1 Resolving Forces, Calculating Resultants

Ref: Hibbeler § 2.4-2.6, Bedford & Fowler: Statics § 2.3-2.5

Resolving forces refers to the process of finding two or more forces which, when combined, will produce a force with the same magnitude and direction as the original. The most common use of the process is finding the components of the original force in the Cartesian coordinate directions: x, y, and z.

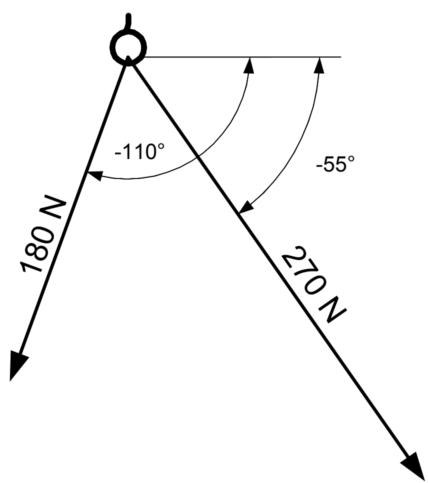
A *resultant* force is the force (magnitude and direction) obtained when two or more forces are combined (i.e., added as vectors).

Breaking down a force into its Cartesian coordinate components (e.g., F_x, F_y) and using Cartesian components to determine the force and direction of a resultant force are common tasks when solving statics problems. These will be demonstrated here using a two-dimensional problem involving co-planar forces.

Example: Co-Planar Forces

Two boys are playing by pulling on ropes connected to a hook in a rafter. The bigger one pulls on the rope with a force of 270 N (about 60 lb_f) at an angle of 55° from horizontal. The smaller boy pulls with a force of 180 N (about 40 lb_f) at an angle of 110° from horizontal.

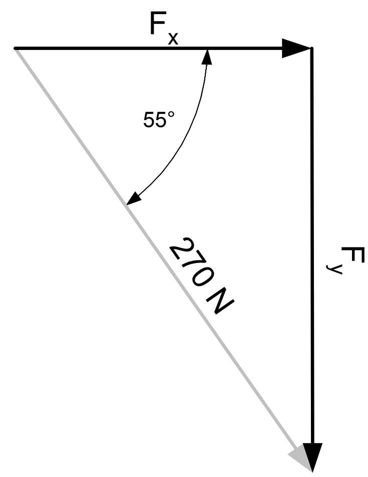
a. Which boy is exerting the greatest vertical force (downward) on the hook?
b. What is the net force (magnitude and direction) on the hook – that is, calculate the resultant force.



> **Note:** The angles in this figure have been indicated as *coordinate direction angles*. That is, each angle has been measured from the positive x axis.

Solution

First, consider the 270 N force acting at 55° from horizontal. The x- and y-components of force are indicated schematically, as



The x- and y-components of the first force (270 N) can be calculated using a little trigonometry involving the included angle, 55°:

$$\cos(55°) = \frac{F_{x1}}{270\ \mathrm{N}}, \quad \text{or} \quad F_{x1} = (270\ \mathrm{N})\cos(55°)$$

and

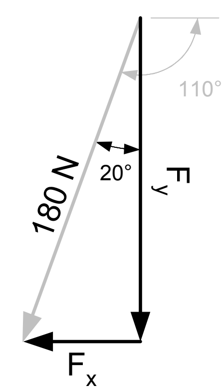
$$\sin(55°) = \frac{F_{y1}}{270\ \mathrm{N}}, \quad \text{or} \quad F_{y1} = (270\ \mathrm{N})\sin(55°).$$

MATLAB can be used to solve for F_{x1} and F_{y1} using its built-in sin() and cos() functions, but these functions assume that the angle will be expressed as radians, not degrees. The factor pi/180 is used to convert the angle from degrees to radians. Note that pi is a predefined variable in MATLAB.

```
» F_x1 = 270 * cos( 55 * pi/180 )
 F_x1 =
   154.8656
» F_x1 = 270 * sin( 55 * pi/180 )
 F_x1 =
   221.1711
```

Your Turn

Show that the x- and y-components of the second force (180 N acting at 110° from the x-axis) are 61.5 N (-x direction) and 169 N (-y direction), respectively. Note that trigonometry relationships are based on the included angle of the triangle (20°, as shown at the right), not the coordinate angle (-110° from the x-axis).

