Solutions - Homework 1

(Due date: September 15\textsuperscript{th} @ 5:30 pm)
Presentation and clarity are very important!

**Problem 1 (25 pts)**
a) Simplify the following functions using ONLY Boolean Algebra Theorems. For each resulting simplified function, sketch the logic circuit using AND, OR, XOR, and NOT gates. (12 pts)

\[ F(X, Y, Z) = \prod(M_1, M_2, M_4, M_6) \]
\[ F = (X \oplus \bar{Y})Z + XYZ \]

\[ F(X, Y, Z) = \prod(M_1, M_2, M_4, M_6) = \Sigma(m_9, m_3, m_5, m_7) = XYZ + YZX + XYZ + XYZ = \bar{Y}Z + Z(Y + X + Y) \]
\[ = XYZ + Z(\bar{Y} + X) = \bar{Y}Z + Z(\bar{Y} + X)(Y + X) = \bar{Y}Z + Zy + zx \]

\[ F = (X \oplus \bar{Y})Z + XYZ = (XY + \bar{Y}Z)Z + XYZ = \bar{Y}Z + XYZ + XYZ = \bar{Y}Z + \bar{Y}Z = \bar{Y}Z \]
\[ = (\bar{Y} + Z)(X + Y + Z) = \bar{Y}X + XZ + YX + Z = \bar{Y}X +ZY + ZY + YZ = \bar{Y}X +ZY + ZY + YX \]

\[ F = (A + \bar{C} + \bar{D})(B + \bar{C} + D)(A + B + \bar{C}) = (D + \bar{B} + \bar{C})(\bar{D} + A + \bar{C})(A + B + \bar{C}) = (D + B + \bar{C})(D + A + \bar{C}) \]
\[ = (D + \bar{B} + \bar{C})(D + A + \bar{C}) = D(A + \bar{C}) + \bar{D}(B + \bar{C}) + (A + \bar{C})(D + \bar{C}) = D(A + \bar{C}) + \bar{D}(D + \bar{C}) \]
\[ = \bar{D}B + DA + \bar{C} \]

\[ F = \overline{B(C + A)} + AB = BC + BA + A + \bar{B} = BC + \bar{A} + \bar{B} = \bar{A} + (B + B)(\bar{B} + \bar{C}) = \bar{A} + \bar{B} + \bar{C} = \overline{ABC} \]
b) Using ONLY Boolean Algebra Theorems, determine whether or not the following expression is valid, i.e., whether the left- and right-hand sides represent the same function: (5 pts)
\[ x_1\bar{x}_2 + x_2x_3 + \bar{x}_2x_3 = (x_1 + \bar{x}_2 + x_3)(x_1 + x_2 + \bar{x}_3)(\bar{x}_1 + x_2 + x_3) \]

\[ (x_1 + \bar{x}_2 + x_3)(x_1 + x_2 + \bar{x}_3) = (x_1 + (\bar{x}_2 + x_3))(x_1 + x_2 + \bar{x}_3) = (x_1 + x_2x_3 + x_2x_3)(\bar{x}_1 + x_2 + x_3) \]
\[ = x_1x_2 + x_1\bar{x}_3 + x_2x_3 + \bar{x}_2x_3 + x_2x_3x_1 + x_2x_3 = x_1x_2 + x_1x_3 + x_2x_3 + x_2x_3 = x_1x_2 + x_3x_1 + x_2x_3 + x_2x_3 \]

\[ = x_1x_2 + x_3x_1 + x_2x_3 + x_2x_3 \]

c) For the following Truth table with two outputs: (8 pts)
\[ \begin{array}{c|c|c|c} x & y & z & f_1, f_2 \\ \hline 0 & 0 & 0 & 0, 1 \\ 0 & 0 & 1 & 0, 1 \\ 0 & 1 & 0 & 1, 1 \\ 0 & 1 & 1 & 0, 1 \\ 1 & 0 & 0 & 1, 0 \\ 1 & 0 & 1 & 0, 0 \\ 1 & 1 & 0 & 1, 1 \\ 1 & 1 & 1 & 1, 0 \end{array} \]

Sketch the logic circuits as Canonical Sum of Products and Product of Sums.

\[ \text{Minterms and maxterms: } f_1 = \sum(m_2, m_4, m_5, m_7), \quad f_2 = \sum(m_0, m_3, m_6, m_7) \]

\[ \text{Sum of Products} \\
\begin{array}{c} f_1 = \bar{X}Y\bar{Z} + X\bar{Y}\bar{Z} + X\bar{Y}Z + XYZ \\ f_2 = \bar{X}\bar{Y}Z + X\bar{Y}Z + XYZ + XY\bar{Z} \end{array} \]

\[ \text{Product of Sums} \\
\begin{array}{c} f_1 = (X + Y + Z)(X + Y + \bar{Z})(X + \bar{Y} + \bar{Z})(\bar{X} + Y + \bar{Z}) \\ f_2 = (\bar{X} + \bar{Y} + Z)(\bar{X} + Y + \bar{Z})(\bar{X} + \bar{Y} + Z) \end{array} \]

