Chapter 16

16-1  Given: \( r = 300/2 = 150 \text{ mm}, \ a = R = 125 \text{ mm}, \ b = 40 \text{ mm}, \ f = 0.28, \ F = 2.2 \text{ kN}, \ \theta_1 = 0^\circ, \ \theta_2 = 120^\circ, \ \text{and} \ \theta_a = 90^\circ. \) From which, \( \sin \theta_a = \sin 90^\circ = 1. \)

Eq. (16-2):

\[
M_f = \frac{0.28 p_a (0.040)(0.150)}{1} \int_{\theta_1}^{\theta_2} \sin \theta (0.150 - 0.125 \cos \theta) \, d\theta = 2.993 \left(10^{-4}\right) p_a \text{ N} \cdot \text{m}
\]

Eq. (16-3):

\[
M_N = \frac{p_a (0.040)(0.150)(0.125)}{1} \int_{\theta_1}^{\theta_2} \sin^2 \theta \, d\theta = 9.478 \left(10^{-4}\right) p_a \text{ N} \cdot \text{m}
\]

\[
c = 2(0.125 \cos 30^\circ) = 0.2165 \text{ m}
\]

Eq. (16-4):

\[
F = \frac{9.478 \left(10^{-4}\right) p_a - 2.993 \left(10^{-4}\right) p_a}{0.2165} = 2.995 \left(10^{-3}\right) p_a
\]

\[
p_a = F/ [2.995(10^{-3})] = 2200/[2.995(10^{-3})]
\]

\[
= 734.5 \left(10^3\right) \text{ Pa} \quad \text{for cw rotation}
\]

Eq. (16-7):

\[
2200 = \frac{9.478 \left(10^{-4}\right) p_a + 2.993 \left(10^{-4}\right) p_a}{0.2165}
\]

\[
p_a = 381.9 \left(10^3\right) \text{ Pa for ccw rotation}
\]

A maximum pressure of 734.5 kPa occurs on the RH shoe for cw rotation.  \text{Ans.}

(b) \textit{RH shoe:}

Eq. (16-6):

\[
T_r = \frac{0.28(734.5)10^3(0.040)(0.150)(\cos 0^\circ - \cos 120^\circ)}{1} = 277.6 \text{ N} \cdot \text{m} \quad \text{Ans.}
\]

\textit{LH shoe:}

\[
T_c = \frac{381.9}{734.5} = 144.4 \text{ N} \cdot \text{m} \quad \text{Ans.}
\]

\[
T_{\text{total}} = 277.6 + 144.4 = 422 \text{ N} \cdot \text{m} \quad \text{Ans.}
\]
RH shoe: \( F_x = 2200 \sin 30^\circ = 1100 \text{ N}, \quad F_y = 2200 \cos 30^\circ = 1905 \text{ N} \)

Eqs. (16-8): \[
A = \left( \frac{1}{2} \sin^2 \theta \right)^{120^\circ} = 0.375, \quad B = \left( \frac{\theta}{2} - \frac{1}{4} \sin 2\theta \right) = 1.264
\]

Eqs. (16-9): \[
R_x = \frac{734.5 (10^3) 0.040 (0.150)}{1} [0.375 - 0.28 (1.264)] - 1100 = -1007 \text{ N}
R_y = \frac{734.5 (10^3) 0.040 (0.150)}{1} [1.264 + 0.28 (0.375)] - 1905 = 4128 \text{ N}
R = \left( (-1007)^2 + 4128^2 \right)^{1/2} = 4249 \text{ N} \quad \text{Ans.}

LH shoe: \( F_x = 1100 \text{ N}, \quad F_y = 1905 \text{ N} \)

Eqs. (16-10): \[
R_x = \frac{381.9 (10^3) 0.040 (0.150)}{1} [0.375 + 0.28 (1.264)] - 1100 = 570 \text{ N}
R_y = \frac{381.9 (10^3) 0.040 (0.150)}{1} [1.264 - 0.28 (0.375)] - 1905 = 751 \text{ N}
R = \left( 597^2 + 751^2 \right)^{1/2} = 959 \text{ N} \quad \text{Ans.}

16-2 Given: \( r = 300/2 = 150 \text{ mm}, \quad a = R = 125 \text{ mm}, \quad b = 40 \text{ mm}, \quad f = 0.28, \quad F = 2.2 \text{ kN}, \quad \theta_1 = 15^\circ, \quad \theta_2 = 105^\circ, \quad \text{and} \quad \theta_3 = 90^\circ. \) From which, \( \sin \theta_3 = \sin 90^\circ = 1. \)

Eq. (16-2):
\[
M_f = \frac{0.28 p_a (0.040)(0.150)}{1} \int_{15^\circ}^{105^\circ} \sin \theta (0.150 - 0.125 \cos \theta) \, d\theta = 2.177 \left( 10^{-4} \right) p_a
\]
Eq. (16-3): \[ M_N = \frac{p_a(0.040)(0.150)(0.125)}{1} \int_{15^\circ}^{105^\circ} \sin^2 \theta \, d\theta = 7.765 \left(10^{-4}\right) p_a \]

\[ c = 2(0.125) \cos 30^\circ = 0.2165 \text{ m} \]

Eq. (16-4): \[ F = \frac{7.765 \left(10^{-4}\right) p_a - 2.177 \left(10^{-4}\right) p_a}{0.2165} = 2.581 \left(10^{-3}\right) p_a \]

**RH shoe:** \[ p_a = 2200/[2.581(10^{-3})] = 852.4 \left(10^3\right) \text{ Pa} \]
\[ = 852.4 \text{ kPa on RH shoe for cw rotation} \quad \text{Ans.} \]

Eq. (16-6): \[ T_R = \frac{0.28(852.4)10^3(0.040)(0.150^2)(\cos 15^\circ - \cos 105^\circ)}{1} = 263 \text{ N} \cdot \text{m} \]

**LH shoe:**
\[ 2200 = \frac{7.765 \left(10^{-4}\right) p_a + 2.177 \left(10^{-4}\right) p_a}{0.2165} \]
\[ p_a = 479.1 \left(10^3\right) \text{ Pa} = 479.1 \text{ kPa on LH shoe for cw rotation} \quad \text{Ans.} \]
\[ T_{\text{LH}} = \frac{0.28(479.1)10^3(0.040)(0.150^2)(\cos 15^\circ - \cos 105^\circ)}{1} = 148 \text{ N} \cdot \text{m} \]
\[ T_{\text{total}} = 263 + 148 = 411 \text{ N} \cdot \text{m} \quad \text{Ans.} \]

Comparing this result with that of Prob. 16-1, a 2.6% reduction in torque is obtained by using 25% less braking material.

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16-3 Given: \( \theta_1 = 0^\circ, \theta_2 = 120^\circ, \theta_a = 90^\circ, \sin \theta_a = 1, a = R = 3.5 \text{ in}, b = 1.25 \text{ in}, f = 0.30, \) \( F = 225 \text{ lbf}, r = 11/2 = 5.5 \text{ in}, \) counter-clockwise rotation.

