left(94.4^2 + 421.3^2\right)^{1/2} = 431.7$ lbf

$F_A(\text{thrust}) = 64.2$ lbf

13-50 $d_2 = 18/10 = 1.8$ in, $d_3 = 30/10 = 3.0$ in

$\gamma = \tan^{-1}\left(\dfrac{d_2/2}{d_3/2}\right) = \tan^{-1}\left(\dfrac{0.9}{1.5}\right) = 30.96^\circ$

$\Gamma = 90^\circ - \gamma = 59.04^\circ$

$r_{av} = 3.0/2 - \left(0.5\sin 59.04^\circ\right)/2 = 1.286$ in

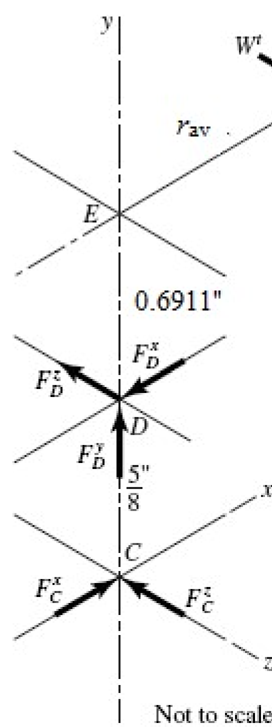
$DE = \dfrac{9}{16} + \left(0.5\cos 59.04^\circ\right)/2 = 0.6911$ in

$W^t = 25$ lbf

$W^r = 25\tan 20^\circ \cos 59.04^\circ = 4.681$ lbf

$W^a = 25\tan 20^\circ \sin 59.04^\circ = 7.803$ lbf

W = –4.681**i** – 7.803**j** +25**k**



$\mathbf{R}_{DG} = 1.286\mathbf{i} + 0.6911\mathbf{j}$

$\mathbf{R}_{DC} = -0.625\mathbf{j}$

$$\Sigma\mathbf{M}_D = \mathbf{R}_{DG} \times \mathbf{W} + \mathbf{R}_{DC} \times \mathbf{F}_C + \mathbf{T} = \mathbf{0}$$

$$\mathbf{R}_{DG} \times \mathbf{W} = 17.28\mathbf{i} - 32.15\mathbf{j} - 6.800\mathbf{k}$$

$$\mathbf{R}_{DC} \times \mathbf{F}_C = -0.625F_C^z\mathbf{i} + 0.625F_C^x\mathbf{k}$$

$$(17.28\mathbf{i} - 32.15\mathbf{j} - 6.800\mathbf{k}) + (-0.625F_C^z\mathbf{i} + 0.625F_C^x\mathbf{k}) + T\mathbf{j} = \mathbf{0}$$

$T = 32.15 \text{ lbf} \cdot \text{in}$ *Ans.*

$\mathbf{F}_C = 10.88\mathbf{i} + 27.65\mathbf{k} \text{ lbf}$ *Ans.*

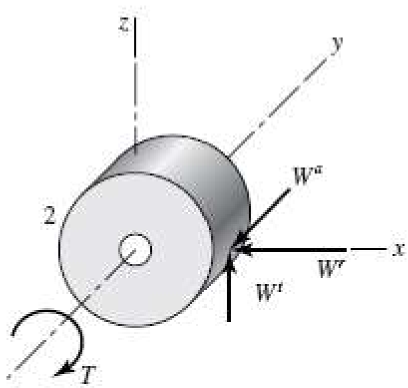
$F_C = (10.88^2 + 27.65^2)^{1/2} = 29.7 \text{ lbf}$ *Ans.*

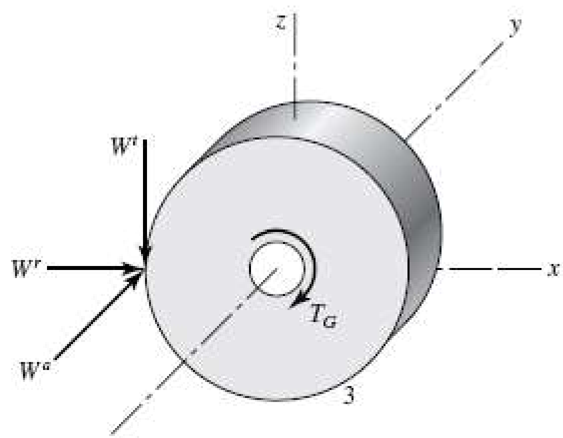
$\Sigma\mathbf{F} = 0 \qquad \mathbf{F}_D = -6.20\mathbf{i} + 7.80\mathbf{j} - 52.65\mathbf{k} \text{ lbf}$

$F_D(\text{radial}) = \left[(-6.20)^2 + (-52.65)^2\right]^{1/2} = 53.0 \text{ lbf}$ *Ans.*

$F_D(\text{thrust}) = W^a = 7.80 \text{ lbf}$ *Ans.*

13-51





NOTE: The shaft forces exerted on the gears are not shown in the figures above.

$$P_t = P_n \cos\psi = 5\cos 30^\circ = 4.330 \text{ teeth/in}$$

$$\phi_t = \tan^{-1}\frac{\tan\phi_n}{\cos\psi} = \tan^{-1}\frac{\tan 20^\circ}{\cos 30^\circ} = 22.80^\circ$$

$$d_P = \frac{18}{4.330} = 4.157 \text{ in}$$

The forces on the shafts will be equal to the forces transmitted to the gears through the meshing teeth.

