

Chapter 6

Note to the instructor: Many of the problems in this chapter are carried over from the previous edition. The solutions have changed slightly due to some minor changes. First, the calculation of the endurance limit of a rotating-beam specimen S'_e is given by $S'_e = 0.5S_{ut}$ instead of $S'_e = 0.504S_{ut}$. Second, when the fatigue stress calculation is made for deterministic problems, only one approach is given, which uses the notch sensitivity factor, q , together with Eq. (6-32). *Neuber's* equation, Eq. (6-33), is simply another form of this. These changes were made to hopefully make the calculations less confusing, and diminish the idea that stress life calculations are precise.

6-1 $H_B = 490$

$$\text{Eq. (2-17): } S_{ut} = 0.495(490) = 242.6 \text{ kpsi} > 212 \text{ kpsi}$$

$$\text{Eq. (6-8): } S'_e = 100 \text{ kpsi}$$

$$\text{Table 6-2: } a = 1.34, \quad b = -0.085$$

$$\text{Eq. (6-19): } k_a = 1.34(242.6)^{-0.085} = 0.840$$

$$\text{Eq. (6-20): } k_b = \left(\frac{1/4}{0.3}\right)^{-0.107} = 1.02$$

$$\text{Eq. (6-18): } S_e = k_a k_b S'_e = 0.840(1.02)(100) = 85.7 \text{ kpsi} \quad \text{Ans.}$$

6-2

$$\text{(a) } S_{ut} = 68 \text{ kpsi}, \quad S'_e = 0.5(68) = 34 \text{ kpsi} \quad \text{Ans.}$$

$$\text{(b) } S_{ut} = 112 \text{ kpsi}, \quad S'_e = 0.5(112) = 56 \text{ kpsi} \quad \text{Ans.}$$

$$\text{(c) 2024T3 has no endurance limit} \quad \text{Ans.}$$

$$\text{(d) Eq. (6-8): } S'_e = 100 \text{ kpsi} \quad \text{Ans.}$$

6-3

$$\text{Eq. (2-11): } \sigma'_F = \sigma_0 \varepsilon^m = 115(0.90)^{0.22} = 112.4 \text{ kpsi}$$

$$\text{Eq. (6-8): } S'_e = 0.5(66.2) = 33.1 \text{ kpsi}$$

$$\text{Eq. (6-12): } b = -\frac{\log(112.4/33.1)}{\log(2 \cdot 10^6)} = -0.08426$$

$$\text{Eq. (6-10): } f = \frac{112.4}{66.2}(2 \cdot 10^3)^{-0.08426} = 0.8949$$

$$\text{Eq. (6-14): } a = \frac{[0.8949(66.2)]^2}{33.1} = 106.0 \text{ kpsi}$$

$$\text{Eq. (6-13): } S_f = aN^b = 106.0(12\,500)^{-0.08426} = 47.9 \text{ kpsi} \quad \text{Ans.}$$

$$\text{Eq. (6-16): } N = \left(\frac{\sigma_a}{a}\right)^{1/b} = \left(\frac{36}{106.0}\right)^{-1/0.08426} = 368\,250 \text{ cycles} \quad \text{Ans.}$$

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6-4 From $S_f = aN^b$

$$\log S_f = \log a + b \log N$$

Substituting (1, S_{ut})

$$\log S_{ut} = \log a + b \log (1)$$

From which

$$a = S_{ut}$$

Substituting (10^3 , $f S_{ut}$) and $a = S_{ut}$

$$\log f S_{ut} = \log S_{ut} + b \log 10^3$$

From which

$$b = \frac{1}{3} \log f$$

$$\therefore S_f = S_{ut} N^{(\log f)/3} \quad 1 \leq N \leq 10^3$$

For 500 cycles as in Prob. 6-3

$$S_f \geq 66.2(500)^{(\log 0.8949)/3} = 59.9 \text{ kpsi} \quad \text{Ans.}$$

6-5 Read from graph: (10^3 , 90) and (10^6 , 50). From $S = aN^b$

$$\log S_1 = \log a + b \log N_1$$

$$\log S_2 = \log a + b \log N_2$$

From which

$$\log a = \frac{\log S_1 \log N_2 - \log S_2 \log N_1}{\log N_2 / N_1}$$

$$= \frac{\log 90 \log 10^6 - \log 50 \log 10^3}{\log 10^6 / 10^3}$$

$$= 2.2095$$

$$a = 10^{\log a} = 10^{2.2095} = 162.0$$

$$b = \frac{\log 50/90}{3} = -0.08509$$

$$(S_f)_{ax} = 162^{-0.08509} \quad 10^3 \leq N \leq 10^6 \text{ in kpsi} \quad \text{Ans.}$$

Check:

$$10^3(S_f)_{ax} = 162(10^3)^{-0.08509} = 90 \text{ kpsi}$$

$$10^6(S_f)_{ax} = 162(10^6)^{-0.08509} = 50 \text{ kpsi}$$

The end points agree.

