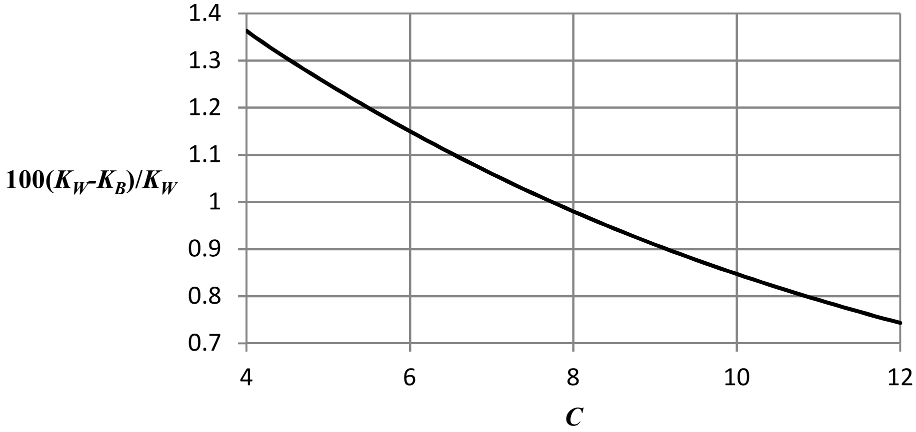
Chapter 10

10-1 From Eqs. (10-4) and (10-5)

$$K_W - K_B = \frac{4C-1}{4C-4} + \frac{0.615}{C} - \frac{4C+2}{4C-3}$$

Plot $100(K_W - K_B)/K_W$ vs. C for $4 \le C \le 12$ obtaining



We see the maximum and minimum occur at $C = 4$ and 12 respectively where

Maximum = 1.36 % *Ans.*, and Minimum = 0.743 % *Ans.*

10-2 $A = Sd^m$

$\dim(A_{\text{uscu}}) = [\dim(S)\dim(d^{\text{m}})]_{\text{uscu}} = \text{kpsi}\cdot\text{in}^m$

$\dim(A_{\text{SI}}) = [\dim(S)\dim(d^m)]_{\text{SI}} = \text{MPa}\cdot\text{mm}^m$

$$A_{\text{SI}} = \frac{\text{MPa}}{\text{kpsi}} \cdot \frac{\text{mm}^m}{\text{in}^m} A_{\text{uscu}} = 6.894\,757(25.4)^m A_{\text{uscu}} \square 6.895(25.4)^m A_{\text{uscu}} \quad \textit{Ans.}$$

For music wire, from Table 10-4:

$A_{\text{uscu}} = 201$ kpsi·in^m, $m = 0.145$; what is A_{SI}?

$A_{\text{SI}} = 6.895(25.4)^{0.145}(201) = 2215$ MPa·mm^m *Ans.*

10-3 Given: Music wire, $d = 2.5$ mm, OD = 31 mm, plain ground ends, $N_t = 14$ coils.

(**a**) Table 10-1: $N_a = N_t - 1 = 14 - 1 = 13$ coils

$D = \text{OD} - d = 31 - 2.5 = 28.5$ mm

$C = D/d = 28.5/2.5 = 11.4$

Table 10-5: $d = 2.5/25.4 = 0.098$ in $\Rightarrow$ $G = 81.0(10^3)$ MPa

Eq. (10-9): $$k = \frac{d^4 G}{8D^3 N_a} = \frac{2.5^4 (81) 10^3}{8(28.5^3)13} = 1.314 \text{ N/mm} \qquad \textit{Ans.}$$

(**b**) Table 10-1: $L_s = d\,N_t = 2.5(14) = 35$ mm

Table 10-4: $m = 0.145$, $A = 2211$ MPa·mm^m

Eq. (10-14): $$S_{ut} = \frac{A}{d^m} = \frac{2211}{2.5^{0.145}} = 1936 \text{ MPa}$$

Table 10-6: $S_{sy} = 0.45(1936) = 871.2$ MPa

Eq. (10-5): $$K_B = \frac{4C+2}{4C-3} = \frac{4(11.4)+2}{4(11.4)-3} = 1.117$$

Eq. (10-7): $$F_s = \frac{\pi d^3 S_{sy}}{8K_B D} = \frac{\pi(2.5^3)871.2}{8(1.117)28.5} = 167.9 \text{ N} \qquad \textit{Ans.}$$

(c) $$L_0 = \frac{F_s}{k} + L_s = \frac{167.9}{1.314} + 35 = 162.8 \text{ mm} \qquad \textit{Ans.}$$

(**d**) $$\left(L_0\right)_{\text{cr}} = \frac{2.63(28.5)}{0.5} = 149.9 \text{ mm}$$. Spring needs to be supported. *Ans.*

10-4 Given: Design load, $F_1 = 130$ N.

Referring to Prob. 10-3 solution, $C = 11.4$, $N_a = 13$ coils, $S_{sy} = 871.2$ MPa, $F_s = 167.9$ N, $L_0 = 162.8$ mm and $(L_0)_{\text{cr}} = 149.9$ mm.

Eq. (10-18): $4 \le C \le 12$ $C = 11.4$ *O.K.*

Eq. (10-19): $3 \le N_a \le 15$ $N_a = 13$ *O.K.*

Eq. (10-17): $$\xi = \frac{F_s}{F_1} - 1 = \frac{167.9}{130} - 1 = 0.29$$

Eq. (10-20): $\xi \geq 0.15, \quad \xi = 0.29$ *O.K.*

From Eq. (10-7) for static service

$$\tau_1 = K_B\left(\frac{8F_1D}{\pi d^3}\right) = 1.117\frac{8(130)(28.5)}{\pi(2.5)^3} = 674 \text{ MPa}$$

$$n = \frac{S_{sy}}{\tau_1} = \frac{871.2}{674} = 1.29$$

Eq. (10-21): $n_s \geq 1.2, \quad n = 1.29$ *O.K.*

$$\tau_s = \tau_1\left(\frac{167.9}{130}\right) = 674\left(\frac{167.9}{130}\right) = 870.5 \text{ MPa}$$

$$S_{sy} \,/\, \tau_s = 871.2 \,/\, 870.5 \;\square\; 1$$

$S_{sy}/\tau_s \geq (n_s)_d$: Not solid-safe (but was the basis of the design). *Not O.K.*

$L_0 \leq (L_0)_{\text{cr}}$: 162.8 ≥ 149.9 *Not O.K.*

Design is unsatisfactory. Operate over a rod? *Ans.*

10-5 Given: Oil-tempered wire, $d = 0.2$ in, $D = 2$ in, $N_t = 12$ coils, $L_0 = 5$ in, squared ends.

(**a**) Table 10-1: $L_s = d\,(N_t + 1) = 0.2(12 + 1) = 2.6$ in *Ans.*

(**b**) Table 10-1: $N_a = N_t - 2 = 12 - 2 = 10$ coils

Table 10-5: $G = 11.2$ Mpsi

Eq. (10-9):
$$k = \frac{d^4G}{8D^3N_a} = \frac{0.2^4\left(11.2\right)10^6}{8\left(2^3\right)10} = 28 \text{ lbf/in}$$

$F_s = k\,y_s = k\,(L_0 - L_s) = 28(5 - 2.6) = 67.2$ lbf *Ans.*

(**c**) Eq. (10-1): $C = D/d = 2/0.2 = 10$

Eq. (10-5):
$$K_B = \frac{4C+2}{4C-3} = \frac{4\left(10\right)+2}{4\left(10\right)-3} = 1.135$$

Eq. (10-7):
$$\tau_s = K_B\frac{8F_sD}{\pi d^3} = 1.135\frac{8\left(67.2\right)2}{\pi\left(0.2^3\right)} = 48.56\left(10^3\right) \text{ psi}$$

Table 10-4: $m = 0.187$, $A = 147$ kpsi·in^m

Eq. (10-14): $$S_{ut} = \frac{A}{d^m} = \frac{147}{0.2^{0.187}} = 198.6 \text{ kpsi}$$

Table 10-6: $S_{sy} = 0.50\, S_{ut} = 0.50(198.6) = 99.3$ kpsi

$$n_s = \frac{S_{sy}}{\tau_s} = \frac{99.3}{48.56} = 2.04 \qquad \textit{Ans.}$$

10-6 Given: Oil-tempered wire, $d = 4$ mm, $C = 10$, plain ends, $L_0 = 80$ mm, and at $F = 50$ N, $y = 15$ mm.

(**a**) $k = F/y = 50/15 = 3.333$ N/mm *Ans.*

(**b**) $D = Cd = 10(4) = 40$ mm

OD $= D + d = 40 + 4 = 44$ mm *Ans.*

(**c**) From Table 10-5, $G = 77.2$ GPa

Eq. (10-9): $$N_a = \frac{d^4 G}{8kD^3} = \frac{4^4\left(77.2\right)10^3}{8\left(3.333\right)40^3} = 11.6 \text{ coils}$$

Table 10-1: $N_t = N_a = 11.6$ coils *Ans.*

(**d**) Table 10-1: $L_s = d\,(N_t + 1) = 4(11.6 + 1) = 50.4$ mm *Ans.*

(**e**) Table 10-4: $m = 0.187$, $A = 1855 \text{ MPa}\cdot\text{mm}^m$

Eq. (10-14): $$S_{ut} = \frac{A}{d^m} = \frac{1855}{4^{0.187}} = 1431 \text{ MPa}$$

Table 10-6: $S_{sy} = 0.50\, S_{ut} = 0.50(1431) = 715.5$ MPa

$y_s = L_0 - L_s = 80 - 50.4 = 29.6$ mm

$F_s = k\, y_s = 3.333(29.6) = 98.66$ N

Eq. (10-5): $$K_B = \frac{4C+2}{4C-3} = \frac{4(10)+2}{4(10)-3} = 1.135$$

Eq. (10-7): $$\tau_s = K_B \frac{8F_s D}{\pi d^3} = 1.135 \frac{8\left(98.66\right)40}{\pi\left(4^3\right)} = 178.2 \text{ MPa}$$

$$n_s = \frac{S_{sy}}{\tau_s} = \frac{715.5}{178.2} = 4.02 \qquad \textit{Ans.}$$

10-7 Static service spring with: HD steel wire, $d = 0.080$ in, OD = 0.880 in, $N_t = 8$ coils, plain and ground ends.

Preliminaries

Table 10-5: $A = 140 \text{ kpsi} \cdot \text{in}^m, \quad m = 0.190$

Eq. (10-14): $S_{ut} = \dfrac{A}{d^m} = \dfrac{140}{0.080^{0.190}} = 226.2$ kpsi

Table 10-6: $S_{sy} = 0.45(226.2) = 101.8$ kpsi

Then,

$D = \text{OD} - d = 0.880 - 0.080 = 0.8$ in

Eq. (10-1): $C = D/d = 0.8/0.08 = 10$

Eq. (10-5): $K_B = \dfrac{4C+2}{4C-3} = \dfrac{4(10)+2}{4(10)-3} = 1.135$

Table 10-1: $N_a = N_t - 1 = 8 - 1 = 7$ coils

$L_s = dN_t = 0.08(8) = 0.64$ in

Eq. (10-7) For solid-safe, $n_s = 1.2$:

$$F_s = \frac{\pi d^3 S_{sy} / n_s}{8K_B D} = \frac{\pi\left(0.08^3\right)\left[101.8\left(10^3\right)/1.2\right]}{8(1.135)(0.8)} = 18.78 \text{ lbf}$$

Eq. (10-9): $k = \dfrac{d^4 G}{8D^3 N_a} = \dfrac{0.08^4(11.5)10^6}{8\left(0.8^3\right)7} = 16.43$ lbf/in

$$y_s = \frac{F_s}{k} = \frac{18.78}{16.43} = 1.14 \text{ in}$$

(a) $L_0 = y_s + L_s = 1.14 + 0.64 = 1.78$ in *Ans.*

(b) Table 10-1: $p = \dfrac{L_0}{N_t} = \dfrac{1.78}{8} = 0.223$ in *Ans.*

(c) From above: $F_s = 18.78$ lbf *Ans.*

(d) From above: $k = 16.43$ lbf/in *Ans.*

(e) Table 10-2 and Eq. (10-13): $(L_0)_{\text{cr}} = \dfrac{2.63D}{\alpha} = \dfrac{2.63(0.8)}{0.5} = 4.21$ in

Since $L_0 < (L_0)_{\text{cr}}$, buckling is unlikely *Ans.*

10-8 Given: Design load, $F_1 = 16.5$ lbf.

Referring to Prob. 10-7 solution, $C = 10$, $N_a = 7$ coils, $S_{sy} = 101.8$ kpsi, $F_s = 18.78$ lbf, $y_s = 1.14$ in, $L_0 = 1.78$ in, and $(L_0)_{\text{cr}} = 4.21$ in.

Eq. (10-18): $4 \le C \le 12 \qquad C = 10 \qquad$ *O.K.*

Eq. (10-19): $3 \le N_a \le 15 \quad N_a = 7 \qquad$ *O.K.*

Eq. (10-17): $\xi = \dfrac{F_s}{F_1} - 1 = \dfrac{18.78}{16.5} - 1 = 0.14$

Eq. (10-20): $\xi \ge 0.15, \quad \xi = 0.14$ *not O.K.*, but probably acceptable.

From Eq. (10-7) for static service

$$\tau_1 = K_B\left(\frac{8F_1D}{\pi d^3}\right) = 1.135\frac{8(16.5)(0.8)}{\pi(0.080)^3} = 74.5\left(10^3\right) \text{ psi} = 74.5 \text{ kpsi}$$

$$n = \frac{S_{sy}}{\tau_1} = \frac{101.8}{74.5} = 1.37$$

Eq. (10-21): $n_s \ge 1.2, \quad n = 1.37$ *O.K.*

$$\tau_s = \tau_1\left(\frac{18.78}{16.5}\right) = 74.5\left(\frac{18.78}{16.5}\right) = 84.8 \text{ kpsi}$$

$$n_s = S_{sy} / \tau_s = 101.8 / 84.8 = 1.20$$

Eq. (10-21): $n_s \ge 1.2, \quad n_s = 1.2$ It is solid-safe (basis of design). *O.K.*

Eq. (10-13) and Table 10-2: $L_0 \le (L_0)_{\text{cr}} \qquad 1.78 \text{ in} \le 4.21 \text{ in}$ *O.K.*

10-9 Given: A228 music wire, squared and ground ends, $d = 0.007$ in, OD = 0.038 in, $L_0 = 0.58$ in, $N_t = 38$ coils.

$D = \text{OD} - d = 0.038 - 0.007 = 0.031$ in

Eq. (10-1): $C = D/d = 0.031/0.007 = 4.429$

Eq. (10-5): $K_B = \dfrac{4C+2}{4C-3} = \dfrac{4(4.429)+2}{4(4.429)-3} = 1.340$

Table 10-1: $N_a = N_t - 2 = 38 - 2 = 36$ coils (high)

Table 10-5: $G = 12.0$ Mpsi

Eq. (10-9): $k = \dfrac{d^4G}{8D^3N_a} = \dfrac{0.007^4(12.0)10^6}{8\left(0.031^3\right)36} = 3.358$ lbf/in

Table 10-1: $L_s = dN_t = 0.007(38) = 0.266$ in

$y_s = L_0 - L_s = 0.58 - 0.266 = 0.314$ in

$F_s = ky_s = 3.358(0.314) = 1.054$ lbf

Eq. (10-7): $\tau_s = K_B\dfrac{8F_sD}{\pi d^3} = 1.340\dfrac{8(1.054)0.031}{\pi\left(0.007^3\right)} = 325.1\left(10^3\right)$ psi (1)

Table 10-4: $A = 201 \text{ kpsi}\cdot\text{in}^m, \ m = 0.145$

Eq. (10-14): $S_{ut} = \dfrac{A}{d^m} = \dfrac{201}{0.007^{0.145}} = 412.7$ kpsi

Table 10-6: $S_{sy} = 0.45\ S_{ut} = 0.45(412.7) = 185.7$ kpsi

$\tau_s > S_{sy}$, that is, 325.1 > 185.7 kpsi, the spring is not solid-safe. Return to Eq. (1) with $F_s = ky_s$ and $\tau_s = S_{sy}/n_s$, and solve for y_s, giving

$$y_s = \frac{\left(S_{sy}/n_s\right)\pi d^3}{8K_B kD} = \frac{\left[185.7\left(10^3\right)/1.2\right]\pi\left(0.007^3\right)}{8\left(1.340\right)3.358\left(0.031\right)} = 0.149 \text{ in}$$

The free length should be wound to

$$L_0 = L_s + y_s = 0.266 + 0.149 = 0.415 \text{ in} \qquad \textit{Ans.}$$

This only addresses the solid-safe criteria. There are additional problems.