**LH shoe:**
Eq. (16-2), with \( \theta_i = 0^\circ: \)
\[ M_f = \frac{f p_b r}{\sin \theta_a} \int \sin \theta (r - a \cos \theta) \, d\theta = \frac{f p_b r}{\sin \theta_a} \left[ r(1 - \cos \theta_2) - \frac{a}{2} \sin^2 \theta_2 \right] \]
\[ = \frac{0.30 p_a (1.25)5.5}{1} \left[ 5.5(1 - \cos 120^\circ) - \frac{3.5}{2} \sin^2 120^\circ \right] \]
\[ = 14.31 p_a \text{ lbf} \cdot \text{in} \]

Eq. (16-3), with \( \theta_i = 0^\circ: \)
\[ M_N = \frac{p_a b r a}{\sin \theta_a} \int \sin^2 \theta \, d\theta = \frac{p_a b r a}{\sin \theta_a} \left[ \frac{\theta_2}{2} - \frac{1}{4} \sin 2\theta_2 \right] \]
\[ = \frac{p_a (1.25)5.5(3.5)}{1} \left[ \frac{120^\circ}{2} \left( \frac{\pi}{180^\circ} \right) - \frac{1}{4} \sin (2(120^\circ)) \right] \]
\[ = 30.41 p_a \text{ lbf} \cdot \text{in} \]
\[ c = 2r \cos \left( \frac{180^\circ - \theta_2}{2} \right) = 2(5.5) \cos 30^\circ = 9.526 \text{ in} \]

\[ F = 225 \frac{30.41p_a - 14.31p_a}{9.526} = 1.690 p_a \]

\[ p_a = \frac{225}{1.690} = 133.1 \text{ psi} \]

Eq. (16-6):

\[ T_k = \frac{f p_b r^2 (\cos \theta_1 - \cos \theta_2)}{\sin \theta_a} = \frac{0.30(133.1)1.25(5.5)}{1}[1 - (0.5)] \]

\[ = 2265 \text{ lbf} \cdot \text{in} = 2.265 \text{ kip} \cdot \text{in} \quad \text{Ans.} \]

RH shoe:

\[ F = 225 \frac{30.41p_a + 14.31p_a}{9.526} = 4.694 p_a \]

\[ p_a = \frac{225}{4.694} = 47.93 \text{ psi} \]

\[ T_k = \frac{47.93}{133.1} = 816 \text{ lbf} \cdot \text{in} = 0.816 \text{ kip} \cdot \text{in} \]

\[ T_{\text{total}} = 2.27 + 0.82 = 3.09 \text{ kip} \cdot \text{in} \quad \text{Ans.} \]

16-4 \( \text{(a)} \) Given: \( \theta_1 = 10^\circ, \theta_2 = 75^\circ, \theta_a = 75^\circ, p_a = 10^6 \text{ Pa}, f = 0.24, b = 0.075 \text{ m (shoe width)}, a = 0.150 \text{ m}, r = 0.200 \text{ m}, d = 0.050 \text{ m}, c = 0.165 \text{ m}. \)

Some of the terms needed are evaluated here:

\[ A = \left[ r \int_{\theta_1}^{\theta_2} \sin \theta \, d\theta - a \int_{\theta_1}^{\theta_2} \sin \theta \cos \theta \, d\theta \right] = r \left[ -\cos \theta \right]_{\theta_1}^{\theta_2} - a \left[ \frac{1}{2} \sin^2 \theta \right]_{\theta_1}^{\theta_2} \]

\[ = 200 \left[ -\cos 75^\circ \right]_{10^\circ}^{75^\circ} - 150 \left[ \frac{1}{2} \sin^2 \theta \right]_{10^\circ}^{75^\circ} = 77.5 \text{ mm} \]

\[ B = \int_{\theta_1}^{\theta_2} \sin^2 \theta \, d\theta = \left[ \frac{\theta}{2} - \frac{1}{4} \sin 2\theta \right]_{\theta_1}^{\theta_2} = 0.528 \]

\[ C = \int_{\theta_1}^{\theta_2} \sin \theta \cos \theta \, d\theta = 0.4514 \]

Now converting to Pascals and meters, we have from Eq. (16-2),

\[ M_f = \frac{f p_b r^2 A}{\sin \theta_a} = \frac{0.24 \left(10^6\right)(0.075)(0.200)}{\sin 75^\circ} = 289 \text{ N} \cdot \text{m} \]
From Eq. (16-3),

\[ M_N = \frac{p_{bra}}{\sin \theta_a} B = \frac{10^6(0.075)(0.200)(0.150)}{\sin 75^\circ} (0.528) = 1230 \text{ N} \cdot \text{m} \]

Finally, using Eq. (16-4), we have

\[ F = \frac{M_N - M_f}{c} = \frac{1230 - 289}{165} = 5.70 \text{ kN} \quad \text{Ans.} \]

(b) Use Eq. (16-6) for the primary shoe.

\[ T = \frac{fp_{bra}r^2(\cos \theta_1 - \cos \theta_2)}{\sin \theta_a} \]

\[ = \frac{0.24\left(10^6\right)(0.075)(0.200)^2(\cos 10^\circ - \cos 75^\circ)}{\sin 75^\circ} = 541 \text{ N} \cdot \text{m} \]

For the secondary shoe, we must first find \( p_a \). Substituting

\[ M_N = \frac{1230}{10^6} p_a \text{ and } M_f = \frac{289}{10^6} p_a \text{ into Eq. (16-7),} \]

\[ 5.70 = \frac{(1230 / 10^6)p_a + (289 / 10^6)p_a}{165} , \text{ solving gives } p_a = 619(10^3) \text{ Pa} \]

Then

\[ T = \frac{0.24\left[619(10^3)\right]0.075(0.200^2)(\cos 10^\circ - \cos 75^\circ)}{\sin 75^\circ} = 335 \text{ N} \cdot \text{m} \]

so the braking capacity is \( T_{total} = 2(541) + 2(335) = 1750 \text{ N} \cdot \text{m} \quad \text{Ans.} \]

(c) Primary shoes:

\[ R_x = \frac{p_{bra}r}{\sin \theta_a} (C - fB) - F_x \]

\[ = \frac{10^6(0.075)(0.200)}{\sin 75^\circ}[0.4514 - 0.24(0.528)](10^{-3}) - 5.70 = -0.658 \text{ kN} \]

\[ R_y = \frac{p_{bra}r}{\sin \theta_a} (B + fC) - F_y \]

\[ = \frac{10^6(0.075)(0.200)}{\sin 75^\circ}[0.528 + 0.24(0.4514)](10^{-3}) - 0 = 9.88 \text{ kN} \]
Secondary shoes:

\[ R_x = \frac{p_{br}}{\sin \theta_a} (C + f B) - F_x \]
\[ = \frac{0.619 \left(10^6\right)0.075(0.200)}{\sin 75^\circ} [0.4514 + 0.24(0.528)](10^{-3}) - 5.70 \]
\[ = -0.143 \text{kN} \]

\[ R_y = \frac{p_{br}}{\sin \theta_a} (B - f C) - F_y \]
\[ = \frac{0.619 \left(10^6\right)0.075(0.200)}{\sin 75^\circ} [0.528 - 0.24(0.4514)](10^{-3}) - 0 \]
\[ = 4.03 \text{kN} \]

Note from figure that +y for secondary shoe is opposite to +y for primary shoe.

Combining horizontal and vertical components,
\[ R_H = -0.658 - 0.143 = -0.801 \text{kN} \]
\[ R_V = 9.88 - 4.03 = 5.85 \text{kN} \]
\[ R = \sqrt{(-0.801)^2 + 5.85^2} \]
\[ = 5.90 \text{kN} \quad \text{Ans.} \]

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16-5 Given: Face width \( b = 1.25 \text{ in} \), \( F = 90 \text{ lbf} \), \( f = 0.25 \).