6-6

Eq. (6-8): $S'_e = 0.5(710) = 355 \text{ MPa}$

Table 6-2: $a = 4.51, \quad b = -0.265$

Eq. (6-19): $k_a = 4.51(710)^{-0.265} = 0.792$

$$\text{Eq. (6-20):} \quad k_b = \left(\frac{d}{7.62} \right)^{-0.107} = \left(\frac{32}{7.62} \right)^{-0.107} = 0.858$$

$$\text{Eq. (6-18):} \quad S_e = k_a k_b S'_e = 0.792(0.858)(355) = 241 \text{ MPa} \quad \text{Ans.}$$

6-7 For AISI 4340 as forged steel,

$$\text{Eq. (6-8):} \quad S_e = 100 \text{ kpsi}$$

$$\text{Table 6-2:} \quad a = 39.9, \quad b = -0.995$$

$$\text{Eq. (6-19):} \quad k_a = 39.9(260)^{-0.995} = 0.158$$

$$\text{Eq. (6-20):} \quad k_b = \left(\frac{0.75}{0.30} \right)^{-0.107} = 0.907$$

Each of the other Marin factors is unity.

$$S_e = 0.158(0.907)(100) = 14.3 \text{ kpsi}$$

For AISI 1040:

$$S'_e = 0.5(113) = 56.5 \text{ kpsi}$$

$$k_a = 39.9(113)^{-0.995} = 0.362$$

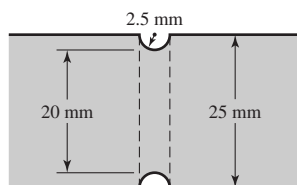
$$k_b = 0.907 \text{ (same as 4340)}$$

Each of the other Marin factors is unity.

$$S_e = 0.362(0.907)(56.5) = 18.6 \text{ kpsi}$$

Not only is AISI 1040 steel a contender, it has a superior endurance strength. Can you see why?

6-8



(a) For an AISI 1018 CD-machined steel, the strengths are

$$\text{Eq. (2-17):} \quad S_{ut} = 440 \text{ MPa} \Rightarrow H_B = \frac{440}{3.41} = 129$$

$$S_y = 370 \text{ MPa}$$

$$S_{su} = 0.67(440) = 295 \text{ MPa}$$

$$\text{Fig. A-15-15:} \quad \frac{r}{d} = \frac{2.5}{20} = 0.125, \quad \frac{D}{d} = \frac{25}{20} = 1.25, \quad K_{ts} = 1.4$$

$$\text{Fig. 6-21:} \quad q_s = 0.94$$

$$\text{Eq. (6-32):} \quad K_{fs} = 1 + 0.94(1.4 - 1) = 1.376$$

For a purely reversing torque of 200 N · m

$$\tau_{\max} = \frac{K_{fs} 16T}{\pi d^3} = \frac{1.376(16)(200 \times 10^3 \text{ N} \cdot \text{mm})}{\pi(20 \text{ mm})^3}$$

$$\tau_{\max} = 175.2 \text{ MPa} = \tau_a$$

$$S'_e = 0.5(440) = 220 \text{ MPa}$$

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The Marin factors are

$$k_a = 4.51(440)^{-0.265} = 0.899$$

$$k_b = \left(\frac{20}{7.62}\right)^{-0.107} = 0.902$$

$$k_c = 0.59, \quad k_d = 1, \quad k_e = 1$$

Eq. (6-18): $S_e = 0.899(0.902)(0.59)(220) = 105.3 \text{ MPa}$

Eq. (6-14): $a = \frac{[0.9(295)]^2}{105.3} = 669.4$

Eq. (6-15): $b = -\frac{1}{3} \log \frac{0.9(295)}{105.3} = -0.13388$

Eq. (6-16): $N = \left(\frac{175.2}{669.4}\right)^{1/-0.13388}$

$$N = 22\,300 \text{ cycles} \quad \text{Ans.}$$

(b) For an operating temperature of 450°C , the temperature modification factor, from Table 6-4, is

$$k_d = 0.843$$

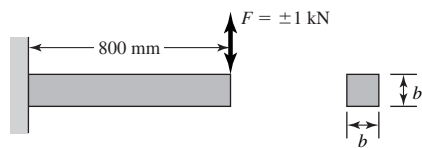
Thus $S_e = 0.899(0.902)(0.59)(0.843)(220) = 88.7 \text{ MPa}$

