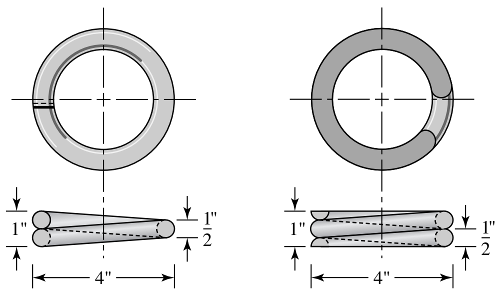
Chapter 10

10-1



10-2 $A = Sd^m$

$$\dim(A_{\text{uscu}}) = \dim(S)\dim(d^m) = \text{kpsi}\cdot\text{in}^m$$

$$\dim(A_{\text{SI}}) = \dim(S_1)\dim\left(d_1^m\right) = \text{MPa}\cdot\text{mm}^m$$

$$A_{\text{SI}} = \frac{\text{MPa}}{\text{kpsi}}\cdot\frac{\text{mm}^m}{\text{in}^m}A_{\text{uscu}} = 6.894\,757(25.40)^m A_{\text{uscu}} \doteq 6.895(25.4)^m A_{\text{uscu}} \quad \textit{Ans.}$$

For music wire, from Table 10-4:

$$A_{\text{uscu}} = 201, \quad m = 0.145; \quad \text{what is } A_{\text{SI}}?$$

$$A_{\text{SI}} = 6.89(25.4)^{0.145}(201) = 2214\ \text{MPa}\cdot\text{mm}^m \quad \textit{Ans.}$$

10-3 Given: Music wire, $d = 0.105$ in, OD = 1.225 in, plain ground ends, $N_t = 12$ coils.

Table 10-1: $N_a = N_t - 1 = 12 - 1 = 11$

$$L_s = dN_t = 0.105(12) = 1.26\text{ in}$$

Table 10-4: $A = 201, \quad m = 0.145$

(a) Eq. (10-14): $$S_{ut} = \frac{201}{(0.105)^{0.145}} = 278.7\text{ kpsi}$$

Table 10-6: $$S_{sy} = 0.45(278.7) = 125.4\text{ kpsi}$$

$$D = 1.225 - 0.105 = 1.120\text{ in}$$

$$C = \frac{D}{d} = \frac{1.120}{0.105} = 10.67$$

Eq. (10-6): $$K_B = \frac{4(10.67)+2}{4(10.67)-3} = 1.126$$

Eq. (10-3): $$F|_{S_{sy}} = \frac{\pi d^3 S_{sy}}{8K_B D} = \frac{\pi(0.105)^3(125.4)(10^3)}{8(1.126)(1.120)} = 45.2\text{ lbf}$$

Eq. (10-9): $$k = \frac{d^4 G}{8D^3 N_a} = \frac{(0.105)^4(11.75)(10^6)}{8(1.120)^3(11)} = 11.55\text{ lbf/in}$$

$$L_0 = \frac{F|_{S_{sy}}}{k} + L_s = \frac{45.2}{11.55} + 1.26 = 5.17\text{ in} \quad \textit{Ans.}$$

(b) $F|_{S_{sy}} = 45.2$ lbf *Ans.*

(c) $k = 11.55$ lbf/in *Ans.*

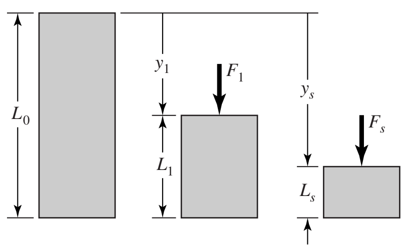
(d) $(L_0)_{cr} = \dfrac{2.63D}{\alpha} = \dfrac{2.63(1.120)}{0.5} = 5.89$ in

Many designers provide $(L_0)_{cr}/L_0 \geq 5$ or more; therefore, plain ground ends are not often used in machinery due to buckling uncertainty.

10-4 Referring to Prob. 10-3 solution, $C = 10.67$, $N_a = 11$, $S_{sy} = 125.4$ kpsi, $(L_0)_{cr} = 5.89$ in and $F = 45.2$ lbf (at yield).

Eq. (10-18): $4 \leq C \leq 12$ $\quad C = 10.67$ *O.K.*

Eq. (10-19): $3 \leq N_a \leq 15$ $\quad N_a = 11$ *O.K.*



$$L_0 = 5.17 \text{ in}, \quad L_s = 1.26 \text{ in}$$

$$y_1 = \frac{F_1}{k} = \frac{30}{11.55} = 2.60 \text{ in}$$

$$L_1 = L_0 - y_1 = 5.17 - 2.60 = 2.57 \text{ in}$$

$$\xi = \frac{y_s}{y_1} - 1 = \frac{5.17 - 1.26}{2.60} - 1 = 0.50$$

Eq. (10-20): $\xi \geq 0.15, \quad \xi = 0.50$ *O.K.*

From Eq. (10-3) for static service

$$\tau_1 = K_B\left(\frac{8F_1D}{\pi d^3}\right) = 1.126\left[\frac{8(30)(1.120)}{\pi(0.105)^3}\right] = 83\,224 \text{ psi}$$

$$n_s = \frac{S_{sy}}{\tau_1} = \frac{125.4(10^3)}{83\,224} = 1.51$$

Eq. (10-21): $n_s \geq 1.2, \quad n_s = 1.51$ *O.K.*

$$\tau_s = \tau_1\left(\frac{45.2}{30}\right) = 83\,224\left(\frac{45.2}{30}\right) = 125\,391 \text{ psi}$$

$$S_{sy}/\tau_s = 125.4(10^3)/125\,391 \doteq 1$$

$S_{sy}/\tau_s \geq (n_s)_d$: Not solid-safe. *Not O.K.*

$L_0 \leq (L_0)_{cr}$: $5.17 \leq 5.89$ Margin could be higher, *Not O.K.*

Design is unsatisfactory. Operate over a rod? *Ans.*

10-5 Static service spring with: HD steel wire, $d = 2$ mm, OD $= 22$ mm, $N_t = 8.5$ turns plain and ground ends.

Preliminaries

Table 10-5: $A = 1783 \text{ MPa} \cdot \text{mm}^m, \quad m = 0.190$

Eq. (10-14): $S_{ut} = \dfrac{1783}{(2)^{0.190}} = 1563$ MPa

Table 10-6: $S_{sy} = 0.45(1563) = 703.4$ MPa

Then,

$$D = \text{OD} - d = 22 - 2 = 20 \text{ mm}$$

$$C = 20/2 = 10$$

$$K_B = \frac{4C + 2}{4C - 3} = \frac{4(10) + 2}{4(10) - 3} = 1.135$$

$$N_a = 8.5 - 1 = 7.5 \text{ turns}$$

$$L_s = 2(8.5) = 17 \text{ mm}$$

Eq. (10-21): Use $n_s = 1.2$ for solid-safe property.

$$F_s = \frac{\pi d^3 S_{sy}/n_s}{8K_B D} = \frac{\pi(2)^3(703.4/1.2)}{8(1.135)(20)}\left[\frac{(10^{-3})^3(10^6)}{10^{-3}}\right] = 81.12 \text{ N}$$

$$k = \frac{d^4 G}{8D^3 N_a} = \frac{(2)^4(79.3)}{8(20)^3(7.5)}\left[\frac{(10^{-3})^4(10^9)}{(10^{-3})^3}\right] = 0.002\,643(10^6) = 2643 \text{ N/m}$$

$$y_s = \frac{F_s}{k} = \frac{81.12}{2643(10^{-3})} = 30.69 \text{ mm}$$

(a) $L_0 = y + L_s = 30.69 + 17 = 47.7$ mm *Ans.*

(b) Table 10-1: $p = \dfrac{L_0}{N_t} = \dfrac{47.7}{8.5} = 5.61$ mm *Ans.*

(c) $F_s = 81.12$ N (from above) *Ans.*

(d) $k = 2643$ N/m (from above) *Ans.*

(e) Table 10-2 and Eq. (10-13):

$$(L_0)_{\text{cr}} = \frac{2.63D}{\alpha} = \frac{2.63(20)}{0.5} = 105.2 \text{ mm}$$

$$(L_0)_{\text{cr}}/L_0 = 105.2/47.7 = 2.21$$

This is less than 5. Operate over a rod?

Plain and ground ends have a poor eccentric footprint. *Ans.*

10-6 Referring to Prob. 10-5 solution: $C = 10$, $N_a = 7.5$, $k = 2643$ N/m, $d = 2$ mm, $D = 20$ mm, $F_s = 81.12$ N and $N_t = 8.5$ turns.

Eq. (10-18): $4 \le C \le 12, \quad C = 10 \quad O.K.$

Eq. (10-19): $3 \le N_a \le 15, \quad N_a = 7.5 \quad O.K.$

$$y_1 = \frac{F_1}{k} = \frac{75}{2643(10^{-3})} = 28.4 \text{ mm}$$

$$(y)_{\text{for yield}} = \frac{81.12(1.2)}{2643(10^{-3})} = 36.8 \text{ mm}$$

$$y_s = \frac{81.12}{2643(10^{-3})} = 30.69 \text{ mm}$$

$$\xi = \frac{(y)_{\text{for yield}}}{y_1} - 1 = \frac{36.8}{28.4} - 1 = 0.296$$

Eq. (10-20): $\xi \ge 0.15, \quad \xi = 0.296 \quad O.K.$

Table 10-6: $S_{sy} = 0.45 S_{ut} \quad O.K.$

As-wound

$$\tau_s = K_B \left(\frac{8F_s D}{\pi d^3}\right) = 1.135 \left[\frac{8(81.12)(20)}{\pi(2)^3}\right]\left[\frac{10^{-3}}{(10^{-3})^3(10^6)}\right] = 586 \text{ MPa}$$

Eq. (10-21): $\frac{S_{sy}}{\tau_s} = \frac{703.4}{586} = 1.2 \quad O.K.$ (Basis for Prob. 10-5 solution)

Table 10-1: $L_s = N_t d = 8.5(2) = 17$ mm

$$L_0 = \frac{F_s}{k} + L_s = \frac{81.12}{2.643} + 17 = 47.7 \text{ mm}$$

$$\frac{2.63D}{\alpha} = \frac{2.63(20)}{0.5} = 105.2 \text{ mm}$$

$$\frac{(L_0)_{\text{cr}}}{L_0} = \frac{105.2}{47.7} = 2.21$$

which is less than 5. Operate over a rod? *Not O.K.*

Plain and ground ends have a poor eccentric footprint. *Ans.*

10-7 Given: A228 (music wire), SQ&GRD ends, $d = 0.006$ in, OD $= 0.036$ in, $L_0 = 0.63$ in, $N_t = 40$ turns.