10-10 Given: B159 phosphor-bronze, squared and ground. ends, $d = 0.014$ in, OD = 0.128 in, $L_0 = 0.50$ in, $N_t = 16$ coils.

$D = \text{OD} - d = 0.128 - 0.014 = 0.114$ in

Eq. (10-1): $C = D/d = 0.114/0.014 = 8.143$

Eq. (10-5): $K_B = \dfrac{4C+2}{4C-3} = \dfrac{4\left(8.143\right)+2}{4\left(8.143\right)-3} = 1.169$

Table 10-1: $N_a = N_t - 2 = 16 - 2 = 14$ coils

Table 10-5: $G = 6$ Mpsi

Eq. (10-9): $k = \dfrac{d^4 G}{8D^3 N_a} = \dfrac{0.014^4\left(6\right)10^6}{8\left(0.114^3\right)14} = 1.389$ lbf/in

Table 10-1: $L_s = dN_t = 0.014(16) = 0.224$ in

$y_s = L_0 - L_s = 0.50 - 0.224 = 0.276$ in

$F_s = ky_s = 1.389(0.276) = 0.3834$ lbf

Eq. (10-7): $\tau_s = K_B \dfrac{8F_s D}{\pi d^3} = 1.169 \dfrac{8\left(0.3834\right)0.114}{\pi\left(0.014^3\right)} = 47.42\left(10^3\right)$ psi (1)

Table 10-4: $A = 145$ kpsi·in^m, $m = 0$

Eq. (10-14): $S_{ut} = \dfrac{A}{d^m} = \dfrac{145}{0.014^0} = 145$ kpsi

Table 10-6: $S_{sy} = 0.35\ S_{ut} = 0.35(135) = 47.25$ kpsi

$\tau_s > S_{sy}$, that is, 47.42 > 47.25 kpsi, the spring is not solid-safe. Return to Eq. (1) with $F_s = ky_s$ and $\tau_s = S_{sy}/n_s$, and solve for y_s, giving

$$y_s = \frac{\left(S_{sy}/n_s\right)\pi d^3}{8K_B kD} = \frac{\left[47.25\left(10^3\right)/1.2\right]\pi\left(0.014^3\right)}{8\left(1.169\right)1.389\left(0.114\right)} = 0.229 \text{ in}$$

The free length should be wound to

$L_0 = L_s + y_s = 0.224 + 0.229 = 0.453$ in *Ans.*

10-11 Given: A313 stainless steel, squared and ground ends, $d = 0.050$ in, OD = 0.250 in, $L_0 = 0.68$ in, $N_t = 11.2$ coils.

$D = \text{OD} - d = 0.250 - 0.050 = 0.200$ in

Eq. (10-1): $C = D/d = 0.200/0.050 = 4$

Eq. (10-5): $$K_B = \frac{4C+2}{4C-3} = \frac{4(4)+2}{4(4)-3} = 1.385$$

Table 10-1: $N_a = N_t - 2 = 11.2 - 2 = 9.2$ coils

Table 10-5: $G = 10$ Mpsi

Eq. (10-9): $$k = \frac{d^4 G}{8D^3 N_a} = \frac{0.050^4 (10)10^6}{8(0.2^3)9.2} = 106.1 \text{ lbf/in}$$

Table 10-1: $L_s = dN_t = 0.050(11.2) = 0.56$ in

$y_s = L_0 - L_s = 0.68 - 0.56 = 0.12$ in

$F_s = ky_s = 106.1(0.12) = 12.73$ lbf

Eq. (10-7): $$\tau_s = K_B \frac{8F_s D}{\pi d^3} = 1.385 \frac{8(12.73)0.2}{\pi(0.050^3)} = 71.8(10^3) \text{ psi}$$

Table 10-4: $A = 169$ kpsi·in^m, $m = 0.146$

Eq. (10-14): $$S_{ut} = \frac{A}{d^m} = \frac{169}{0.050^{0.146}} = 261.7 \text{ kpsi}$$

Table 10-6: $S_{sy} = 0.35\, S_{ut} = 0.35(261.7) = 91.6$ kpsi

$$n_s = \frac{S_{sy}}{\tau_s} = \frac{91.6}{71.8} = 1.28$$ Spring is solid-safe ($n_s > 1.2$) *Ans.*

10-12 Given: A227 hard-drawn wire, squared and ground ends, $d = 0.148$ in, OD = 2.12 in, $L_0 = 2.5$ in, $N_t = 5.75$ coils.

$D = \text{OD} - d = 2.12 - 0.148 = 1.972$ in

Eq. (10-1): $C = D/d = 1.972/0.148 = 13.32$ (high)

Eq. (10-5): $$K_B = \frac{4C+2}{4C-3} = \frac{4(13.32)+2}{4(13.32)-3} = 1.099$$

Table 10-1: $N_a = N_t - 2 = 5.75 - 2 = 3.75$ coils

Table 10-5: $G = 11.4$ Mpsi

Eq. (10-9): $$k = \frac{d^4 G}{8D^3 N_a} = \frac{0.148^4 (11.4)10^6}{8(1.972^3)3.75} = 23.77 \text{ lbf/in}$$

Table 10-1: $L_s = dN_t = 0.148(5.75) = 0.851$ in

$y_s = L_0 - L_s = 2.5 - 0.851 = 1.649$ in

$F_s = ky_s = 23.77(1.649) = 39.20$ lbf

Eq. (10-7): $\tau_s = K_B \dfrac{8F_s D}{\pi d^3} = 1.099 \dfrac{8(39.20)1.972}{\pi(0.148^3)} = 66.7(10^3)$ psi

Table 10-4: $A = 140$ kpsi·in^m, $m = 0.190$

Eq. (10-14): $S_{ut} = \dfrac{A}{d^m} = \dfrac{140}{0.148^{0.190}} = 201.3$ kpsi

Table 10-6: $S_{sy} = 0.35\ S_{ut} = 0.45(201.3) = 90.6$ kpsi

$$n_s = \frac{S_{sy}}{\tau_s} = \frac{90.6}{66.7} = 1.36 \quad \text{Spring is solid-safe } (n_s > 1.2) \qquad \textit{Ans.}$$

10-13 Given: A229 OQ&T steel, squared and ground ends, $d = 0.138$ in, OD = 0.92 in, $L_0 = 2.86$ in, $N_t = 12$ coils.

$D = \text{OD} - d = 0.92 - 0.138 = 0.782$ in

Eq. (10-1): $C = D/d = 0.782/0.138 = 5.667$

Eq. (10-5): $K_B = \dfrac{4C+2}{4C-3} = \dfrac{4(5.667)+2}{4(5.667)-3} = 1.254$

Table 10-1: $N_a = N_t - 2 = 12 - 2 = 10$ coils

A229 OQ&T steel is not given in Table 10-5. From Table A-5, for carbon steels, $G = 11.5$ Mpsi.

Eq. (10-9): $k = \dfrac{d^4 G}{8D^3 N_a} = \dfrac{0.138^4(11.5)10^6}{8(0.782^3)10} = 109.0$ lbf/in

Table 10-1: $L_s = dN_t = 0.138(12) = 1.656$ in

$y_s = L_0 - L_s = 2.86 - 1.656 = 1.204$ in

$F_s = ky_s = 109.0(1.204) = 131.2$ lbf

Eq. (10-7): $\tau_s = K_B \dfrac{8F_s D}{\pi d^3} = 1.254 \dfrac{8(131.2)0.782}{\pi(0.138^3)} = 124.7(10^3)$ psi (1)

Table 10-4: $A = 147$ kpsi·in^m, $m = 0.187$

Eq. (10-14): $S_{ut} = \dfrac{A}{d^m} = \dfrac{147}{0.138^{0.187}} = 212.9$ kpsi

Table 10-6: $S_{sy} = 0.50\ S_{ut} = 0.50(212.9) = 106.5$ kpsi

$\tau_s > S_{sy}$, that is, 124.7 > 106.5 kpsi, the spring is not solid-safe. Return to Eq. (1) with $F_s = ky_s$ and $\tau_s = S_{sy}/n_s$, and solve for y_s, giving

$$y_s = \frac{(S_{sy}/n_s)\pi d^3}{8K_B kD} = \frac{\left[106.5(10^3)/1.2\right]\pi(0.138^3)}{8(1.254)109.0(0.782)} = 0.857 \text{ in}$$

The free length should be wound to

$L_0 = L_s + y_s = 1.656 + 0.857 = 2.51$ in *Ans.*

10-14 Given: A232 chrome-vanadium steel, squared and ground ends, $d = 0.185$ in, OD = 2.75 in, $L_0 = 7.5$ in, $N_t = 8$ coils.

$D = \text{OD} - d = 2.75 - 0.185 = 2.565$ in

Eq. (10-1): $C = D/d = 2.565/0.185 = 13.86$ (high)

Eq. (10-5): $$K_B = \frac{4C+2}{4C-3} = \frac{4(13.86)+2}{4(13.86)-3} = 1.095$$

Table 10-1: $N_a = N_t - 2 = 8 - 2 = 6$ coils

Table 10-5: $G = 11.2$ Mpsi.

Eq. (10-9): $$k = \frac{d^4G}{8D^3N_a} = \frac{0.185^4(11.2)10^6}{8(2.565^3)6} = 16.20 \text{ lbf/in}$$

Table 10-1: $L_s = dN_t = 0.185(8) = 1.48$ in

$y_s = L_0 - L_s = 7.5 - 1.48 = 6.02$ in

$F_s = ky_s = 16.20(6.02) = 97.5$ lbf

Eq. (10-7): $$\tau_s = K_B\frac{8F_sD}{\pi d^3} = 1.095\frac{8(97.5)2.565}{\pi(0.185^3)} = 110.1(10^3) \text{ psi} \qquad (1)$$

Table 10-4: $A = 169$ kpsi·in^m, $m = 0.168$

Eq. (10-14): $$S_{ut} = \frac{A}{d^m} = \frac{169}{0.185^{0.168}} = 224.4 \text{ kpsi}$$

Table 10-6: $S_{sy} = 0.50\, S_{ut} = 0.50(224.4) = 112.2$ kpsi

$$n_s = \frac{S_{sy}}{\tau_s} = \frac{112.2}{110.1} = 1.02$$ Spring is not solid-safe ($n_s < 1.2$)

Return to Eq. (1) with $F_s = ky_s$ and $\tau_s = S_{sy}/n_s$, and solve for y_s, giving

$$y_s = \frac{(S_{sy}/n_s)\pi d^3}{8K_BkD} = \frac{\left[112.2(10^3)/1.2\right]\pi(0.185^3)}{8(1.095)16.20(2.565)} = 5.109 \text{ in}$$

The free length should be wound to

$L_0 = L_s + y_s = 1.48 + 5.109 = 6.59$ in *Ans.*

10-15 Given: A313 stainless steel, squared and ground ends, $d = 0.25$ mm, OD = 0.95 mm, $L_0 = 12.1$ mm, $N_t = 38$ coils.

$D = \text{OD} - d = 0.95 - 0.25 = 0.7$ mm

Eq. (10-1): $C = D/d = 0.7/0.25 = 2.8$ (low)

Eq. (10-5): $$K_B = \frac{4C+2}{4C-3} = \frac{4(2.8)+2}{4(2.8)-3} = 1.610$$

Table 10-1: $N_a = N_t - 2 = 38 - 2 = 36$ coils (high)

Table 10-5: $G = 69.0(10^3)$ MPa.

Eq. (10-9): $$k = \frac{d^4G}{8D^3N_a} = \frac{0.25^4\left(69.0\right)10^3}{8\left(0.7^3\right)36} = 2.728 \text{ N/mm}$$

Table 10-1: $L_s = dN_t = 0.25(38) = 9.5$ mm
$y_s = L_0 - L_s = 12.1 - 9.5 = 2.6$ mm
$F_s = ky_s = 2.728(2.6) = 7.093$ N

Eq. (10-7): $$\tau_s = K_B\frac{8F_sD}{\pi d^3} = 1.610\frac{8\left(7.093\right)0.7}{\pi\left(0.25^3\right)} = 1303 \text{ MPa} \qquad (1)$$

Table 10-4 (dia. less than table): $A = 1867$ MPa·mm^m, $m = 0.146$

Eq. (10-14): $$S_{ut} = \frac{A}{d^m} = \frac{1867}{0.25^{0.146}} = 2286 \text{ MPa}$$

Table 10-6: $S_{sy} = 0.35\, S_{ut} = 0.35(2286) = 734$ MPa

$\tau_s > S_{sy}$, that is, 1303 > 734 MPa, the spring is not solid-safe. Return to Eq. (1) with $F_s = ky_s$ and $\tau_s = S_{sy}/n_s$, and solve for y_s, giving

$$y_s = \frac{\left(S_{sy}/n_s\right)\pi d^3}{8K_BkD} = \frac{\left(734/1.2\right)\pi\left(0.25^3\right)}{8\left(1.610\right)2.728\left(0.7\right)} = 1.22 \text{ mm}$$

The free length should be wound to

$L_0 = L_s + y_s = 9.5 + 1.22 = 10.72$ mm *Ans.*

This only addresses the solid-safe criteria. There are additional problems.

10-16 Given: A228 music wire, squared and ground ends, $d = 1.2$ mm, OD = 6.5 mm, $L_0 = 15.7$ mm, $N_t = 10.2$ coils.

$D = \text{OD} - d = 6.5 - 1.2 = 5.3$ mm

Eq. (10-1): $C = D/d = 5.3/1.2 = 4.417$

Eq. (10-5): $$K_B = \frac{4C+2}{4C-3} = \frac{4\left(4.417\right)+2}{4\left(4.417\right)-3} = 1.368$$

Table (10-1): $N_a = N_t - 2 = 10.2 - 2 = 8.2$ coils

Table 10-5 ($d = 1.2/25.4 = 0.0472$ in): $G = 81.7(10^3)$ MPa.

Eq. (10-9): $$k = \frac{d^4G}{8D^3N_a} = \frac{1.2^4\left(81.7\right)10^3}{8\left(5.3^3\right)8.2} = 17.35 \text{ N/mm}$$

Table 10-1: $L_s = dN_t = 1.2(10.2) = 12.24$ mm
$y_s = L_0 - L_s = 15.7 - 12.24 = 3.46$ mm
$F_s = ky_s = 17.35(3.46) = 60.03$ N

Eq. (10-7): $\tau_s = K_B \dfrac{8F_sD}{\pi d^3} = 1.368 \dfrac{8(60.03)5.3}{\pi(1.2^3)} = 641.4$ MPa (1)

Table 10-4: $A = 2211$ MPa·mm^m, $m = 0.145$

Eq. (10-14): $S_{ut} = \dfrac{A}{d^m} = \dfrac{2211}{1.2^{0.145}} = 2153$ MPa

Table 10-6: $S_{sy} = 0.45\, S_{ut} = 0.45(2153) = 969$ MPa

$n_s = \dfrac{S_{sy}}{\tau_s} = \dfrac{969}{641.4} = 1.51$ Spring is solid-safe ($n_s > 1.2$) *Ans.*

10-17 Given: A229 OQ&T steel, squared and ground ends, $d = 3.5$ mm, OD = 50.6 mm, $L_0 = 75.5$ mm, $N_t = 5.5$ coils.