\[ \text{PROBLEM 2 (15 pts)} \]

a) The following circuit has the following logic function: \( f = \bar{s}a + sb. \)
- Complete the truth table of the circuit, and sketch the logic circuit using ONLY 2-input NAND gates. (5 pts)

\[ \begin{array}{c} x & y & z & f_1, f_2 \\ \hline 0 & 0 & 0 & 0, 0 \\ 0 & 0 & 1 & 0, 0 \\ 0 & 1 & 0 & 1, 0 \\ 0 & 1 & 1 & 1, 1 \\ 1 & 0 & 0 & 0, 0 \\ 1 & 0 & 1 & 0, 1 \\ 1 & 1 & 0 & 0, 1 \\ 1 & 1 & 1 & 1, 1 \end{array} \]

Instructor: Daniel Llamocca
b) The circuit on the right can be used to realize various different functions. (10 pts)

- For example, the following selection of inputs produce the function: \( g = x_1x_2 + x_2x_3 \). Demonstrate that this is the case.

<table>
<thead>
<tr>
<th>in1</th>
<th>in2</th>
<th>in3</th>
<th>in4</th>
<th>in5</th>
<th>in6</th>
<th>in7</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>x_3</td>
<td>0</td>
<td>1</td>
<td>x_2</td>
<td>x_1</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
f_1 &= \overline{x_2}(0) + x_2(x_3) = x_2x_3 \\
f_2 &= \overline{x_2}(0) + x_2(1) = x_2 \\
g &= x_2(x_3x_1 + x_1(x_2)) = x_2(x_1 + \overline{x_1}x_3) = x_2(x_1 + x_3)(x_1 + x_3) \\
g &= x_2(x_1 + x_3) = x_1x_2 + x_2x_3
\end{align*}
\]

- Given the following inputs, provide the resulting function \( g \) (minimize the function).

<table>
<thead>
<tr>
<th>in1</th>
<th>in2</th>
<th>in3</th>
<th>in4</th>
<th>in5</th>
<th>in6</th>
<th>in7</th>
</tr>
</thead>
<tbody>
<tr>
<td>x_1</td>
<td>0</td>
<td>x_3</td>
<td>1</td>
<td>0</td>
<td>x_1</td>
<td>x_2</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
f_1 &= \overline{x_3}(x_1) + x_3(0) = \overline{x_3}(x_1) \\
f_2 &= \overline{x_1}(1) + x_3(0) = \overline{x_1} \\
g &= \overline{x_2}(\overline{x_3}x_1) + x_2(\overline{x_1}) = \overline{x_1}x_2 + \overline{x_2}x_3x_1
\end{align*}
\]

PROBLEM 3 (12 PTS)

- Design a circuit (simplify your circuit) that verifies the logical operation of a 3-input NOR gate. \( f = '1' \) (LED ON) if the NOR gate does NOT work properly. Assumption: when the NOR gate is not working, it generates 1's instead of 0's and vice versa.

\[
\begin{align*}
f &= x_a + x_b + x_c + \overline{x_a}b\overline{c} \\
f &= \overline{x_a}b\overline{c} + x(a + b + c) \\
f &= x(a + b + c) + (a + b + c) \\
f &= x \oplus (a + b + c)
\end{align*}
\]
PROBLEM 4 (20 PTS)

a) Complete the timing diagram of the logic circuit whose VHDL description is shown below: (6 pts)

```vhdl
library ieee;
use ieee.std_logic_1164.all;

entity circ is
  port ( a, b, c: in std_logic;
        f: out std_logic);
end circ;

architecture st of circ is
  signal x, y: std_logic;
begin
  x <= a xor b;
  y <= x nor c;
  f <= y nand (not b);
end st;
```

b) The following is the timing diagram of a logic circuit with 3 inputs. Sketch the logic circuit that generates this waveform. Then, complete the VHDL code. (8 pts)

```vhdl
library ieee;
use ieee.std_logic_1164.all;

entity wav is
  port ( a, b, c: in std_logic;
        f: out std_logic);
end wav;

architecture st of wav is
begin
  f <= c or (a and not (b));
end st;
```

<table>
<thead>
<tr>
<th>a</th>
<th>b</th>
<th>c</th>
<th>f</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

\[ f = c + a\overline{b} \]
c) Complete the timing diagram of the following circuit: (6 pts)

\[ f = abc \]

PROBLEM 5 (28 pts)
- A numeric keypad produces a 4-bit code as shown below. We want to design a logic circuit that converts each 4-bit code to a 7-segment code, where each segment is an LED: A LED is ON if it is given a logic '1'. A LED is OFF if it is given a logic '0'.