Table 10-4: $A = 201 \text{ kpsi} \cdot \text{in}^m, \quad m = 0.145$

$$D = \text{OD} - d = 0.036 - 0.006 = 0.030 \text{ in}$$

$$C = D/d = 0.030/0.006 = 5$$

$$K_B = \frac{4(5) + 2}{4(5) - 3} = 1.294$$

Table 10-1: $N_a = N_t - 2 = 40 - 2 = 38$ turns

$$S_{ut} = \frac{201}{(0.006)^{0.145}} = 422.1 \text{ kpsi}$$

$$S_{sy} = 0.45(422.1) = 189.9 \text{ kpsi}$$

$$k = \frac{Gd^4}{8D^3 N_a} = \frac{12(10^6)(0.006)^4}{8(0.030)^3(38)} = 1.895 \text{ lbf/in}$$

Table 10-1: $L_s = N_t d = 40(0.006) = 0.240$ in

Now $F_s = ky_s$ where $y_s = L_0 - L_s = 0.390$ in. Thus,

$$\tau_s = K_B\left[\frac{8(ky_s)D}{\pi d^3}\right] = 1.294\left[\frac{8(1.895)(0.39)(0.030)}{\pi(0.006)^3}\right](10^{-3}) = 338.2 \text{ kpsi} \qquad (1)$$

$\tau_s > S_{sy}$, that is, $338.2 > 189.9$ kpsi; the spring is not solid-safe. Solving Eq. (1) for y_s gives

$$y'_s = \frac{(\tau_s/n_s)(\pi d^3)}{8K_B kD} = \frac{(189\,900/1.2)(\pi)(0.006)^3}{8(1.294)(1.895)(0.030)} = 0.182 \text{ in}$$

Using a design factor of 1.2,

$$L'_0 = L_s + y'_s = 0.240 + 0.182 = 0.422 \text{ in}$$

The spring should be wound to a free length of 0.422 in. *Ans.*

10-8 Given: B159 (phosphor bronze), SQ&GRD ends, $d = 0.012$ in, OD $= 0.120$ in, $L_0 = 0.81$ in, $N_t = 15.1$ turns.

Table 10-4: $A = 145 \text{ kpsi} \cdot \text{in}^m, \quad m = 0$

Table 10-5: $G = 6$ Mpsi

$$D = \text{OD} - d = 0.120 - 0.012 = 0.108 \text{ in}$$

$$C = D/d = 0.108/0.012 = 9$$

$$K_B = \frac{4(9)+2}{4(9)-3} = 1.152$$

Table 10-1: $N_a = N_t - 2 = 15.1 - 2 = 13.1$ turns

$$S_{ut} = \frac{145}{0.012^0} = 145 \text{ kpsi}$$

Table 10-6: $S_{sy} = 0.35(145) = 50.8$ kpsi

$$k = \frac{Gd^4}{8D^3 N_a} = \frac{6(10^6)(0.012)^4}{8(0.108)^3(13.1)} = 0.942 \text{ lbf/in}$$

Table 10-1: $L_s = dN_t = 0.012(15.1) = 0.181$ in

Now $F_s = ky_s$, $y_s = L_0 - L_s = 0.81 - 0.181 = 0.629$ in

$$\tau_s = K_B\left[\frac{8(ky_s)D}{\pi d^3}\right] = 1.152\left[\frac{8(0.942)(0.6)(0.108)}{\pi(0.012)^3}\right](10^{-3}) = 108.6 \text{ kpsi} \qquad (1)$$

$\tau_s > S_{sy}$, that is, $108.6 > 50.8$ kpsi; the spring is not solid safe. Solving Eq. (1) for y'_s gives

$$y'_s = \frac{(S_{sy}/n)\pi d^3}{8K_B kD} = \frac{(50.8/1.2)(\pi)(0.012)^3(10^3)}{8(1.152)(0.942)(0.108)} = 0.245 \text{ in}$$

$$L'_0 = L_s + y'_s = 0.181 + 0.245 = 0.426 \text{ in}$$

Wind the spring to a free length of 0.426 in. *Ans.*

10-9 Given: A313 (stainless steel), SQ&GRD ends, $d = 0.040$ in, OD $= 0.240$ in, $L_0 = 0.75$ in, $N_t = 10.4$ turns.

Table 10-4: $A = 169\ \text{kpsi} \cdot \text{in}^m, \quad m = 0.146$

Table 10-5: $G = 10(10^6)$ psi

$$D = \text{OD} - d = 0.240 - 0.040 = 0.200 \text{ in}$$

$$C = D/d = 0.200/0.040 = 5$$

$$K_B = \frac{4(5) + 2}{4(5) - 3} = 1.294$$

Table 10-6: $N_a = N_t - 2 = 10.4 - 2 = 8.4$ turns

$$S_{ut} = \frac{169}{(0.040)^{0.146}} = 270.4 \text{ kpsi}$$

Table 10-13: $S_{sy} = 0.35(270.4) = 94.6$ kpsi

$$k = \frac{Gd^4}{8D^3N_a} = \frac{10(10^6)(0.040)^4}{8(0.2)^3(8.4)} = 47.62 \text{ lbf/in}$$

Table 10-6: $L_s = dN_t = 0.040(10.4) = 0.416$ in

Now $F_s = ky_s$, $y_s = L_0 - L_s = 0.75 - 0.416 = 0.334$ in

$$\tau_s = K_B \left[\frac{8(ky_s)D}{\pi d^3}\right] = 1.294 \left[\frac{8(47.62)(0.334)(0.2)}{\pi(0.040)^3}\right](10^{-3}) = 163.8 \text{ kpsi} \qquad (1)$$

$\tau_s > S_{sy}$, that is, $163.8 > 94.6$ kpsi; the spring is not solid-safe. Solving Eq. (1) for y_s gives

$$y_s' = \frac{(S_{sy}/n)(\pi d^3)}{8K_B kD} = \frac{(94\,600/1.2)(\pi)(0.040)^3}{8(1.294)(47.62)(0.2)} = 0.161 \text{ in}$$

$$L_0' = L_s + y_s' = 0.416 + 0.161 = 0.577 \text{ in}$$

Wind the spring to a free length 0.577 in. *Ans.*

10-10 Given: A227 (hard drawn steel), $d = 0.135$ in, OD $= 2.0$ in, $L_0 = 2.94$ in, $N_t = 5.25$ turns.

Table 10-4: $A = 140\ \text{kpsi} \cdot \text{in}^m, \quad m = 0.190$

Table 10-5: $G = 11.4(10^6)$ psi

$$D = \text{OD} - d = 2 - 0.135 = 1.865 \text{ in}$$

$$C = D/d = 1.865/0.135 = 13.81$$

$$K_B = \frac{4(13.81) + 2}{4(13.81) - 3} = 1.096$$

$$N_a = N_t - 2 = 5.25 - 2 = 3.25 \text{ turns}$$

$$S_{ut} = \frac{140}{(0.135)^{0.190}} = 204.8 \text{ kpsi}$$

Table 10-6: $S_{sy} = 0.45(204.8) = 92.2$ kpsi

$$k = \frac{Gd^4}{8D^3N_a} = \frac{11.4(10^6)(0.135)^4}{8(1.865)^3(3.25)} = 22.45 \text{ lbf/in}$$

Table 10-1: $L_s = dN_t = 0.135(5.25) = 0.709$ in

Now $F_s = ky_s$, $y_s = L_0 - L_s = 2.94 - 0.709 = 2.231$ in

$$\tau_s = K_B\left[\frac{8(ky_s)D}{\pi d^3}\right] = 1.096\left[\frac{8(22.45)(2.231)(1.865)}{\pi(0.135)^3}\right](10^{-3}) = 106.0 \text{ kpsi} \quad (1)$$

$\tau_s > S_{sy}$, that is, $106 > 92.2$ kpsi; the spring is not solid-safe. Solving Eq. (1) for y_s gives

$$y_s' = \frac{(S_{sy}/n)(\pi d^3)}{8K_BkD} = \frac{(92\,200/1.2)(\pi)(0.135)^3}{8(1.096)(22.45)(1.865)} = 1.612 \text{ in}$$

$$L_0' = L_s + y_s' = 0.709 + 1.612 = 2.321 \text{ in}$$

Wind the spring to a free length of 2.32 in. *Ans.*

10-11 Given: A229 (OQ&T steel), SQ&GRD ends, $d = 0.144$ in, OD = 1.0 in, $L_0 = 3.75$ in, $N_t = 13$ turns.

Table 10-4: $A = 147 \text{ kpsi} \cdot \text{in}^m$, $m = 0.187$

Table 10-5: $G = 11.4(10^6)$ psi

$$D = \text{OD} - d = 1.0 - 0.144 = 0.856 \text{ in}$$

$$C = D/d = 0.856/0.144 = 5.944$$

$$K_B = \frac{4(5.944) + 2}{4(5.944) - 3} = 1.241$$

Table 10-1: $N_a = N_t - 2 = 13 - 2 = 11$ turns

$$S_{ut} = \frac{147}{(0.144)^{0.187}} = 211.2 \text{ kpsi}$$

Table 10-6: $S_{sy} = 0.50(211.2) = 105.6$ kpsi

$$k = \frac{Gd^4}{8D^3N_a} = \frac{11.4(10^6)(0.144)^4}{8(0.856)^3(11)} = 88.8 \text{ lbf/in}$$

Table 10-1: $L_s = dN_t = 0.144(13) = 1.872$ in

Now $F_s = ky_s$, $y_s = L_0 - L_s = 3.75 - 1.872 = 1.878$ in

$$\tau_s = K_B\left[\frac{8(ky_s)D}{\pi d^3}\right] = 1.241\left[\frac{8(88.8)(1.878)(0.856)}{\pi(0.144)^3}\right](10^{-3}) = 151.1 \text{ kpsi} \quad (1)$$

$\tau_s > S_{sy}$, that is, $151.1 > 105.6$ kpsi; the spring is not solid-safe. Solving Eq. (1) for y_s gives

$$y_s' = \frac{(S_{sy}/n)(\pi d^3)}{8K_BkD} = \frac{(105\,600/1.2)(\pi)(0.144)^3}{8(1.241)(88.8)(0.856)} = 1.094 \text{ in}$$

$$L_0' = L_s + y_s' = 1.878 + 1.094 = 2.972 \text{ in}$$

Wind the spring to a free length 2.972 in. *Ans.*

10-12 Given: A232 (Cr-V steel), SQ&GRD ends, $d = 0.192$ in, OD = 3 in, $L_0 = 9$ in, $N_t =$ 8 turns.