$D = \text{OD} - d = 50.6 - 3.5 = 47.1$ mm

Eq. (10-1): $C = D/d = 47.1/3.5 = 13.46$ (high)

Eq. (10-5): $K_B = \dfrac{4C+2}{4C-3} = \dfrac{4(13.46)+2}{4(13.46)-3} = 1.098$

Table 10-1: $N_a = N_t - 2 = 5.5 - 2 = 3.5$ coils

A229 OQ&T steel is not given in Table 10-5. From Table A-5, for carbon steels, $G = 79.3(10^3)$ MPa.

Eq. (10-9): $k = \dfrac{d^4G}{8D^3N_a} = \dfrac{3.5^4(79.3)10^3}{8(47.1^3)3.5} = 4.067$ N/mm

Table 10-1: $L_s = dN_t = 3.5(5.5) = 19.25$ mm

$y_s = L_0 - L_s = 75.5 - 19.25 = 56.25$ mm

$F_s = ky_s = 4.067(56.25) = 228.8$ N

Eq. (10-7): $\tau_s = K_B \dfrac{8F_sD}{\pi d^3} = 1.098 \dfrac{8(228.8)47.1}{\pi(3.5^3)} = 702.8$ MPa (1)

Table 10-4: $A = 1855$ MPa·mm^m, $m = 0.187$

Eq. (10-14): $S_{ut} = \dfrac{A}{d^m} = \dfrac{1855}{3.5^{0.187}} = 1468$ MPa

Table 10-6: $S_{sy} = 0.50\, S_{ut} = 0.50(1468) = 734$ MPa

$n_s = \dfrac{S_{sy}}{\tau_s} = \dfrac{734}{702.8} = 1.04$ Spring is not solid-safe ($n_s < 1.2$)

Return to Eq. (1) with $F_s = ky_s$ and $\tau_s = S_{sy}/n_s$, and solve for y_s, giving

$$y_s = \frac{(S_{sy}/n_s)\pi d^3}{8K_BkD} = \frac{(734/1.2)\pi(3.5^3)}{8(1.098)4.067(47.1)} = 48.96 \text{ mm}$$

The free length should be wound to

$L_0 = L_s + y_s = 19.25 + 48.96 = 68.2$ mm *Ans.*

10-18 Given: B159 phosphor-bronze, squared and ground ends, $d = 3.8$ mm, OD = 31.4 mm, $L_0 = 71.4$ mm, $N_t = 12.8$ coils.

$D = \text{OD} - d = 31.4 - 3.8 = 27.6$ mm

Eq. (10-1): $C = D/d = 27.6/3.8 = 7.263$

Eq. (10-5): $K_B = \dfrac{4C+2}{4C-3} = \dfrac{4(7.263)+2}{4(7.263)-3} = 1.192$

Table 10-1: $N_a = N_t - 2 = 12.8 - 2 = 10.8$ coils

Table 10-5: $G = 41.4(10^3)$ MPa.

Eq. (10-9): $k = \dfrac{d^4 G}{8D^3 N_a} = \dfrac{3.8^4(41.4)10^3}{8(27.6^3)10.8} = 4.752 \text{ N/mm}$

Table 10-1: $L_s = dN_t = 3.8(12.8) = 48.64$ mm

$y_s = L_0 - L_s = 71.4 - 48.64 = 22.76$ mm

$F_s = ky_s = 4.752(22.76) = 108.2$ N

Eq. (10-7): $\tau_s = K_B \dfrac{8F_s D}{\pi d^3} = 1.192 \dfrac{8(108.2)27.6}{\pi(3.8^3)} = 165.2 \text{ MPa}$ (1)

Table 10-4 ($d = 3.8/25.4 = 0.150$ in): $A = 932$ MPa·mm^m, $m = 0.064$

Eq. (10-14): $S_{ut} = \dfrac{A}{d^m} = \dfrac{932}{3.8^{0.064}} = 855.7 \text{ MPa}$

Table 10-6: $S_{sy} = 0.35\, S_{ut} = 0.35(855.7) = 299.5$ MPa

$n_s = \dfrac{S_{sy}}{\tau_s} = \dfrac{299.5}{165.2} = 1.81$ Spring is solid-safe ($n_s > 1.2$) *Ans.*

10-19 Given: A232 chrome-vanadium steel, squared and ground ends, $d = 4.5$ mm, OD = 69.2 mm, $L_0 = 215.6$ mm, $N_t = 8.2$ coils.

$D = \text{OD} - d = 69.2 - 4.5 = 64.7$ mm

Eq. (10-1): $C = D/d = 64.7/4.5 = 14.38$ (high)

Eq. (10-5): $K_B = \dfrac{4C+2}{4C-3} = \dfrac{4(14.38)+2}{4(14.38)-3} = 1.092$

Table 10-1: $N_a = N_t - 2 = 8.2 - 2 = 6.2$ coils

Table 10-5: $G = 77.2(10^3)$ MPa.

Eq. (10-9): $k = \dfrac{d^4 G}{8D^3 N_a} = \dfrac{4.5^4(77.2)10^3}{8(64.7^3)6.2} = 2.357 \text{ N/mm}$

Table 10-1: $L_s = dN_t = 4.5(8.2) = 36.9$ mm

$y_s = L_0 - L_s = 215.6 - 36.9 = 178.7$ mm
$F_s = ky_s = 2.357(178.7) = 421.2$ N

Eq. (10-7): $\tau_s = K_B \dfrac{8F_s D}{\pi d^3} = 1.092 \dfrac{8\left(421.2\right)64.7}{\pi\left(4.5^3\right)} = 832 \text{ MPa}$ (1)

Table 10-4: $A = 2005$ MPa·mm^m, $m = 0.168$

Eq. (10-14): $S_{ut} = \dfrac{A}{d^m} = \dfrac{2005}{4.5^{0.168}} = 1557 \text{ MPa}$

Table 10-6: $S_{sy} = 0.50\, S_{ut} = 0.50(1557) = 779$ MPa

$\tau_s > S_{sy}$, that is, 832 > 779 MPa, the spring is not solid-safe. Return to Eq. (1) with $F_s = ky_s$ and $\tau_s = S_{sy}/n_s$, and solve for y_s, giving

$$y_s = \frac{\left(S_{sy}/n_s\right)\pi d^3}{8K_B kD} = \frac{\left(779/1.2\right)\pi\left(4.5^3\right)}{8\left(1.092\right)2.357\left(64.7\right)} = 139.5 \text{ mm}$$

The free length should be wound to

$L_0 = L_s + y_s = 36.9 + 139.5 = 176.4$ mm *Ans.*

This only addresses the solid-safe criteria. There are additional problems.

10-20 Given: A227 HD steel.
From the figure: $L_0 = 4.75$ in, OD = 2 in, and $d = 0.135$ in. Thus
$D = \text{OD} - d = 2 - 0.135 = 1.865$ in

(a) By counting, $N_t = 12.5$ coils. Since the ends are squared along 1/4 turn on each end,

$N_a = 12.5 - 0.5 = 12$ turns *Ans.*
$p = 4.75 / 12 = 0.396$ in *Ans.*

The solid stack is 13 wire diameters

$L_s = 13(0.135) = 1.755$ in *Ans.*

(b) From Table 10-5, $G = 11.4$ Mpsi

$$k = \frac{d^4 G}{8D^3 N_a} = \frac{0.135^4(11.4)\left(10^6\right)}{8\left(1.865^3\right)(12)} = 6.08 \text{ lbf/in}$$ *Ans.*

(c) $F_s = k(L_0 - L_s) = 6.08(4.75 - 1.755) = 18.2$ lbf *Ans.*

(d) $C = D/d = 1.865/0.135 = 13.81$

$$K_B = \frac{4(13.81) + 2}{4(13.81) - 3} = 1.096$$

$$\tau_s = K_B \frac{8F_s D}{\pi d^3} = 1.096 \frac{8(18.2)(1.865)}{\pi\left(0.135^3\right)} = 38.5\left(10^3\right) \text{ psi} = 38.5 \text{ kpsi} \quad \textit{Ans.}$$

10-21 Given: Plain end, hard drawn steel, 12 gauge W & M wire, OD = 0.75 in, $N_t = 20$ coils, $L_0 = 3.75$ in.

Table A-28: $d = 0.1055$ in

(a) $D = \text{OD} - d = 0.75 - 0.1055 = 0.6445$ in. $C = D / d = 0.6445/0.1055 = 6.109$ *Ans.*

(b) Table 10-1: $N_a = N_t = 20$ coils,

$p = (L_0 - d)/ N_a = (3.75 - 0.1055)/20 = 0.1822$ in/coil *Ans.*

(c) Table 10-1: $L_s = d\,(N_t + 1) = 0.1055\,(20 + 1) = 2.2155$ in,

$y_s = L_0 - L_s = 3.75 - 2.2155 = 1.5345$ in *Ans.*

(d) Eq. (10-8):

$$F_s = \frac{d^4 G y_s}{8D^3 N\left(1+\dfrac{1}{2C^2}\right)} = \frac{0.1055^4\left(11.5\right)10^6\left(1.5345\right)}{8\left(0.6445\right)^3\left(20\right)\left[1+\dfrac{1}{2\left(6.109\right)^2}\right]} = 50.36 \text{ lbf} \qquad \textit{Ans.}$$

(e) Eq. (10-5): $K_B = \dfrac{4C+2}{4C-3} = \dfrac{4\left(6.109\right)+2}{4\left(6.109\right)-3} = 1.233$

Eq. (10-7):

$$\tau_s = K_B \frac{8F_s D}{\pi d^3} = 1.233 \frac{8\left(50.36\right)0.6445}{\pi\left(0.1055\right)^3} = 86.8\left(10^3\right) \text{ psi} = 86.8 \text{ kpsi} \quad \textit{Ans.}$$

(f) Table 10-4 and Eq. (10-14): $S_{ut} = \dfrac{A}{d^m} = \dfrac{140}{\left(0.1055\right)^{0.190}} = 214.6$ kpsi

Table 10-6: $S_{sy} = 0.45\, S_{ut} = 0.45(214.6) = 96.57$ kpsi.

$$n_s = \frac{S_{sy}}{\tau_s} = \frac{96.57}{86.8} = 1.11 \qquad \textit{Ans.}$$

(g) Exact, $k = F_s / y_s = 50.36/1.5345 = 32.82$ lbf/in *Ans.*

Approximate using Eq. (10-9): $k \approx \dfrac{d^4 G}{8D^3 N} = \dfrac{0.1055^4\left(11.5\right)10^6}{8\left(0.6445\right)^3 20} = 33.26$ lbf/in *Ans.*

Approximate is 1.34 percent higher than the exact. *Ans.*

10-22 Given: Squared and ground, oil tempered steel, $d = 3$ mm, OD = 30 mm, $N_t = 32$ coils, $L_0 = 240$ mm.

Table A-28: $d = 0.1055$ in

(a) $D = \text{OD} - d = 30 - 3 = 27$ mm. $C = D / d = 27/3 = 9$ *Ans.*

(b) Table 10-1: $N_a = N_t - 2 = 32 - 2 = 30$ coils,

$p = (L_0 - 2d)/ N_a = [240 - 2(3)] / 30 = 7.8$ mm/coil *Ans.*

(c) Table 10-1: $L_s = d\,N_t = 3\,(32) = 96$ mm,

$y_s = L_0 - L_s = 240 - 96 = 144$ mm *Ans.*

(d) Eq. (10-8):

$$F_s = \frac{d^4 G y_s}{8D^3 N\left(1+\frac{1}{2C^2}\right)} = \frac{(0.003)^4\, 77.2\left(10^9\right)0.144}{8(0.027)^3(30)\left[1+\frac{1}{2(9)^2}\right]} = 189.45 \text{ N} \qquad \textit{Ans.}$$

(e) Eq. (10-5): $K_B = \frac{4C+2}{4C-3} = \frac{4(9)+2}{4(9)-3} = 1.152$

Eq. (10-7):

$$\tau_s = K_B \frac{8F_s D}{\pi d^3} = 1.152 \frac{8(189.45)0.027}{\pi(0.003)^3}\left(10^{-6}\right) = 555.8 \text{ MPa} \qquad \textit{Ans.}$$

(f) Table 10-4 and Eq. (10-14): $S_{ut} = \frac{A}{d^m} = \frac{1855}{(3)^{0.187}} = 1510.5 \text{ MPa}$

Table 10-5: $S_{sy} = 0.45\, S_{ut} = 0.45(1510.5) = 679.7$ MPa.

$$n_s = \frac{S_{sy}}{\tau_s} = \frac{679.7}{555.8} = 1.22 \qquad \textit{Ans.}$$

(g) Exact, $k = F_s / y_s = 189.45/144 = 1.316$ N/mm *Ans.*
Approximate using Eq. (10-9):

$$k \approx \frac{d^4 G}{8D^3 N} = \frac{(0.003)^4\, 77.2\left(10^9\right)}{8(0.027)^3\, 30}\left(10^{-3}\right) = 1.324 \text{ N/mm} \qquad \textit{Ans.}$$

Approximate is 0.61 percent higher than the exact. *Ans.*

10-23 $y = 50$ mm, $F = 90$ N, $k = F / y = 90/50 = 1.8$ N/mm *Ans.*
$y_s = 60$ mm, $F_s = k\, y_s = 1.8(60) = 108$ N.

Eq. (10-14), Table 10-4, assume $2.5 \le d \le 5$ mm: $S_{ut} = \frac{A}{d^m} = \frac{2065}{d^{0.263}}$

Table 10-6 (includes K_B): $S_{ys} = 0.35 S_{ut} = 0.35\frac{2065}{d^{0.263}} = \frac{722.75}{d^{0.263}}$ (1)

Eq. (10-7) (with $n_s = 1.2$ but without K_B):

$$\tau_{\max} = \frac{8n_s F_s D}{\pi d^3} = \frac{8 n_s F_s C}{\pi d^2} = \frac{8(1.2)108(10)}{\pi d^2} = \frac{3300}{d^2} \text{ MPa} \qquad (2)$$

Equate Eqs. (1) and (2), $\frac{722.75}{d^{0.263}} = \frac{3300}{d^2} \Rightarrow d^{1.737} = \frac{3300}{722.75} \Rightarrow d = 2.40$ mm

Since this is less than 2.5 mm, return to Eq. (10-14), Table 10-4, for $0.3 \le d \le 2.5$ mm:

$$S_{ut} = \frac{A}{d^m} = \frac{1867}{d^{0.146}} \quad \Rightarrow \quad S_{ys} = \frac{653.45}{d^{0.146}} \qquad (1)$$

Again, equate Eqs. (1) and (2),

$$\frac{653.45}{d^{0.146}} = \frac{3300}{d^2} \Rightarrow d^{1.854} = \frac{3300}{653.45} \Rightarrow d = 2.40 \text{ mm} \qquad \textit{Ans.}$$

The final factor of safety is

$$n_s = \frac{S_{ys}}{\tau_s} = \frac{653.45/d^{0.146}}{\left(\frac{8F_sC}{\pi d^2}\right)} = \frac{653.45\pi\left(2.40\right)^{1.854}}{8\left(108\right)10} = 1.20 \qquad Ans.$$

OD $= Cd + d = (C + 1)\,d = 11(2.40) = 26.4$ mm *Ans.*

ID $= (C - 1)\,d = 9(2.40) = 21.6$ mm *Ans.*

$k = 1.8$ N/mm *Ans.* (found earlier)

Table 10-5, $G = 69.0$ GPa, Eq. (10-9):

$$N_a = \frac{d^4G}{8D^3k} = \frac{dG}{8C^3k} = \frac{2.4\left(10^{-3}\right)69\left(10^9\right)}{8\left(10^3\right)1.8\left(10^3\right)} = 11.5 \text{ coils}$$

Table 10-1: $N_t = N_a + 2 = 13.5$ coils *Ans.*

$L_s = d\,(N_t + 1) = 2.4(13.5 + 1) = 34.8$ mm *Ans.*

$L_0 = L_s + y_s = 34.8 + 60 = 94.8$ mm *Ans.*

Eq. (10-13): $\alpha = 2.63\dfrac{D}{L_0} = 2.63\dfrac{10\left(2.4\right)}{94.8} = 0.666$

Stable if supported between fixed-fixed ends. Otherwise would need to be supported by hole or rod.