$$a = \frac{[0.9(295)]^2}{88.7} = 794.7$$

$$b = -\frac{1}{3} \log \frac{0.9(295)}{88.7} = -0.15871$$

$$N = \left(\frac{175.2}{794.7}\right)^{1/-0.15871}$$

$$N = 13\,700 \text{ cycles} \quad \text{Ans.}$$

6-9


$$f = 0.9$$

$$n = 1.5$$

$$N = 10^4 \text{ cycles}$$

For AISI 1045 HR steel, $S_{ut} = 570 \text{ MPa}$ and $S_y = 310 \text{ MPa}$

$$S'_e = 0.5(570 \text{ MPa}) = 285 \text{ MPa}$$

Find an initial guess based on yielding:

$$\sigma_a = \sigma_{\max} = \frac{Mc}{I} = \frac{M(b/2)}{b(b^3)/12} = \frac{6M}{b^3}$$

$$M_{\max} = (1 \text{ kN})(800 \text{ mm}) = 800 \text{ N} \cdot \text{m}$$

$$\sigma_{\max} = \frac{S_y}{n} \Rightarrow \frac{6(800 \times 10^3 \text{ N} \cdot \text{mm})}{b^3} = \frac{310 \text{ N/mm}^2}{1.5}$$

$$b = 28.5 \text{ mm}$$

$$\text{Eq. (6-25): } d_e = 0.808b$$

$$\text{Eq. (6-20): } k_b = \left(\frac{0.808b}{7.62} \right)^{-0.107} = 1.2714b^{-0.107}$$

$$k_b = 0.888$$

The remaining Marin factors are

$$k_a = 57.7(570)^{-0.718} = 0.606$$

$$k_c = k_d = k_e = k_f = 1$$

$$\text{Eq. (6-18): } S_e = 0.606(0.888)(285) = 153.4 \text{ MPa}$$

$$\text{Eq. (6-14): } a = \frac{[0.9(570)]^2}{153.4} = 1715.6$$

$$\text{Eq. (6-15): } b = -\frac{1}{3} \log \frac{0.9(570)}{153.4} = -0.17476$$

$$\text{Eq. (6-13): } S_f = aN^b = 1715.6[(10^4)^{-0.17476}] = 343.1 \text{ MPa}$$

$$n = \frac{S_f}{\sigma_a} \quad \text{or} \quad \sigma_a = \frac{S_f}{n}$$

$$\frac{6(800 \times 10^3)}{b^3} = \frac{343.1}{1.5} \Rightarrow b = 27.6 \text{ mm}$$

Check values for k_b , S_e , etc.

$$k_b = 1.2714(27.6)^{-0.107} = 0.891$$

$$S_e = 0.606(0.891)(285) = 153.9 \text{ MPa}$$

$$a = \frac{[0.9(570)]^2}{153.9} = 1710$$

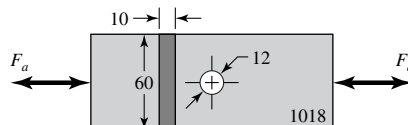
$$b = -\frac{1}{3} \log \frac{0.9(570)}{153.9} = -0.17429$$

$$S_f = 1710[(10^4)^{-0.17429}] = 343.4 \text{ MPa}$$

$$\frac{6(800 \times 10^3)}{b^3} = \frac{343.4}{1.5}$$

$$b = 27.6 \text{ mm} \quad \text{Ans.}$$

6-10



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Table A-20: $S_{ut} = 440 \text{ MPa}$, $S_y = 370 \text{ MPa}$

$$S'_e = 0.5(440) = 220 \text{ MPa}$$

Table 6-2: $k_a = 4.51(440)^{-0.265} = 0.899$

$$k_b = 1 \quad (\text{axial loading})$$

Eq. (6-26): $k_c = 0.85$

$$S_e = 0.899(1)(0.85)(220) = 168.1 \text{ MPa}$$

Table A-15-1: $d/w = 12/60 = 0.2$, $K_t = 2.5$

From Fig. 6-20, $q \doteq 0.82$

Eq. (6-32): $K_f = 1 + 0.82(2.5 - 1) = 2.23$

$$\sigma_a = K_f \frac{F_a}{A} \Rightarrow \frac{S_e}{n_f} = \frac{2.23 F_a}{10(60 - 12)} = \frac{168.1}{1.8}$$

$$F_a = 20\,100 \text{ N} = 20.1 \text{ kN} \quad \text{Ans.}$$

$$\frac{F_a}{A} = \frac{S_y}{n_y} \Rightarrow \frac{F_a}{10(60 - 12)} = \frac{370}{1.8}$$

$$F_a = 98\,700 \text{ N} = 98.7 \text{ kN} \quad \text{Ans.}$$

Largest force amplitude is 20.1 kN. *Ans.*

6-11 A priori design decisions:

The design decision will be: d

Material and condition: 1095 HR and from Table A-20 $S_{ut} = 120$, $S_y = 66 \text{ kpsi}$.