Preliminaries: \( \theta_1 = 45^\circ - \tan^{-1}(6/8) = 8.13^\circ \), \( \theta_2 = 98.13^\circ \), \( \theta_a = 90^\circ \),
\( a = (6^2 + 8^2)^{1/2} = 10 \text{ in} \)

Eq. (16-2):
\[ M_f = \frac{f}{\sin \theta_a \theta_b} \int \sin \theta (r - a \cos \theta) d\theta = \frac{0.25 p_a(1.25)6}{1} \int_{0.813^\circ}^{98.13^\circ} \sin \theta (6 - 10 \cos \theta) d\theta \]
\[ = 3.728 p_a \text{ lbf \cdot in} \]

Eq. (16-3):
\[ M_N = \frac{p_a r_b}{\sin \theta_a \theta_b} \int \sin^2 \theta d\theta = \frac{p_a(1.25)6(10)}{1} \int_{0.813^\circ}^{98.13^\circ} \sin^2 \theta d\theta \]
\[ = 69.405 p_a \text{ lbf \cdot in} \]

Eq. (16-4): Using \( F_c = M_N - M_f \), we obtain
\[ 90(20) = (69.405 - 3.728)p_a \quad \Rightarrow \quad p_a = 27.4 \text{ psi} \quad \text{Ans.} \]
Eq. (16-6):
\[
T = \frac{fp_br^2(\cos \theta_f - \cos \theta_o)}{\sin \theta_a} = \frac{0.25(27.4)(1.25)(6^2)(\cos 8.13^\circ - \cos 98.13^\circ)}{1} = 348.7 \text{ lbf \cdot in} \quad \text{Ans.}
\]

16-6 For \( \hat{\sigma}_f \):
\[
f = \bar{f} + 3\hat{\sigma}_f = 0.25 + 3(0.025) = 0.325
\]
From Prob. 16-5, with \( f = 0.25, M_f = 3.728 \ p_a \). Thus, \( M_f = (0.325/0.25) \ 3.728 \ p_a = 4.846 \ p_a \). From Prob. 16-5, \( M_N = 69.405 \ p_a \).

Eq. (16-4): Using \( F_c = M_N - M_f \), we obtain
\[
90(20) = (69.405 - 4.846) \ p_a \quad \Rightarrow \quad \ p_a = 27.88 \text{ psi} \quad \text{Ans.}
\]
From Prob. 16-5, \( p_a = 27.4 \text{ psi} \) and \( T = 348.7 \text{ lbf \cdot in} \). Thus,
\[
T = \left( \frac{0.325}{0.25} \right) \left( \frac{27.88}{27.4} \right) 348.7 = 461.3 \text{ lbf \cdot in} \quad \text{Ans.}
\]

Similarly, for \( -3\hat{\sigma}_f \):
\[
f = \bar{f} - 3\hat{\sigma}_f = 0.25 - 3(0.025) = 0.175
\]
\[
M_f = (0.175 / 0.25) \ 3.728 \ p_a = 2.610 \ p_a
\]
\[
90(20) = (69.405 - 2.610) \ p_a \quad \Rightarrow \quad \ p_a = 26.95 \text{ psi}
\]
\[
T = \left( \frac{0.175}{0.25} \right) \left( \frac{26.95}{27.4} \right) 348.7 = 240.1 \text{ lbf \cdot in} \quad \text{Ans.}
\]

16-7 Preliminaries: \( \theta_f = 180^\circ - 30^\circ - \tan^{-1}(3/12) = 136^\circ, \theta_e = 20^\circ - \tan^{-1}(3/12) = 6^\circ, \theta_o = 90^\circ, \sin \theta_a = 1, a = (3^2 + 12^2)^{1/2} = 12.37 \text{ in}, r = 10 \text{ in}, f = 0.30, b = 2 \text{ in}, p_a = 150 \text{ psi} \).

Eq. (16-2):
\[
M_f = \frac{0.30(150)(2)(10)}{\sin 90^\circ} \int_{6^\circ}^{136^\circ} \sin \theta(10 - 12.37 \cos \theta) \ d\theta = 12800 \text{ lbf \cdot in}
\]

Eq. (16-3):
\[
M_N = \frac{150(2)(10)(12.37)}{\sin 90^\circ} \int_{6^\circ}^{136^\circ} \sin^2 \theta \ d\theta = 53300 \text{ lbf \cdot in}
\]

LH shoe:
\[
c_f = 12 + 12 + 4 = 28 \text{ in}
\]
Now note that \( M_f \) is cw and \( M_N \) is ccw. Thus,
\[ F_L = \frac{53\,300 - 12\,800}{28} = 1446 \text{ lbf} \]

\[ F_L = 1446 \text{ lbf} \]

\[ F_S = 1491 \text{ lbf} \]

\[ F_{act} = 361 \text{ lbf} \]

Eq. (16-6): \[ T_L = \frac{0.30(150)(2)(10)^2(\cos 6^\circ - \cos 136^\circ)}{\sin 90^\circ} = 15\,420 \text{ lbf} \cdot \text{in} \]

RH shoe:

\[ M_N = 53\,300 \frac{P_a}{150} = 355.3 P_a, \quad M_f = 12\,800 \frac{P_a}{150} = 85.3 P_a \]

On this shoe, both \( M_N \) and \( M_f \) are ccw. Also,

\[ c_R = (24 - 2 \tan 14^\circ) \cos 14^\circ = 22.8 \text{ in} \]

\[ F_{act} = F_L \sin 14^\circ = 361 \text{ lbf} \quad \text{Ans.} \]

\[ F_R = F_L / \cos 14^\circ = 1491 \text{ lbf} \]

Thus,

\[ 1491 = \frac{355.3 + 85.3}{22.8} P_a \Rightarrow P_a = 77.2 \text{ psi} \]

Then,

\[ T_R = \frac{0.30(77.2)(2)(10)^2(\cos 6^\circ - \cos 136^\circ)}{\sin 90^\circ} = 7940 \text{ lbf} \cdot \text{in} \]

\[ T_{total} = 15\,420 + 7940 = 23\,400 \text{ lbf} \cdot \text{in} \quad \text{Ans.} \]

16-8

\[ M_f = 2 \int_0^\theta (f dN)(a' \cos \theta - r) \quad \text{where} \ dN = pbr \ d\theta \]

\[ = 2 fpbr \int_0^\theta (a' \cos \theta - r) \ d\theta = 0 \]

From which

\[ a' \int_0^\theta \cos \theta \ d\theta = r \int_0^\theta d\theta \]

\[ a' = \frac{r \theta_2}{\sin \theta_2} = \frac{r(60^\circ)(\pi / 180)}{\sin 60^\circ} = 1.209 r \quad \text{Ans.} \]

Eq. (16-15):
\[
a = \frac{4r \sin 60^\circ}{2(60)(\pi / 180) + \sin[2(60)]} = 1.170r \quad \text{Ans.}
\]

\[
a \quad \text{differs with} \quad a' \quad \text{by} \quad 100(1.170 - 1.209)/1.209 = -3.23 \% \quad \text{Ans.}
\]

**16-9** (a) Counter-clockwise rotation, $\theta_2 = \pi/4$ rad, $r = 13.5/2 = 6.75$ in

\[
ea = \frac{4r \sin \theta_2}{2 \theta_2 + \sin 2 \theta_2} = \frac{4(6.75) \sin(\pi/4)}{2\pi/4 + \sin(2\pi/4)} = 7.426 \text{ in}
\]

\[
e = 2a = 2(7.426) = 14.85 \text{ in} \quad \text{Ans.}
\]

(b) Left shoe lever.

\[
\sum M_R = 0 = 7.78S^y - 15.28F^x
\]

\[
S^y = \frac{15.28}{7.78} (2.125P) = 4.174P
\]

\[
\sum F_y = 0 = R^x + S^y + F^y
\]

\[
F^y = -F^x \tan 11.4^\circ = 0.428P
\]

\[
\sum F_y = -P - F^y + R^y
\]

\[
R^y = P + 0.428P = 1.428P
\]
The direction of brake pulley rotation affects the sense of \( S^x \), which has no effect on the brake shoe lever moment and hence, no effect on \( S^x \) or the brake torque.