10-24 Phosphor-bronze, closed ends, $C = 10$, at $y = 2$ in $F = 15$ lbf, $y_s = 3$ in, $n_s = 1.2$.

$k = F / y = 15/2 = 7.5$ lbf/in, $F_s = k\,y_s = 7.5\ (3) = 22.5$ lbf.

Eq. (10-14) with Table 10-4 assuming $0.022 \le d \le 0.075$ in, $A = 121$ kpsi·in^m and $m = 0.028$. Then, from Table 10-5, $S_{ys} = 0.45\ S_{ut}$:

$$S_{ys} = 0.45\frac{121}{d^{0.028}} = \frac{54.45}{d^{0.028}} \qquad (1)$$

Eq. (10-5): $K_B = \dfrac{4C+2}{4C-3} = \dfrac{4\left(10\right)+2}{4\left(10\right)-3} = 1.135$

Eq. (10-7) with $n_s = 1.2$:

$$\tau_{\max} = n_sK_B\frac{8F_sC}{\pi d^2} = 1.2\left(1.135\right)\frac{8\left(22.5\right)10}{\pi d^2}\left(10^{-3}\right) = \frac{0.7804}{d^2} \qquad (2)$$

Where $\tau_{\max}$ is in kpsi. Equating (1) and (2) gives

$$\frac{54.45}{d^{0.028}} = \frac{0.7804}{d^2} \quad \Rightarrow \quad d^{1.972} = \frac{0.7804}{54.45} \quad \Rightarrow \quad d = 0.116 \text{ in}$$

Returning to Table 10-4, use $A = 110$ kpsi·in^m and $m = 0.028$ for $0.075 \le d \le 0.3$ in,

$$S_{ys} = 0.45\frac{110}{d^{0.064}} = \frac{49.5}{d^{0.064}} = \frac{0.7804}{d^2} \quad \Rightarrow \quad d^{1.936} = \frac{0.7804}{49.5} \quad \Rightarrow \quad d = 0.117 \text{ in}$$

Table A-17, select the preferred size of: $d = 0.12$ in *Ans.*

Check n_s, $n_s = \dfrac{S_{ys}}{\tau_s} = \dfrac{49.5/\left(d^{0.064}\right)}{K_B\left(8\dfrac{F_sC}{\pi d^2}10^{-3}\right)} = \dfrac{49.5\left(10^3\right)\pi d^{1.936}}{8K_BF_sC}$ (3)

$$n_s = \frac{49.5\left(10^3\right)\pi\left(0.12\right)^{1.936}}{8\left(1.135\right)22.5\left(10\right)} = 1.26 \qquad \textit{Ans.}$$

OD = $Cd + d = (C + 1)\,d = (10 + 1)0.12 = 1.32$ in *Ans.*

ID = $(C - 1)\,d = (10 - 1)0.12 = 1.08$ in

Found earlier, $k = 7.5$ lbf/in *Ans.*

Table 10-5, $G = 6$ Mpsi. Eq. (10-9): $N_a = \dfrac{dG}{8C^3k} = \dfrac{0.12\left(6\right)10^6}{8\left(10^3\right)7.5} = 12$ coils

Table 10-1: $N_t = N_a + 2 = 14$ coils *Ans.*

$L_s = d\,(N_t + 1) = 0.12(14 + 1) = 1.8$ in *Ans.*

$L_0 = L_s + y_s = 1.8 + 3 = 4.8$ in *Ans.*

Eq. (10-13): $\alpha = 2.63\,D\,/\,L_0 = 2.63\,(10)0.12/4.8 = 0.658$
Table 10-2: Stable if supported between fixed-fixed ends. Otherwise would need to be supported by hole or rod.

10-25 From Prob. 10-24, $d = 0.12$ in and from Eq. (3),

$$C = \frac{49.5\left(10^3\right)\pi d^{1.936}}{8n_sK_BF_s} = \frac{49.5\left(10^3\right)\pi\left(0.12\right)^{1.936}}{8\left(1.2\right)1.135\left(22.5\right)} = 10.46 \qquad \textit{Ans.}$$

OD = $(C + 1)\,d = 11.46(0.12) = 1.375$ in *Ans.*
ID = $(C - 1)\,d = 9.46(0.12) = 1.135$ in *Ans.*

Table 10-5, $G = 6$ Mpsi, Eq. (10-9): $N_a = \dfrac{dG}{8C^3k} = \dfrac{0.12\left(6\right)10^6}{8\left(10.46^3\right)7.5} = 10.5$ coils

Table 10-1: $N_t = N_a + 2 = 12.5$ coils *Ans.*

$L_s = d\,(N_t + 1) = 0.12(12.5 + 1) = 1.62$ in *Ans.*

$L_0 = L_s + y_s = 1.62 + 3 = 4.62$ in *Ans.*

Eq. (10-13): $\alpha = 2.63\,D\,/\,L_0 = 2.63\,(10.46)0.12/4.62 = 0.715$

Table 10-2: Stable if supported between fixed-fixed ends, or one end on flat surface and other end hinged. Otherwise would need to be supported by hole or rod.

10-26 For the wire diameter analyzed, $G = 11.75$ Mpsi per Table 10-5. Use squared and ground ends. The following is a spread-sheet study using Fig. 10-3 for parts (*a*) and (*b*). For N_a, $k = F_{\max}/y = 20/2 = 10$ lbf/in. For τ_s, $F = F_s = 20(1 + \xi) = 20(1 + 0.15) = 23$ lbf.

(a) Spring over a Rod					(b) Spring in a Hole				
Source		Parameter	Values		Source		Parameter	Values	
	d	0.075	0.080	0.085		d	0.075	0.080	0.085
	ID	0.800	0.800	0.800		OD	0.950	0.950	0.950
	D	0.875	0.880	0.885		D	0.875	0.870	0.865
Eq. (10-1)	C	11.667	11.000	10.412	Eq. (10-1)	C	11.667	10.875	10.176
Eq. (10-9)	N_a	6.937	8.828	11.061	Eq. (10-9)	N_a	6.937	9.136	11.846
Table 10-1	N_t	8.937	10.828	13.061	Table 10-1	N_t	8.937	11.136	13.846
Table 10-1	L_s	0.670	0.866	1.110	Table 10-1	L_s	0.670	0.891	1.177
$1.15y + L_s$	L_0	2.970	3.166	3.410	$1.15y + L_s$	L_0	2.970	3.191	3.477
Eq. (10-13)	$(L_0)_{\mathrm{cr}}$	4.603	4.629	4.655	Eq. (10-13)	$(L_0)_{\mathrm{cr}}$	4.603	4.576	4.550
Table 10-4	A	201.000	201.000	201.000	Table 10-4	A	201.000	201.000	201.000
Table 10-4	m	0.145	0.145	0.145	Table 10-4	m	0.145	0.145	0.145
Eq. (10-14)	S_{ut}	292.626	289.900	287.363	Eq. (10-14)	S_{ut}	292.626	289.900	287.363
Table 10-6	S_{sy}	131.681	130.455	129.313	Table 10-6	S_{sy}	131.681	130.455	129.313
Eq. (10-5)	K_B	1.115	1.122	1.129	Eq. (10-5)	K_B	1.115	1.123	1.133
Eq. (10-7)	τ_s	135.335	112.948	95.293	Eq. (10-7)	τ_s	135.335	111.787	93.434
Eq. (10-3)	n_s	0.973	1.155	1.357	Eq. (10-3)	n_s	0.973	1.167	1.384
Eq. (10-22)	fom	–0.282	–0.391	–0.536	Eq. (10-22)	fom	–0.282	–0.398	–0.555

For $n_s \geq 1.2$, the optimal size is $d = 0.085$ in for both cases.

10-27 In Prob. 10-26, there is an advantage of first selecting d as one can select from the available sizes (Table A-28). Selecting C first requires a calculation of d where then a size must be selected from Table A-28.

Consider part (a) of the problem. It is required that

$$\text{ID} = D - d = 0.800 \text{ in.} \qquad (1)$$

From Eq. (10-1), $D = Cd$. Substituting this into the first equation yields

$$d = \frac{0.800}{C-1} \qquad (2)$$

Starting with $C = 10$, from Eq. (2) we find that $d = 0.089$ in. From Table A-28, the closest diameter is $d = 0.090$ in. Substituting this back into Eq. (1) gives $D = 0.890$ in, with $C = 0.890/0.090 = 9.889$, which are acceptable. From this point the solution is the same as Prob. 10-26. For part (b), use

$$\text{OD} = D + d = 0.950 \text{ in.} \qquad (3)$$

and, $$d = \frac{0.800}{C-1} \qquad (4)$$

(a) Spring over a rod				(b) Spring in a Hole		
Source		Parameter	Values	Source	Parameter	Values
	C	10.000	10.5		C	10.000
Eq. (2)	d	0.089	0.084	Eq. (4)	d	0.086
Table A-28	d	0.090	0.085	Table A-28	d	0.085
Eq. (1)	D	0.890	0.885	Eq. (3)	D	0.865
Eq. (10-1)	C	9.889	10.412	Eq. (10-1)	C	10.176
Eq. (10-9)	N_a	13.669	11.061	Eq. (10-9)	N_a	11.846
Table 10-1	N_t	15.669	13.061	Table 10-1	N_t	13.846
Table 10-1	L_s	1.410	1.110	Table 10-1	L_s	1.177
$1.15y + L_s$	L_0	3.710	3.410	$1.15y + L_s$	L_0	3.477
Eq. (10-13)	$(L_0)_{cr}$	4.681	4.655	Eq. (10-13)	$(L_0)_{cr}$	4.550
Table 10-4	A	201.000	201.000	Table 10-4	A	201.000
Table 10-4	m	0.145	0.145	Table 10-4	m	0.145
Eq. (10-14)	S_{ut}	284.991	287.363	Eq. (10-14)	S_{ut}	287.363
Table 10-6	S_{sy}	128.246	129.313	Table 10-6	S_{sy}	129.313
Eq. (10-5)	K_B	1.135	1.128	Eq. (10-5)	K_B	1.135
Eq. (10-7)	τ_s	81.167	95.223	Eq. (10-7)	τ_s	93.643
$n_s = S_{sy}/\tau_s$	n_s	1.580	1.358	$n_s = S_{sy}/\tau_s$	n_s	1.381
Eq. (10-22)	fom	-0.725	-0.536	Eq. (10-22)	fom	-0.555

Again, for $n_s \geq 1.2$, the optimal size is = 0.085 in.

Although this approach used less iterations than in Prob. 10-26, this was due to the initial values picked and not the approach.

10-28 One approach is to select A227 HD steel for its low cost. Try $L_0 = 48$ mm, then for $y = 48 - 37.5 = 10.5$ mm when $F = 45$ N. The spring rate is $k = F/y = 45/10.5 = 4.286$ N/mm.

For a clearance of 1.25 mm with screw, ID = 10 + 1.25 = 11.25 mm. Starting with $d = 2$ mm,

$$D = \text{ID} + d = 11.25 + 2 = 13.25 \text{ mm}$$

$$C = D/d = 13.25/2 = 6.625 \quad \text{(acceptable)}$$

Table 10-5 ($d = 2/25.4 = 0.0787$ in): $G = 79.3$ GPa

Eq. (10-9): $$N_a = \frac{d^4 G}{8kD^3} = \frac{2^4(79.3)10^3}{8(4.286)13.25^3} = 15.9 \text{ coils}$$

Assume squared and closed.

Table 10-1: $N_t = N_a + 2 = 15.9 + 2 = 17.9$ coils

$L_s = dN_t = 2(17.9) = 35.8$ mm

$y_s = L_0 - L_s = 48 - 35.8 = 12.2$ mm
$F_s = ky_s = 4.286(12.2) = 52.29$ N

Eq. (10-5): $$K_B = \frac{4C+2}{4C-3} = \frac{4(6.625)+2}{4(6.625)-3} = 1.213$$

Eq. (10-7): $$\tau_s = K_B \frac{8F_sD}{\pi d^3} = 1.213\left[\frac{8(52.29)13.25}{\pi(2^3)}\right] = 267.5 \text{ MPa}$$

Table 10-4: $A = 1783 \text{ MPa} \cdot \text{mm}^m, \quad m = 0.190$

Eq. (10-14): $$S_{ut} = \frac{A}{d^m} = \frac{1783}{2^{0.190}} = 1563 \text{ MPa}$$

Table 10-6: $S_{sy} = 0.45S_{ut} = 0.45(1563) = 703.3$ MPa

$$n_s = \frac{S_{sy}}{\tau_s} = \frac{703.3}{267.5} = 2.63 > 1.2 \qquad O.K.$$

No other diameters in the given range work. So specify

A227-47 HD steel, $d = 2$ mm, $D = 13.25$ mm, ID = 11.25 mm, OD = 15.25 mm, squared and closed, $N_t = 17.9$ coils, $N_a = 15.9$ coils, $k = 4.286$ N/mm, $L_s = 35.8$ mm, and $L_0 = 48$ mm. *Ans.*

10-29 Select A227 HD steel for its low cost. Try $L_0 = 48$ mm, then for $y = 48 - 37.5 = 10.5$ mm when $F = 45$ N. The spring rate is $k = F/y = 45/10.5 = 4.286$ N/mm.

For a clearance of 1.25 mm with screw, ID = 10 + 1.25 = 11.25 mm.

$$D - d = 11.25 \qquad (1)$$

and, $$D = Cd \qquad (2)$$

Starting with $C = 8$, gives $D = 8d$. Substitute into Eq. (1) resulting in $d = 1.607$ mm. Selecting the nearest diameter in the given range, $d = 1.6$ mm. From this point, the calculations are shown in the third column of the spreadsheet output shown. We see that for $d = 1.6$ mm, the spring is not solid safe. Iterating on C we find that $C = 6.5$ provides acceptable results with the specifications

A227-47 HD steel, $d = 2$ mm, $D = 13.25$ mm, ID = 11.25 mm, OD = 15.25 mm, squared and closed, $N_t = 17.9$ coils, $N_a = 15.9$ coils, $k = 4.286$ N/mm, $L_s = 35.8$ mm, and $L_0 = 48$ mm. *Ans.*

Source		Parameter Values		
	C	8.000	7	6.500
Eq. (2)	d	1.607	1.875	2.045
Table A-28	d	1.600	1.800	2.000
Eq. (1)	D	12.850	13.050	13.250
Eq. (10-1)	C	8.031	7.250	6.625
Eq. (10-9)	N_a	7.206	10.924	15.908
Table 10-1	N_t	9.206	12.924	17.908
Table 10-1	L_s	14.730	23.264	35.815
$L_0 - L_s$	y_s	33.270	24.736	12.185
$F_s = ky_s$	F_s	142.594	106.020	52.224
Table 10-4	A	1783.000	1783.000	1783.000
Table 10-4	m	0.190	0.190	0.190
Eq. (10-14)	S_{ut}	1630.679	1594.592	1562.988
Table 10-6	S_{sy}	733.806	717.566	703.345
Eq. (10-5)	K_B	1.172	1.200	1.217
Eq. (10-7)	τ_s	1335.568	724.943	268.145
$n_s = S_{sy}/\tau_s$	n_s	0.549	0.990	2.623

The only difference between selecting *C* first rather than *d* as was done in Prob. 10-28, is that once *d* is calculated, the closest wire size must be selected. Iterating on *d* uses available wire sizes from the beginning.