Design factor: $n_f = 1.6$ per problem statement.

Life: $(1150)(3) = 3450$ cycles

Function: carry 10 000 lbf load

Preliminaries to iterative solution:

$$S'_e = 0.5(120) = 60 \text{ kpsi}$$

$$k_a = 2.70(120)^{-0.265} = 0.759$$

$$\frac{I}{c} = \frac{\pi d^3}{32} = 0.098\,17 d^3$$

$$M(\text{crit.}) = \left(\frac{6}{24} \right) (10\,000)(12) = 30\,000 \text{ lbf} \cdot \text{in}$$

The critical location is in the middle of the shaft at the shoulder. From Fig. A-15-9: $D/d = 1.5$, $r/d = 0.10$, and $K_t = 1.68$. With no direct information concerning f , use $f = 0.9$.

For an initial trial, set $d = 2.00 \text{ in}$

$$k_b = \left(\frac{2.00}{0.30} \right)^{-0.107} = 0.816$$

$$S_e = 0.759(0.816)(60) = 37.2 \text{ kpsi}$$

$$a = \frac{[0.9(120)]^2}{37.2} = 313.5$$

$$b = -\frac{1}{3} \log \frac{0.9(120)}{37.2} = -0.15429$$

$$S_f = 313.5(3450)^{-0.15429} = 89.2 \text{ kpsi}$$

$$\sigma_0 = \frac{M}{I/c} = \frac{30}{0.09817d^3} = \frac{305.6}{d^3}$$

$$= \frac{305.6}{2^3} = 38.2 \text{ kpsi}$$

$$r = \frac{d}{10} = \frac{2}{10} = 0.2$$

Fig. 6-20: $q \doteq 0.87$

Eq. (6-32): $K_f \doteq 1 + 0.87(1.68 - 1) = 1.59$

$$\sigma_a = K_f \sigma_0 = 1.59(38.2) = 60.7 \text{ kpsi}$$

$$n_f = \frac{S_f}{\sigma_a} = \frac{89.2}{60.7} = 1.47$$

Design is adequate unless more uncertainty prevails.

Choose $d = 2.00$ in *Ans.*

6-12

Yield: $\sigma'_{\max} = [172^2 + 3(103^2)]^{1/2} = 247.8 \text{ kpsi}$

$$n_y = S_y / \sigma'_{\max} = 413 / 247.8 = 1.67 \text{ Ans.}$$

$$\sigma'_a = 172 \text{ MPa} \quad \sigma'_m = \sqrt{3} \tau_m = \sqrt{3}(103) = 178.4 \text{ MPa}$$

(a) Modified Goodman, Table 6-6

$$n_f = \frac{1}{(172/276) + (178.4/551)} = 1.06 \text{ Ans.}$$

(b) Gerber, Table 6-7

$$n_f = \frac{1}{2} \left(\frac{551}{178.4} \right)^2 \left(\frac{172}{276} \right) \left\{ -1 + \sqrt{1 + \left[\frac{2(178.4)(276)}{551(172)} \right]^2} \right\} = 1.31 \text{ Ans.}$$

(c) ASME-Elliptic, Table 6-8

$$n_f = \left[\frac{1}{(172/276)^2 + (178.4/413)^2} \right]^{1/2} = 1.32 \text{ Ans.}$$

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6-13

Yield: $\sigma'_{\max} = [69^2 + 3(138)^2]^{1/2} = 248.8 \text{ MPa}$

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{413}{248.8} = 1.66 \text{ Ans.}$$

$$\sigma'_a = 69 \text{ MPa}, \quad \sigma'_m = \sqrt{3}(138) = 239 \text{ MPa}$$

(a) Modified Goodman, Table 6-6

$$n_f = \frac{1}{(69/276) + (239/551)} = 1.46 \text{ Ans.}$$

(b) Gerber, Table 6-7

$$n_f = \frac{1}{2} \left(\frac{551}{239} \right)^2 \left(\frac{69}{276} \right) \left\{ -1 + \sqrt{1 + \left[\frac{2(239)(276)}{551(69)} \right]^2} \right\} = 1.73 \text{ Ans.}$$

(c) ASME-Elliptic, Table 6-8

$$n_f = \left[\frac{1}{(69/276)^2 + (239/413)^2} \right]^{1/2} = 1.59 \text{ Ans.}$$

6-14

Yield: $\sigma'_{\max} = [83^2 + 3(103 + 69)^2]^{1/2} = 309.2 \text{ MPa}$

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{413}{309.3} = 1.34 \text{ Ans.}$$

$$\sigma'_a = \sqrt{\sigma_a^2 + 3\tau_a^2} = \sqrt{83^2 + 3(69)^2} = 145.5 \text{ MPa}, \quad \sigma'_m = \sqrt{3}(103) = 178.4 \text{ MPa}$$