Pinion (*Gear* 2)

$$W^r = W^t \tan\phi_t = 800 \tan 22.80^\circ = 336 \text{ lbf}$$

$$W^a = W^t \tan\psi = 800 \tan 30^\circ = 462 \text{ lbf}$$

$$\mathbf{W} = -336\mathbf{i} - 462\mathbf{j} + 800\mathbf{k} \text{ lbf} \quad \textit{Ans.}$$

$$W = \left[(-336)^2 + (-462)^2 + 800^2\right]^{1/2} = 983 \text{ lbf} \quad \textit{Ans.}$$

Gear 3

$$\mathbf{W} = 336\mathbf{i} + 462\mathbf{j} - 800\mathbf{k} \text{ lbf} \quad \textit{Ans.}$$

$$W = 983 \text{ lbf} \quad \textit{Ans.}$$

$$d_G = \frac{32}{4.330} = 7.390 \text{ in}$$

$$T_G = W^t r = 800(7.390) = 5912 \text{ lbf} \cdot \text{in}$$

13-52 $$\phi_t = \tan^{-1}\frac{\tan\phi_n}{\cos\psi} = \tan^{-1}\frac{\tan 20^\circ}{\cos 30^\circ} = 22.80^\circ$$

Pinion (*Gear* 2)

$$W^r = W^t \tan\phi_t = 800 \tan 22.80^\circ = 336 \text{ lbf}$$

$$W^a = W^t \tan\psi = 800 \tan 30^\circ = 462 \text{ lbf}$$

$$\mathbf{W} = -336\mathbf{i} - 462\mathbf{j} - 800\mathbf{k} \text{ lbf} \quad \textit{Ans.}$$

$$W = \left[(-336)^2 + (-462)^2 + (-800)^2\right]^{1/2} = 983 \text{ lbf} \quad \textit{Ans.}$$

Idler (*Gear* 3)

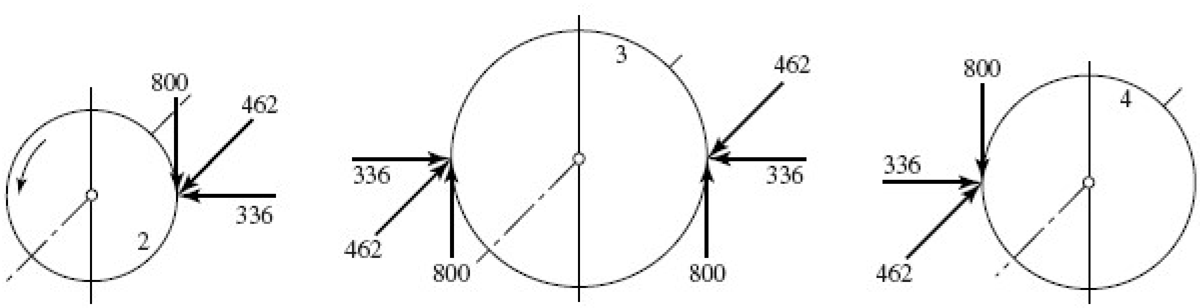
From the diagram for the idler, noting that the radial and axial forces from gears 2 and 4 cancel each other, the force acting on the shaft is

$$\mathbf{W} = +1600\mathbf{k} \text{ lbf} \quad \textit{Ans.}$$

Output gear (*Gear* 4)

$$\mathbf{W} = 336\mathbf{i} - 462\mathbf{j} - 800\mathbf{k} \text{ lbf} \quad \textit{Ans.}$$