Table 10-4: $A = 169 \text{ kpsi} \cdot \text{in}^m, \quad m = 0.168$

Table 10-5: $G = 11.2(10^6)$ psi

$$D = \text{OD} - d = 3 - 0.192 = 2.808 \text{ in}$$

$$C = D/d = 2.808/0.192 = 14.625 \quad \text{(large)}$$

$$K_B = \frac{4(14.625) + 2}{4(14.625) - 3} = 1.090$$

Table 10-1: $N_a = N_t - 2 = 8 - 2 = 6$ turns

$$S_{ut} = \frac{169}{(0.192)^{0.168}} = 223.0 \text{ kpsi}$$

Table 10-6: $S_{sy} = 0.50(223.0) = 111.5$ kpsi

$$k = \frac{Gd^4}{8D^3N_a} = \frac{11.2(10^6)(0.192)^4}{8(2.808)^3(6)} = 14.32 \text{ lbf/in}$$

Table 10-1: $L_s = dN_t = 0.192(8) = 1.536$ in

Now $F_s = ky_s,\ y_s = L_0 - L_s = 9 - 1.536 = 7.464$ in

$$\tau_s = K_B\left[\frac{8(ky_s)D}{\pi d^3}\right] = 1.090\left[\frac{8(14.32)(7.464)(2.808)}{\pi(0.192)^3}\right](10^{-3}) = 117.7 \text{ kpsi} \qquad (1)$$

$\tau_s > S_{sy}$, that is, $117.7 > 111.5$ kpsi; the spring is not solid safe. Solving Eq. (1) for y_s gives

$$y_s' = \frac{(S_{sy}/n)(\pi d^3)}{8K_BkD} = \frac{(111\,500/1.2)(\pi)(0.192)^3}{8(1.090)(14.32)(2.808)} = 5.892 \text{ in}$$

$$L_0' = L_s + y_s' = 1.536 + 5.892 = 7.428 \text{ in}$$

Wind the spring to a free length of 7.428 in. *Ans.*

10-13 Given: A313 (stainless steel) SQ&GRD ends, $d = 0.2$ mm, OD = 0.91 mm, $L_0 =$ 15.9 mm, $N_t = 40$ turns.

Table 10-4: $A = 1867 \text{ MPa} \cdot \text{mm}^m, \quad m = 0.146$

Table 10-5: $G = 69.0$ GPa

$$D = \text{OD} - d = 0.91 - 0.2 = 0.71 \text{ mm}$$

$$C = D/d = 0.71/0.2 = 3.55 \quad \text{(small)}$$

$$K_B = \frac{4(3.55) + 2}{4(3.55) - 3} = 1.446$$

$$N_a = N_t - 2 = 40 - 2 = 38 \text{ turns}$$

$$S_{ut} = \frac{1867}{(0.2)^{0.146}} = 2361.5 \text{ MPa}$$

Table 10-6:

$$S_{sy} = 0.35(2361.5) = 826.5 \text{ MPa}$$

$$k = \frac{d^4 G}{8D^3 N_a} = \frac{(0.2)^4(69.0)}{8(0.71)^3(38)}\left[\frac{(10^{-3})^4(10^9)}{(10^{-3})^3}\right]$$

$$= 1.0147(10^{-3})(10^6) = 1014.7 \text{ N/m} \quad \text{or} \quad 1.0147 \text{ N/mm}$$

$$L_s = dN_t = 0.2(40) = 8 \text{ mm}$$

$$F_s = ky_s$$

$$y_s = L_0 - L_s = 15.9 - 8 = 7.9$$

$$\tau_s = K_B\left[\frac{8(ky_s)D}{\pi d^3}\right] = 1.446\left[\frac{8(1.0147)(7.9)(0.71)}{\pi(0.2)^3}\right]\left[\frac{10^{-3}(10^{-3})(10^{-3})}{(10^{-3})^3}\right]$$

$$= 2620(1) = 2620 \text{ MPa} \qquad (1)$$

$\tau_s > S_{sy}$, that is, $2620 > 826.5$ MPa; the spring is not solid safe. Solve Eq. (1) for y_s giving

$$y'_s = \frac{(S_{sy}/n)(\pi d^3)}{8K_B kD} = \frac{(826.5/1.2)(\pi)(0.2)^3}{8(1.446)(1.0147)(0.71)} = 2.08 \text{ mm}$$

$$L'_0 = L_s + y'_s = 8.0 + 2.08 = 10.08 \text{ mm}$$

Wind the spring to a free length of 10.08 mm. This only addresses the solid-safe criteria. There are additional problems. *Ans.*

10-14 Given: A228 (music wire), SQ&GRD ends, $d = 1$ mm, OD $= 6.10$ mm, $L_0 = 19.1$ mm, $N_t = 10.4$ turns.

Table 10-4: $A = 2211 \text{ MPa} \cdot \text{mm}^m$, $m = 0.145$

Table 10-5: $G = 81.7$ GPa

$$D = \text{OD} - d = 6.10 - 1 = 5.1 \text{ mm}$$

$$C = D/d = 5.1/1 = 5.1$$

$$N_a = N_t - 2 = 10.4 - 2 = 8.4 \text{ turns}$$

$$K_B = \frac{4(5.1) + 2}{4(5.1) - 3} = 1.287$$

$$S_{ut} = \frac{2211}{(1)^{0.145}} = 2211 \text{ MPa}$$

Table 10-6: $S_{sy} = 0.45(2211) = 995$ MPa

$$k = \frac{d^4 G}{8D^3 N_a} = \frac{(1)^4(81.7)}{8(5.1)^3(8.4)}\left[\frac{(10^{-3})^4(10^9)}{(10^{-3})^3}\right] = 0.009\,165(10^6)$$

$$= 9165 \text{ N/m} \quad \text{or} \quad 9.165 \text{ N/mm}$$

$$L_s = dN_t = 1(10.4) = 10.4 \text{ mm}$$

$$F_s = ky_s$$

$$y_s = L_0 - L_s = 19.1 - 10.4 = 8.7 \text{ mm}$$

$$\tau_s = K_B\left[\frac{8(ky_s)D}{\pi d^3}\right] = 1.287\left[\frac{8(9.165)(8.7)(5.1)}{\pi(1)^3}\right] = 1333 \text{ MPa} \qquad (1)$$

$\tau_s > S_{sy}$, that is, $1333 > 995$ MPa; the spring is not solid safe. Solve Eq. (1) for y_s giving

$$y_s' = \frac{(S_{sy}/n)(\pi d^3)}{8K_B kD} = \frac{(995/1.2)(\pi)(1)^3}{8(1.287)(9.165)(5.1)} = 5.43 \text{ mm}$$

$$L_0' = L_s + y_s' = 10.4 + 5.43 = 15.83 \text{ mm}$$

Wind the spring to a free length of 15.83 mm. *Ans.*

10-15 Given: A229 (OQ&T spring steel), SQ&GRD ends, $d = 3.4$ mm, OD $= 50.8$ mm, $L_0 = 74.6$ mm, $N_t = 5.25$.

Table 10-4: $A = 1855 \text{ MPa}\cdot\text{mm}^m, \quad m = 0.187$

Table 10-5: $G = 77.2$ GPa

$$D = \text{OD} - d = 50.8 - 3.4 = 47.4 \text{ mm}$$

$$C = D/d = 47.4/3.4 = 13.94 \quad \text{(large)}$$

$$N_a = N_t - 2 = 5.25 - 2 = 3.25 \text{ turns}$$

$$K_B = \frac{4(13.94) + 2}{4(13.94) - 3} = 1.095$$

$$S_{ut} = \frac{1855}{(3.4)^{0.187}} = 1476 \text{ MPa}$$

Table 10-6: $S_{sy} = 0.50(1476) = 737.8$ MPa

$$k = \frac{d^4 G}{8D^3 N_a} = \frac{(3.4)^4(77.2)}{8(47.4)^3(3.25)}\left[\frac{(10^{-3})^4(10^9)}{(10^{-3})^3}\right] = 0.003\,75(10^6)$$

$$= 3750 \text{ N/m} \quad \text{or} \quad 3.750 \text{ N/mm}$$

$$L_s = dN_t = 3.4(5.25) = 17.85$$

$$F_s = ky_s$$

$$y_s = L_0 - L_s = 74.6 - 17.85 = 56.75 \text{ mm}$$

$$\tau_s = K_B\left[\frac{8(ky_s)D}{\pi d^3}\right]$$

$$= 1.095\left[\frac{8(3.750)(56.75)(47.4)}{\pi(3.4)^3}\right] = 720.2 \text{ MPa} \qquad (1)$$

$\tau_s < S_{sy}$, that is, $720.2 < 737.8$ MPa

$\therefore$ The spring is solid safe. With $n_s = 1.2$,

$$y'_s = \frac{(S_{sy}/n)(\pi d^3)}{8K_B k D} = \frac{(737.8/1.2)(\pi)(3.4)^3}{8(1.095)(3.75)(47.4)} = 48.76 \text{ mm}$$

$$L'_0 = L_s + y'_s = 17.85 + 48.76 = 66.61 \text{ mm}$$

Wind the spring to a free length of 66.61 mm. *Ans.*

10-16 Given: B159 (phosphor bronze), SQ&GRD ends, $d = 3.7$ mm, OD $= 25.4$ mm, $L_0 = 95.3$ mm, $N_t = 13$ turns.