(a) Modified Goodman, Table 6-6

$$n_f = \frac{1}{(145.5/276) + (178.4/551)} = 1.18 \text{ Ans.}$$

(b) Gerber, Table 6-7

$$n_f = \frac{1}{2} \left(\frac{551}{178.4} \right)^2 \left(\frac{145.5}{276} \right) \left\{ -1 + \sqrt{1 + \left[\frac{2(178.4)(276)}{551(145.5)} \right]^2} \right\} = 1.47 \text{ Ans.}$$

(c) ASME-Elliptic, Table 6-8

$$n_f = \left[\frac{1}{(145.5/276)^2 + (178.4/413)^2} \right]^{1/2} = 1.47 \text{ Ans.}$$

6-15

$$\sigma'_{\max} = \sigma'_a = \sqrt{3}(207) = 358.5 \text{ MPa}, \quad \sigma'_m = 0$$

Yield: $358.5 = \frac{413}{n_y} \Rightarrow n_y = 1.15 \text{ Ans.}$

(a) Modified Goodman, Table 6-6

$$n_f = \frac{1}{(358.5/276)} = 0.77 \text{ Ans.}$$

(b) Gerber criterion of Table 6-7 does not work; therefore use Eq. (6-47).

$$n_f \frac{\sigma_a}{S_e} = 1 \Rightarrow n_f = \frac{S_e}{\sigma_a} = \frac{276}{358.5} = 0.77 \text{ Ans.}$$

(c) ASME-Elliptic, Table 6-8

$$n_f = \sqrt{\left(\frac{1}{358.5/276}\right)^2} = 0.77 \text{ Ans.}$$

Let $f = 0.9$ to assess the cycles to failure by fatigue

Eq. (6-14): $a = \frac{[0.9(551)]^2}{276} = 891.0 \text{ MPa}$

Eq. (6-15): $b = -\frac{1}{3} \log \frac{0.9(551)}{276} = -0.084828$

Eq. (6-16): $N = \left(\frac{358.5}{891.0}\right)^{-1/0.084828} = 45\,800 \text{ cycles Ans.}$

6-16

Yield: $\sigma'_{\max} = [103^2 + 3(103)^2]^{1/2} = 206 \text{ MPa}$

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{413}{206} = 2.00 \text{ Ans.}$$

$$\sigma'_a = \sqrt{3}(103) = 178.4 \text{ MPa}, \quad \sigma'_m = 103 \text{ MPa}$$

(a) Modified Goodman, Table 7-9

$$n_f = \frac{1}{(178.4/276) + (103/551)} = 1.20 \text{ Ans.}$$

(b) Gerber, Table 7-10

$$n_f = \frac{1}{2} \left(\frac{551}{103}\right)^2 \left(\frac{178.4}{276}\right) \left\{ -1 + \sqrt{1 + \left[\frac{2(103)(276)}{551(178.4)}\right]^2} \right\} = 1.44 \text{ Ans.}$$

(c) ASME-Elliptic, Table 7-11

$$n_f = \left[\frac{1}{(178.4/276)^2 + (103/413)^2} \right]^{1/2} = 1.44 \text{ Ans.}$$

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6-17 Table A-20: $S_{ut} = 64$ kpsi, $S_y = 54$ kpsi

$$A = 0.375(1 - 0.25) = 0.2813 \text{ in}^2$$

$$\sigma_{\max} = \frac{F_{\max}}{A} = \frac{3000}{0.2813}(10^{-3}) = 10.67 \text{ kpsi}$$

$$n_y = \frac{54}{10.67} = 5.06 \text{ Ans.}$$

$$S'_e = 0.5(64) = 32 \text{ kpsi}$$

$$k_a = 2.70(64)^{-0.265} = 0.897$$

$$k_b = 1, \quad k_c = 0.85$$

$$S_e = 0.897(1)(0.85)(32) = 24.4 \text{ kpsi}$$

Table A-15-1: $w = 1$ in, $d = 1/4$ in, $d/w = 0.25 \therefore K_t = 2.45$.