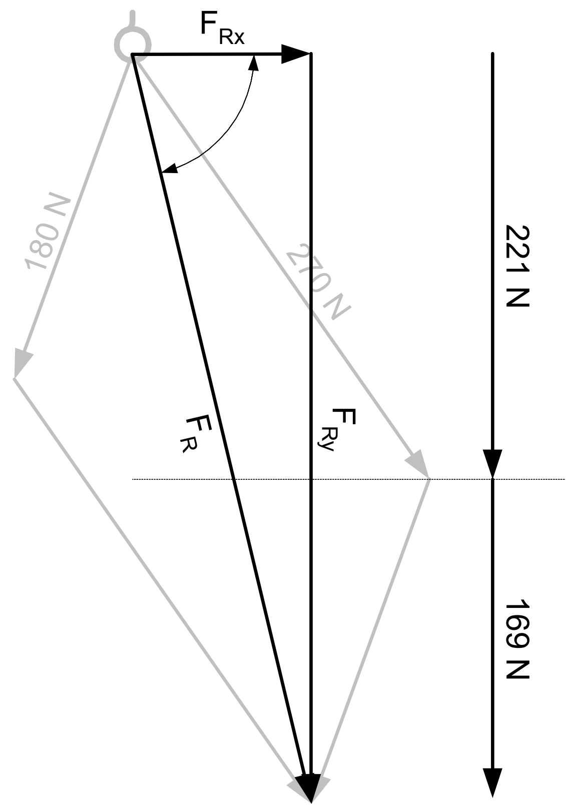


Answer, part a)

The larger boy exerts the greatest vertical force (221 N) on the hook. The vertical force exerted by the smaller boy is only 169 N.

Solution, continued

To determine the combined force on the hook, F_R, first add the two y-components calculated above, to determine the combined y-directed force, F_{Ry}, on the hook:



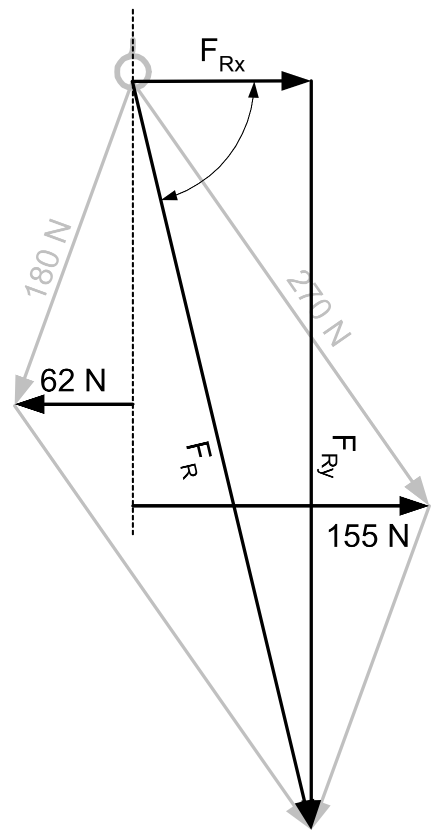
» F_Ry = F_y1 + F_y2

F_Ry =

390.3157

The y-component of the resultant force is 390 N (directed down, or in the –y direction). Note that the direction has not been accounted for in this calculation.