Table 10-4: $A = 932 \text{ MPa} \cdot \text{mm}^m, \quad m = 0.064$

Table 10-5: $G = 41.4$ GPa

$$D = \text{OD} - d = 25.4 - 3.7 = 21.7 \text{ mm}$$

$$C = D/d = 21.7/3.7 = 5.865$$

$$K_B = \frac{4(5.865) + 2}{4(5.865) - 3} = 1.244$$

$$N_a = N_t - 2 = 13 - 2 = 11 \text{ turns}$$

$$S_{ut} = \frac{932}{(3.7)^{0.064}} = 857.1 \text{ MPa}$$

Table 10-6: $S_{sy} = 0.35(857.1) = 300$ MPa

$$k = \frac{d^4 G}{8D^3 N_a} = \frac{(3.7)^4(41.4)}{8(21.7)^3(11)}\left[\frac{(10^{-3})^4(10^9)}{(10^{-3})^3}\right] = 0.008\,629(10^6)$$

$$= 8629 \text{ N/m} \quad \text{or} \quad 8.629 \text{ N/mm}$$

$$L_s = dN_t = 3.7(13) = 48.1 \text{ mm}$$

$$F_s = k y_s$$

$$y_s = L_0 - L_s = 95.3 - 48.1 = 47.2 \text{ mm}$$

$$\tau_s = K_B\left[\frac{8(k y_s) D}{\pi d^3}\right]$$

$$= 1.244\left[\frac{8(8.629)(47.2)(21.7)}{\pi(3.7)^3}\right] = 553 \text{ MPa} \qquad (1)$$

$\tau_s > S_{sy}$, that is, $553 > 300$ MPa; the spring is not solid-safe. Solving Eq. (1) for y_s gives

$$y'_s = \frac{(S_{sy}/n)(\pi d^3)}{8K_B k D} = \frac{(300/1.2)(\pi)(3.7)^3}{8(1.244)(8.629)(21.7)} = 21.35 \text{ mm}$$

$$L'_0 = L_s + y'_s = 48.1 + 21.35 = 69.45 \text{ mm}$$

Wind the spring to a free length of 69.45 mm. *Ans.*

10-17 Given: A232 (Cr-V steel), SQ&GRD ends, $d = 4.3$ mm, OD $= 76.2$ mm, $L_0 = 228.6$ mm, $N_t = 8$ turns.

Table 10-4: $A = 2005\ \text{MPa}\cdot\text{mm}^m,\quad m = 0.168$

Table 10-5: $G = 77.2$ GPa

$$D = \text{OD} - d = 76.2 - 4.3 = 71.9 \text{ mm}$$

$$C = D/d = 71.9/4.3 = 16.72 \quad \text{(large)}$$

$$K_B = \frac{4(16.72) + 2}{4(16.72) - 3} = 1.078$$

$$N_a = N_t - 2 = 8 - 2 = 6 \text{ turns}$$

$$S_{ut} = \frac{2005}{(4.3)^{0.168}} = 1569 \text{ MPa}$$

Table 10-6:

$$S_{sy} = 0.50(1569) = 784.5 \text{ MPa}$$

$$k = \frac{d^4 G}{8D^3 N_a} = \frac{(4.3)^4(77.2)}{8(71.9)^3(6)}\left[\frac{(10^{-3})^4(10^9)}{(10^{-3})^3}\right] = 0.001\,479(10^6)$$

$$= 1479 \text{ N/m} \quad \text{or} \quad 1.479 \text{ N/mm}$$

$$L_s = dN_t = 4.3(8) = 34.4 \text{ mm}$$

$$F_s = ky_s$$

$$y_s = L_0 - L_s = 228.6 - 34.4 = 194.2 \text{ mm}$$

$$\tau_s = K_B\left[\frac{8(ky_s)D}{\pi d^3}\right] = 1.078\left[\frac{8(1.479)(194.2)(71.9)}{\pi(4.3)^3}\right] = 713.0 \text{ MPa} \qquad (1)$$

$\tau_s < S_{sy}$, that is, $713.0 < 784.5$; the spring is solid safe. With $n_s = 1.2$

Eq. (1) becomes

$$y_s' = \frac{(S_{sy}/n)(\pi d^3)}{8K_B kD} = \frac{(784.5/1.2)(\pi)(4.3)^3}{8(1.078)(1.479)(71.9)} = 178.1 \text{ mm}$$

$$L_0' = L_s + y_s' = 34.4 + 178.1 = 212.5 \text{ mm}$$

Wind the spring to a free length of $L_0' = 212.5$ mm. *Ans.*

10-18 For the wire diameter analyzed, $G = 11.75$ Mpsi per Table 10-5. Use squared and ground ends. The following is a spread-sheet study using Fig. 10-3 for parts (*a*) and (*b*). For N_a, $k = 20/2 = 10$ lbf/in.

(a) Spring over a Rod					(b) Spring in a Hole				
Source		Parameter Values			Source		Parameter Values		
	d	0.075	0.08	0.085		d	0.075	0.08	0.085
	D	0.875	0.88	0.885		D	0.875	0.870	0.865
	ID	0.800	0.800	0.800		ID	0.800	0.790	0.780
	OD	0.950	0.960	0.970		OD	0.950	0.950	0.950
Eq. (10-2)	C	11.667	11.000	10.412	Eq. (10-2)	C	11.667	10.875	10.176
Eq. (10-9)	N_a	6.937	8.828	11.061	Eq. (10-9)	N_a	6.937	9.136	11.846
Table 10-1	N_t	8.937	10.828	13.061	Table 10-1	N_t	8.937	11.136	13.846
Table 10-1	L_s	0.670	0.866	1.110	Table 10-1	L_s	0.670	0.891	1.177
$1.15y + L_s$	L_0	2.970	3.166	3.410	$1.15y + L_s$	L_0	2.970	3.191	3.477
Eq. (10-13)	$(L_0)_{cr}$	4.603	4.629	4.655	Eq. (10-13)	$(L_0)_{cr}$	4.603	4.576	4.550
Table 10-4	A	201.000	201.000	201.000	Table 10-4	A	201.000	201.000	201.000
Table 10-4	m	0.145	0.145	0.145	Table 10-4	m	0.145	0.145	0.145
Eq. (10-14)	S_{ut}	292.626	289.900	287.363	Eq. (10-14)	S_{ut}	292.626	289.900	287.363
Table 10-6	S_{sy}	131.681	130.455	129.313	Table 10-6	S_{sy}	131.681	130.455	129.313
Eq. (10-6)	K_B	1.115	1.122	1.129	Eq. (10-6)	K_B	1.115	1.123	1.133
Eq. (10-3)	n_s	0.973	1.155	1.357	Eq. (10-3)	n_s	0.973	1.167	1.384
Eq. (10-22)	fom	−0.282	−0.391	−0.536	Eq. (10-22)	fom	−0.282	−0.398	−0.555

For $n_s \geq 1.2$, the optimal size is $d = 0.085$ in for both cases.

10-19 From the figure: $L_0 = 120$ mm, OD = 50 mm, and $d = 3.4$ mm. Thus

$$D = \text{OD} - d = 50 - 3.4 = 46.6 \text{ mm}$$

(a) By counting, $N_t = 12.5$ turns. Since the ends are squared along 1/4 turn on each end,

$$N_a = 12.5 - 0.5 = 12 \text{ turns} \quad \textit{Ans.}$$

$$p = 120/12 = 10 \text{ mm} \quad \textit{Ans.}$$

The solid stack is 13 diameters across the top and 12 across the bottom.

$$L_s = 13(3.4) = 44.2 \text{ mm} \quad \textit{Ans.}$$

(b) $d = 3.4/25.4 = 0.1339$ in and from Table 10-5, $G = 78.6$ GPa

$$k = \frac{d^4 G}{8D^3 N_a} = \frac{(3.4)^4(78.6)(10^9)}{8(46.6)^3(12)}(10^{-3}) = 1080 \text{ N/m} \quad \textit{Ans.}$$

(c) $F_s = k(L_0 - L_s) = 1080(120 - 44.2)(10^{-3}) = 81.9$ N *Ans.*

(d) $C = D/d = 46.6/3.4 = 13.71$

$$K_B = \frac{4(13.71) + 2}{4(13.71) - 3} = 1.096$$

$$\tau_s = \frac{8K_B F_s D}{\pi d^3} = \frac{8(1.096)(81.9)(46.6)}{\pi(3.4)^3} = 271 \text{ MPa} \quad \textit{Ans.}$$

10-20 One approach is to select A227-47 HD steel for its low cost. Then, for $y_1 \leq 3/8$ at $F_1 = 10$ lbf, $k \geq 10/0.375 = 26.67$ lbf/in. Try $d = 0.080$ in #14 gauge

For a clearance of 0.05 in: ID = (7/16) + 0.05 = 0.4875 in; OD = 0.4875 + 0.16 = 0.6475 in

$$D = 0.4875 + 0.080 = 0.5675 \text{ in}$$

$$C = 0.5675/0.08 = 7.094$$

$$G = 11.5 \text{ Mpsi}$$

$$N_a = \frac{d^4 G}{8kD^3} = \frac{(0.08)^4(11.5)(10^6)}{8(26.67)(0.5675)^3} = 12.0 \text{ turns}$$

$$N_t = 12 + 2 = 14 \text{ turns}, \quad L_s = dN_t = 0.08(14) = 1.12 \text{ in} \quad O.K.$$

$$L_0 = 1.875 \text{ in}, \quad y_s = 1.875 - 1.12 = 0.755 \text{ in}$$

$$F_s = ky_s = 26.67(0.755) = 20.14 \text{ lbf}$$

$$K_B = \frac{4(7.094) + 2}{4(7.094) - 3} = 1.197$$

$$\tau_s = K_B \left(\frac{8F_s D}{\pi d^3}\right) = 1.197 \left[\frac{8(20.14)(0.5675)}{\pi(0.08)^3}\right] = 68\,046 \text{ psi}$$

Table 10-4: $A = 140 \text{ kpsi} \cdot \text{in}^m, \quad m = 0.190$

$$S_{sy} = 0.45 \frac{140}{(0.080)^{0.190}} = 101.8 \text{ kpsi}$$

$$n = \frac{101.8}{68.05} = 1.50 > 1.2 \quad O.K.$$

$$\tau_1 = \frac{F_1}{F_s}\tau_s = \frac{10}{20.14}(68.05) = 33.79 \text{ kpsi},$$

$$n_1 = \frac{101.8}{33.79} = 3.01 > 1.5 \quad O.K.$$

There is much latitude for reducing the amount of material. Iterate on y_1 using a spread sheet. The final results are: $y_1 = 0.32$ in, $k = 31.25$ lbf/in, $N_a = 10.3$ turns, $N_t = 12.3$ turns, $L_s = 0.985$ in, $L_0 = 1.820$ in, $y_s = 0.835$ in, $F_s = 26.1$ lbf, $K_B = 1.197$, $\tau_s = 88\,190$ kpsi, $n_s = 1.15$, and $n_1 = 3.01$.

$$\text{ID} = 0.4875 \text{ in}, \quad \text{OD} = 0.6475 \text{ in}, \quad d = 0.080 \text{ in}$$

Try other sizes and/or materials.

10-21 A stock spring catalog may have over two hundred pages of compression springs with up to 80 springs per page listed.

- Students should be aware that such catalogs exist.
- Many springs are selected from catalogs rather than designed.
- The wire size you want may not be listed.
- Catalogs may also be available on disk or the web through search routines. For example, disks are available from Century Spring at

  1 − (800) − 237 − 5225

  www.centuryspring.com
- It is better to familiarize yourself with vendor resources rather than invent them yourself.
- Sample catalog pages can be given to students for study.