Fig. 6-20, with $r = 0.125$ in, $q \doteq 0.8$

Eq. (6-32): $K_f = 1 + 0.8(2.45 - 1) = 2.16$

$$\begin{aligned} \sigma_a &= K_f \left| \frac{F_{\max} - F_{\min}}{2A} \right| \\ &= 2.16 \left| \frac{3.000 - 0.800}{2(0.2813)} \right| = 8.45 \text{ kpsi} \end{aligned}$$

$$\begin{aligned} \sigma_m &= K_f \frac{F_{\max} + F_{\min}}{2A} \\ &= 2.16 \left[\frac{3.000 + 0.800}{2(0.2813)} \right] = 14.6 \text{ kpsi} \end{aligned}$$

(a) Gerber, Table 6-7

$$\begin{aligned} n_f &= \frac{1}{2} \left(\frac{64}{14.6} \right)^2 \left(\frac{8.45}{24.4} \right) \left[-1 + \sqrt{1 + \left(\frac{2(14.6)(24.4)}{8.45(64)} \right)^2} \right] \\ &= 2.17 \text{ Ans.} \end{aligned}$$

(b) ASME-Elliptic, Table 6-8

$$n_f = \sqrt{\frac{1}{(8.45/24.4)^2 + (14.6/54)^2}} = 2.28 \text{ Ans.}$$

6-18 Referring to the solution of Prob. 6-17, for load fluctuations of -800 to 3000 lbf

$$\sigma_a = 2.16 \left| \frac{3.000 - (-0.800)}{2(0.2813)} \right| = 14.59 \text{ kpsi}$$

$$\sigma_m = 2.16 \left| \frac{3.000 + (-0.800)}{2(0.2813)} \right| = 8.45 \text{ kpsi}$$

(a) Table 6-7, DE-Gerber

$$n_f = \frac{1}{2} \left(\frac{64}{8.45} \right)^2 \left(\frac{14.59}{24.4} \right) \left[-1 + \sqrt{1 + \left(\frac{2(8.45)(24.4)}{64(14.59)} \right)^2} \right] = 1.60 \quad \text{Ans.}$$

(b) Table 6-8, DE-Elliptic

$$n_f = \sqrt{\frac{1}{(14.59/24.4)^2 + (8.45/54)^2}} = 1.62 \quad \text{Ans.}$$

6-19 Referring to the solution of Prob. 6-17, for load fluctuations of 800 to -3000 lbf

$$\sigma_a = 2.16 \left| \frac{0.800 - (-3.000)}{2(0.2813)} \right| = 14.59 \text{ kpsi}$$

$$\sigma_m = 2.16 \left[\frac{0.800 + (-3.000)}{2(0.2813)} \right] = -8.45 \text{ kpsi}$$

(a) We have a compressive midrange stress for which the failure locus is horizontal at the S_e level.

$$n_f = \frac{S_e}{\sigma_a} = \frac{24.4}{14.59} = 1.67 \quad \text{Ans.}$$

(b) Same as (a)

$$n_f = \frac{S_e}{\sigma_a} = \frac{24.4}{14.59} = 1.67 \quad \text{Ans.}$$

6-20

$$S_{ut} = 0.495(380) = 188.1 \text{ kpsi}$$

$$S'_e = 0.5(188.1) = 94.05 \text{ kpsi}$$

$$k_a = 14.4(188.1)^{-0.718} = 0.335$$

For a non-rotating round bar in bending, Eq. (6-24) gives: $d_e = 0.370d = 0.370(3/8) = 0.1388$ in

$$k_b = \left(\frac{0.1388}{0.3} \right)^{-0.107} = 1.086$$

$$S_e = 0.335(1.086)(94.05) = 34.22 \text{ kpsi}$$

$$F_a = \frac{30 - 15}{2} = 7.5 \text{ lbf}, \quad F_m = \frac{30 + 15}{2} = 22.5 \text{ lbf}$$

$$\sigma_m = \frac{32M_m}{\pi d^3} = \frac{32(22.5)(16)}{\pi(0.375^3)}(10^{-3}) = 69.54 \text{ kpsi}$$

$$\sigma_a = \frac{32(7.5)(16)}{\pi(0.375^3)}(10^{-3}) = 23.18 \text{ kpsi}$$

$$r = \frac{23.18}{69.54} = 0.333$$

(a) Modified Goodman, Table 6-6

$$n_f = \frac{1}{(23.18/34.22) + (69.54/188.1)} = 0.955$$

Since finite failure is predicted, proceed to calculate N

From Fig. 6-18, for $S_{ut} = 188.1$ kpsi, $f = 0.778$

$$\text{Eq. (6-14):} \quad a = \frac{[0.7781(188.1)]^2}{34.22} = 625.8 \text{ kpsi}$$

$$\text{Eq. (6-15):} \quad b = -\frac{1}{3} \log \frac{0.778(188.1)}{34.22} = -0.21036$$

$$\frac{\sigma_a}{S_f} + \frac{\sigma_m}{S_{ut}} = 1 \Rightarrow S_f = \frac{\sigma_a}{1 - (\sigma_m/S_{ut})} = \frac{23.18}{1 - (69.54/188.1)} = 36.78 \text{ kpsi}$$

Eq. (6-16) with $\sigma_a = S_f$

$$N = \left(\frac{36.78}{625.8} \right)^{1/-0.21036} = 710\,000 \text{ cycles} \quad \text{Ans.}$$