The brake shoe levers carry identical bending moments but the left lever carries a tension while the right carries compression (column loading). The right lever is designed and used as a left lever, producing interchangeable levers (identical levers). But do not infer from these identical loadings.

16-10 \( r = 13.5/2 = 6.75 \text{ in}, \ b = 6 \text{ in}, \ \theta_2 = 45^\circ = \pi/4 \text{ rad.} \)

From Table 16-3 for a rigid, molded non-asbestos lining use a conservative estimate of \( p_a = 100 \text{ psi}, \ f = 0.33 \).

Equation (16-16) gives the horizontal brake hinge pin reaction which corresponds to \( S^x \) in Prob. 16-9. Thus,

\[
N = S^x = \frac{p_{br}}{2} \left( 2\theta_2 + \sin 2\theta_2 \right) = \frac{100(6.75)}{2} \left[ 2 \left( \frac{\pi}{4} \right) + \sin \left( 2 \left( 45^\circ \right) \right) \right]
\]

\[
= 5206 \text{ lbf}
\]

which, from Prob. 6-9 is 4.174 \( P \). Therefore,

\[ 4.174 \; P = 5206 \quad \Rightarrow \quad P = 1250 \text{ lbf} = 1.25 \text{ kip} \quad \text{Ans}. \]

Applying Eq. (16-18) for two shoes, where from Prob. 16-9, \( a = 7.426 \text{ in} \)

\[
T = 2af \; N = 2(7.426)(0.33)(5206)
\]

\[
= 25 \; 520 \text{ lbf \cdot in} = 25.52 \text{ kip \cdot in} \quad \text{Ans}. \]
16-11 Given: \( D = 350 \text{ mm}, \ b = 100 \text{ mm}, \ p_a = 620 \text{ kPa}, \ f = 0.30, \ \phi = 270^\circ \).

Eq. (16-22):

\[
P_1 = \frac{p_a b D}{2} = \frac{620(0.100)0.350}{2} = 10.85 \text{ kN} \quad \text{Ans.}
\]

\[
f \phi = 0.30(270^\circ)(\pi / 180^\circ) = 1.414
\]

Eq. (16-19):

\[
P_2 = P_1 \exp(-f \phi) = 10.85 \exp(-1.414) = 2.64 \text{ kN} \quad \text{Ans.}
\]

\[
T = (P_1 - P_2)(D / 2) = (10.85 - 2.64)(0.350 / 2) = 1.437 \text{ kN} \cdot \text{m} \quad \text{Ans.}
\]

16-12 Given: \( D = 12 \text{ in}, \ f = 0.28, \ b = 3.25 \text{ in}, \ \phi = 270^\circ, \ P_1 = 1800 \text{ lbf} \).

Eq. (16-22):

\[
p_a = \frac{2P_1}{b D} = \frac{2(1800)}{3.25(12)} = 92.3 \text{ psi} \quad \text{Ans.}
\]

\[
f \phi = 0.28(270^\circ)(\pi / 180^\circ) = 1.319
\]

\[
P_2 = P_1 \exp(-f \phi) = 1800 \exp(-1.319) = 481 \text{ lbf}
\]

\[
T = (P_1 - P_2)(D / 2) = (1800 - 481)(12 / 2)
\]

\[
= 7910 \text{ lbf} \cdot \text{in} = 7.91 \text{ kip} \cdot \text{in} \quad \text{Ans.}
\]

16-13

\[
\Sigma M_O = 0 = 100 P_2 - 325 F \quad \Rightarrow \quad P_2 = \frac{325(300)}{100} = 975 \text{ N} \quad \text{Ans.}
\]
\[ \alpha = \cos^{-1}\left(\frac{100}{160}\right) = 51.32^\circ \]
\[ \phi = 270^\circ - 51.32^\circ = 218.7^\circ \]
\[ f \phi = 0.30(218.7)(\pi / 180^\circ) = 1.145 \]
\[ P_1 = P_2 \exp(f \phi) = 975 \exp(1.145) = 3064 \text{ N} \quad \text{Ans.} \]
\[ T = \left( P_1 - P_2 \right) \left( D / 2 \right) = (3064 - 975)(200 / 2) \]
\[ = 209 \left(10^3\right) \text{ N} \cdot \text{mm} = 209 \text{ N} \cdot \text{m} \quad \text{Ans.} \]

16-14 (a) \( D = 16 \text{ in} \), \( b = 3 \text{ in} \)
\( n = 200 \text{ rev/min} \)
\( f = 0.20, \quad p_a = 70 \text{ psi} \)

Eq. (16-22):
\[ P_1 = \frac{p_a b D}{2} = \frac{70(3)(16)}{2} = 1680 \text{ lbf} \]
\[ f \phi = 0.20(3\pi / 2) = 0.942 \]

Eq. (16-14):
\[ P_2 = P_1 \exp(-f \phi) = 1680 \exp(-0.942) = 655 \text{ lbf} \]
\[ T = \left( P_1 - P_2 \right) \frac{D}{2} = (1680 - 655) \frac{16}{2} \]
\[ = 8200 \text{ lbf} \cdot \text{in} \quad \text{Ans.} \]
\[ H = \frac{Tn}{63 025} = \frac{8200(200)}{63 025} = 26.0 \text{ hp} \quad \text{Ans.} \]
\[ P = \frac{3P_1}{10} = \frac{3(1680)}{10} = 504 \text{ lbf} \quad \text{Ans.} \]
(b) Force of belt on the drum:

\[ R = (1680^2 + 655^2)^{1/2} = 1803 \text{ lbf} \]

Force of shaft on the drum: 1680 and 655 lbf

\[ T_R = 1680(8) = 13440 \text{ lbf} \cdot \text{ in} \]
\[ T_F = 655(8) = 5240 \text{ lbf} \cdot \text{ in} \]

Net torque on drum due to brake band:

\[ T = T_R - T_F \]
\[ = 13440 - 5240 \]
\[ = 8200 \text{ lbf} \cdot \text{ in} \]

The radial load on the bearing pair is 1803 lbf. If the bearing is straddle mounted with the drum at center span, the bearing radial load is \( 1803/2 = 901 \text{ lbf} \).
(c) Eq. (16-21):

\[ p = \frac{2P}{bD} \]

\[ p \big|_{\phi=0^\circ} = \frac{2P_1}{3(16)} = \frac{2(1680)}{3(16)} = 70 \text{ psi} \quad \text{Ans.} \]

\[ p \big|_{\phi=270^\circ} = \frac{2P_2}{3(16)} = \frac{2(655)}{3(16)} = 27.3 \text{ psi} \quad \text{Ans.} \]

16-15 Given: \( \phi = 270^\circ, b = 2.125 \text{ in}, f = 0.20, T = 150 \text{ lb} \cdot \text{ft}, D = 8.25 \text{ in}, c_2 = 2.25 \text{ in} \) (see figure). Notice that the pivoting rocker is not located on the vertical centerline of the drum.