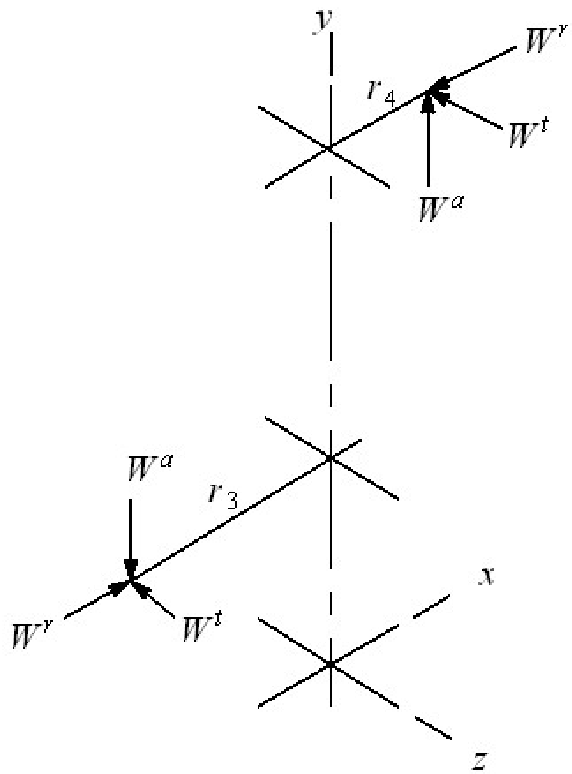
$$W = \left[(-336)^2 + (-462)^2 + (-800)^2\right]^{1/2} = 983 \text{ lbf} \quad \textit{Ans.}$$



NOTE: For simplicity, the above figures only show the gear contact forces.

Also, notice that the idler shaft reaction contains a couple tending to turn the shaft end-over-end. Also the idler teeth are bent both ways. Idlers are more severely loaded than other gears, belying their name. Thus, be cautious.

13-53 *Gear* 3:



$$P_t = P_n \cos\psi = 7\cos 30^\circ = 6.062 \text{ teeth/in}$$

$$\tan\phi_t = \frac{\tan 20^\circ}{\cos 30^\circ} = 0.4203, \quad \phi_t = 22.8^\circ$$

$$d_3 = \frac{54}{6.062} = 8.908 \text{ in}$$

$$W^t = 500 \text{ lbf}$$

$$W^a = 500\tan 30^\circ = 288.7 \text{ lbf}$$

$$W^r = 500\tan 22.8^\circ = 210.2 \text{ lbf}$$

$$\mathbf{W}_3 = 210.2\mathbf{i} - 288.7\mathbf{j} - 500\mathbf{k} \text{ lbf} \quad \textit{Ans.}$$

Gear 4:

$$d_4 = \frac{14}{6.062} = 2.309 \text{ in}$$

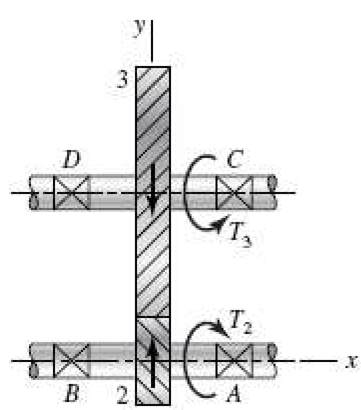
$$W^t = 500\frac{8.908}{2.309} = 1929 \text{ lbf}$$

$$W^a = 1929\tan 30^\circ = 1114 \text{ lbf}$$

$$W^r = 1929\tan 22.8^\circ = 811 \text{ lbf}$$

$$\mathbf{W}_4 = -811\mathbf{i} + 1114\mathbf{j} - 1929\mathbf{k} \text{ lbf} \quad \textit{Ans.}$$

13-54



$$P_t = 6\cos 30^\circ = 5.196 \text{ teeth/in}$$

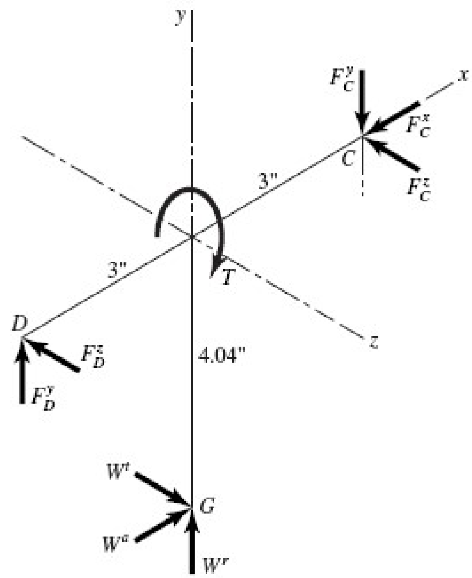
$$d_3 = \frac{42}{5.196} = 8.083 \text{ in}$$

$$\phi_t = 22.8^\circ$$

$$d_2 = \frac{16}{5.196} = 3.079 \text{ in}$$

$$T_2 = \frac{63\,025(25)}{1720} = 916 \text{ lbf}\cdot\text{in}$$

$$W^t = \frac{T}{r} = \frac{916}{3.079/2} = 595 \text{ lbf}$$
$$W^a = 595 \tan 30^\circ = 344 \text{ lbf}$$
$$W^r = 595 \tan 22.8^\circ = 250 \text{ lbf}$$



$$\mathbf{W} = 344\mathbf{i} + 250\mathbf{j} + 595\mathbf{k} \text{ lbf}$$

$$\mathbf{R}_{DC} = 6\mathbf{i}, \quad \mathbf{R}_{DG} = 3\mathbf{i} - 4.04\mathbf{j}$$

$$\Sigma \mathbf{M}_D = \mathbf{R}_{DC} \times \mathbf{F}_C + \mathbf{R}_{DG} \times \mathbf{W} + \mathbf{T} = \mathbf{0} \quad (1)$$

$$\mathbf{R}_{DG} \times \mathbf{W} = -2404\mathbf{i} - 1785\mathbf{j} + 2140\mathbf{k}$$
$$\mathbf{R}_{DC} \times \mathbf{F}_C = -6F_C^z\mathbf{j} + 6F_C^y\mathbf{k}$$