10-22 For a coil radius given by:

$$R = R_1 + \frac{R_2 - R_1}{2\pi N}\theta$$

The torsion of a section is $T = PR$ where $dL = R\,d\theta$

$$\delta_p = \frac{\partial U}{\partial P} = \frac{1}{GJ}\int T\frac{\partial T}{\partial P}\,dL = \frac{1}{GJ}\int_0^{2\pi N} PR^3\,d\theta$$

$$= \frac{P}{GJ}\int_0^{2\pi N}\left(R_1 + \frac{R_2 - R_1}{2\pi N}\theta\right)^3 d\theta$$

$$= \frac{P}{GJ}\left(\frac{1}{4}\right)\left(\frac{2\pi N}{R_2 - R_1}\right)\left[\left(R_1 + \frac{R_2 - R_1}{2\pi N}\theta\right)^4\right]\Bigg|_0^{2\pi N}$$

$$= \frac{\pi PN}{2GJ(R_2 - R_1)}\left(R_2^4 - R_1^4\right) = \frac{\pi PN}{2GJ}(R_1 + R_2)\left(R_1^2 + R_2^2\right)$$

$$J = \frac{\pi}{32}d^4 \qquad \therefore \delta_p = \frac{16PN}{Gd^4}(R_1 + R_2)\left(R_1^2 + R_2^2\right)$$

$$k = \frac{P}{\delta_p} = \frac{d^4 G}{16N(R_1 + R_2)\left(R_1^2 + R_2^2\right)} \quad \textit{Ans.}$$

10-23 For a food service machinery application select A313 Stainless wire.

$$G = 10(10^6)\text{ psi}$$

Note that for $0.013 \le d \le 0.10$ in $\quad A = 169, \quad m = 0.146$

$\qquad\qquad\qquad 0.10 < d \le 0.20$ in $\quad A = 128, \quad m = 0.263$

$$F_a = \frac{18 - 4}{2} = 7\text{ lbf}, \quad F_m = \frac{18 + 4}{2} = 11\text{ lbf}, \quad r = 7/11$$

Try $\quad d = 0.080$ in, $\quad S_{ut} = \dfrac{169}{(0.08)^{0.146}} = 244.4$ kpsi

$$S_{su} = 0.67S_{ut} = 163.7\text{ kpsi}, \qquad S_{sy} = 0.35S_{ut} = 85.5\text{ kpsi}$$

Try unpeened using Zimmerli's endurance data: $S_{sa} = 35$ kpsi, $S_{sm} = 55$ kpsi

Gerber: $\quad S_{se} = \dfrac{S_{sa}}{1 - (S_{sm}/S_{su})^2} = \dfrac{35}{1 - (55/163.7)^2} = 39.5$ kpsi

$$S_{sa} = \frac{(7/11)^2(163.7)^2}{2(39.5)}\left\{-1 + \sqrt{1 + \left[\frac{2(39.5)}{(7/11)(163.7)}\right]^2}\right\} = 35.0\text{ kpsi}$$

$$\alpha = S_{sa}/n_f = 35.0/1.5 = 23.3\text{ kpsi}$$

$$\beta = \frac{8F_a}{\pi d^2}(10^{-3}) = \left[\frac{8(7)}{\pi(0.08^2)}\right](10^{-3}) = 2.785\text{ kpsi}$$

$$C = \frac{2(23.3) - 2.785}{4(2.785)} + \sqrt{\left[\frac{2(23.3) - 2.785}{4(2.785)}\right]^2 - \frac{3(23.3)}{4(2.785)}} = 6.97$$

$$D = Cd = 6.97(0.08) = 0.558\text{ in}$$

$$K_B = \frac{4(6.97)+2}{4(6.97)-3} = 1.201$$

$$\tau_a = K_B\left(\frac{8F_aD}{\pi d^3}\right) = 1.201\left[\frac{8(7)(0.558)}{\pi(0.08^3)}(10^{-3})\right] = 23.3 \text{ kpsi}$$

$$n_f = 35/23.3 = 1.50 \quad \text{checks}$$

$$N_a = \frac{Gd^4}{8kD^3} = \frac{10(10^6)(0.08)^4}{8(9.5)(0.558)^3} = 31.02 \text{ turns}$$

$$N_t = 31+2 = 33 \text{ turns}, \qquad L_s = dN_t = 0.08(33) = 2.64 \text{ in}$$

$$y_{\max} = F_{\max}/k = 18/9.5 = 1.895 \text{ in},$$

$$y_s = (1+\xi)y_{\max} = (1+0.15)(1.895) = 2.179 \text{ in}$$

$$L_0 = 2.64 + 2.179 = 4.819 \text{ in}$$

$$(L_0)_{\text{cr}} = 2.63\frac{D}{\alpha} = \frac{2.63(0.558)}{0.5} = 2.935 \text{ in}$$

$$\tau_s = 1.15(18/7)\tau_a = 1.15(18/7)(23.3) = 68.9 \text{ kpsi}$$

$$n_s = S_{sy}/\tau_s = 85.5/68.9 = 1.24$$

$$f = \sqrt{\frac{kg}{\pi^2 d^2 D N_a \gamma}} = \sqrt{\frac{9.5(386)}{\pi^2(0.08^2)(0.558)(31.02)(0.283)}} = 109 \text{ Hz}$$

These steps are easily implemented on a spreadsheet, as shown below, for different diameters.

	d_1	d_2	d_3	d_4
d	0.080	0.0915	0.1055	0.1205
m	0.146	0.146	0.263	0.263
A	169.000	169.000	128.000	128.000
S_{ut}	244.363	239.618	231.257	223.311
S_{su}	163.723	160.544	154.942	149.618
S_{sy}	85.527	83.866	80.940	78.159
S_{se}	39.452	39.654	40.046	40.469
S_{sa}	35.000	35.000	35.000	35.000
α	23.333	23.333	23.333	23.333
β	2.785	2.129	1.602	1.228
C	6.977	9.603	13.244	17.702
D	0.558	0.879	1.397	2.133
K_B	1.201	1.141	1.100	1.074
τ_a	23.333	23.333	23.333	23.333
n_f	1.500	1.500	1.500	1.500
N_a	30.993	13.594	5.975	2.858
N_t	32.993	15.594	7.975	4.858
L_s	2.639	1.427	0.841	0.585
y_s	2.179	2.179	2.179	2.179
L_0	4.818	3.606	3.020	2.764
$(L_0)_{\text{cr}}$	2.936	4.622	7.350	11.220
τ_s	69.000	69.000	69.000	69.000
n_s	1.240	1.215	1.173	1.133
f (Hz)	108.895	114.578	118.863	121.775

The shaded areas depict conditions outside the recommended design conditions. Thus, one spring is satisfactory–A313, as wound, unpeened, squared and ground,

$$d = 0.0915 \text{ in}, \quad \text{OD} = 0.879 + 0.092 = 0.971 \text{ in}, \quad N_t = 15.59 \text{ turns}$$

10-24 The steps are the same as in Prob. 10-23 except that the Gerber-Zimmerli criterion is replaced with Goodman-Zimmerli:

$$S_{se} = \frac{S_{sa}}{1 - (S_{sm}/S_{su})}$$

The problem then proceeds as in Prob. 10-23. The results for the wire sizes are shown below (see solution to Prob. 10-23 for additional details).

Iteration of d for the first trial

	d_1	d_2	d_3	d_4		d_1	d_2	d_3	d_4
d	0.080	0.0915	0.1055	0.1205	d	0.080	0.0915	0.1055	0.1205
m	0.146	0.146	0.263	0.263	K_B	1.151	1.108	1.078	1.058
A	169.000	169.000	128.000	128.000	τ_a	29.008	29.040	29.090	29.127
S_{ut}	244.363	239.618	231.257	223.311	n_f	1.500	1.500	1.500	1.500
S_{su}	163.723	160.544	154.942	149.618	N_a	14.191	6.456	2.899	1.404
S_{sy}	85.527	83.866	80.940	78.159	N_t	16.191	8.456	4.899	3.404
S_{se}	52.706	53.239	54.261	55.345	L_s	1.295	0.774	0.517	0.410
S_{sa}	43.513	43.560	43.634	43.691	y_s	2.179	2.179	2.179	2.179
α	29.008	29.040	29.090	29.127	L_0	3.474	2.953	2.696	2.589
β	2.785	2.129	1.602	1.228	$(L_0)_{cr}$	3.809	5.924	9.354	14.219
C	9.052	12.309	16.856	22.433	τ_s	85.782	85.876	86.022	86.133
D	0.724	1.126	1.778	2.703	n_s	0.997	0.977	0.941	0.907
					f (Hz)	141.284	146.853	151.271	154.326

Without checking all of the design conditions, it is obvious that none of the wire sizes satisfy $n_s \geq 1.2$. Also, the Gerber line is closer to the yield line than the Goodman. Setting $n_f = 1.5$ for Goodman makes it impossible to reach the yield line $(n_s < 1)$. The table below uses $n_f = 2$.

Iteration of d for the second trial

	d_1	d_2	d_3	d_4		d_1	d_2	d_3	d_4
d	0.080	0.0915	0.1055	0.1205	d	0.080	0.0915	0.1055	0.1205
m	0.146	0.146	0.263	0.263	K_B	1.221	1.154	1.108	1.079
A	169.000	169.000	128.000	128.000	τ_a	21.756	21.780	21.817	21.845
S_{ut}	244.363	239.618	231.257	223.311	n_f	2.000	2.000	2.000	2.000
S_{su}	163.723	160.544	154.942	149.618	N_a	40.243	17.286	7.475	3.539
S_{sy}	85.527	83.866	80.940	78.159	N_t	42.243	19.286	9.475	5.539
S_{se}	52.706	53.239	54.261	55.345	L_s	3.379	1.765	1.000	0.667
S_{sa}	43.513	43.560	43.634	43.691	y_s	2.179	2.179	2.179	2.179
α	21.756	21.780	21.817	21.845	L_0	5.558	3.944	3.179	2.846
β	2.785	2.129	1.602	1.228	$(L_0)_{cr}$	2.691	4.266	6.821	10.449
C	6.395	8.864	12.292	16.485	τ_s	64.336	64.407	64.517	64.600
D	0.512	0.811	1.297	1.986	n_s	1.329	1.302	1.255	1.210
					f (Hz)	99.816	105.759	110.312	113.408

The satisfactory spring has design specifications of: A313, as wound, unpeened, squared and ground, $d = 0.0915$ in, OD $= 0.811 + 0.092 = 0.903$ in, $N_t = 19.3$ turns.