(a) To have the band tighten for ccw rotation, it is necessary to have \( c_1 < c_2 \). When friction is fully developed,

\[ \frac{P_1}{P_2} = \exp(f \phi) = \exp[0.2(3\pi / 2)] = 2.566 \]

If friction is not fully developed,

\[ \frac{P_1}{P_2} \leq \exp(f \phi) \]

To help visualize what is going on let’s add a force \( W \) parallel to \( P_1 \), at a lever arm of \( c_3 \). Now sum moments about the rocker pivot.

\[ \sum M = 0 = c_3 W + c_1 P_1 - c_2 P_2 \]

From which

\[ W = \frac{c_2 P_2 - c_1 P_1}{c_3} \]

The device is self locking for ccw rotation if \( W \) is no longer needed, that is, \( W \leq 0 \). It follows from the equation above

\[ \frac{P_1}{P_2} \geq \frac{c_2}{c_1} \]

When friction is fully developed

\[ 2.566 = 2.25 / c_1 \]

\[ c_1 = \frac{2.25}{2.566} = 0.877 \text{ in} \]

When \( P_1/P_2 \) is less than 2.566, friction is not fully developed. Suppose \( P_1/P_2 = 2.25 \), then
c_1 = \frac{2.25}{2.25} = 1 \text{ in}

We don’t want to be at the point of slip, and we need the band to tighten.

\[ \frac{c_2}{P_1 / P_2} \leq c_1 \leq c_2 \]

When the developed friction is very small, \( P_1/P_2 \to 1 \) and \( c_1 \to c_2 \) \( \text{Ans.} \)

(b) Rocker has \( c_1 = 1 \text{ in} \)

\[ \frac{P_1}{P_2} = \frac{c_2}{c_1} = \frac{2.25}{1} = 2.25 \]

\[ f = \frac{\ln(P_1 / P_2)}{\phi} = \frac{\ln 2.25}{3\pi/2} = 0.172 \]

Friction is not fully developed, no slip.

\[ T = (P_1 - P_2) \frac{D}{2} = P_2 \left( \frac{P_1}{P_2} - 1 \right) \frac{D}{2} \]

Solve for \( P_2 \)

\[ P_2 = \frac{2T}{[(P_1 / P_2) - 1]D} = \frac{2(150)(12)}{(2.25 - 1)(8.25)} = 349 \text{ lbf} \]

\[ P_1 = 2.25P_2 = 2.25(349) = 785 \text{ lbf} \]

\[ p = \frac{2P_1}{bD} = \frac{2(785)}{2.125(8.25)} = 89.6 \text{ psi} \quad \text{Ans.} \]

(c) The torque ratio is \( 150(12)/100 \) or 18-fold.

\[ P_2 = \frac{349}{18} = 19.4 \text{ lbf} \]

\[ P_1 = 2.25P_2 = 2.25(19.4) = 43.6 \text{ lbf} \]

\[ p = \frac{89.6}{18} = 4.98 \text{ psi} \quad \text{Ans.} \]

Comment:

As the torque opposed by the locked brake increases, \( P_2 \) and \( P_1 \) increase (although ratio is still 2.25), then \( p \) follows. The brake can self-destruct. Protection could be provided by a shear key.

16-16 Given: OD = 250 mm, ID = 175 mm, \( f = 0.30 \), \( F = 4 \text{ kN} \).
(a) From Eq. (16-23),
\[
p_a = \frac{2F}{\pi d(D - d)} = \frac{2(4000)}{\pi(175)(250 - 175)} = 0.194 \text{ N/mm}^2 = 194 \text{ kPa} \quad \text{Ans.}
\]

Eq. (16-25):
\[
T = \frac{F f}{4} (D + d) = \frac{4000(0.30)}{4}(250 + 175)10^{-3} = 127.5 \text{ N} \cdot \text{m} \quad \text{Ans.}
\]

(b) From Eq. (16-26),
\[
p_a = \frac{4F}{\pi (D^2 - d^2)} = \frac{4(4000)}{\pi(250^2 - 175^2)} = 0.159 \text{ N/mm}^2 = 159 \text{ kPa} \quad \text{Ans.}
\]

Eq. (16-27):
\[
T = \frac{\pi f}{12} p_a (D^3 - d^3) = \frac{\pi}{12} (0.30)159 \left(10^3\right) \left(250^3 - 175^3\right) \left(10^{-3}\right)^3
\]
\[
= 128 \text{ N} \cdot \text{m} \quad \text{Ans.}
\]

---

16-17 Given: OD = 6.5 in, ID = 4 in, \( f = 0.24 \), \( p_a = 120 \text{ psi} \).

(a) Eq. (16-23):
\[
F = \frac{\pi p_a d (D - d)}{2} = \frac{\pi(120)(4)}{2}(6.5 - 4) = 1885 \text{ lbf} \quad \text{Ans.}
\]

Eq. (16-24) with \( N \) sliding planes:
\[
T = \frac{\pi f p_a d}{8} (D^2 - d^2) N = \frac{\pi(0.24)(120)(4)}{8} (6.5^2 - 4^2)(6)
\]
\[
= 7125 \text{ lbf} \cdot \text{in} \quad \text{Ans.}
\]

(b) \[
T = \frac{\pi(0.24)(120d)}{8} (6.5^2 - d^2)(6)
\]
\[
\begin{array}{c|c}
d, \text{ in} & T, \text{ lbf} \cdot \text{in} \\
\hline
2 & 5191 \\
3 & 6769 \\
4 & 7125 \quad \text{Ans.} \\
5 & 5853 \\
6 & 2545 \\
\end{array}
\]

(c) The torque-diameter curve exhibits a stationary point maximum in the range of diameter \( d \). The clutch has nearly optimal proportions.

---

16-18 (a) Eq. (16-24) with \( N \) sliding planes:
Differentiating with respect to \( d \) and equating to zero gives

\[
\frac{dT}{dd} = \frac{\pi f p_s N}{8} (D^2 - 3d^2) = 0
\]

\[d^* = \frac{D}{\sqrt{3}} \quad \text{Ans.}
\]

\[
\frac{d^2T}{dd^2} = -6\frac{\pi f p_s N}{8} d = -3\frac{\pi f p_s N}{4} d
\]

which is negative for all positive \( d \). We have a stationary point maximum.

(b) \[d^* = \frac{6.5}{\sqrt{3}} = 3.75 \text{ in} \quad \text{Ans.}
\]

Eq. (16-24):

\[
T^* = \frac{\pi(0.24)(120)(6.5 / \sqrt{3})}{8} \left[ 6.5^2 - \left(6.5 / \sqrt{3}\right)^2 \right] (6) = 7173 \text{ lbf \cdot in}
\]

(c) The table indicates a maximum within the range: \(3 \leq d \leq 5 \text{ in}\)

(d) Consider: \[0.45 = \frac{d}{D} = 0.80
\]

Multiply through by \( D \),

\[0.45D \leq d \leq 0.80D
\]

\[0.45(6.5) \leq d \leq 0.80(6.5)
\]

\[2.925 \leq d \leq 5.2 \text{ in}
\]

\[
\left(\frac{d}{D}\right)^* = d^* / D = \frac{1}{\sqrt{3}} = 0.577
\]

which lies within the common range of clutches.