Substituting and solving Eq. (1) gives

$$\mathbf{T} = 2404\mathbf{i} \text{ lbf} \cdot \text{in}$$
$$F_C^z = -297.5 \text{ lbf}$$
$$F_C^y = -365.7 \text{ lbf}$$

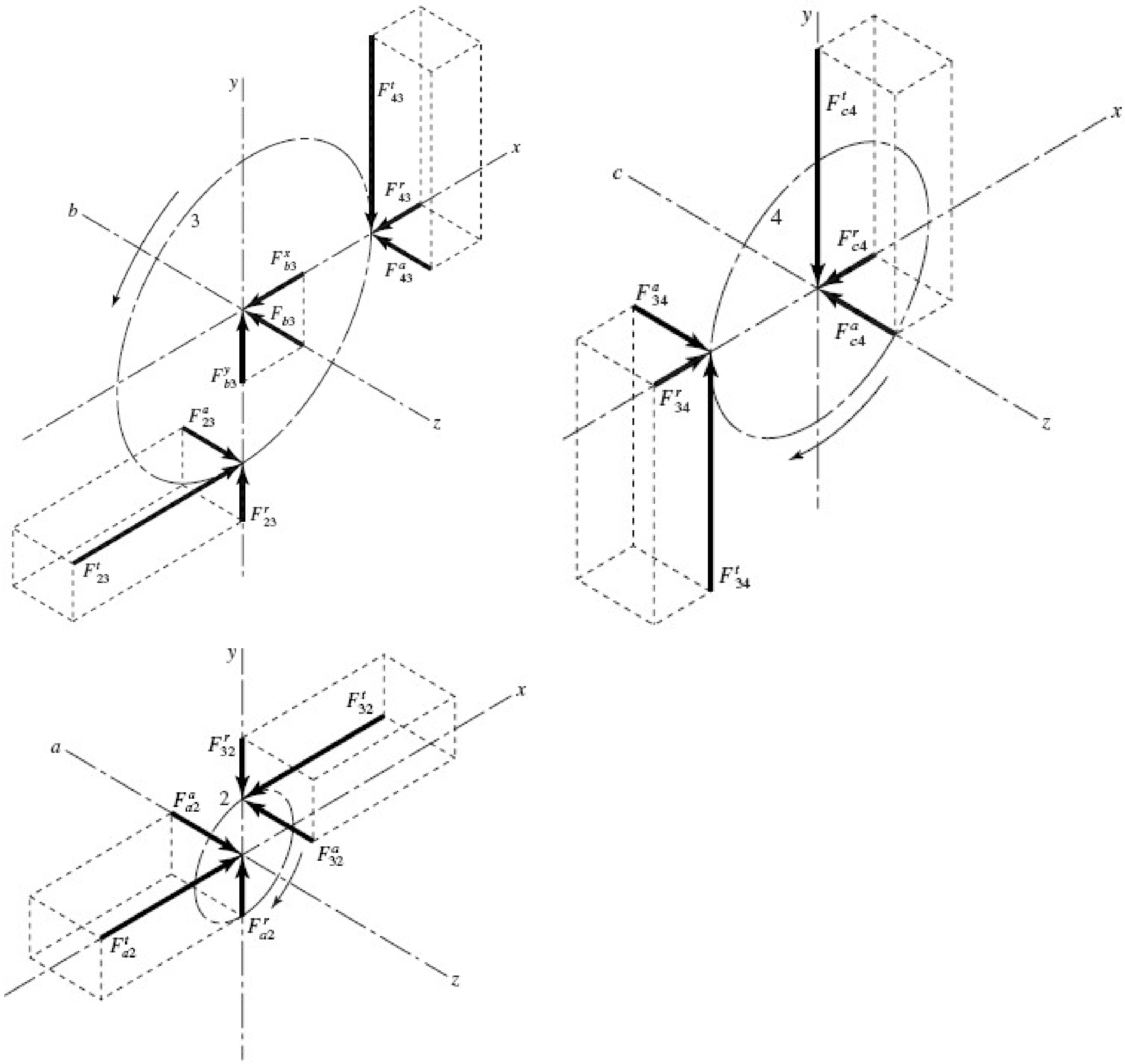
$$\Sigma \mathbf{F} = \mathbf{F}_D + \mathbf{F}_C + \mathbf{W} = \mathbf{0}$$