10-30 A stock spring catalog may have over two hundred pages of compression springs with up to 80 springs per page listed.

- Students should be made aware that such catalogs exist.
- Many springs are selected from catalogs rather than designed.
- The wire size you want may not be listed.
- Catalogs may also be available on disk or the web through search routines.
- It is better to familiarize yourself with vendor resources rather than invent them yourself.
- Sample catalog pages can be given to students for study.

10-31 Given: ID = 0.6 in, $C = 10$, $L_0 = 5$ in, $L_s = 5 - 3 = 2$ in, sq. & grd ends, unpeened, HD A227 wire.

(**a**) With ID $= D - d = 0.6$ in and $C = D/d = 10 \Rightarrow 10\,d - d = 0.6 \;\Rightarrow d = 0.0667$ in *Ans.*, and $D = 0.667$ in.

(**b**) Table 10-1: $L_s = dN_t = 2$ in $\Rightarrow N_t = 2/0.0667 = 30$ coils *Ans.*

(**c**) Table 10-1: $N_a = N_t - 2 = 30 - 2 = 28$ coils

Table 10-5: $G = 11.5$ Mpsi

Eq. (10-9):
$$k = \frac{d^4 G}{8D^3 N_a} = \frac{0.0667^4 \left(11.5\right)10^6}{8\left(0.667^3\right)28} = 3.424 \text{ lbf/in} \qquad \textit{Ans.}$$

(**d**) Table 10-4: $A = 140$ kpsi·in^m, $m = 0.190$

Eq. (10-14): $S_{ut} = \dfrac{A}{d^m} = \dfrac{140}{0.0667^{0.190}} = 234.2$ kpsi

Table 10-6: $S_{sy} = 0.45\,S_{ut} = 0.45\,(234.2) = 105.4$ kpsi

$F_s = ky_s = 3.424(3) = 10.27$ lbf

Eq. (10-5): $K_B = \dfrac{4C+2}{4C-3} = \dfrac{4(10)+2}{4(10)-3} = 1.135$

Eq. (10-7):

$$\tau_s = K_B \frac{8F_sD}{\pi d^3} = 1.135 \frac{8(10.27)0.667}{\pi\left(0.0667^3\right)}$$

$$= 66.72\left(10^3\right) \text{ psi} = 66.72 \text{ kpsi}$$

$$n_s = \frac{S_{sy}}{\tau_s} = \frac{105.4}{66.72} = 1.58 \qquad \textit{Ans.}$$

(**e**) $\tau_a = \tau_m = 0.5\,\tau_s = 0.5(66.72) = 33.36$ kpsi, $r = \tau_a / \tau_m = 1$. Using the Gerber fatigue failure criterion with Zimmerli data,

Eq. (10-30): $S_{su} = 0.67\,S_{ut} = 0.67(234.2) = 156.9$ kpsi

The Gerber ordinate intercept for the Zimmerli data is obtained using Eqs. (10-28) and (10-29b).

$$S_{se} = \frac{S_{sa}}{1-\left(S_{sm}/S_{su}\right)^2} = \frac{35}{1-\left(55/156.9\right)^2} = 39.9 \text{ kpsi}$$

The Gerber fatigue criterion from Eq. (6-48), adapted for shear,

$$n_f = \frac{1}{2}\left(\frac{S_{su}}{\tau_m}\right)^2 \frac{\tau_a}{S_{se}}\left[-1+\sqrt{1+\left(\frac{2\tau_m S_{se}}{S_{su}\tau_a}\right)^2}\right]$$

$$= \frac{1}{2}\left(\frac{156.9}{33.36}\right)^2 \frac{33.36}{39.9}\left[-1+\sqrt{1+\left(\frac{2(33.36)39.9}{156.9(33.36)}\right)^2}\right]$$

$$= 1.13 \qquad \textit{Ans.}$$

10-32 Given: OD ≤ 0.9 in, $C = 8$, $L_0 = 3$ in, $L_s = 1$ in, $y_s = 3 - 1 = 2$ in, sq. ends, unpeened, music wire.

(**a**) Try OD $= D + d = 0.9$ in, $C = D/d = 8 \Rightarrow D = 8d \Rightarrow 9d = 0.9 \Rightarrow d = 0.1$ *Ans.*
$D = 8(0.1) = 0.8$ in

(**b**) Table 10-1: $L_s = d\,(N_t + 1) \Rightarrow N_t = L_s / d - 1 = 1/0.1 - 1 = 9$ coils *Ans.*

Table 10-1: $N_a = N_t - 2 = 9 - 2 = 7$ coils (**c**) Table 10-5: $G = 11.75$ Mpsi

Eq. (10-9): $$k = \frac{d^4 G}{8D^3 N_a} = \frac{0.1^4 \left(11.75\right)10^6}{8\left(0.8^3\right)7} = 40.98 \text{ lbf/in} \qquad \textit{Ans.}$$

(**d**) $F_s = ky_s = 40.98(2) = 81.96$ lbf

Eq. (10-5): $$K_B = \frac{4C+2}{4C-3} = \frac{4\left(8\right)+2}{4\left(8\right)-3} = 1.172$$

Eq. (10-7): $$\tau_s = K_B \frac{8F_s D}{\pi d^3} = 1.172 \frac{8\left(81.96\right)0.8}{\pi\left(0.1^3\right)} = 195.7\left(10^3\right) \text{ psi} = 195.7 \text{ kpsi}$$

Table 10-4: $A = 201$ kpsi·in^m, $m = 0.145$

Eq. (10-14): $$S_{ut} = \frac{A}{d^m} = \frac{201}{0.1^{0.145}} = 280.7 \text{ kpsi}$$

Table 10-6: $S_{sy} = 0.45\, S_{ut} = 0.45(280.7) = 126.3$ kpsi

$$n_s = \frac{S_{sy}}{\tau_s} = \frac{126.3}{195.7} = 0.645 \qquad \textit{Ans.}$$

(**e**) $\tau_a = \tau_m = \tau_s/2 = 195.7/2 = 97.85$ kpsi. Using the Gerber fatigue failure criterion with Zimmerli data,

Eq. (10-30): $S_{su} = 0.67\, S_{ut} = 0.67(280.7) = 188.1$ kpsi

The Gerber ordinate intercept for the Zimmerli data is obtained using Eqs. (10-28) and (10-29b).

$$S_{se} = \frac{S_{sa}}{1-\left(S_{sm}/S_{su}\right)^2} = \frac{35}{1-\left(55/188.1\right)^2} = 38.3 \text{ kpsi}$$

The Gerber fatigue criterion from Eq. (6-48), adapted for shear,

$$n_f = \frac{1}{2}\left(\frac{S_{su}}{\tau_m}\right)^2 \frac{\tau_a}{S_{se}}\left[-1+\sqrt{1+\left(\frac{2\tau_m S_{se}}{S_{su}\tau_a}\right)^2}\right]$$

$$= \frac{1}{2}\left(\frac{188.1}{97.85}\right)^2 \frac{97.85}{38.3}\left[-1+\sqrt{1+\left(\frac{2\left(97.85\right)38.3}{188.1\left(97.85\right)}\right)^2}\right]$$

$$= 0.38 \qquad \textit{Ans.}$$

Obviously, the spring is severely under designed and will fail statically and in fatigue. Increasing C would improve matters. Try $C = 12$. This yields $n_s = 1.83$ and $n_f = 1.00$.

10-33 Given: $F_{\max} = 300$ lbf, $F_{\min} = 150$ lbf, $\Delta y = 1$ in, OD = 2.1 − 0.2 = 1.9 in, $C = 7$, unpeened, squared & ground, oil-tempered wire.

(**a**) $D = \text{OD} - d = 1.9 - d \qquad (1)$

$C = D/d = 7 \;\Rightarrow\; D = 7d \qquad (2)$

Substitute Eq. (2) into (1)

$7d = 1.9 - d \;\Rightarrow\; d = 1.9/8 = 0.2375$ in *Ans.*

(**b**) From Eq. (2): $D = 7d = 7(0.2375) = 1.663$ in *Ans.*

(**c**) $$k = \frac{\Delta F}{\Delta y} = \frac{300-150}{1} = 150 \text{ lbf/in} \qquad \textit{Ans.}$$

(**d**) Table 10-5: $G = 11.6$ Mpsi

Eq. (10-9): $$N_a = \frac{d^4G}{8D^3k} = \frac{0.2375^4\left(11.6\right)10^6}{8\left(1.663^3\right)150} = 6.69 \text{ coils}$$

Table 10-1: $N_t = N_a + 2 = 8.69$ coils *Ans.*

(**e**) Table 10-4: $A = 147$ kpsi·in^m, $m = 0.187$

Eq. (10-14): $$S_{ut} = \frac{A}{d^m} = \frac{147}{0.2375^{0.187}} = 192.3 \text{ kpsi}$$

Table 10-6: $S_{sy} = 0.5\, S_{ut} = 0.5(192.3) = 96.15$ kpsi

Eq. (10-5): $$K_B = \frac{4C+2}{4C-3} = \frac{4\left(7\right)+2}{4\left(7\right)-3} = 1.2$$

Eq. (10-7): $$\tau_s = K_B\frac{8F_sD}{\pi d^3} = S_{sy}$$

$$F_s = \frac{\pi d^3 S_{sy}}{8K_BD} = \frac{\pi\left(0.2375^3\right)96.15\left(10^3\right)}{8\left(1.2\right)1.663} = 253.5 \text{ lbf}$$

$y_s = F_s / k = 253.5/150 = 1.69$ in

Table 10-1: $L_s = N_t\, d = 8.46(0.2375) = 2.01$ in

$$L_0 = L_s + y_s = 2.01 + 1.69 = 3.70 \text{ in} \qquad \textit{Ans.}$$

10-34 For a coil radius given by:

$$R = R_1 + \frac{R_2 - R_1}{2\pi N}\theta$$

The torsion of a section is $T = PR$ where $dL = R\,d\theta$

$$\begin{aligned}
\delta_P &= \frac{\partial U}{\partial P} = \frac{1}{GJ}\int T\frac{\partial T}{\partial P}dL = \frac{1}{GJ}\int_0^{2\pi N} PR^3\,d\theta \\
&= \frac{P}{GJ}\int_0^{2\pi N}\left(R_1 + \frac{R_2 - R_1}{2\pi N}\theta\right)^3 d\theta \\
&= \frac{P}{GJ}\left(\frac{1}{4}\right)\left(\frac{2\pi N}{R_2 - R_1}\right)\left[\left(R_1 + \frac{R_2 - R_1}{2\pi N}\theta\right)^4\right]\Bigg|_0^{2\pi N} \\
&= \frac{\pi PN}{2GJ(R_2 - R_1)}\left(R_2^4 - R_1^4\right) = \frac{\pi PN}{2GJ}(R_1 + R_2)\left(R_1^2 + R_2^2\right)
\end{aligned}$$

$$J = \frac{\pi}{32}d^4 \qquad \therefore\ \delta_p = \frac{16PN}{Gd^4}(R_1 + R_2)\left(R_1^2 + R_2^2\right)$$

$$k = \frac{P}{\delta_P} = \frac{d^4 G}{16N(R_1 + R_2)\left(R_1^2 + R_2^2\right)} \qquad \textit{Ans.}$$

10-35 Given: $F_{\min} = 4$ lbf, $F_{\max} = 18$ lbf, $k = 9.5$ lbf/in, OD ≤ 2.5 in, $n_f = 1.5$.

For a food service machinery application select A313 Stainless wire.
Table 10-5: $G = 10(10^6)$ psi
Note that for $0.013 \le d \le 0.10$ in $A = 169$, $m = 0.146$
$0.10 < d \le 0.20$ in $A = 128$, $m = 0.263$

$$F_a = \frac{18 - 4}{2} = 7 \text{ lbf}, \quad F_m = \frac{18 + 4}{2} = 11 \text{ lbf}, \quad r = 7/11$$

Try, $d = 0.080$ in, $S_{ut} = \dfrac{169}{(0.08)^{0.146}} = 244.4$ kpsi

$S_{su} = 0.67S_{ut} = 163.7$ kpsi, $\quad S_{sy} = 0.35S_{ut} = 85.5$ kpsi

The Gerber ordinate intercept for the Zimmerli data is obtained using Eqs. (10-28) and (10-29b).

$$S_{se} = \frac{S_{sa}}{1 - (S_{sm}/S_{su})^2} = \frac{35}{1 - (55/163.7)^2} = 39.5 \text{ kpsi}$$

Let $r = \tau_a / \tau_m = 7/11$. The Gerber fatigue criterion from Eq. (6-48), adapted for shear,

$$n_f = \frac{1}{2}\frac{S_{su}^2}{\tau_m}\frac{r}{S_{se}}\left[-1+\sqrt{1+\left(\frac{2S_{se}}{S_{su}r}\right)^2}\right]$$

Solving for τ_m gives,

$$\tau_m = \frac{1}{2}\frac{S_{su}^2}{n_f}\frac{r}{S_{se}}\left[-1+\sqrt{1+\left(\frac{2S_{se}}{S_{su}r}\right)^2}\right] = \frac{1}{2}\frac{(163.7)^2}{(1.5)}\frac{(7/11)}{39.5}\left[-1+\sqrt{1+\left(\frac{2(39.5)}{163.7(7/11)}\right)^2}\right]$$

$$= 36.70 \text{ kpsi}$$

But,

$$\tau_m = K_B\frac{8F_mC}{\pi d^2} = \frac{4C+2}{4C-3}\left(\frac{8F_mC}{\pi d^2}\right)$$

Let $\alpha = \tau_m = 36.70$ kpsi, and $\beta = \frac{8F_m}{\pi d^2} = \frac{8(11)10^{-3}}{\pi(0.08)^2} = 4.377$ kpsi From Eq. (10-23),

$$C = \frac{2\alpha-\beta}{4\beta}+\sqrt{\left(\frac{2\alpha-\beta}{4\beta}\right)^2-\frac{3\alpha}{4\beta}}$$

$$= \frac{2(36.70)-4.377}{4(4.377)}+\sqrt{\left(\frac{2(36.70)-4.377}{4(4.377)}\right)^2-\frac{3(36.70)}{4(4.377)}} = 6.98$$

$D = Cd = 6.98(0.08) = 0.558$ in

$$K_B = \frac{4C+2}{4C-3} = \frac{4(6.98)+2}{4(6.98)-3} = 1.201$$

$$\tau_a = K_B\left(\frac{8F_aD}{\pi d^3}\right) = 1.201\left[\frac{8(7)(0.558)}{\pi(0.08^3)}(10^{-3})\right] = 23.3 \text{ kpsi}$$

The Gerber fatigue criterion from Eq. (6-48), adapted for shear,

$$n_f = \frac{1}{2}\left(\frac{S_{su}}{\tau_m}\right)^2\frac{\tau_a}{S_{se}}\left[-1+\sqrt{1+\left(\frac{2\tau_mS_{se}}{S_{su}\tau_a}\right)^2}\right]$$

$$= \frac{1}{2}\left(\frac{163.7}{36.6}\right)^2\frac{23.3}{39.5}\left\{-1+\sqrt{1+\left[2\left(\frac{11}{7}\right)\frac{39.5}{163.7}\right]^2}\right\}$$

$$= 1.50 \qquad \text{checks}$$

$$N_a = \frac{Gd^4}{8kD^3} = \frac{10(10^6)(0.08)^4}{8(9.5)(0.558)^3} = 31.02 \text{ coils}$$

$N_t = 31.02 + 2 = 33$ coils

$L_s = dN_t = 0.08(33) = 2.64$ in

$y_{\max} = F_{\max} / k = 18 / 9.5 = 1.895$ in

$y_s = (1 + \xi)\, y_{\max} = (1 + 0.15)(1.895) = 2.179$ in

$L_0 = L_s + y_{\max} = 2.64 + 2.179 = 4.819$ in

$(L_0)_{\text{cr}} = 2.63\, D / \alpha = 2.63(0.558) / 0.5 = 2.935$ in

$\tau_s = (1 + \xi)(\, F_{\max} / F_a)\ \tau_a\ = 1.15(18/7)23.3 = 68.8$ kpsi

$n_s = S_{sy} / \tau_s = 85.5/68.9 = 1.24$

$$f = \sqrt{\frac{kg}{\pi^2 d^2 D N_a \gamma}} = \sqrt{\frac{9.5(386)}{\pi^2 (0.08^2)(0.558)(31.02)(0.283)}} = 109 \text{ Hz}$$