✓ Complete the truth table for each output \((a, b, c, d, e, f, g)\).
✓ Provide the simplified expression for each output \((a, b, c, d, e, f, g)\). Use Karnaugh maps for \(c, d, e, f, g\) and the Quine-McCluskey algorithm for \(a, b\). Note that it is safe to assume that the codes \(1100\) to \(1111\) will not be produced by the keypad.

\[
\begin{array}{cccccccc|cccccccc}
\text{Value} & \text{X} & \text{Y} & \text{Z} & \text{W} & \text{a} & \text{b} & \text{c} & \text{d} & \text{e} & \text{f} & \text{g} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\
2 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\
3 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\
4 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\
5 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 1 \\
6 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 \\
7 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
8 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
9 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
P & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
H & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
6 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
7 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
8 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
9 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{array}
\]
\[ c = y + \bar{z} + w \]
\[ d = x\bar{z} + x\bar{wy} + \bar{y}yz + \bar{z}wy + z\bar{w}y \]
\[ e = \bar{w}y + z\bar{w} + xz \]
\[ f = x + z\bar{w} + y\bar{z} + y\bar{w} \]
\[ g = x + z\bar{w} + y\bar{z} + y\bar{w} \]

\[ a = \sum m(0, 2, 3, 5, 6, 7, 8, 9, 10) + \sum d(12, 13, 14, 15) \]

Too many minterms. We better optimize: \( \bar{a} = \sum m(1, 4, 11) + \sum d(12, 13, 14, 15) \)

<table>
<thead>
<tr>
<th>Number of ones</th>
<th>4-literal implicants</th>
<th>3-literal implicants</th>
<th>2-literal implicants</th>
<th>1-literal implicants</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( m_1 = 0001 )</td>
<td>( m_4, 12 = -100 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>( m_4 = 0100 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>( m_{12} = 1100 )</td>
<td>( m_{13, 14} = 110- )</td>
<td>( m_{12, 14} = 11-0 )</td>
<td>( m_{12, 13, 14, 15} = 11-- )</td>
</tr>
<tr>
<td>3</td>
<td>( m_{11} = 1011 )</td>
<td>( m_{13, 15} = 11-1 )</td>
<td>( m_{14, 15} = 111- )</td>
<td>( m_{11, 15} = 1-11 )</td>
</tr>
<tr>
<td></td>
<td>( m_{13} = 1111 )</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\( \bar{a} = \bar{x}\bar{y}\bar{z}w + y\bar{z}\bar{w} + xzw + xy \)

<table>
<thead>
<tr>
<th>Prime Implicants</th>
<th>Minterms</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m_1 )</td>
<td>( \bar{x}\bar{y}\bar{z}w )</td>
</tr>
<tr>
<td>( m_{4, 12} )</td>
<td>( y\bar{z}\bar{w} )</td>
</tr>
<tr>
<td>( m_{11, 15} )</td>
<td>( xzw )</td>
</tr>
<tr>
<td>( m_{12, 13, 14, 15} )</td>
<td>( xy )</td>
</tr>
</tbody>
</table>

\( \bar{a} = \bar{x}\bar{y}\bar{z}w + y\bar{z}\bar{w} + xzw \Rightarrow \)

\[ a = (x + y + z + \bar{w})(\bar{y} + z + w)(\bar{x} + \bar{z} + \bar{w}) \]
\[ b = \sum m(0,1,2,3,4,7,8,9,10,11) + \sum d(12,13,14,15). \]

Too many minterms. We better optimize: \( \overline{b} = \sum m(5,6) + \sum d(12,13,14,15) \)

<table>
<thead>
<tr>
<th>Number of ones</th>
<th>4-literal implicants</th>
<th>3-literal implicants</th>
<th>2-literal implicants</th>
<th>1-literal implicants</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>( m_5 = 0101 ) ✓</td>
<td>( m_{5,13} = 101 )</td>
<td>( m_{5,13} = 110- ) ✓</td>
<td>( m_{12,13,14,15} = 11-- ) ✓</td>
</tr>
<tr>
<td></td>
<td>( m_6 = 0110 ) ✓</td>
<td>( m_{6,14} = 110 )</td>
<td>( m_{6,14} = 11-0 ) ✓</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( m_{12} = 1100 ) ✓</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>( m_{13} = 1101 ) ✓</td>
<td>( m_{3,15} = 11-1 ) ✓</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>( m_{14} = 1110 ) ✓</td>
<td>( m_{4,15} = 111- ) ✓</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>( m_{15} = 1111 ) ✓</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ \overline{b} = \overline{x} \overline{y} \overline{z} \overline{w} + y \overline{z} \overline{w} + xzw + xy \]

### Prime Implicants

<table>
<thead>
<tr>
<th>Prime Implicants</th>
<th>Minterms</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m_{5,13} )</td>
<td>( y \overline{z} \overline{w} ) ✓</td>
</tr>
<tr>
<td>( m_{6,14} )</td>
<td>( yz \overline{w} ) X</td>
</tr>
<tr>
<td>( m_{12,13,14,15} )</td>
<td>( x \overline{y} ) ✓</td>
</tr>
</tbody>
</table>

\[ \overline{b} = y \overline{z} \overline{w} + yz \overline{w} \]

\[ \Rightarrow \quad b = (\overline{y} + z + \overline{w})(\overline{y} + \overline{z} + w) \]