Substituting and solving gives

$$F_C^x = -344 \text{ lbf}$$
$$F_D^y = 106.7 \text{ lbf}$$
$$F_D^z = -297.5 \text{ lbf}$$

$$\mathbf{F}_C = -344\mathbf{i} - 356.7\mathbf{j} - 297.5\mathbf{k} \text{ lbf} \quad \textit{Ans.}$$
$$\mathbf{F}_D = 106.7\mathbf{j} - 297.5\mathbf{k} \text{ lbf} \quad \textit{Ans.}$$

13-55



Since the transverse pressure angle is specified, we will assume the given module is also in terms of the transverse orientation.

$$d_2 = mN_2 = 4(16) = 64 \text{ mm}$$
$$d_3 = mN_3 = 4(36) = 144 \text{ mm}$$
$$d_4 = mN_4 = 4(28) = 112 \text{ mm}$$
$$T = \frac{H}{\omega} = \frac{6 \text{ kW}}{1600 \text{ rev/min}}\left(\frac{1000 \text{ W}}{\text{kW}}\right)\left(\frac{\text{rev}}{2\pi \text{ rad}}\right)\left(\frac{60 \text{ s}}{\text{min}}\right) = 35.81 \text{ N}\cdot\text{m}$$
$$W^t = \frac{T}{d_2/2} = \frac{35.81}{0.064/2} = 1119 \text{ N}$$

$W^r = W^t \tan\phi_t = 1119\tan 20^\circ = 407.3\text{ N}$

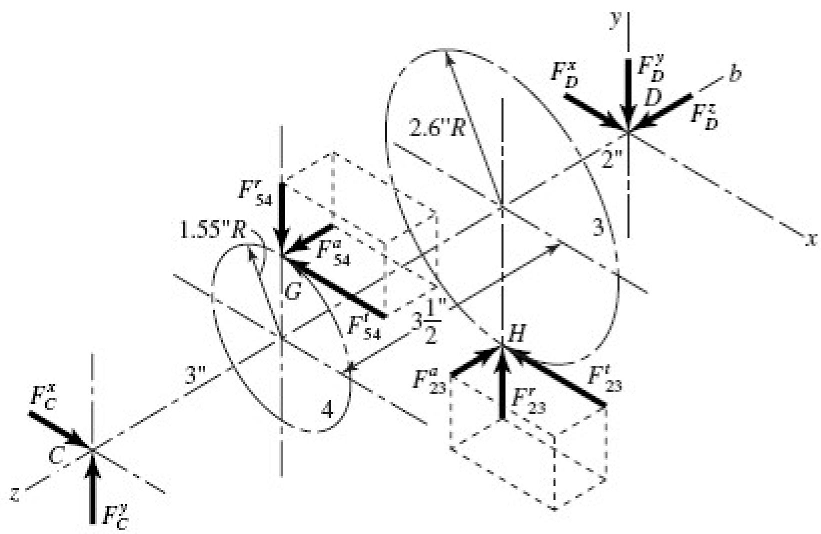
$W^a = W^t \tan\psi = 1119\tan 15^\circ = 299.8\text{ N}$

$\mathbf{F}_{2a} = -1119\mathbf{i} - 407.3\mathbf{j} - 299.8\mathbf{k}\text{ N}$ *Ans.*

$$\mathbf{F}_{3b} = (1119 - 407.3)\mathbf{i} - (1119 - 407.3)\mathbf{j}$$
$$= 711.7\mathbf{i} - 711.7\mathbf{j}\text{ N}$$ *Ans.*

$\mathbf{F}_{4c} = 407.3\mathbf{i} + 1119\mathbf{j} + 299.8\mathbf{k}\text{ N}$ *Ans.*

13-56



$$d_2 = \frac{N}{P_n\cos\psi} = \frac{14}{8\cos 30^\circ} = 2.021\text{ in},\quad d_3 = \frac{36}{8\cos 30^\circ} = 5.196\text{ in}$$

$$d_4 = \frac{15}{5\cos 15^\circ} = 3.106\text{ in},\quad d_5 = \frac{45}{5\cos 15^\circ} = 9.317\text{ in}$$

For gears 2 and 3: $\phi_t = \tan^{-1}(\tan\phi_n / \cos\psi) = \tan^{-1}(\tan 20^\circ / \cos 30^\circ) = 22.8^\circ$