(b) Gerber, Table 6-7

$$n_f = \frac{1}{2} \left(\frac{188.1}{69.54} \right)^2 \left(\frac{23.18}{34.22} \right) \left\{ -1 + \sqrt{1 + \left[\frac{2(69.54)(34.22)}{188.1(23.18)} \right]^2} \right\}$$

$$= 1.20 \quad \text{Thus, infinite life is predicted } (N \geq 10^6 \text{ cycles}). \quad \text{Ans.}$$

6-21

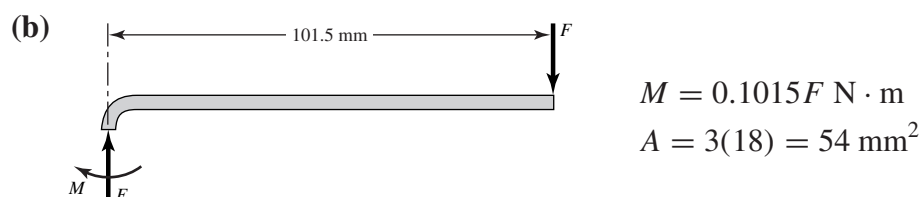
(a)

$$I = \frac{1}{12}(18)(3^3) = 40.5 \text{ mm}^4$$

$$y = \frac{Fl^3}{3EI} \Rightarrow F = \frac{3EIy}{l^3}$$

$$F_{\min} = \frac{3(207)(10^9)(40.5)(10^{-12})(2)(10^{-3})}{(100^3)(10^{-9})} = 50.3 \text{ N} \quad \text{Ans.}$$

$$F_{\max} = \frac{6}{2}(50.3) = 150.9 \text{ N} \quad \text{Ans.}$$



Curved beam: $r_n = \frac{h}{\ln(r_o/r_i)} = \frac{3}{\ln(6/3)} = 4.3281 \text{ mm}$

$$r_c = 4.5 \text{ mm}, \quad e = r_c - r_n = 4.5 - 4.3281 = 0.1719 \text{ mm}$$

$$\sigma_i = -\frac{Mc_i}{Aer_i} - \frac{F}{A} = -\frac{(0.1015F)(1.5 - 0.1719)}{54(0.1719)(3)(10^{-3})} - \frac{F}{54} = -4.859F \text{ MPa}$$

$$\sigma_o = \frac{Mc_o}{Aer_o} - \frac{F}{A} = \frac{(0.1015F)(1.5 + 0.1719)}{54(0.1719)(6)(10^{-3})} - \frac{F}{54} = 3.028F \text{ MPa}$$

$$(\sigma_i)_{\min} = -4.859(150.9) = -733.2 \text{ MPa}$$

$$(\sigma_i)_{\max} = -4.859(50.3) = -244.4 \text{ MPa}$$

$$(\sigma_o)_{\max} = 3.028(150.9) = 456.9 \text{ MPa}$$

$$(\sigma_o)_{\min} = 3.028(50.3) = 152.3 \text{ MPa}$$

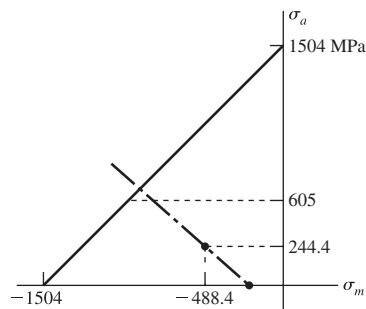
Eq. (2-17) $S_{ut} = 3.41(490) = 1671 \text{ MPa}$

Per the problem statement, estimate the yield as $S_y = 0.9S_{ut} = 0.9(1671) = 1504 \text{ MPa}$. Then from Eq. (6-8), $S'_e = 700 \text{ MPa}$; Eq. (6-19), $k_a = 1.58(1671)^{-0.085} = 0.841$; Eq. (6-25) $d_e = 0.808[18(3)]^{1/2} = 5.938 \text{ mm}$; and Eq. (6-20), $k_b = (5.938/7.62)^{-0.107} = 1.027$.

$$S_e = 0.841(1.027)(700) = 605 \text{ MPa}$$

At Inner Radius $(\sigma_i)_a = \left| \frac{-733.2 + 244.4}{2} \right| = 244.4 \text{ MPa}$

$$(\sigma_i)_m = \frac{-733.2 - 244.4}{2} = -488.8 \text{ MPa}$$



Load line: $\sigma_m = -244.4 - \sigma_a$

Langer (yield) line: $\sigma_m = \sigma_a - 1504 = -244.4 - \sigma_a$

Intersection: $\sigma_a = 629.8 \text{ MPa}, \quad \sigma_m = -874.2 \text{ MPa}$
(Note that σ_a is more than 605 MPa)