These steps are easily implemented on a spreadsheet, as shown below, for different diameters.

	d_1	d_2	d_3	d_4
d	0.080	0.092	0.106	0.121
m	0.146	0.146	0.263	0.263
A	169.000	169.000	128.000	128.000
S_{ut}	244.363	239.618	231.257	223.311
S_{su}	163.723	160.544	154.942	149.618
S_{sy}	85.527	83.866	80.940	78.159
S_{sa}	35.000	35.000	35.000	35.000
S_{se}	39.452	39.654	40.046	40.469
τ_m	36.667	36.667	36.667	36.667
α	36.667	36.667	36.667	36.667
β	4.377	3.346	2.517	1.929
C	6.977	9.603	13.244	17.702
D	0.558	0.879	1.397	2.133
K_B	1.201	1.141	1.100	1.074
τ_a	23.333	23.333	23.333	23.333
n_f	1.500	1.500	1.500	1.500
N_a	30.993	13.594	5.975	2.858
N_t	32.993	15.594	7.975	4.858
L_s	2.639	1.427	0.841	0.585
y_s	2.179	2.179	2.179	2.179

L_0	4.818	3.606	3.020	2.764
(L_0)cr	2.936	4.622	7.350	11.220
τ_s	69.000	69.000	69.000	69.000
n_s	1.240	1.215	1.173	1.133
f, (Hz)	108.895	114.578	118.863	121.775

The shaded areas depict conditions outside the recommended design conditions. Thus, one spring is satisfactory. The specifications are: A313 stainless wire, unpeened, squared and ground, $d = 0.0915$ in, OD = 0.879 + 0.092 = 0.971 in, $L_0 = 3.606$ in, and $N_t = 15.59$ turns *Ans.*

10-36 The steps are the same as in Prob. 10-35 except that the Gerber-Zimmerli criterion is replaced with the Goodman-Zimmerli relationship of Eq. (10-29*a*) :

$$S_{se} = \frac{S_{sa}}{1 - \left(S_{sm}/S_{su}\right)}$$

The problem then proceeds as in Prob. 10-30. The results for the wire sizes are shown below (see solution to Prob. 10-35 for additional details).

Iteration of d for the first trial

	d_1	d_2	d_3	d_4		d_1	d_2	d_3	d_4
d	0.080	0.0915	0.1055	0.1205	d	0.080	0.0915	0.1055	0.1205
m	0.146	0.146	0.263	0.263	K_B	1.151	1.108	1.078	1.058
A	169.000	169.000	128.000	128.000	τ_a	29.008	29.040	29.090	29.127
S_{ut}	244.363	239.618	231.257	223.311	n_f	1.500	1.500	1.500	1.500
S_{su}	163.723	160.544	154.942	149.618	N_a	14.191	6.456	2.899	1.404
S_{sy}	85.527	83.866	80.940	78.159	N_t	16.191	8.456	4.899	3.404
S_{sa}	35.000	35.000	35.000	35.000	L_s	1.295	0.774	0.517	0.410
S_{se}	52.706	53.239	54.261	55.345	y_s	2.179	2.179	2.179	2.179
τ_m	45.585	45.635	45.712	45.771	L_0	3.474	2.953	2.696	2.589
α	45.585	45.635	45.712	45.771	$(L_0)_{cr}$	3.809	5.924	9.354	14.219
β	4.377	3.346	2.517	1.929	τ_s	85.782	85.876	86.022	86.133
C	9.052	12.309	16.856	22.433	n_s	0.997	0.977	0.941	0.907
D	0.724	1.126	1.778	2.703	f, (Hz)	141.284	146.853	151.271	154.326

Without checking all of the design conditions, it is obvious that none of the wire sizes satisfy $n_s \geq 1.2$. Also, the Gerber line is closer to the yield line than the Goodman. Setting $n_f = 1.5$ for Goodman makes it impossible to reach the yield line ($n_s < 1$) . The table below uses $n_f = 2$.

Iteration of d for the second trial									
	d_1	d_2	d_3	d_4		d_1	d_2	d_3	d_4
d	0.080	0.0915	0.1055	0.1205	d	0.080	0.0915	0.1055	0.1205
m	0.146	0.146	0.263	0.263	K_B	1.221	1.154	1.108	1.079
A	169.000	169.000	128.000	128.000	τ_a	21.756	21.780	21.817	21.845
S_{ut}	244.363	239.618	231.257	223.311	n_f	2.000	2.000	2.000	2.000
S_{su}	163.723	160.544	154.942	149.618	N_a	40.243	17.286	7.475	3.539
S_{sy}	85.527	83.866	80.940	78.159	N_t	42.243	19.286	9.475	5.539
S_{sa}	35.000	35.000	35.000	35.000	L_s	3.379	1.765	1.000	0.667
S_{se}	52.706	53.239	54.261	55.345	y_s	2.179	2.179	2.179	2.179
τ_m	34.188	34.226	34.284	34.329	L_0	5.558	3.944	3.179	2.846
α	34.188	34.226	34.284	34.329	$(L_0)_{\text{cr}}$	2.691	4.266	6.821	10.449
β	4.377	3.346	2.517	1.929	τ_s	64.336	64.407	64.517	64.600
C	6.395	8.864	12.292	16.485	n_s	1.329	1.302	1.255	1.210
D	0.512	0.811	1.297	1.986	f, (Hz)	99.816	105.759	110.312	113.408

The satisfactory spring has design specifications of: A313 stainless wire, unpeened, squared and ground, $d = 0.0915$ in, OD = 0.811 + 0.092 = 0.903 in, $L_0 = 3.944$ in, and .$N_t = 19.3$ turns. *Ans.*

10-37 This is the same as Prob. 10-35 since $S_{sa} = 35$ kpsi. Therefore, the specifications are: A313 stainless wire, unpeened, squared and ground, $d = 0.0915$ in, OD = 0.879 + 0.092 = 0.971 in, $L_0 = 3.606$ in, and $N_t = 15.59$ turns *Ans.*

10-38 For the Gerber-Zimmerli fatigue-failure criterion, $S_{su} = 0.67S_{ut}$,

$$S_{se} = \frac{S_{sa}}{1 - (S_{sm} / S_{su})^2} \ , \qquad \tau_m = \frac{1}{2}\frac{S_{su}^2}{n_f}\frac{r}{S_{se}}\left[-1 + \sqrt{1 + \left(\frac{2S_{se}}{S_{su}r}\right)^2}\,\right]$$

See the process used in Prob. 10-36. The last 2 columns of diameters of Ex. 10-5 are presented below with additional calculations.

d	0.105	0.112	d	0.105	0.112
S_{ut}	278.691	276.096	N_t	10.915	8.190
S_{su}	186.723	184.984	L_s	1.146	0.917
S_{se}	38.325	38.394	L_0	3.446	3.217
τ_m	38.508	38.502	$(L_0)_{cr}$	6.630	8.160
α	38.508	38.502	K_B	1.111	1.095
β	2.887	2.538	τ_a	23.105	23.101
C	12.004	13.851	n_f	1.500	1.500
D	1.260	1.551	τ_s	70.855	70.844
ID	1.155	1.439	n_s	1.770	1.754
OD	1.365	1.663	f_n	105.433	106.922
N_a	8.915	6.190	fom	-0.973	-1.022

There are only slight changes in the results.

10-39 As in Prob. 10-38, the basic change is S_{sa}.

For Goodman, using Eq. (10-29*a*): $S_{se} = \dfrac{S_{sa}}{1-(S_{sm}/S_{su})}$

Recalculate τ_m using Eq. (6-41) for shear. That is,

$$n_f = \left(\frac{\tau_a}{S_{se}} + \frac{\tau_m}{S_{su}}\right)^{-1} = \frac{S_{se}S_{su}}{\tau_a S_{su} + \tau_m S_{se}} = \frac{S_{se}S_{su}}{\tau_m\left(rS_{su} + S_{se}\right)}$$

Where $r = \tau_a / \tau_m$. Thus,

$$\tau_m = \frac{S_{se}S_{su}}{n_f\left(rS_{su} + S_{se}\right)}$$

See the process used in Prob. 10-36. Calculations for the last 2 diameters of Ex. 10-5 are given below.

d	0.105	0.112	d	0.105	0.112
S_{ut}	278.691	276.096	N_t	11.153	8.353
S_{su}	186.723	184.984	L_s	1.171	0.936
S_{se}	49.614	49.810	L_0	3.471	3.236
τ_m	38.207	38.201	$(L_0)_{cr}$	6.572	8.090
α	38.207	38.201	K_B	1.112	1.096
β	2.887	2.538	τ_a	22.924	22.920
C	11.899	13.732	n_f	1.500	1.500
D	1.249	1.538	τ_s	70.301	70.289
ID	1.144	1.426	n_s	1.784	1.768
OD	1.354	1.650	f_n	104.509	106.000
N_a	9.153	6.353	fom	-0.986	-1.034

There are only slight differences in the results.

10-40 Use: $E = 28.6$ Mpsi, $G = 11.5$ Mpsi, $A = 140$ kpsi · in^m , $m = 0.190$, rel cost = 1.

Try $$d = 0.067\ in, \quad S_{ut} = \frac{140}{(0.067)^{0.190}} = 234.0 \text{ kpsi}$$

Table 10-6: $S_{sy} = 0.45S_{ut} = 105.3$ kpsi
Table 10-7: $S_y = 0.75S_{ut} = 175.5$ kpsi
Eq. (10-34) with $D/d = C$ and $C_1 = C$

$$\sigma_A = \frac{F_{\max}}{\pi d^2}[(K)_A(16C) + 4] = \frac{S_y}{n_y}$$

$$\frac{4C^2 - C - 1}{4C(C-1)}(16C) + 4 = \frac{\pi d^2 S_y}{n_y F_{\max}}$$

$$4C^2 - C - 1 = (C-1)\left(\frac{\pi d^2 S_y}{4n_y F_{\max}} - 1\right)$$

$$C^2 - \frac{1}{4}\left(1 + \frac{\pi d^2 S_y}{4n_y F_{\max}} - 1\right)C + \frac{1}{4}\left(\frac{\pi d^2 S_y}{4n_y F_{\max}} - 2\right) = 0$$

$$C = \frac{1}{2}\left[\frac{\pi d^2 S_y}{16 n_y F_{\max}} \pm \sqrt{\left(\frac{\pi d^2 S_y}{16 n_y F_{\max}}\right)^2 - \frac{\pi d^2 S_y}{4 n_y F_{\max}} + 2}\right] \text{ take positive root}$$

$$= \frac{1}{2}\left\{\frac{\pi(0.067^2)(175.5)(10^3)}{16(1.5)(18)} + \sqrt{\left[\frac{\pi(0.067)^2(175.5)(10^3)}{16(1.5)(18)}\right]^2 - \frac{\pi(0.067)^2(175.5)(10^3)}{4(1.5)(18)} + 2}\right\} = 4.590$$

$$D = Cd = 4.59\left(0.067\right) = 0.3075 \text{ in}$$

$$F_i = \frac{\pi d^3 \tau_i}{8D} = \frac{\pi d^3}{8D}\left[\frac{33\ 500}{\exp(0.105C)} \pm 1000\left(4 - \frac{C-3}{6.5}\right)\right]$$

Use the lowest F_i in the preferred range. This results in the best fom.

$$F_i = \frac{\pi(0.067)^3}{8(0.3075)}\left\{\frac{33\ 500}{\exp[0.105(4.590)]} - 1000\left(4 - \frac{4.590-3}{6.5}\right)\right\} = 6.505 \text{ lbf}$$

For simplicity, we will round up to the next integer or half integer. Therefore, use $F_i = 7$ lbf

$$k = \frac{18-7}{0.5} = 22 \text{ lbf/in}$$

$$N_a = \frac{d^4 G}{8kD^3} = \frac{(0.067)^4(11.5)(10^6)}{8(22)(0.3075)^3} = 45.28 \text{ turns}$$

$$N_b = N_a - \frac{G}{E} = 45.28 - \frac{11.5}{28.6} = 44.88 \text{ turns}$$

$$L_0 = (2C - 1 + N_b)d = [2(4.590) - 1 + 44.88](0.067) = 3.555 \text{ in}$$

$$L_{18\text{ lbf}} = 3.555 + 0.5 = 4.055 \text{ in}$$

Body: $K_B = \dfrac{4C+2}{4C-3} = \dfrac{4(4.590)+2}{4(4.590)-3} = 1.326$

$$\tau_{\max} = \frac{8K_B F_{\max} D}{\pi d^3} = \frac{8(1.326)(18)(0.3075)}{\pi(0.067)^3}(10^{-3}) = 62.1 \text{ kpsi}$$

$$(n_y)_{\text{body}} = \frac{S_{sy}}{\tau_{\max}} = \frac{105.3}{62.1} = 1.70$$

$$r_2 = 2d = 2(0.067) = 0.134 \text{ in}, \quad C_2 = \frac{2r_2}{d} = \frac{2(0.134)}{0.067} = 4$$

$$(K)_B = \frac{4C_2 - 1}{4C_2 - 4} = \frac{4(4)-1}{4(4)-4} = 1.25$$

$$\tau_B = (K)_B\left[\frac{8F_{\max}D}{\pi d^3}\right] = 1.25\left[\frac{8(18)(0.3075)}{\pi(0.067)^3}\right](10^{-3}) = 58.58\text{ kpsi}$$

$$(n_y)_B = \frac{S_{sy}}{\tau_B} = \frac{105.3}{58.58} = 1.80$$

$$\text{fom} = -(1)\frac{\pi^2 d^2(N_b+2)D}{4} = -\frac{\pi^2(0.067)^2(44.88+2)(0.3075)}{4} = -0.160$$

Several diameters, evaluated using a spreadsheet, are shown below.

d	0.067	0.072	0.076	0.081	0.085	0.09	0.095	0.104
S_{ut}	233.977	230.799	228.441	225.692	223.634	221.219	218.958	215.224
S_{sy}	105.290	103.860	102.798	101.561	100.635	99.548	98.531	96.851
S_y	175.483	173.100	171.331	169.269	167.726	165.914	164.218	161.418
C	4.589	5.412	6.099	6.993	7.738	8.708	9.721	11.650
D	0.307	0.390	0.463	0.566	0.658	0.784	0.923	1.212
F_i (calc)	6.505	5.773	5.257	4.675	4.251	3.764	3.320	2.621
F_i (rd)	7.0	6.0	5.5	5.0	4.5	4.0	3.5	3.0
k	22.000	24.000	25.000	26.000	27.000	28.000	29.000	30.000
N_a	45.29	27.20	19.27	13.10	9.77	7.00	5.13	3.15
N_b	44.89	26.80	18.86	12.69	9.36	6.59	4.72	2.75
L_0	3.556	2.637	2.285	2.080	2.026	2.071	2.201	2.605
$L_{18\text{ lbf}}$	4.056	3.137	2.785	2.580	2.526	2.571	2.701	3.105
K_B	1.326	1.268	1.234	1.200	1.179	1.157	1.139	1.115
$\tau_{\max}$	62.118	60.686	59.707	58.636	57.875	57.019	56.249	55.031
$(n_y)_{\text{body}}$	1.695	1.711	1.722	1.732	1.739	1.746	1.752	1.760
τ_B	58.576	59.820	60.495	61.067	61.367	61.598	61.712	61.712
$(n_y)_B$	1.797	1.736	1.699	1.663	1.640	1.616	1.597	1.569
$(n_y)_A$	1.500	1.500	1.500	1.500	1.500	1.500	1.500	1.500
fom	−0.160	−0.144	-0.138	−0.135	−0.133	−0.135	−0.138	−0.154

Except for the 0.067 in wire, all springs satisfy the requirements of length and number of coils. The 0.085 in wire has the highest fom.