Yes. \quad \text{Ans.}

---

16-19 Given: \( d = 11 \text{ in} \), \( l = 2.25 \text{ in} \), \( T = 1800 \text{ lbf \cdot in} \), \( D = 12 \text{ in} \), \( f = 0.28 \).

\[
\alpha = \tan^{-1}\left(\frac{0.5}{2.25}\right) = 12.53^\circ
\]
Uniform wear
Eq. (16-45):
\[ T = \frac{\pi f^2 p_a d}{8 \sin \alpha} (D^2 - d^2) \]
\[ 1800 = \frac{\pi(0.28)p_a(11)}{8 \sin 12.53^\circ}(12^2 - 11^2) = 128.2 p_a \]
\[ p_a = \frac{1800}{128.2} = 14.04 \text{ psi} \quad \text{Ans.} \]

Eq. (16-44):
\[ F = \frac{\pi p_a d}{2} (D - d) = \frac{\pi(14.04)11}{2} (12 - 11) = 243 \text{ lbf} \quad \text{Ans.} \]

Uniform pressure
Eq. (16-48):
\[ T = \frac{\pi f^2 p_a}{12 \sin \alpha} (D^3 - d^3) \]
\[ 1800 = \frac{\pi(0.28)p_a}{12 \sin 12.53^\circ}(12^3 - 11^3) = 134.1 p_a \]
\[ p_a = \frac{1800}{134.1} = 13.42 \text{ psi} \quad \text{Ans.} \]

Eq. (16-47):
\[ F = \frac{\pi p_a}{4} (D^3 - d^3) = \frac{\pi(13.42)}{4} (12^2 - 11^2) = 242 \text{ lbf} \quad \text{Ans.} \]

16-20 Uniform wear
Eq. (16-34):
\[ T = \frac{1}{2} (\theta_2 - \theta_1) f p_a r_i (r_o^2 - r_i^3) \]
Eq. (16-33):
\[ F = (\theta_1 - \theta_2) p_a r_i (r_o - r_i) \]

Thus,
\[ \frac{T}{F D} = \frac{(1/2)(\theta_2 - \theta_1) f p_a r_i (r_o^2 - r_i^2)}{f(\theta_2 - \theta_1) p_a r_i (r_o - r_i) D} \]
\[ = \frac{r_o + r_i}{2D} = \frac{D/2 + d/2}{2D} = \frac{1}{4} \left(1 + \frac{d}{D}\right) \quad \text{O.K.} \quad \text{Ans.} \]

Uniform pressure
Eq. (16-38):
\[ T = \frac{1}{3} (\theta_2 - \theta_1) f p_a (r_o^3 - r_i^3) \]
Eq. (16-37): \[ F = \frac{1}{2} (\theta_2 - \theta_1) p_a \left( r_o^3 - r_i^3 \right) \]

Thus,

\[
\frac{T}{f \ FD} = \frac{(1 / 3)(\theta_2 - \theta_1) f \ p_a \left( r_o^3 - r_i^3 \right)}{(1 / 2) f (\theta_2 - \theta_1) p_a \left( r_o^3 - r_i^3 \right) D} = \frac{2}{3} \left\{ \frac{(D / 2)^3 - (d / 2)^3}{(D / 2)^2 - (d / 2)^2 D} \right\} = \frac{2(D / 2)^3 \left[ 1 - (d / D)^3 \right]}{3(D / 2)^3 \left[ 1 - (d / D)^3 \right]} D = \frac{1}{3} \left[ 1 - (d / D)^3 \right] O.K. \text{ Ans.}
\]

16-21

\[ \omega = 2\pi n / 60 = 2\pi 500 / 60 = 52.4 \text{ rad/s} \]

\[ T = \frac{H}{\omega} = \frac{2(10^3)}{52.4} = 38.2 \text{ N} \cdot \text{m} \]

Key:

\[ F = \frac{T}{r} = \frac{38.2}{12} = 3.18 \text{ kN} \]

Average shear stress in key is

\[ \tau = \frac{3.18(10^3)}{6(40)} = 13.2 \text{ MPa} \text{ Ans.} \]

Average bearing stress is

\[ \sigma_b = -\frac{F}{A_b} = -\frac{3.18(10^3)}{3(40)} = -26.5 \text{ MPa} \text{ Ans.} \]

Let one jaw carry the entire load.

\[ r_{av} = \frac{1}{2} \left( \frac{26}{2} + \frac{45}{2} \right) = 17.75 \text{ mm} \]

\[ F = \frac{T}{r_{av}} = \frac{38.2}{17.75} = 2.15 \text{ kN} \]

The bearing and shear stress estimates are

\[ \sigma_b = \frac{-2.15(10^3)}{10(22.5 - 13)} = -22.6 \text{ MPa} \text{ Ans.} \]

\[ \tau = \frac{2.15(10^3)}{10[0.25\pi(17.75)^2]} = 0.869 \text{ MPa} \text{ Ans.} \]
16-22
\[ \omega_1 = 2\pi n / 60 = 2\pi(1600) / 60 = 167.6 \text{ rad/s} \]
\[ \omega_2 = 0 \]

From Eq. (16-51),
\[ \frac{I_1 / I_2}{I_1 + I_2} = \frac{T_1}{\omega_1 - \omega_2} = \frac{2800(8)}{167.6 - 0} = 133.7 \text{ lbf \cdot in \cdot s}^2 \]

Eq. (16-52):
\[ E = \frac{I_1 I_2}{2(I_1 + I_2)} (\omega_1 - \omega_2)^2 = \frac{133.7}{2} (167.6 - 0)^2 = 1.877 \left(10^6\right) \text{ lbf \cdot in} \]

In Btu, Eq. (16-53): \[ H = E / 9336 = 1.877(10^6) / 9336 = 201 \text{ Btu} \]

Eq. (16-54):
\[ \Delta T = \frac{H}{C_p W} = \frac{201}{0.12(40)} = 41.9^\circ F \quad \text{Ans.} \]

16-23
\[ n = \frac{n_1 + n_2}{2} = \frac{260 + 240}{2} = 250 \text{ rev/min} \]

Eq. (16-62): \[ C_s = (\omega_2 - \omega_1) / \omega = (n_2 - n_1) / n = (260 - 240) / 250 = 0.08 \quad \text{Ans.} \]
\[ \omega = 2\pi(250) / 60 = 26.18 \text{ rad/s} \]

From Eq. (16-64):
\[ I = \frac{E_2 - E_1}{C_s \omega^2} = \frac{6.75(10^3)}{0.08(26.18)^2} = 123.1 \text{ N} \cdot \text{m} \cdot \text{s}^2 \]
\[ I = \frac{m}{8} \left(d_o^2 + d_i^2\right) \Leftrightarrow m = \frac{8I}{d_o^2 + d_i^2} = \frac{8(123.1)}{1.5^2 + 1.4^2} = 233.9 \text{ kg} \]

Table A-5, cast iron unit weight = 70.6 kN/m^3 \[ \Rightarrow \rho = 70.6(10^3) / 9.81 = 7197 \text{ kg} / \text{m}^3. \]

Volume: \[ V = m / \rho = 233.9 / 7197 = 0.0325 \text{ m}^3 \]
\[ V = \pi t \left(d_o^2 - d_i^2\right) / 4 = \pi t \left(1.5^2 - 1.4^2\right) / 4 = 0.2278t \]