Then add the two x-components to determine the combined x-directed force, F_{Rx}, on the hook. Note that the two x-component forces are acting in opposite directions, so the combined x-directed force, F_{Rx}, is smaller than either of the components, and directed in the +x direction.




```
» F_Rx = F_x1 + ( -F_x2 )
F_Rx =
  93.3020
```

The minus sign was included before F_{x2} because it is directed in the –x direction. The result is an x-component of the resultant force of 93 N in the +x direction.

Once the x- and y-components of the resultant force have been determined, the magnitude can be calculated using

$$F_R = \sqrt{{F_{Rx}}^2 + {F_{Ry}}^2}$$

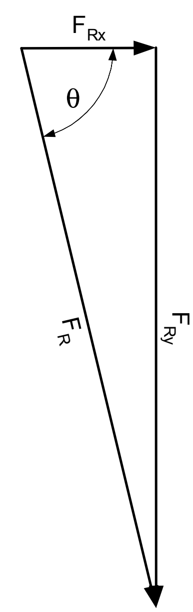
The MATLAB calculation uses the built-in square-root function sqrt().

```
» F_R  = sqrt( F_Rx ^ 2 + F_Ry ^ 2 )
F_R =
  401.3124
```

The angle of the resultant force can be calculated using any of three functions in MATLAB:

Function	*Argument(s)*	*Notes*
atan(abs(Fx / Fy))	one argument: abs(F_x / F_y)	Returns the included angle
atan2(F_y, F_x)	two arguments: F_x and F_y	Returns the coordinate direction angle Angle value is always between 0 and π radians (0 and 180°) A negative sign on the angle indicates a result in one of the lower quadrants of the Cartesian coordinate system
cart2pol (F_x, F_y)	two arguments: F_x and F_y	Returns the positive angle from the positive x-axis to the vector Angle value always between 0 and 2π radians (0 and 360°) An angle value greater than 180° (π radians) indicates a result in one of the lower quadrants of the Cartesian coordinate system

The atan2() function is used here, and F_{Ry} is negative because it is acting in the –y direction.




```
» F_Rx  =   93.302;
» F_Ry  = -390.316;
» theta   = 180/pi * atan2( F_Ry, F_Rx )
theta =
  -76.5562
```

Answer, part b)

The net force (magnitude and direction) on the hook is now known:

F_R = 401 N (about 90 lb_f) acting at an angle 76.6° below the x-axis.

Annotated MATLAB Script Solution

```
% Determine the x- and y-components of the two forces
% (270 N at -55°, and 180 N at -110°)
%
%   Note: These trig. Calculations use the included angles
%   (55° & 20°), with minus signs added to both y-component
%   equations to indicate the forces act in the -y direction,
%   and the F_x2 equation to show that this force acts in
%   the -x direction.

% Calculate the x- and y- components of the first force (270 N)
F_x1 =  270 * cos( 55 * pi/180 );
F_y1 = -270 * sin( 55 * pi/180 );
fprintf('\nF_x1 = %8.3f N\t F_y1 = %+9.3f N\n',F_x1,F_y1);

% Calculate the x- and y- components of the first force (180 N)
F_x2 = -180 * sin( 20 * pi/180 );
F_y2 = -180 * cos( 20 * pi/180 );
fprintf('F_x2 =  %7.3f N\t F_y2 = %9.3f N\n',F_x2,F_y2);

% Sum the y-components of the two forces to determine the
% y-component of the resultant force
F_Ry = F_y1 + F_y2;

% Sum the x-components of the two forces to determine the
% x-component of the resultant force
F_Rx = F_x1 + F_x2;
fprintf('F_Rx =  %7.3f N\t F_Ry = %9.3f N\n\n',F_Rx,F_Ry);

% Calculate the magnitude of the resultant force
F_R  = sqrt( F_Rx ^ 2 + F_Ry ^ 2 );
fprintf('F_R  = %8.3f N\n',F_R);

% Calculate the angle of the resultant force
% (in degrees from the x-axis)
theta = atan2( F_Ry, F_Rx ) * 180/pi;
fprintf('theta = %7.3f N\n\n',theta);
```