For gears 4 and 5: $\phi_t = \tan^{-1}(\tan 20^\circ / \cos 15^\circ) = 20.6^\circ$

$$F_{23}^t = T_2 / r_2 = 1200/(2.021/2) = 1188\text{ lbf}$$

$$F_{54}^t = 1188\frac{5.196}{3.106} = 1987\text{ lbf}$$

$$F_{23}^r = F_{23}^t \tan\phi_t = 1188\tan 22.8^\circ = 499\text{ lbf}$$

$$F_{54}^r = 1986\tan 20.6^\circ = 746\text{ lbf}$$

$$F_{23}^a = F_{23}^t \tan\psi = 1188\tan 30^\circ = 686\text{ lbf}$$

$$F_{54}^a = 1986\tan 15^\circ = 532\text{ lbf}$$

Next, designate the points of action on gears 4 and 3, respectively, as points *G* and *H*, as shown. Position vectors are

$$\mathbf{R}_{CG} = 1.553\mathbf{j} - 3\mathbf{k}$$
$$\mathbf{R}_{CH} = -2.598\mathbf{j} - 6.5\mathbf{k}$$
$$\mathbf{R}_{CD} = -8.5\mathbf{k}$$

Force vectors are

$$\mathbf{F}_{54} = -1986\mathbf{i} - 748\mathbf{j} + 532\mathbf{k}$$
$$\mathbf{F}_{23} = -1188\mathbf{i} + 500\mathbf{j} - 686\mathbf{k}$$
$$\mathbf{F}_{C} = F_C^x\mathbf{i} + F_C^y\mathbf{j}$$
$$\mathbf{F}_{D} = F_D^x\mathbf{i} + F_D^y\mathbf{j} + F_D^z\mathbf{k}$$

Now, a summation of moments about bearing C gives

$$\Sigma\mathbf{M}_C = \mathbf{R}_{CG} \times \mathbf{F}_{54} + \mathbf{R}_{CH} \times \mathbf{F}_{23} + \mathbf{R}_{CD} \times \mathbf{F}_{D} = \mathbf{0}$$

The terms for this equation are found to be

$$\mathbf{R}_{CG} \times \mathbf{F}_{54} = -1412\mathbf{i} + 5961\mathbf{j} + 3086\mathbf{k}$$
$$\mathbf{R}_{CH} \times \mathbf{F}_{23} = 5026\mathbf{i} + 7722\mathbf{j} - 3086\mathbf{k}$$
$$\mathbf{R}_{CD} \times \mathbf{F}_{D} = 8.5F_D^y\mathbf{i} - 8.5F_D^x\mathbf{j}$$

When these terms are placed back into the moment equation, the **k** terms, representing the shaft torque, cancel. The **i** and **j** terms give

$$F_D^y = -\frac{3614}{8.5} = -425 \text{ lbf}$$
$$F_D^x = \frac{13\,683}{8.5} = 1610 \text{ lbf}$$

Next, we sum the forces to zero.

$$\Sigma\mathbf{F} = \mathbf{F}_C + \mathbf{F}_{54} + \mathbf{F}_{23} + \mathbf{F}_D = \mathbf{0}$$

Substituting, gives

$$\left(F_C^x\mathbf{i} + F_C^y\mathbf{j}\right) + \left(-1987\mathbf{i} - 746\mathbf{j} + 532\mathbf{k}\right) + \left(-1188\mathbf{i} + 499\mathbf{j} - 686\mathbf{k}\right)$$
$$+\left(1610\mathbf{i} - 425\mathbf{j} + F_D^z\mathbf{k}\right) = \mathbf{0}$$

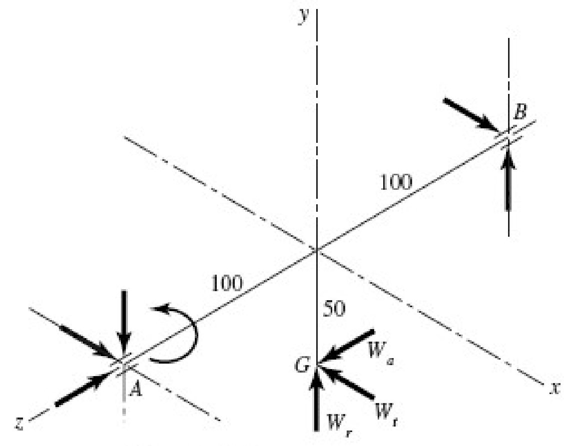
Solving gives

$$F_C^x = 1987 + 1188 - 1610 = 1565 \text{ lbf}$$
$$F_C^y = 746 - 499 + 425 = 672 \text{ lbf}$$
$$F_D^z = -532 + 686 = 154 \text{ lbf}$$

So, $\mathbf{F}_C = 1565\mathbf{i} + 672\mathbf{j}$ lbf *Ans.*

$\mathbf{F}_D = 1610\mathbf{i} - 425\mathbf{j} + 154\mathbf{k}$ lbf *Ans.*

13-57 $V_W = \frac{\pi d_W n_W}{60} = \frac{\pi(0.100)(600)}{60} = \pi \text{ m/s}$