Equating the expressions for volume and solving for \( t \),
\[ t = \frac{0.0325}{0.2278} = 0.143 \text{ m} = 143 \text{ mm} \quad \text{Ans.} \]
16-24 (a) The useful work performed in one revolution of the crank shaft is

\[ U = 320 \times (10^3) \times 200 \times (10^{-3}) \times 0.15 = 9.6 \times (10^3) \text{ J} \]

Accounting for friction, the total work done in one revolution is

\[ U = 9.6 \times (10^3) \times (1 - 0.20) = 12.0 \times (10^3) \text{ J} \]

Since 15% of the crank shaft stroke accounts for 7.5% of a crank shaft revolution, the energy fluctuation is

\[ E_2 - E_1 = 9.6 \times (10^3) - 12.0 \times (10^3) \times (0.075) = 8.70 \times (10^3) \text{ J} \quad \text{Ans.} \]

(b) For the flywheel,

\[ n = 6(90) = 540 \text{ rev/min} \]

\[ \omega = \frac{2\pi n}{60} = \frac{2\pi(540)}{60} = 56.5 \text{ rad/s} \]

Since \( C_s = 0.10 \)

Eq. (16-64):

\[ I = \frac{E_2 - E_1}{C_s \omega^2} = \frac{8.70 \times (10^3)}{0.10(56.5)^2} = 27.25 \text{ N} \cdot \text{m} \cdot \text{s}^2 \]

Assuming all the mass is concentrated at the effective diameter, \( d \),

\[ I = mr^2 = \frac{md^2}{4} \]

\[ m = \frac{4I}{d^2} = \frac{4(27.25)}{1.2^2} = 75.7 \text{ kg} \quad \text{Ans.} \]

16-25 Use Ex. 16-6 and Table 16-6 data for one cylinder of a 3-cylinder engine.

\[ C_s = 0.30 \]

\[ n = 2400 \text{ rev/min} \quad \text{or} \quad 251 \text{ rad/s} \]

\[ T_m = \frac{3(3368)}{4\pi} = 804 \text{ lbf \cdot in} \quad \text{Ans.} \]

\[ E_2 - E_1 = 3(3531) = 10590 \text{ in} \cdot \text{lbf} \]

\[ I = \frac{E_2 - E_1}{C_s \omega^2} = \frac{10590}{0.30(251^2)} = 0.560 \text{ in} \cdot \text{lbf} \cdot \text{s}^2 \quad \text{Ans.} \]
16-26 (a) 

(1) \[ (T_2)_i = -F_{21}r_p = -\frac{T_2}{r_G}r_p = \frac{T_2}{-n} \quad \text{Ans.} \]

(2) Equivalent energy

\[ (1 / 2)I_2\omega_i^2 = (1 / 2)(I_2)_i\left(\alpha^2\right) \]

\[ (I_2)_i = \frac{\alpha^2}{\omega_i^2}I_2 = \frac{I_2}{n^2} \quad \text{Ans.} \]

\[ \frac{I_G}{I_p} = \left(\frac{r_G}{r_p}\right)^2 \left(\frac{m_G}{m_p}\right) = \left(\frac{r_G}{r_p}\right)^2 \left(\frac{r_G}{r_p}\right)^2 = n^4 \]

From (2) \[ (I_2)_i = \frac{I_G}{n^2} = \frac{n^4I_p}{n^2} = n^2I_p \quad \text{Ans.} \]

(b) \[ I_c = I_M + I_p + n^2I_p + \frac{I_G}{n^2} \quad \text{Ans.} \]

(c) \[ I_c = 10 + 1 + 10^2(1) + \frac{100}{10^2} = 112 \]

16-27 (a) Reflect \( I_L, I_{G2} \) to the center shaft

\[ I_L = I_p + m^2I_p + \frac{I_G}{m^2} \]
Reflect the center shaft to the motor shaft

\[ I_e = I_M + I_P + n^2 I_P + \frac{I_P}{n^2} + \frac{m^2 I_P}{n^2} + \frac{I_L}{m^2 n^2} \]  

\[ I_e = I_M + I_P + n^2 I_P + \frac{I_P}{n^2} + \frac{m^2 I_P}{n^2} + \frac{I_L}{m^2 n^2} \quad \text{Ans.} \]

(b) For \( R = \text{constant} = nm \),  
\[ I_e = I_M + I_P + n^2 I_P + \frac{I_P}{n^2} + \frac{R^2 I_P}{n^4} + \frac{I_L}{R^2} \quad \text{Ans.} \]

(c) For \( R = 10 \),  
\[ \frac{df}{dn} = 0 + 0 + 2n(1) - \frac{2(1)}{n^3} - \frac{4(10^2)(1)}{n^5} + 0 = 0 \]

\[ n^6 - n^2 - 200 = 0 \]

From which

\[ n^* = 2.430 \quad \text{Ans.} \]

\[ m^* = \frac{10}{2.430} = 4.115 \quad \text{Ans.} \]

Notice that \( n^* \) and \( m^* \) are independent of \( I_L \).

\[ \textbf{16-28} \quad \text{From Prob. 16-27,} \]

\[ I_e = I_M + I_P + n^2 I_P + \frac{I_P}{n^2} + \frac{R^2 I_P}{n^4} + \frac{I_L}{R^2} \]

\[ = 10 + 1 + n^2(1) + \frac{1}{n^2} + \frac{100(1)}{n^4} + \frac{100}{10^2} \]

\[ = 12 + n^2 + \frac{1}{n^2} + \frac{100}{n^4} \]
Optimizing the partitioning of a double reduction lowered the gear-train inertia to 20.9/112 = 0.187, or to 19% of that of a single reduction. This includes the two additional gears.

<table>
<thead>
<tr>
<th>( n )</th>
<th>( I_e )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>114.00</td>
</tr>
<tr>
<td>1.50</td>
<td>34.40</td>
</tr>
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<td>2.00</td>
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<td>93.00</td>
</tr>
<tr>
<td>10.00</td>
<td>112.02</td>
</tr>
</tbody>
</table>

16-29 Figure 16-29 applies,
\[
\begin{align*}
    t_2 &= 10 \text{ s}, \quad t_1 = 0.5 \text{ s} \\
    \frac{t_2 - t_1}{t_1} &= \frac{10 - 0.5}{0.5} = 19
\end{align*}
\]

The load torque, as seen by the motor shaft (Rule 1, Prob. 16-26), is
\[
T_L = \frac{1300(12)}{10} = 1560 \text{ lbf \cdot in}
\]

The rated motor torque \( T_r \) is
\[
T_r = \frac{63 025(3)}{1125} = 168.07 \text{ lbf \cdot in}
\]

For Eqs. (16-65):
\[
\begin{align*}
    \omega_s &= \frac{2\pi}{60} (1125) = 117.81 \text{ rad/s} \\
    \omega_z &= \frac{2\pi}{60} (1200) = 125.66 \text{ rad/s} \\
    a &= \frac{-T_r}{\omega_z - \omega_s} = -\frac{168.07}{125.66 - 117.81} = -21.41 \text{ lbf \cdot in/s/rad} \\
    b &= \frac{T_r \omega_s}{\omega_z - \omega_s} = \frac{168.07(125.66)}{125.66 - 117.81} = 2690.4 \text{ lbf \cdot in}
\end{align*}
\]
The linear portion of the squirrel-cage motor characteristic can now be expressed as

\[ T_M = -21.41 \omega + 2690.4 \text{ lbf \cdot in} \]

Eq. (16-68):

\[ T_2 = 168.07 \left( \frac{1560 - 168.07}{1560 - T_2} \right)^{19} \]

One root is 168.07 which is for infinite time. The root for 10 s is desired. Use a successive substitution method