10-41 Given: $N_b = 84$ coils, $F_i = 16$ lbf, OQ&T steel, OD = 1.5 in, $d = 0.162$ in.

$$D = \text{OD} - d = 1.5 - 0.162 = 1.338\text{ in}$$

(a) Eq. (10-39):

$$L_0 = 2(D-d) + (N_b+1)d$$
$$= 2(1.338 - 0.162) + (84+1)(0.162) = 16.12\text{ in} \quad \textit{Ans.}$$

or $\quad 2d + L_0 = 2(0.162) + 16.12 = 16.45$ in overall

(b)

$$C = \frac{D}{d} = \frac{1.338}{0.162} = 8.26$$

$$K_B = \frac{4C + 2}{4C - 3} = \frac{4(8.26) + 2}{4(8.26) - 3} = 1.166$$

$$\tau_i = K_B \left[\frac{8F_i D}{\pi d^3} \right] = 1.166 \frac{8(16)(1.338)}{\pi (0.162)^3} = 14\ 950 \text{ psi} \qquad \textit{Ans.}$$

(c) From Table 10-5 use: $G = 11.4(10^6)$ psi and $E = 28.5(10^6)$ psi

$$N_a = N_b + \frac{G}{E} = 84 + \frac{11.4}{28.5} = 84.4 \text{ turns}$$

$$k = \frac{d^4 G}{8D^3 N_a} = \frac{(0.162)^4 (11.4)(10^6)}{8(1.338)^3 (84.4)} = 4.855 \text{ lbf/in} \qquad \textit{Ans.}$$

(d) Table 10-4: $A = 147 \text{ psi} \cdot \text{in}^m$, $m = 0.187$

$$S_{ut} = \frac{147}{(0.162)^{0.187}} = 207.1 \text{ kpsi}$$

$$S_y = 0.75(207.1) = 155.3 \text{ kpsi}$$

$$S_{sy} = 0.50(207.1) = 103.5 \text{ kpsi}$$

Body

$$F = \frac{\pi d^3 S_{sy}}{\pi K_B D}$$

$$= \frac{\pi (0.162)^3 (103.5)(10^3)}{8(1.166)(1.338)} = 110.8 \text{ lbf}$$

Torsional stress on hook point B

$$C_2 = \frac{2r_2}{d} = \frac{2(0.25 + 0.162 / 2)}{0.162} = 4.086$$

$$(K)_B = \frac{4C_2 - 1}{4C_2 - 4} = \frac{4(4.086) - 1}{4(4.086) - 4} = 1.243$$

$$F = \frac{\pi (0.162)^3 (103.5)(10^3)}{8(1.243)(1.338)} = 103.9 \text{ lbf}$$

Normal stress on hook point A

$$C_1 = \frac{2r_1}{d} = \frac{1.338}{0.162} = 8.26$$

$$(K)_A = \frac{4C_1^2 - C_1 - 1}{4C_1(C_1 - 1)} = \frac{4(8.26)^2 - 8.26 - 1}{4(8.26)(8.26 - 1)} = 1.099$$

$$S_{yt} = \sigma = F\left[\frac{16(K)_A D}{\pi d^3} + \frac{4}{\pi d^2}\right]$$

$$F = \frac{155.3(10^3)}{\left[16(1.099)(1.338)\right] / \left[\pi(0.162)^3\right] + \left\{4 / \left[\pi(0.162)^2\right]\right\}} = 85.8 \text{ lbf}$$

$$= \min(110.8,\ 103.9,\ 85.8) = 85.8 \text{ lbf} \quad \textit{Ans.}$$

(e) Eq. (10-48):

$$y = \frac{F - F_i}{k} = \frac{85.8 - 16}{4.855} = 14.4 \text{ in} \quad \textit{Ans.}$$

10-42 $F_{\min} = 9$ lbf, $F_{\max} = 18$ lbf

$$F_a = \frac{18 - 9}{2} = 4.5 \text{ lbf}, \quad F_m = \frac{18 + 9}{2} = 13.5 \text{ lbf}$$

A313 stainless: $0.013 \le d \le 0.1$ $A = 169$ kpsi · in^m, $m = 0.146$
$0.1 \le d \le 0.2$ $A = 128$ kpsi · in^m, $m = 0.263$
$E = 28$ Mpsi, $G = 10$ Gpsi

Try $d = 0.081$ in and refer to the discussion following Ex. 10-7

$$S_{ut} = \frac{169}{(0.081)^{0.146}} = 243.9 \text{ kpsi}$$

$$S_{su} = 0.67 S_{ut} = 163.4 \text{ kpsi}$$

$$S_{sy} = 0.35 S_{ut} = 85.4 \text{ kpsi}$$

$$S_y = 0.55 S_{ut} = 134.2 \text{ kpsi}$$

Table 10-8: $S_r = 0.45 S_{ut} = 109.8$ kpsi

$$S_e = \frac{S_r / 2}{1 - [S_r / (2S_{ut})]^2} = \frac{109.8 / 2}{1 - [(109.8 / 2) / 243.9]^2} = 57.8 \text{ kpsi}$$

$$r = \tau_a / \tau_m = F_a / F_m = 4.5 / 13.5$$

For Gerber, Eq. (6-48), solving for $(\sigma_m)_A$ gives

$$(\sigma_m)_A = \frac{1}{2}\frac{S_{ut}^2 r}{n_f S_e}\left[-1 + \sqrt{1 + \left(\frac{2S_e}{S_{ut} r}\right)^2}\right]$$

$$= \frac{1}{2}\frac{243.9^2(4.5/13.5)}{2(57.8)}\left\{-1 + \sqrt{1 + \left[\frac{2(57.8)}{243.9(4.5/13.5)}\right]^2}\right\} = 63.3 \text{ kpsi}$$

Hook bending

$$(\sigma_m)_A = F_m\left[(K)_A \frac{16C}{\pi d^2} + \frac{4}{\pi d^2}\right] = \frac{13.5}{\pi d^2}\left[\frac{(4C^2 - C - 1)16C}{4C(C - 1)} + 4\right] \quad (1)$$

Let $\alpha = \dfrac{(\sigma_a)_A 10^3 \pi d^2}{13.5} = \dfrac{63.3\left(10^3\right)\pi\left(0.081\right)^2}{13.5} = 96.65$

Equation (1) reduces to

$$C^2 - \frac{\alpha}{16}C + \frac{\alpha - 8}{16} = 0$$

The useable root for C is

$$C = \frac{1}{4}\left(\frac{\alpha}{8} + \sqrt{\left(\frac{\alpha}{8}\right)^2 - \alpha + 8}\right) = \frac{1}{4}\left(\frac{96.647}{8} + \sqrt{\left(\frac{96.647}{8}\right)^2 - 96.647 + 8}\right)$$

$$= 4.91$$

$$(\sigma_a)_A = \frac{F_a}{F_m}(\sigma_m)_A = \frac{4.5}{13.5}63.3 = 21.1 \text{ kpsi}$$

$$\left(n_f\right)_A = \frac{1}{2}\left(\frac{S_{ut}}{\sigma_m}\right)^2\left(\frac{\sigma_a}{S_e}\right)\left[-1 + \sqrt{1 + \left(\frac{2\sigma_m S_e}{S_{ut}\sigma_a}\right)^2}\right]$$

$$= \frac{1}{2}\left(\frac{243.9}{63.3}\right)^2\left(\frac{21.1}{57.8}\right)\left\{-1 + \sqrt{1 + \left[\frac{2(63.3)(57.8)}{243.9(21.1)}\right]^2}\right\} = 2.00 \qquad \text{checks}$$

$$D = Cd = 0.398 \text{ in}$$

Using Eq. (10-4) for τ_i

$$F_i = \frac{\pi d^3 \tau_i}{8D} = \frac{\pi d^3}{8D}\left[\frac{33\,500}{\exp(0.105C)} \pm 1000\left(4 - \frac{C-3}{6.5}\right)\right]$$

Use the lowest F_i in the preferred range.

$$F_i = \frac{\pi(0.081)^3}{8(0.398)}\left[\frac{33\,500}{\exp[0.105(4.91)]} - 1000\left(4 - \frac{4.91-3}{6.5}\right)\right]$$

$$= 8.55 \text{ lbf}$$

For simplicity we will round F_i up to next 1/4 integer. Let $F_i = 8.75$ lbf.

$$k = \frac{18-9}{0.25} = 36 \text{ lbf/in}$$

$$N_a = \frac{d^4 G}{8kD^3} = \frac{(0.081)^4(10)(10^6)}{8(36)(0.398)^3} = 23.7 \text{ turns}$$

$$N_b = N_a - \frac{G}{E} = 23.7 - \frac{10}{28} = 23.3 \text{ turns}$$

$$L_0 = (2C - 1 + N_b)d = [2(4.91) - 1 + 23.3](0.081) = 2.602 \text{ in}$$

$$L_{\max} = L_0 + (F_{\max} - F_i)\,/\,k = 2.602 + (18 - 8.75)\,/\,36 = 2.859 \text{ in}$$

$$\left(\sigma_a\right)_A = \frac{F_a}{F_m}\left(\sigma_m\right)_A = \frac{4.5}{13.5}63.3 = 21.1 \text{ kpsi}$$

Body: $$K_B = \frac{4C+2}{4C-3} = \frac{4(4.91)+2}{4(4.91)-3} = 1.300$$

$$\left(\tau_a\right)_{\text{body}} = \frac{8(1.300)(4.5)(0.398)}{\pi(0.081)^3}(10^{-3}) = 11.16 \text{ kpsi}$$

$$\left(\tau_m\right)_{\text{body}} = \frac{F_m}{F_a}\left(\tau_a\right)_{\text{body}} = \frac{13.5}{4.5}(11.16) = 33.48 \text{ kpsi}$$

The repeating allowable stress from Table 10-8 is

$$S_{sr} = 0.30S_{ut} = 0.30(243.9) = 73.17 \text{ kpsi}$$

The Gerber intercept is given by Eq. (10-42) as

$$S_{se} = \frac{73.17 / 2}{1 - [(73.17/2)/163.4]^2} = 38.5 \text{ kpsi}$$

From Eq. (6-48),

$$(n_f)_{\text{body}} = \frac{1}{2}\left(\frac{S_{su}}{\tau_m}\right)^2\left(\frac{\tau_a}{S_{se}}\right)\left[-1 + \sqrt{1 + \left(\frac{2\tau_m S_{se}}{S_{su}\tau_a}\right)^2}\right]$$

$$= \frac{1}{2}\left(\frac{163.4}{33.47}\right)^2\left(\frac{11.16}{38.5}\right)\left\{-1 + \sqrt{1 + \left[\frac{2(33.47)(38.5)}{163.4(11.16)}\right]^2}\right\} = 2.53$$

Let $r_2 = 2d = 2(0.081) = 0.162$

$$C_2 = \frac{2r_2}{d} = 4, \quad (K)_B = \frac{4(4)-1}{4(4)-4} = 1.25$$

$$(\tau_a)_B = \frac{(K)_B}{K_B}\tau_a = \frac{1.25}{1.30}(11.16) = 10.73 \text{ kpsi}$$

$$(\tau_m)_B = \frac{(K)_B}{K_B}\tau_m = \frac{1.25}{1.30}(33.48) = 32.19 \text{ kpsi}$$

Table 10-8: $(S_{sr})_B = 0.28S_{ut} = 0.28(243.9) = 68.3$ kpsi

$$(S_{se})_B = \frac{68.3/2}{1 - [(68.3/2)/163.4]^2} = 35.7 \text{ kpsi}$$

$$(n_f)_B = \frac{1}{2}\left(\frac{163.4}{32.18}\right)^2\left(\frac{10.73}{35.7}\right)\left\{-1 + \sqrt{1 + \left[\frac{2(32.18)(35.7)}{163.4(10.73)}\right]^2}\right\} = 2.51$$

Yield

Bending:

$$(\sigma_A)_{\max} = \frac{4F_{\max}}{\pi d^2}\left[\frac{(4C^2 - C - 1)}{C - 1} + 1\right]$$

$$= \frac{4(18)}{\pi(0.081^2)}\left[\frac{4(4.91)^2 - 4.91 - 1}{4.91 - 1} + 1\right](10^{-3}) = 84.4 \text{ kpsi}$$

$$(n_y)_A = \frac{134.2}{84.4} = 1.59$$

Body:

$$\tau_i = (F_i \,/\, F_a)\tau_a = (8.75 \,/\, 4.5)(11.16) = 21.7 \text{ kpsi}$$

$$r = \tau_a/(\tau_m - \tau_i) = 11.16 \,/\, (33.47 - 21.7) = 0.948$$

$$(S_{sa})_y = \frac{r}{r+1}(S_{sy} - \tau_i) = \frac{0.948}{0.948+1}(85.4 - 21.7) = 31.0 \text{ kpsi}$$

$$(n_y)_{\text{body}} = \frac{(S_{sa})_y}{\tau_a} = \frac{31.0}{11.16} = 2.78$$

Hook shear:

$$S_{sy} = 0.3S_{ut} = 0.3(243.9) = 73.2 \text{ kpsi}$$

$$\tau_{\max} = (\tau_a)_B + (\tau_m)_B = 10.73 + 32.18 = 42.9 \text{ kpsi}$$

$$(n_y)_B = \frac{73.2}{42.9} = 1.71$$

$$\text{fom} = -\frac{7.6\pi^2 d^2(N_b + 2)D}{4} = -\frac{7.6\pi^2(0.081)^2(23.3 + 2)(0.398)}{4} = -1.239$$

A tabulation of several wire sizes follow

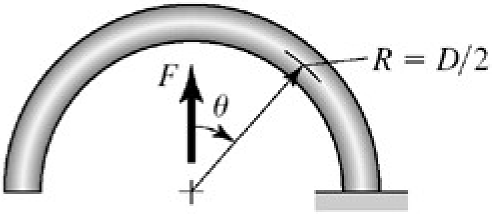
d	0.081	0.085	0.092	0.098	0.105	0.120
S_{ut}	243.920	242.210	239.427	237.229	234.851	230.317
S_{su}	163.427	162.281	160.416	158.943	157.350	154.312
S_{sy}	85.372	84.773	83.800	83.030	82.198	80.611
S_r	109.764	108.994	107.742	106.753	105.683	103.643
S_e	57.809	57.403	56.744	56.223	55.659	54.585
$(\sigma_m)_A$	63.331	62.887	62.164	61.594	60.976	59.799
α	96.695	105.734	122.443	137.659	156.443	200.389
C	4.916	5.497	6.563	7.527	8.713	11.477
$(\sigma_a)_A$	21.110	20.962	20.721	20.531	20.325	19.933
$(n_f)_A$	2.000	2.000	2.000	2.000	2.000	2.000
D	0.398	0.467	0.604	0.738	0.915	1.377
OD	0.479	0.552	0.696	0.836	1.020	1.497
F_i (calc)	8.537	7.842	6.769	5.960	5.117	3.618
F_i (rd)	8.750	8.750	8.750	8.750	8.750	8.750

k	36.000	36.000	36.000	36.000	36.000	36.000
N_a	23.677	17.767	11.301	7.979	5.512	2.756
N_b	23.320	17.410	10.944	7.622	5.155	2.399
L_0	2.604	2.329	2.122	2.124	2.266	2.922
$L_{\max}$	2.861	2.586	2.379	2.381	2.523	3.179
K_B	1.300	1.263	1.215	1.184	1.157	1.117
$(\tau_a)_{\text{body}}$	11.162	11.015	10.796	10.638	10.478	10.197
$(\tau_m)_{\text{body}}$	33.486	33.044	32.388	31.913	31.433	30.591
S_{sr}	73.176	72.663	71.828	71.169	70.455	69.095
S_{se}	38.519	38.249	37.809	37.462	37.087	36.371
$(n_f)_{\text{body}}$	2.526	2.542	2.564	2.578	2.591	2.611
$(K_B)_B$	4.000	4.000	4.000	4.000	4.000	4.000
$(\tau_a)_B$	10.732	10.898	11.106	11.226	11.320	11.416
$(\tau_m)_B$	32.196	32.695	33.319	33.679	33.960	34.248
$(S_{sr})_B$	68.298	67.819	67.040	66.424	65.758	64.489
$(S_{se})_B$	35.708	35.458	35.050	34.728	34.380	33.717
$(n_f)_B$	2.512	2.457	2.383	2.336	2.293	2.230
S_y	134.156	133.215	131.685	130.476	129.168	126.674
$(\sigma_A)_{\max}$	84.441	83.849	82.886	82.125	81.302	79.732
$(n_y)_A$	1.589	1.589	1.589	1.589	1.589	1.589
τ_i	21.704	21.417	20.992	20.684	20.373	19.828
r	0.947	0.947	0.947	0.947	0.947	0.947
$(S_{sa})_y$	30.974	30.822	30.555	30.330	30.077	29.570
$(n_y)_{\text{body}}$	2.775	2.798	2.830	2.851	2.871	2.900
$(S_{sy})_B$	73.176	72.663	71.828	71.169	70.455	69.095
$(\tau_B)_{\max}$	42.928	43.594	44.426	44.905	45.280	45.664
$(n_y)_B$	1.705	1.667	1.617	1.585	1.556	1.513
fom	-1.240	-1.229	-1.240	-1.278	-1.353	-1.636

optimal fom

The shaded areas show the conditions not satisfied.