Worm shaft diagram

$$W_{Wt} = \frac{H}{V_W} = \frac{2000}{\pi} = 637 \text{ N}$$

$$L = p_x N_W = 25(1) = 25 \text{ mm}$$

$$\lambda = \tan^{-1}\frac{L}{\pi d_W}$$

$$= \tan^{-1}\frac{25}{\pi(100)} = 4.550^\circ \text{ lead angle}$$

$$W = \frac{W_{Wt}}{\cos\phi_n \sin\lambda + f\cos\lambda}$$

$$V_S = \frac{V_W}{\cos\lambda} = \frac{\pi}{\cos 4.550^\circ} = 3.152 \text{ m/s}$$

In ft/min: V_S = 3.28(3.152) = 10.33 ft/s = 620 ft/min
Use f = 0.043 from curve A of Fig. 13-38. Then, from the first of Eq. (13-43)

$$W = \frac{637}{\cos 14.5^\circ\left(\sin 4.55^\circ\right) + 0.043\cos 4.55^\circ} = 5323 \text{ N}$$

$$W^y = W\sin\phi_n = 5323\sin 14.5^\circ = 1333 \text{ N}$$

$$W^z = 5323\left[\cos 14.5^\circ\left(\cos 4.55^\circ\right) - 0.043\sin 4.55^\circ\right] = 5119 \text{ N}$$

The force acting against the worm is

$$\mathbf{W} = -637\mathbf{i} + 1333\mathbf{j} + 5119\mathbf{k} \text{ N}$$

Thus, A is the thrust bearing. *Ans.*

$$\mathbf{R}_{AG} = -0.05\mathbf{j} - 0.10\mathbf{k} \text{ m}, \quad \mathbf{R}_{AB} = -0.20\mathbf{k} \text{ m}$$

$$\Sigma\mathbf{M}_A = \mathbf{R}_{AG}\times\mathbf{W} + \mathbf{R}_{AB}\times\mathbf{F}_B + \mathbf{T} = \mathbf{0}$$

$$\mathbf{R}_{AG}\times\mathbf{W} = -122.6\mathbf{i} + 63.7\mathbf{j} - 31.85\mathbf{k} \quad \text{N}\cdot\text{m}$$

$$\mathbf{R}_{AB}\times\mathbf{F}_B = 0.2F_B^y\mathbf{i} - 0.2F_B^x\mathbf{j}$$

Substituting and solving gives

$$T = 31.85 \text{ N}\cdot\text{m} \quad \textit{Ans.}$$

$$F_B^x = 318.5 \text{ N}, \quad F_B^y = 613 \text{ N}$$

So $\mathbf{F}_B = 318.5\mathbf{i} + 613\mathbf{j}$ N *Ans.*

Or $F_B = \left[(613)^2 + (318.5)^2\right]^{1/2} = 691 \text{ N radial}$

$$\Sigma\mathbf{F} = \mathbf{F}_A + \mathbf{W} + \mathbf{R}_B = \mathbf{0}$$

$$\mathbf{F}_A = -(\mathbf{W} + \mathbf{F}_B) = -(-637\mathbf{i} + 1333\mathbf{j} + 5119\mathbf{k} + 318.5\mathbf{i} + 613\mathbf{j})$$

$$= 318.5\mathbf{i} - 1946\mathbf{j} - 5119\mathbf{k} \quad \textit{Ans.}$$

Radial $\mathbf{F}_A^r = 318.5\mathbf{i} - 1946\mathbf{j} \text{ N}$

$$F_A^r = \left[(318.5)^2 + (-1946)^2\right]^{1/2} = 1972 \text{ N}$$

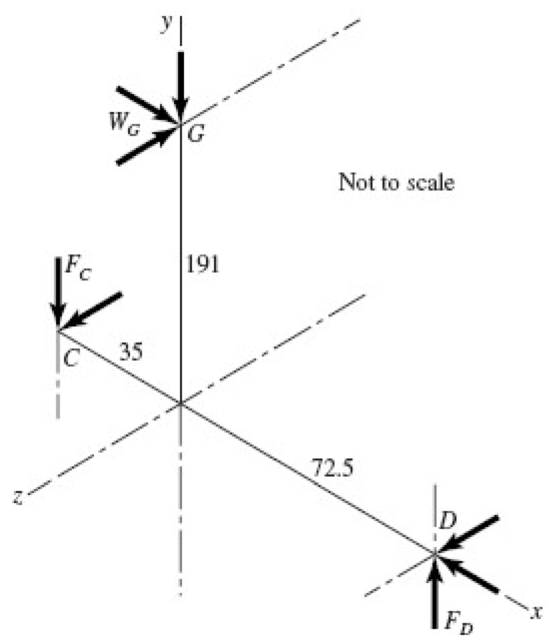
Thrust $F_A^a = -5119 \text{ N}$

13-58 From Prob. 13-57,

$$\mathbf{W}_G = 637\mathbf{i} - 1333\mathbf{j} - 5119\mathbf{k} \text{ N}$$

$$p_t = p_x$$

So $d_G = \dfrac{N_G p_x}{\pi} = \dfrac{48(25)}{\pi} = 382 \text{ mm}$



Bearing D takes the thrust load.