10-25 This is the same as Prob. 10-23 since $S_{se} = S_{sa} = 35$ kpsi. Therefore, design the spring using: A313, as wound, un-peened, squared and ground, $d = 0.915$ in, OD $= 0.971$ in, $N_t = 15.59$ turns.

10-26 For the Gerber fatigue-failure criterion, $S_{su} = 0.67 S_{ut}$,

$$S_{se} = \frac{S_{sa}}{1 - (S_{sm}/S_{su})^2}, \qquad S_{sa} = \frac{r^2 S_{su}^2}{2S_{se}}\left[-1 + \sqrt{1 + \left(\frac{2S_{se}}{r S_{su}}\right)^2}\right]$$

The equation for S_{sa} is the basic difference. The last 2 columns of diameters of Ex. 10-5 are presented below with additional calculations.

	$d = 0.105$	$d = 0.112$		$d = 0.105$	$d = 0.112$
S_{ut}	278.691	276.096	N_a	8.915	6.190
S_{su}	186.723	184.984	L_s	1.146	0.917
S_{se}	38.325	38.394	L_0	3.446	3.217
S_{sy}	125.411	124.243	$(L_0)_{cr}$	6.630	8.160
S_{sa}	34.658	34.652	K_B	1.111	1.095
α	23.105	23.101	τ_a	23.105	23.101
β	1.732	1.523	n_f	1.500	1.500
C	12.004	13.851	τ_s	70.855	70.844
D	1.260	1.551	n_s	1.770	1.754
ID	1.155	1.439	f_n	105.433	106.922
OD	1.365	1.663	fom	−0.973	−1.022

There are only slight changes in the results.

10-27 As in Prob. 10-26, the basic change is S_{sa}.

For Goodman,
$$S_{se} = \frac{S_{sa}}{1 - (S_{sm}/S_{su})}$$

Recalculate S_{sa} with
$$S_{sa} = \frac{r S_{se} S_{su}}{r S_{su} + S_{se}}$$

Calculations for the last 2 diameters of Ex. 10-5 are given below.

	$d = 0.105$	$d = 0.112$		$d = 0.105$	$d = 0.112$
S_{ut}	278.691	276.096	N_a	9.153	6.353
S_{su}	186.723	184.984	L_s	1.171	0.936
S_{se}	49.614	49.810	L_0	3.471	3.236
S_{sy}	125.411	124.243	$(L_0)_{cr}$	6.572	8.090
S_{sa}	34.386	34.380	K_B	1.112	1.096
α	22.924	22.920	τ_a	22.924	22.920
β	1.732	1.523	n_f	1.500	1.500
C	11.899	13.732	τ_s	70.301	70.289
D	1.249	1.538	n_s	1.784	1.768
ID	1.144	1.426	f_n	104.509	106.000
OD	1.354	1.650	fom	−0.986	−1.034

There are only slight differences in the results.

10-28 Use: $E = 28.6$ Mpsi, $G = 11.5$ Mpsi, $A = 140$ kpsi $\cdot$ in^m, $m = 0.190$, rel cost = 1.

Try $$d = 0.067 \text{ in}, \quad S_{ut} = \frac{140}{(0.067)^{0.190}} = 234.0 \text{ kpsi}$$

Table 10-6: $$S_{sy} = 0.45 S_{ut} = 105.3 \text{ kpsi}$$

Table 10-7: $$S_y = 0.75 S_{ut} = 175.5 \text{ kpsi}$$

Eq. (10-34) with $D/d = C$ and $C_1 = C$

$$\sigma_A = \frac{F_{max}}{\pi d^2}[(K)_A(16C) + 4] = \frac{S_y}{n_y}$$

$$\frac{4C^2 - C - 1}{4C(C-1)}(16C) + 4 = \frac{\pi d^2 S_y}{n_y F_{max}}$$

$$4C^2 - C - 1 = (C-1)\left(\frac{\pi d^2 S_y}{4 n_y F_{max}} - 1\right)$$

$$C^2 - \frac{1}{4}\left(1 + \frac{\pi d^2 S_y}{4 n_y F_{max}} - 1\right) C + \frac{1}{4}\left(\frac{\pi d^2 S_y}{4 n_y F_{max}} - 2\right) = 0$$

$$C = \frac{1}{2}\left[\frac{\pi d^2 S_y}{16 n_y F_{max}} \pm \sqrt{\left(\frac{\pi d^2 S_y}{16 n_y F_{max}}\right)^2 - \frac{\pi d^2 S_y}{4 n_y F_{max}} + 2}\right] \quad \text{take positive root}$$

$$= \frac{1}{2}\left\{\frac{\pi(0.067^2)(175.5)(10^3)}{16(1.5)(18)} + \sqrt{\left[\frac{\pi(0.067)^2(175.5)(10^3)}{16(1.5)(18)}\right]^2 - \frac{\pi(0.067)^2(175.5)(10^3)}{4(1.5)(18)} + 2}\right\} = 4.590$$

$$D = Cd = 0.3075 \text{ in}$$

$$F_i = \frac{\pi d^3 \tau_i}{8D} = \frac{\pi d^3}{8D}\left[\frac{33\,500}{\exp(0.105C)} \pm 1000\left(4 - \frac{C-3}{6.5}\right)\right]$$

Use the lowest F_i in the preferred range. This results in the best fom.

$$F_i = \frac{\pi(0.067)^3}{8(0.3075)}\left\{\frac{33\,500}{\exp[0.105(4.590)]} - 1000\left(4 - \frac{4.590 - 3}{6.5}\right)\right\} = 6.505 \text{ lbf}$$

For simplicity, we will round up to the next integer or half integer;
therefore, use $F_i = 7$ lbf

$$k = \frac{18 - 7}{0.5} = 22 \text{ lbf/in}$$

$$N_a = \frac{d^4 G}{8kD^3} = \frac{(0.067)^4(11.5)(10^6)}{8(22)(0.3075)^3} = 45.28 \text{ turns}$$

$$N_b = N_a - \frac{G}{E} = 45.28 - \frac{11.5}{28.6} = 44.88 \text{ turns}$$

$$L_0 = (2C - 1 + N_b)d = [2(4.590) - 1 + 44.88](0.067) = 3.555 \text{ in}$$

$$L_{18\text{ lbf}} = 3.555 + 0.5 = 4.055 \text{ in}$$

Body: $K_B = \dfrac{4C+2}{4C-3} = \dfrac{4(4.590)+2}{4(4.590)-3} = 1.326$

$$\tau_{\max} = \frac{8K_B F_{\max} D}{\pi d^3} = \frac{8(1.326)(18)(0.3075)}{\pi(0.067)^3}(10^{-3}) = 62.1 \text{ kpsi}$$

$$(n_y)_{\text{body}} = \frac{S_{sy}}{\tau_{\max}} = \frac{105.3}{62.1} = 1.70$$

$$r_2 = 2d = 2(0.067) = 0.134 \text{ in}, \quad C_2 = \frac{2r_2}{d} = \frac{2(0.134)}{0.067} = 4$$

$$(K)_B = \frac{4C_2 - 1}{4C_2 - 4} = \frac{4(4)-1}{4(4)-4} = 1.25$$

$$\tau_B = (K)_B\left[\frac{8F_{\max}D}{\pi d^3}\right] = 1.25\left[\frac{8(18)(0.3075)}{\pi(0.067)^3}\right](10^{-3}) = 58.58 \text{ kpsi}$$

$$(n_y)_B = \frac{S_{sy}}{\tau_B} = \frac{105.3}{58.58} = 1.80$$

$$\text{fom} = -(1)\frac{\pi^2 d^2(N_b+2)D}{4} = -\frac{\pi^2(0.067)^2(44.88+2)(0.3075)}{4} = -0.160$$

Several diameters, evaluated using a spreadsheet, are shown below.

d:	0.067	0.072	0.076	0.081	0.085	0.09	0.095	0.104
S_{ut}	233.977	230.799	228.441	225.692	223.634	221.219	218.958	215.224
S_{sy}	105.290	103.860	102.798	101.561	100.635	99.548	98.531	96.851
S_y	175.483	173.100	171.331	169.269	167.726	165.914	164.218	161.418
C	4.589	5.412	6.099	6.993	7.738	8.708	9.721	11.650
D	0.307	0.390	0.463	0.566	0.658	0.784	0.923	1.212
F_i (calc)	6.505	5.773	5.257	4.675	4.251	3.764	3.320	2.621
F_i (rd)	7.0	6.0	5.5	5.0	4.5	4.0	3.5	3.0
k	22.000	24.000	25.000	26.000	27.000	28.000	29.000	30.000
N_a	45.29	27.20	19.27	13.10	9.77	7.00	5.13	3.15
N_b	44.89	26.80	18.86	12.69	9.36	6.59	4.72	2.75
L_0	3.556	2.637	2.285	2.080	2.026	2.071	2.201	2.605
$L_{18\text{ lbf}}$	4.056	3.137	2.785	2.580	2.526	2.571	2.701	3.105
K_B	1.326	1.268	1.234	1.200	1.179	1.157	1.139	1.115
$\tau_{\max}$	62.118	60.686	59.707	58.636	57.875	57.019	56.249	55.031
$(n_y)_{\text{body}}$	1.695	1.711	1.722	1.732	1.739	1.746	1.752	1.760
τ_B	58.576	59.820	60.495	61.067	61.367	61.598	61.712	61.712
$(n_y)_B$	1.797	1.736	1.699	1.663	1.640	1.616	1.597	1.569
$(n_y)_A$	1.500	1.500	1.500	1.500	1.500	1.500	1.500	1.500
fom	−0.160	−0.144	−0.138	−0.135	−0.133	−0.135	−0.138	−0.154

Except for the 0.067 in wire, all springs satisfy the requirements of length and number of coils. The 0.085 in wire has the highest fom.