<table>
<thead>
<tr>
<th>( T_2 )</th>
<th>New ( T_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>19.30</td>
</tr>
<tr>
<td>19.30</td>
<td>24.40</td>
</tr>
<tr>
<td>24.40</td>
<td>26.00</td>
</tr>
<tr>
<td>26.00</td>
<td>26.50</td>
</tr>
<tr>
<td>26.50</td>
<td>26.67</td>
</tr>
</tbody>
</table>

Continue until convergence to

\[ T_2 = 26.771 \text{ lbf \cdot in} \]

Eq. (16-69):

\[
I = \frac{a(t_2 - t_1)}{\ln(T_2 / T_1)} = \frac{-21.41(10 - 0.5)}{\ln(26.771 / 168.07)} = 110.72 \text{ lbf \cdot in \cdot s}^2
\]

\[ \omega = \frac{T - b}{a} \]

\[ \omega_{\text{max}} = \frac{T_2 - b}{a} = \frac{26.771 - 2690.4}{-21.41} = 124.41 \text{ rad/s} \quad \text{Ans.} \]

\[ \omega_{\text{min}} = 117.81 \text{ rad/s} \quad \text{Ans.} \]

\[ \bar{\omega} = \frac{124.41 + 117.81}{2} = 121.11 \text{ rad/s} \]

\[ C_s = \frac{\omega_{\text{max}} - \omega_{\text{min}}}{(\omega_{\text{max}} + \omega_{\text{min}}) / 2} = \frac{124.41 - 117.81}{(124.41 + 117.81) / 2} = 0.0545 \quad \text{Ans.} \]

\[ E_1 = \frac{1}{2} I \omega_1^2 = \frac{1}{2}(110.72)(117.81)^2 = 768 \text{ 352 in} \cdot \text{lbf} \]

\[ E_2 = \frac{1}{2} I \omega_2^2 = \frac{1}{2}(110.72)(124.41)^2 = 856 \text{ 854 in} \cdot \text{lbf} \]

\[ \Delta E = E_2 - E_1 = 856 \text{ 854} - 768 \text{ 352} = 88 \text{ 502 in} \cdot \text{lbf} \]

Eq. (16-64):

\[ \Delta E = C_s I \bar{\omega}^2 = 0.0545(110.72)(121.11)^2 = 88 \text{ 508 in} \cdot \text{lbf}, \quad \text{close enough} \quad \text{Ans.} \]

During the punch
\[ T = \frac{63025H}{n} \]
\[ H = \frac{T \omega (60/2\pi)}{63025} = \frac{1560(121.11)(60/2\pi)}{63025} = 28.6 \text{ hp} \]

The gear train has to be sized for 28.6 hp under shock conditions since the flywheel is on the motor shaft. From Table A-18,

\[ I = \frac{m}{8} (d_o^2 + d_i^2) = \frac{W}{8g} (d_o^2 + d_i^2) \]

\[ W = \frac{8gI}{d_o^2 + d_i^2} = \frac{8(386)(110.72)}{d_o^2 + d_i^2} \]

If a mean diameter of the flywheel rim of 30 in is acceptable, try a rim thickness of 4 in

\[ d_i = 30 - (4 / 2) = 28 \text{ in} \]
\[ d_o = 30 + (4 / 2) = 32 \text{ in} \]
\[ W = \frac{8(386)(110.72)}{32^2 + 28^2} = 189.1 \text{ lbf} \]

Rim volume \( V \) is given by

\[ V = \frac{\pi l}{4} (d_o^2 - d_i^2) = \frac{\pi l}{4} (32^2 - 28^2) = 188.5l \]

where \( l \) is the rim width as shown in Table A-18. The specific weight of cast iron is \( \gamma = 0.260 \text{ lbf/in}^3 \), therefore the volume of cast iron is

\[ V = \frac{W}{\gamma} = \frac{189.1}{0.260} = 727.3 \text{ in}^3 \]

Equating the volumes,

\[ 188.5l = 727.3 \]
\[ l = \frac{727.3}{188.5} = 3.86 \text{ in wide} \]

Proportions can be varied.

16-30 Prob. 16-29 solution has \( I \) for the motor shaft flywheel as

\[ I = 110.72 \text{ lbf} \cdot \text{in} \cdot \text{s}^2 \]
A flywheel located on the crankshaft needs an inertia of $10^2 I$ (Prob. 16-26, rule 2)

$$I = 10^2 (110.72) = 11,072 \text{ lbf} \cdot \text{in} \cdot s^2$$

A 100-fold inertia increase. On the other hand, the gear train has to transmit 3 hp under shock conditions.

Stating the problem is most of the solution. Satisfy yourself that on the crankshaft:

$$T_c = 1300(12) = 15,600 \text{ lbf} \cdot \text{in}$$
$$T_r = 10(168.07) = 1680.7 \text{ lbf} \cdot \text{in}$$
$$\omega_c = 117.81 / 10 = 11.781 \text{ rad/s}$$
$$\omega_r = 125.66 / 10 = 12.566 \text{ rad/s}$$
$$a = -21.41(100) = -2141 \text{ lbf} \cdot \text{in} \cdot s/\text{rad}$$
$$b = 2690.35(10) = 26903.5 \text{ lbf} \cdot \text{in}$$
$$T_M = -2141\omega_c + 26903.5 \text{ lbf} \cdot \text{in}$$

$$T_2 = 1680.6 \left( \frac{15,600 - 1680.5}{15,600 - T_2} \right)^{19}$$

The root is $10(26.67) = 266.7 \text{ lbf} \cdot \text{in}$

$$\bar{\omega} = 121.11 / 10 = 12.111 \text{ rad/s}$$
$$C_s = 0.0549 \text{ (same)}$$
$$\omega_{\text{max}} = 121.11 / 10 = 12.111 \text{ rad/s \ Ans.}$$
$$\omega_{\text{min}} = 117.81 / 10 = 11.781 \text{ rad/s \ Ans.}$$

$E_1, E_2, \Delta E$ and peak power are the same. From Table A-18

$$W = \frac{8gl}{d_o^2 + d_i^2} = \frac{8(386)(11072)}{d_o^2 + d_i^2} = \frac{34.19 \times 10^6}{d_o^2 + d_i^2}$$

Scaling will affect $d_o$ and $d_i$, but the gear ratio changed $I$. Scale up the flywheel in the Prob. 16-29 solution by a factor of 2.5. Thickness becomes $4(2.5) = 10 \text{ in}$. 

$$\bar{d} = 30(2.5) = 75 \text{ in}$$
$$d_o = 75 + (10 / 2) = 80 \text{ in}$$
$$d_i = 75 - (10 / 2) = 70 \text{ in}$$
\[
W = \frac{34.19 \left(10^6\right)}{80^2 + 70^2} = 3026 \text{ lbf}
\]
\[
V = \frac{W}{\gamma} = \frac{3026}{0.260} = 11638 \text{ in}^3
\]
\[
V = \frac{\pi}{4} l(80^2 - 70^2) = 1178 l
\]
\[
l = \frac{11638}{1178} = 9.88 \text{ in}
\]

Proportions can be varied. The weight has increased \(\frac{3026}{189.1}\) or about 16-fold while the moment of inertia \(I\) increased 100-fold. The gear train transmits a steady 3 hp. But the motor armature has its inertia magnified 100-fold, and during the punch there are deceleration stresses in the train. With no motor armature information, we cannot comment.

16-31 This can be the basis for a class discussion.