10-43 For the hook,



$$M = FR\sin\theta,\quad \partial M/\partial F = R\sin\theta$$

$$\delta_F = \frac{1}{EI}\int_0^{\pi/2} F\left(R\sin\theta\right)^2 R\,d\theta = \frac{\pi}{2}\frac{FR^3}{EI}$$

The total deflection of the body and the two hooks

$$\delta = \frac{8FD^3N_b}{d^4G} + 2\left(\frac{\pi}{2}\frac{FR^3}{EI}\right) = \frac{8FD^3N_b}{d^4G} + \frac{\pi F(D/2)^3}{E(\pi/64)(d^4)}$$

$$= \frac{8FD^3}{d^4G}\left(N_b + \frac{G}{E}\right) = \frac{8FD^3N_a}{d^4G}$$

$$\therefore\ N_a = N_b + \frac{G}{E}\qquad \text{Q.E.D.}$$

10-44 Table 10-5 ($d = 4$ mm = 0.1575 in): $E = 196.5$ GPa

Table 10-4 for A227:

$$A = 1783\ \text{MPa}\cdot\text{mm}^m,\qquad m = 0.190$$

Eq. (10-14): $$S_{ut} = \frac{A}{d^m} = \frac{1783}{4^{0.190}} = 1370\ \text{MPa}$$

Eq. (10-57): $$S_y = \sigma_{\text{all}} = 0.78\,S_{ut} = 0.78(1370) = 1069\ \text{MPa}$$

$$D = \text{OD} - d = 32 - 4 = 28\ \text{mm}$$

$$C = D/d = 28/4 = 7$$

Eq. (10-43): $$K_i = \frac{4C^2 - C - 1}{4C(C-1)} = \frac{4\left(7^2\right) - 7 - 1}{4(7)(7-1)} = 1.119$$

Eq. (10-44): $$\sigma = K_i\frac{32Fr}{\pi d^3}$$

At yield, $Fr = M_y$, $\sigma = S_y$. Thus,

$$M_y = \frac{\pi d^3 S_y}{32K_i} = \frac{\pi\left(4^3\right)1069\left(10^{-3}\right)}{32(1.119)} = 6.00\ \text{N}\cdot\text{m}$$

Count the turns when $M = 0$

$$N = 2.5 - \frac{M_y}{k}$$

where from Eq. (10-51): $k = \dfrac{d^4 E}{10.8DN}$

Thus,

$$N = 2.5 - \frac{M_y}{d^4 E \,/\, (10.8DN)}$$

Solving for N gives

$$N = \frac{2.5}{1 + [10.8DM_y \,/\, (d^4 E)]}$$

$$= \frac{2.5}{1 + \left\{\left[10.8(28)(6.00)\right] / \left[4^4(196.5)\right]\right\}} = 2.413 \text{ turns}$$

This means (2.5 – 2.413)(360°) or 31.3° from closed. *Ans.*

Treating the hand force as in the middle of the grip,

$$r = 112.5 - 87.5 + \frac{87.5}{2} = 68.75 \text{ mm}$$

$$F_{\max} = \frac{M_y}{r} = \frac{6.00\left(10^3\right)}{68.75} = 87.3 \text{ N} \quad \textit{Ans.}$$

10-45 The spring material and condition are unknown. Given $d = 0.081$ in and OD = 0.500,

(a) $D = 0.500 - 0.081 = 0.419$ in

Using $E = 28.6$ Mpsi for an estimate

$$k' = \frac{d^4 E}{10.8DN} = \frac{(0.081)^4(28.6)(10^6)}{10.8(0.419)(11)} = 24.7 \text{ lbf} \cdot \text{in/turn}$$

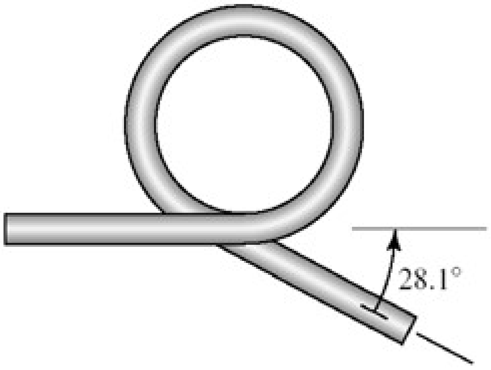
for each spring. The moment corresponding to a force of 8 lbf

$$Fr = (8/2)(3.3125) = 13.25 \text{ lbf} \cdot \text{in/spring}$$

The fraction windup turn is

$$n = \frac{Fr}{k'} = \frac{13.25}{24.7} = 0.536 \text{ turns}$$

The arm swings through an arc of slightly less than 180°, say 165°. This uses up 165/360 or 0.458 turns. So $n = 0.536 - 0.458 = 0.078$ turns are left (or $0.078(360°) = 28.1°$). The original configuration of the spring was



Ans.

(b)

$$C = \frac{D}{d} = \frac{0.419}{0.081} = 5.17$$

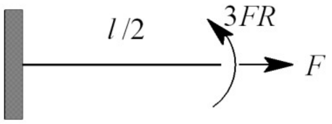
$$K_i = \frac{4C^2 - C - 1}{4C\left(C - 1\right)} = \frac{4(5.17)^2 - 5.17 - 1}{4(5.17)(5.17 - 1)} = 1.168$$

$$\sigma = K_i \frac{32M}{\pi d^3} = 1.168\left[\frac{32(13.25)}{\pi(0.081)^3}\right] = 297\left(10^3\right) \text{ psi} = 297 \text{ kpsi} \qquad \textit{Ans.}$$

To achieve this stress level, the spring had to have set removed.

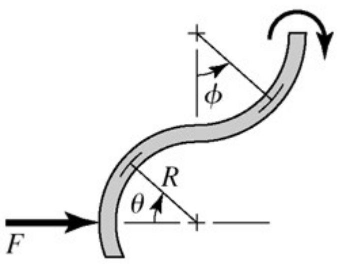
10-46 (**a**) Consider half and double results

Straight section:



$$M = 3FR, \quad \frac{\partial M}{\partial F} = 3R$$

Upper 180° section:



$$\begin{aligned} M &= F[R + R(1 - \cos\phi)] \\ &= FR(2 - \cos\phi), \quad \frac{\partial M}{\partial F} = R(2 - \cos\phi) \end{aligned}$$

Lower section: $M = FR \sin\theta, \quad \dfrac{\partial M}{\partial F} = R\sin\theta$

Considering bending only:

$$\delta = \frac{\partial U}{\partial F} = \frac{2}{EI}\left[\int_0^{l/2} 9FR^2\,dx + \int_0^{\pi} FR^2(2-\cos\phi)^2 R\,d\phi + \int_0^{\pi/2} F(R\sin\theta)^2 R\,d\theta\right]$$

$$= \frac{2F}{EI}\left[\frac{9}{2}R^2 l + R^3\left(4\pi - 4\sin\phi\Big|_0^{\pi} + \frac{\pi}{2}\right) + R^3\left(\frac{\pi}{4}\right)\right]$$

$$= \frac{2FR^2}{EI}\left(\frac{19\pi}{4}R + \frac{9}{2}l\right) = \frac{FR^2}{2EI}(19\pi R + 18l)$$

The spring rate is

$$k = \frac{F}{\delta} = \frac{2EI}{R^2(19\pi R + 18l)} \qquad Ans.$$

(**b**) Given: A227 HD wire, $d = 2$ mm, $R = 6$ mm, and $l = 25$ mm.

Table 10-5 ($d = 2$ mm $= 0.0787$ in): $E = 197.2$ GPa

$$k = \frac{2(197.2)10^9\pi\left(0.002^4\right)/(64)}{0.006^2\left[19\pi(0.006)+18(0.025)\right]} = 10.65\left(10^3\right)\text{ N/m} = 10.65\text{ N/mm} \qquad Ans.$$

(**c**) The maximum stress will occur at the bottom of the top hook where the bending-moment is $3FR$ and the axial fore is F. Using curved beam theory for bending,

Eq. (3-65): $$\sigma_i = \frac{Mc_i}{Aer_i} = \frac{3FRc_i}{\left(\pi d^2/4\right)e\left(R-d/2\right)}$$

Axial: $$\sigma_a = \frac{F}{A} = \frac{F}{\pi d^2/4}$$

Combining, $$\sigma_{\max} = \sigma_i + \sigma_a = \frac{4F}{\pi d^2}\left[\frac{3Rc_i}{e\left(R-d/2\right)} + 1\right] = S_y$$

$$F = \frac{\pi d^2 S_y}{4\left[\dfrac{3Rc_i}{e\left(R-d/2\right)} + 1\right]} \qquad (1) \qquad Ans.$$

For the clip in part (b),

Eq. (10-14) and Table 10-4: $S_{ut} = A/d^m = 1783/2^{0.190} = 1563$ MPa

Eq. (10-57): $S_y = 0.78\,S_{ut} = 0.78(1563) = 1219$ MPa

Table 3-4:

$$r_n = \frac{1^2}{2\left(6-\sqrt{6^2-1^2}\right)} = 5.95804 \text{ mm}$$

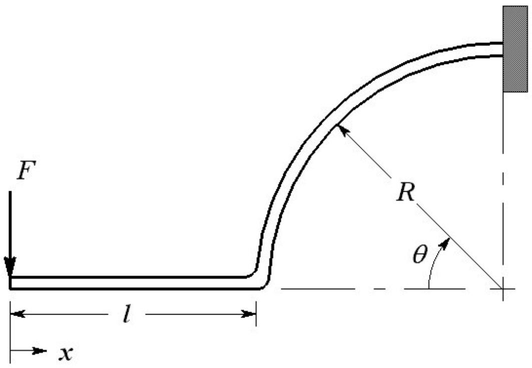
$$e = r_c - r_n = 6 - 5.95804 = 0.04196 \text{ mm}$$

$$c_i = r_n - (R - d/2) = 5.95804 - (6 - 2/2) = 0.95804 \text{ mm}$$

Eq. (1):

$$F = \frac{\pi\left(0.002^2\right)1219\left(10^6\right)}{4\left[\dfrac{3(6)0.95804}{0.04196(6-1)}+1\right]} = 46.0 \text{ N} \qquad Ans.$$

10-47 (**a**)



$$M = -Fx, \quad \frac{\partial M}{\partial F} = -x \quad 0 \le x \le l$$

$$M = Fl + FR(1-\cos\theta), \quad \frac{\partial M}{\partial F} = l + R(1-\cos\theta) \quad 0 \le \theta \le \pi/2$$

$$\delta_F = \frac{1}{EI}\left\{\int_0^l -Fx(-x)dx + \int_0^{\pi/2} F\left[l + R(1-\cos\theta)\right]^2 R d\theta\right\}$$

$$= \frac{F}{12EI}\left\{4l^3 + 3R\left[2\pi l^2 + 4(\pi-2)lR + (3\pi-8)R^2\right]\right\}$$

The spring rate is

$$k = \frac{F}{\delta_F} = \frac{12EI}{4l^3 + 3R\left[2\pi l^2 + 4(\pi-2)lR + (3\pi-8)R^2\right]} \qquad Ans.$$

(**b**) Given: A313 stainless wire, d = 0.063 in, R = 0.625 in, and l = 0.5 in.

Table 10-5: E = 28 Mpsi

$$I = \frac{\pi}{64} d^4 = \frac{\pi}{64}\left(0.063^4\right) = 7.733\left(10^{-7}\right) \text{ in}^4$$

$$k = \frac{12(28)10^6(7.733)10^{-7}}{4\left(0.5^3\right)+3(0.625)\left[2\pi\left(0.5^2\right)+4(\pi-2)0.5(0.625)+(3\pi-8)\left(0.625^2\right)\right]}$$
$$= 36.3 \text{ lbf/in} \qquad \textit{Ans.}$$

(**c**) Table 10-4: A = 169 kpsi·in^m, m = 0.146

Eq. (10-14): $S_{ut} = A/d^m = 169/0.063^{0.146} = 253.0$ kpsi

Eq. (10-57): $S_y = 0.61\, S_{ut} = 0.61(253.0) = 154.4$ kpsi

One can use curved beam theory as in the solution for Prob. 10-41. However, the equations developed in Sec. 10-12 are equally valid.

$$C = D/d = 2(0.625 + 0.063/2)/0.063 = 20.8$$

Eq. (10-43): $$K_i = \frac{4C^2 - C - 1}{4C(C-1)} = \frac{4\left(20.8^2\right)-20.8-1}{4(20.8)(20.8-1)} = 1.037$$

Eq. (10-44), setting $\sigma = S_y$:

$$K_i \frac{32Fr}{\pi d^3} = S_y \quad \Rightarrow \quad 1.037 \frac{32F(0.5+0.625)}{\pi\left(0.063^3\right)} = 154.4\left(10^3\right)$$

Solving for F yields F = 3.25 lbf *Ans.*

Try solving part (c) of this problem using curved beam theory. You should obtain the same answer.

10-48 (**a**) $M = -Fx$

$$\sigma = \left|\frac{M}{I/c}\right| = \frac{Fx}{I/c} = \frac{Fx}{bh^2/6}$$

Constant stress,

$$\frac{bh^2}{6} = \frac{Fx}{\sigma} \quad \Rightarrow \quad h = \sqrt{\frac{6Fx}{b\sigma}} \qquad (1) \qquad Ans.$$

At $x = l$,

$$h_o = \sqrt{\frac{6Fl}{b\sigma}} \qquad \Rightarrow \qquad h = h_o\sqrt{x/l} \qquad Ans.$$

(b) $M = -Fx, \quad \partial M/\partial F = -x$

$$y = \int_0^l \frac{M\left(\partial M/\partial F\right)}{EI}dx = \frac{1}{E}\int_0^l \frac{-Fx\left(-x\right)}{\frac{1}{12}bh_o^3\left(x/l\right)^{3/2}}dx = \frac{12Fl^{3/2}}{bh_o^3E}\int_0^l x^{1/2}dx$$

$$= \frac{2}{3}\frac{12Fl^{3/2}}{bh_o^3E}l^{3/2} = \frac{8Fl^3}{bh_o^3E}$$

$$k = \frac{F}{y} = \frac{bh_o^3E}{8l^3} \qquad Ans.$$

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