10-29 Given: $N_b = 84$ coils, $F_i = 16$ lbf, OQ&T steel, OD $= 1.5$ in, $d = 0.162$ in.

$$D = 1.5 - 0.162 = 1.338 \text{ in}$$

(a) Eq. (10-39):

$$L_0 = 2(D - d) + (N_b + 1)d$$
$$= 2(1.338 - 0.162) + (84 + 1)(0.162) = 16.12 \text{ in} \quad Ans.$$

or
$$2d + L_0 = 2(0.162) + 16.12 = 16.45 \text{ in overall.}$$

(b)
$$C = \frac{D}{d} = \frac{1.338}{0.162} = 8.26$$

$$K_B = \frac{4(8.26) + 2}{4(8.26) - 3} = 1.166$$

$$\tau_i = K_B \left[\frac{8F_i D}{\pi d^3}\right] = 1.166 \left[\frac{8(16)(1.338)}{\pi(0.162)^3}\right] = 14\,950 \text{ psi} \quad Ans.$$

(c) From Table 10-5 use: $G = 11.4(10^6)$ psi and $E = 28.5(10^6)$ psi

$$N_a = N_b + \frac{G}{E} = 84 + \frac{11.4}{28.5} = 84.4 \text{ turns}$$

$$k = \frac{d^4 G}{8D^3 N_a} = \frac{(0.162)^4(11.4)(10^6)}{8(1.338)^3(84.4)} = 4.855 \text{ lbf/in} \quad Ans.$$

(d) Table 10-4: $A = 147 \text{ psi} \cdot \text{in}^m, \quad m = 0.187$

$$S_{ut} = \frac{147}{(0.162)^{0.187}} = 207.1 \text{ kpsi}$$
$$S_y = 0.75(207.1) = 155.3 \text{ kpsi}$$
$$S_{sy} = 0.50(207.1) = 103.5 \text{ kpsi}$$

Body

$$F = \frac{\pi d^3 S_{sy}}{\pi K_B D}$$
$$= \frac{\pi(0.162)^3(103.5)(10^3)}{8(1.166)(1.338)} = 110.8 \text{ lbf}$$

Torsional stress on hook point B

$$C_2 = \frac{2r_2}{d} = \frac{2(0.25 + 0.162/2)}{0.162} = 4.086$$

$$(K)_B = \frac{4C_2 - 1}{4C_2 - 4} = \frac{4(4.086) - 1}{4(4.086) - 4} = 1.243$$

$$F = \frac{\pi(0.162)^3(103.5)(10^3)}{8(1.243)(1.338)} = 103.9 \text{ lbf}$$

Normal stress on hook point A

$$C_1 = \frac{2r_1}{d} = \frac{1.338}{0.162} = 8.26$$

$$(K)_A = \frac{4C_1^2 - C_1 - 1}{4C_1(C_1 - 1)} = \frac{4(8.26)^2 - 8.26 - 1}{4(8.26)(8.26 - 1)} = 1.099$$

$$S_{yt} = \sigma = F\left[\frac{16(K)_A D}{\pi d^3} + \frac{4}{\pi d^2}\right]$$

$$F = \frac{155.3(10^3)}{[16(1.099)(1.338)]/[\pi(0.162)^3] + \{4/[\pi(0.162)^2]\}} = 85.8 \text{ lbf}$$

$$= \min(110.8, 103.9, 85.8) = 85.8 \text{ lbf} \quad Ans.$$

(e) Eq. (10-48):

$$y = \frac{F - F_i}{k} = \frac{85.8 - 16}{4.855} = 14.4 \text{ in} \quad Ans.$$

10-30 $F_{\min} = 9$ lbf, $F_{\max} = 18$ lbf

$$F_a = \frac{18-9}{2} = 4.5 \text{ lbf}, \qquad F_m = \frac{18+9}{2} = 13.5 \text{ lbf}$$

A313 stainless: $0.013 \le d \le 0.1$ $A = 169 \text{ kpsi} \cdot \text{in}^m$, $m = 0.146$

$0.1 \le d \le 0.2$ $A = 128 \text{ kpsi} \cdot \text{in}^m$, $m = 0.263$

$E = 28$ Mpsi, $G = 10$ Gpsi

Try $d = 0.081$ in and refer to the discussion following Ex. 10-7

$$S_{ut} = \frac{169}{(0.081)^{0.146}} = 243.9 \text{ kpsi}$$

$$S_{su} = 0.67 S_{ut} = 163.4 \text{ kpsi}$$

$$S_{sy} = 0.35 S_{ut} = 85.4 \text{ kpsi}$$

$$S_y = 0.55 S_{ut} = 134.2 \text{ kpsi}$$

Table 10-8: $S_r = 0.45 S_{ut} = 109.8$ kpsi

$$S_e = \frac{S_r/2}{1 - [S_r/(2S_{ut})]^2} = \frac{109.8/2}{1 - [(109.8/2)/243.9]^2} = 57.8 \text{ kpsi}$$

$$r = F_a/F_m = 4.5/13.5 = 0.333$$

Table 6-7: $$S_a = \frac{r^2 S_{ut}^2}{2S_e}\left[-1 + \sqrt{1 + \left(\frac{2S_e}{r S_{ut}}\right)^2}\right]$$

$$S_a = \frac{(0.333)^2(243.9^2)}{2(57.8)}\left[-1 + \sqrt{1 + \left[\frac{2(57.8)}{0.333(243.9)}\right]^2}\right] = 42.2 \text{ kpsi}$$

Hook bending

$$(\sigma_a)_A = F_a\left[(K)_A \frac{16C}{\pi d^2} + \frac{4}{\pi d^2}\right] = \frac{S_a}{(n_f)_A} = \frac{S_a}{2}$$

$$\frac{4.5}{\pi d^2}\left[\frac{(4C^2 - C - 1)16C}{4C(C-1)} + 4\right] = \frac{S_a}{2}$$

This equation reduces to a quadratic in C—see Prob. 10-28

The useable root for C is

$$C = 0.5\left[\frac{\pi d^2 S_a}{144} + \sqrt{\left(\frac{\pi d^2 S_a}{144}\right)^2 - \frac{\pi d^2 S_a}{36} + 2}\right]$$

$$= 0.5\left\{\frac{\pi(0.081)^2(42.2)(10^3)}{144} + \sqrt{\left[\frac{\pi(0.081)^2(42.2)(10^3)}{144}\right]^2 - \frac{\pi(0.081)^2(42.2)(10^3)}{36} + 2}\right\}$$

$$= 4.91$$

$$D = Cd = 0.398 \text{ in}$$

$$F_i = \frac{\pi d^3 \tau_i}{8D} = \frac{\pi d^3}{8D}\left[\frac{33\,500}{\exp(0.105C)} \pm 1000\left(4 - \frac{C-3}{6.5}\right)\right]$$

Use the lowest F_i in the preferred range.

$$F_i = \frac{\pi(0.081)^3}{8(0.398)}\left\{\frac{33\,500}{\exp[0.105(4.91)]} - 1000\left(4 - \frac{4.91-3}{6.5}\right)\right\}$$

$$= 8.55 \text{ lbf}$$

For simplicity we will round up to next $1/4$ integer.

$$F_i = 8.75 \text{ lbf}$$

$$k = \frac{18-9}{0.25} = 36 \text{ lbf/in}$$

$$N_a = \frac{d^4 G}{8kD^3} = \frac{(0.081)^4(10)(10^6)}{8(36)(0.398)^3} = 23.7 \text{ turns}$$

$$N_b = N_a - \frac{G}{E} = 23.7 - \frac{10}{28} = 23.3 \text{ turns}$$

$$L_0 = (2C - 1 + N_b)d = [2(4.91) - 1 + 23.3](0.081) = 2.602 \text{ in}$$

$$L_{\max} = L_0 + (F_{\max} - F_i)/k = 2.602 + (18 - 8.75)/36 = 2.859 \text{ in}$$

$$(\sigma_a)_A = \frac{4.5(4)}{\pi d^2}\left(\frac{4C^2 - C - 1}{C-1} + 1\right)$$

$$= \frac{18(10^{-3})}{\pi(0.081^2)}\left[\frac{4(4.91^2) - 4.91 - 1}{4.91 - 1} + 1\right] = 21.1 \text{ kpsi}$$

$$(n_f)_A = \frac{S_a}{(\sigma_a)_A} = \frac{42.2}{21.1} = 2 \quad \text{checks}$$

Body: $$K_B = \frac{4C+2}{4C-3} = \frac{4(4.91)+2}{4(4.91)-3} = 1.300$$

$$\tau_a = \frac{8(1.300)(4.5)(0.398)}{\pi(0.081)^3}(10^{-3}) = 11.16 \text{ kpsi}$$

$$\tau_m = \frac{F_m}{F_a}\tau_a = \frac{13.5}{4.5}(11.16) = 33.47 \text{ kpsi}$$

The repeating allowable stress from Table 7-8 is

$$S_{sr} = 0.30S_{ut} = 0.30(243.9) = 73.17 \text{ kpsi}$$

The Gerber intercept is

$$S_{se} = \frac{73.17/2}{1 - [(73.17/2)/163.4]^2} = 38.5 \text{ kpsi}$$

From Table 6-7,

$$(n_f)_{\text{body}} = \frac{1}{2}\left(\frac{163.4}{33.47}\right)^2 \left(\frac{11.16}{38.5}\right)\left\{-1 + \sqrt{1 + \left[\frac{2(33.47)(38.5)}{163.4(11.16)}\right]^2}\right\} = 2.53$$

Let $r_2 = 2d = 2(0.081) = 0.162$

$$C_2 = \frac{2r_2}{d} = 4, \qquad (K)_B = \frac{4(4) - 1}{4(4) - 4} = 1.25$$

$$(\tau_a)_B = \frac{(K)_B}{K_B}\tau_a = \frac{1.25}{1.30}(11.16) = 10.73 \text{ kpsi}$$

$$(\tau_m)_B = \frac{(K)_B}{K_B}\tau_m = \frac{1.25}{1.30}(33.47) = 32.18 \text{ kpsi}$$

Table 10-8: $(S_{sr})_B = 0.28S_{ut} = 0.28(243.9) = 68.3 \text{ kpsi}$

$$(S_{se})_B = \frac{68.3/2}{1 - [(68.3/2)/163.4]^2} = 35.7 \text{ kpsi}$$

$$(n_f)_B = \frac{1}{2}\left(\frac{163.4}{32.18}\right)^2 \left(\frac{10.73}{35.7}\right)\left\{-1 + \sqrt{1 + \left[\frac{2(32.18)(35.7)}{163.4(10.73)}\right]^2}\right\} = 2.51$$