$$\Sigma\mathbf{M}_D = \mathbf{R}_{DG} \times \mathbf{W}_G + \mathbf{R}_{DC} \times \mathbf{F}_C + \mathbf{T} = \mathbf{0}$$

$$\mathbf{R}_{DG} = -0.0725\mathbf{i} + 0.191\mathbf{j} \text{ m}$$

$$\mathbf{R}_{DC} = -0.1075\mathbf{i} \text{ m}$$

$$\mathbf{R}_{DG} \times \mathbf{W}_G = -977.7\mathbf{i} - 371.1\mathbf{j} - 25.02\mathbf{k} \text{ N}\cdot\text{m}$$

$$\mathbf{R}_{DC} \times \mathbf{F}_C = 0.1075 F_C^z \mathbf{j} - 0.1075 F_C^y \mathbf{k} \text{ N}\cdot\text{m}$$

Putting it together and solving,

$$T = 977.7 \text{ N}\cdot\text{m} \quad \textit{Ans.}$$

$$\mathbf{F}_C = -233\mathbf{j} + 3450\mathbf{k} \text{ N}, \quad F_C = 3460 \text{ N} \quad \textit{Ans.}$$

$$\Sigma\mathbf{F} = \mathbf{F}_C + \mathbf{W}_G + \mathbf{F}_D = \mathbf{0}$$

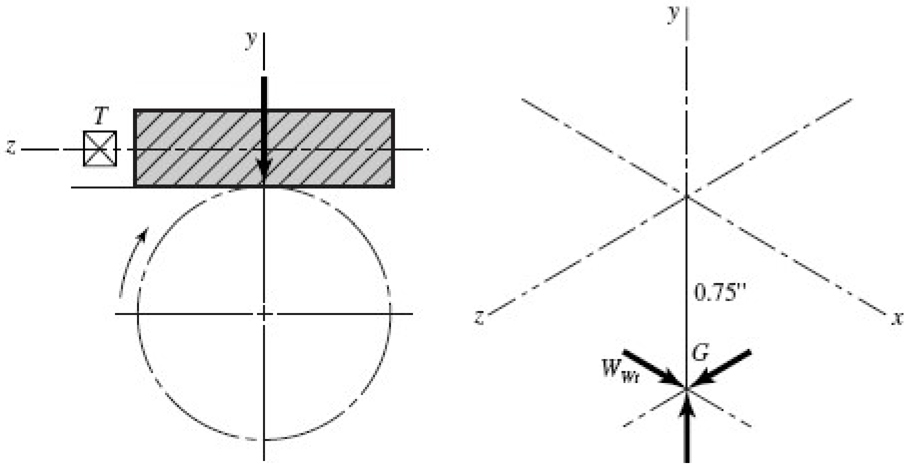
$$\mathbf{F}_D = -(\mathbf{F}_C + \mathbf{W}_G) = -637\mathbf{i} + 1566\mathbf{j} + 1669\mathbf{k} \text{ N} \quad \textit{Ans.}$$

Radial $\mathbf{F}_D^r = 1566\mathbf{j} + 1669\mathbf{k} \text{ N}$

Or $F_D^r = \left(1566^2 + 1669^2\right)^{1/2} = 2289 \text{ N (total radial)}$

$$\mathbf{F}_D^t = -637\mathbf{i} \text{ N} \quad \text{(thrust)}$$

13-59



$$V_W = \frac{\pi(1.5)(600)}{12} = 235.7 \text{ ft/min}$$

$$W^x = W_{Wt} = \frac{33000(0.75)}{235.7} = 105.0 \text{ lbf}$$

$$p_t = p_x = \frac{\pi}{8} = 0.3927 \text{ in}$$

$$L = 0.3927(2) = 0.7854 \text{ in}$$

$$\lambda = \tan^{-1}\frac{0.7854}{\pi(1.5)} = 9.46^\circ$$

$$W = \frac{105.0}{\cos 20^\circ \sin 9.46^\circ + 0.05\cos 9.46^\circ} = 515.3 \text{ lbf}$$

$$W^y = 515.3\sin 20^\circ = 176.2 \text{ lbf}$$

$$W^z = 515.3\left[\cos 20^\circ\left(\cos 9.46^\circ\right) - 0.05\sin 9.46^\circ\right] = 473.4 \text{ lbf}$$

So $\quad \mathbf{W} = 105\mathbf{i} + 176\mathbf{j} + 473\mathbf{k}$ lbf *Ans.*

$$T = 105(0.75) = 78.8 \text{ lbf}\cdot\text{in} \quad \textit{Ans.}$$

13-60 Computer programs will vary.