Yield

Bending:

$$(\sigma_A)_{\max} = \frac{4F_{\max}}{\pi d^2}\left[\frac{(4C^2 - C - 1)}{C - 1} + 1\right]$$

$$= \frac{4(18)}{\pi(0.081^2)}\left[\frac{4(4.91)^2 - 4.91 - 1}{4.91 - 1} + 1\right](10^{-3}) = 84.4 \text{ kpsi}$$

$$(n_y)_A = \frac{134.2}{84.4} = 1.59$$

Body:

$$\tau_i = (F_i/F_a)\tau_a = (8.75/4.5)(11.16) = 21.7 \text{ kpsi}$$

$$r = \tau_a/(\tau_m - \tau_i) = 11.16/(33.47 - 21.7) = 0.948$$

$$(S_{sa})_y = \frac{r}{r + 1}(S_{sy} - \tau_i) = \frac{0.948}{0.948 + 1}(85.4 - 21.7) = 31.0 \text{ kpsi}$$

$$(n_y)_{\text{body}} = \frac{(S_{sa})_y}{\tau_a} = \frac{31.0}{11.16} = 2.78$$

Hook shear:

$$S_{sy} = 0.3S_{ut} = 0.3(243.9) = 73.2 \text{ kpsi}$$

$$\tau_{\max} = (\tau_a)_B + (\tau_m)_B = 10.73 + 32.18 = 42.9 \text{ kpsi}$$

$$(n_y)_B = \frac{73.2}{42.9} = 1.71$$

$$\text{fom} = -\frac{7.6\pi^2 d^2(N_b + 2)D}{4} = -\frac{7.6\pi^2(0.081)^2(23.3 + 2)(0.398)}{4} = -1.239$$

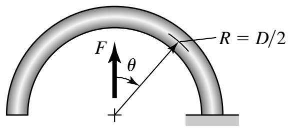
A tabulation of several wire sizes follow

d	0.081	0.085	0.092	0.098	0.105	0.12
S_{ut}	243.920	242.210	239.427	237.229	234.851	230.317
S_{su}	163.427	162.281	160.416	158.943	157.350	154.312
S_r	109.764	108.994	107.742	106.753	105.683	103.643
S_e	57.809	57.403	56.744	56.223	55.659	54.585
S_a	42.136	41.841	41.360	40.980	40.570	39.786
C	4.903	5.484	6.547	7.510	8.693	11.451
D	0.397	0.466	0.602	0.736	0.913	1.374
OD	0.478	0.551	0.694	0.834	1.018	1.494
F_i (calc)	8.572	7.874	6.798	5.987	5.141	3.637
F_i (rd)	8.75	9.75	10.75	11.75	12.75	13.75
k	36.000	36.000	36.000	36.000	36.000	36.000
N_a	23.86	17.90	11.38	8.03	5.55	2.77
N_b	23.50	17.54	11.02	7.68	5.19	2.42
L_0	2.617	2.338	2.127	2.126	2.266	2.918
$L_{18\,\text{lbf}}$	2.874	2.567	2.328	2.300	2.412	3.036
$(\sigma_a)_A$	21.068	20.920	20.680	20.490	20.285	19.893
$(n_f)_A$	2.000	2.000	2.000	2.000	2.000	2.000
K_B	1.301	1.264	1.216	1.185	1.157	1.117
$(\tau_a)_{\text{body}}$	11.141	10.994	10.775	10.617	10.457	10.177
$(\tau_m)_{\text{body}}$	33.424	32.982	32.326	31.852	31.372	30.532
S_{sr}	73.176	72.663	71.828	71.169	70.455	69.095
S_{se}	38.519	38.249	37.809	37.462	37.087	36.371
$(n_f)_{\text{body}}$	2.531	2.547	2.569	2.583	2.596	2.616
$(K)_B$	1.250	1.250	1.250	1.250	1.250	1.250
$(\tau_a)_B$	10.705	10.872	11.080	11.200	11.294	11.391
$(\tau_m)_B$	32.114	32.615	33.240	33.601	33.883	34.173
$(S_{sr})_B$	68.298	67.819	67.040	66.424	65.758	64.489
$(S_{se})_B$	35.708	35.458	35.050	34.728	34.380	33.717
$(n_f)_B$	2.519	2.463	2.388	2.341	2.298	2.235
S_y	134.156	133.215	131.685	130.476	129.168	126.674
$(\sigma_A)_{\max}$	84.273	83.682	82.720	81.961	81.139	79.573
$(n_y)_A$	1.592	1.592	1.592	1.592	1.592	1.592
τ_i	21.663	23.820	25.741	27.723	29.629	31.097
r	0.945	1.157	1.444	1.942	2.906	4.703
$(S_{sy})_{\text{body}}$	85.372	84.773	83.800	83.030	82.198	80.611
$(S_{sa})_y$	30.958	32.688	34.302	36.507	39.109	40.832
$(n_y)_{\text{body}}$	2.779	2.973	3.183	3.438	3.740	4.012
$(S_{sy})_B$	73.176	72.663	71.828	71.169	70.455	69.095
$(\tau_B)_{\max}$	42.819	43.486	44.321	44.801	45.177	45.564
$(n_y)_B$	1.709	1.671	1.621	1.589	1.560	1.516
fom	−1.246	−1.234	−1.245	−1.283	−1.357	−1.639

optimal fom

The shaded areas show the conditions not satisfied.

10-31 For the hook,



$$M = FR\sin\theta, \quad \partial M/\partial F = R\sin\theta$$

$$\delta_F = \frac{1}{EI}\int_0^{\pi/2} FR^2\sin^2 R\,d\theta = \frac{\pi}{2}\frac{PR^3}{EI}$$

The total deflection of the body and the two hooks

$$\delta = \frac{8FD^3N_b}{d^4G} + 2\frac{\pi}{2}\frac{FR^3}{EI} = \frac{8FD^3N_b}{d^4G} + \frac{\pi F(D/2)^3}{E(\pi/64)(d^4)}$$

$$= \frac{8FD^3}{d^4G}\left(N_b + \frac{G}{E}\right) = \frac{8FD^3N_a}{d^4G}$$

$$\therefore \quad N_a = N_b + \frac{G}{E} \quad \text{QED}$$

10-32 Table 10-4 for A227:

$$A = 140 \text{ kpsi} \cdot \text{in}^m, \quad m = 0.190$$

Table 10-5:

$$E = 28.5(10^6) \text{ psi}$$

$$S_{ut} = \frac{140}{(0.162)^{0.190}} = 197.8 \text{ kpsi}$$

Eq. (10-57):

$$S_y = \sigma_{\text{all}} = 0.78(197.8) = 154.3 \text{ kpsi}$$

$$D = 1.25 - 0.162 = 1.088 \text{ in}$$

$$C = D/d = 1.088/0.162 = 6.72$$

$$K_i = \frac{4C^2 - C - 1}{4C(C-1)} = \frac{4(6.72)^2 - 6.72 - 1}{4(6.72)(6.72-1)} = 1.125$$

From

$$\sigma = K_i\frac{32M}{\pi d^3}$$

Solving for M for the yield condition,

$$M_y = \frac{\pi d^3 S_y}{32K_i} = \frac{\pi(0.162)^3(154\,300)}{32(1.125)} = 57.2 \text{ lbf} \cdot \text{in}$$

Count the turns when $M = 0$

$$N = 2.5 - \frac{M_y}{d^4E/(10.8DN)}$$

from which

$$N = \frac{2.5}{1 + [10.8DM_y/(d^4E)]}$$

$$= \frac{2.5}{1 + \{[10.8(1.088)(57.2)]/[(0.162)^4(28.5)(10^6)]\}} = 2.417 \text{ turns}$$

This means $(2.5 - 2.417)(360^\circ)$ or 29.9° from closed. Treating the hand force as in the middle of the grip

$$r = 1 + \frac{3.5}{2} = 2.75 \text{ in}$$

$$F = \frac{M_y}{r} = \frac{57.2}{2.75} = 20.8 \text{ lbf} \quad Ans.$$

10-33 The spring material and condition are unknown. Given $d = 0.081$ in and OD $= 0.500$,

(a) $D = 0.500 - 0.081 = 0.419$ in

Using $E = 28.6$ Mpsi for an estimate

$$k' = \frac{d^4 E}{10.8DN} = \frac{(0.081)^4(28.6)(10^6)}{10.8(0.419)(11)} = 24.7 \text{ lbf} \cdot \text{in/turn}$$

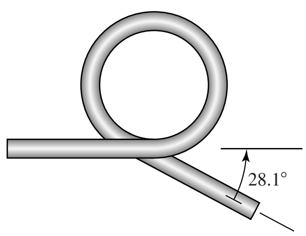
for each spring. The moment corresponding to a force of 8 lbf

$$Fr = (8/2)(3.3125) = 13.25 \text{ lbf} \cdot \text{in/spring}$$

The fraction windup turn is

$$n = \frac{Fr}{k'} = \frac{13.25}{24.7} = 0.536 \text{ turns}$$

The arm swings through an arc of slightly less than 180°, say 165°. This uses up 165/360 or 0.458 turns. So $n = 0.536 - 0.458 = 0.078$ turns are left (or $0.078(360^\circ) = 28.1^\circ$). The original configuration of the spring was



Ans.

(b)

$$C = \frac{0.419}{0.081} = 5.17$$

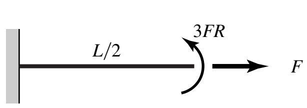
$$K_i = \frac{4(5.17)^2 - 5.17 - 1}{4(5.17)(5.17 - 1)} = 1.168$$

$$\sigma = K_i \frac{32M}{\pi d^3}$$

$$= 1.168 \left[\frac{32(13.25)}{\pi(0.081)^3}\right] = 296\,623 \text{ psi} \quad Ans.$$

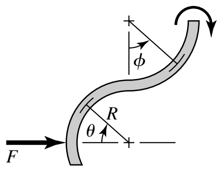
To achieve this stress level, the spring had to have set removed.

10-34 Consider half and double results

Straight section: 

$$M = 3FR, \quad \frac{\partial M}{\partial P} = 3R$$

Upper 180° section:



$$M = F[R + R(1 - \cos\phi)]$$

$$= FR(2 - \cos\phi), \quad \frac{\partial M}{\partial P} = R(2 - \cos\phi)$$

Lower section:

$$M = FR\sin\theta$$

$$\frac{\partial M}{\partial P} = R\sin\theta$$

Considering bending only:

$$\delta = \frac{2}{EI}\left[\int_0^{L/2} 9FR^2\,dx + \int_0^{\pi} FR^2(2-\cos\phi)^2 R\,d\phi + \int_0^{\pi/2} F(R\sin\theta)^2 R\,d\theta\right]$$

$$= \frac{2F}{EI}\left[\frac{9}{2}R^2L + R^3\left(4\pi - 4\sin\phi\Big|_0^{\pi} + \frac{\pi}{2}\right) + R^3\left(\frac{\pi}{4}\right)\right]$$

$$= \frac{2FR^2}{EI}\left(\frac{19\pi}{4}R + \frac{9}{2}L\right) = \frac{FR^2}{2EI}(19\pi R + 18L) \quad \textit{Ans.}$$

10-35 Computer programs will vary.

10-36 Computer programs will vary.