Yield: $n_y = \frac{629.8}{244.4} = 2.58$

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Fatigue: $n_f = \frac{605}{244.4} = 2.48$ Thus, the spring is likely to fail in fatigue at the inner radius. *Ans.*

At Outer Radius

$$(\sigma_o)_a = \frac{456.9 - 152.3}{2} = 152.3 \text{ MPa}$$

$$(\sigma_o)_m = \frac{456.9 + 152.3}{2} = 304.6 \text{ MPa}$$

Yield load line: $\sigma_m = 152.3 + \sigma_a$

Langer line: $\sigma_m = 1504 - \sigma_a = 152.3 + \sigma_a$

Intersection: $\sigma_a = 675.9 \text{ MPa}, \quad \sigma_m = 828.2 \text{ MPa}$

$$n_y = \frac{675.9}{152.3} = 4.44$$

Fatigue line: $\sigma_a = [1 - (\sigma_m/S_{ut})^2]S_e = \sigma_m - 152.3$

$$605 \left[1 - \left(\frac{\sigma_m}{1671} \right)^2 \right] = \sigma_m - 152.3$$

$$\sigma_m^2 + 4615.3\sigma_m - 3.4951(10^6) = 0$$

$$\sigma_m = \frac{-4615.3 + \sqrt{4615.3^2 + 4(3.4951)(10^6)}}{2} = 662.2 \text{ MPa}$$

$$\sigma_a = 662.2 - 152.3 = 509.9 \text{ MPa}$$

$$n_f = \frac{509.9}{152.3} = 3.35$$

Thus, the spring is not likely to fail in fatigue at the outer radius. *Ans.*

6-22 The solution at the inner radius is the same as in Prob. 6-21. At the outer radius, the yield solution is the same.

Fatigue line: $\sigma_a = \left(1 - \frac{\sigma_m}{S_{ut}} \right) S_e = \sigma_m - 152.3$

$$605 \left(1 - \frac{\sigma_m}{1671} \right) = \sigma_m - 152.3$$

$$1.362\sigma_m = 757.3 \Rightarrow \sigma_m = 556.0 \text{ MPa}$$

$$\sigma_a = 556.0 - 152.3 = 403.7 \text{ MPa}$$

$$n_f = \frac{403.7}{152.3} = 2.65 \text{ } Ans.$$

6-23 Preliminaries:

Table A-20:

$$S_{ut} = 64 \text{ kpsi}, \quad S_y = 54 \text{ kpsi}$$

$$S'_e = 0.5(64) = 32 \text{ kpsi}$$

$$k_a = 2.70(64)^{-0.265} = 0.897$$

$$k_b = 1$$

$$k_c = 0.85$$

$$S_e = 0.897(1)(0.85)(32) = 24.4 \text{ kpsi}$$

Fillet:

Fig. A-15-5: $D = 3.75 \text{ in}$, $d = 2.5 \text{ in}$, $D/d = 3.75/2.5 = 1.5$, and $r/d = 0.25/2.5 = 0.10$
 $\therefore K_t = 2.1$. Fig. 6-20 with $r = 0.25 \text{ in}$, $q \doteq 0.82$

$$\text{Eq. (6-32):} \quad K_f = 1 + 0.82(2.1 - 1) = 1.90$$

$$\sigma_{\max} = \frac{4}{2.5(0.5)} = 3.2 \text{ kpsi}$$

$$\sigma_{\min} = \frac{-16}{2.5(0.5)} = -12.8 \text{ kpsi}$$

$$\sigma_a = 1.90 \left| \frac{3.2 - (-12.8)}{2} \right| = 15.2 \text{ kpsi}$$

$$\sigma_m = 1.90 \left[\frac{3.2 + (-12.8)}{2} \right] = -9.12 \text{ kpsi}$$

$$n_y = \left| \frac{S_y}{\sigma_{\min}} \right| = \left| \frac{54}{-12.8} \right| = 4.22$$

Since the midrange stress is negative,

$$S_a = S_e = 24.4 \text{ kpsi}$$

$$n_f = \frac{S_a}{\sigma_a} = \frac{24.4}{15.2} = 1.61$$

Hole:

Fig. A-15-1: $d/w = 0.75/3.75 = 0.20$, $K_t = 2.5$. Fig. 6-20, with $r = 0.375 \text{ in}$, $q \doteq 0.85$

$$\text{Eq. (6-32):} \quad K_f = 1 + 0.85(2.5 - 1) = 2.28$$

$$\sigma_{\max} = \frac{4}{0.5(3.75 - 0.75)} = 2.67 \text{ kpsi}$$

$$\sigma_{\min} = \frac{-16}{0.5(3.75 - 0.75)} = -10.67 \text{ kpsi}$$

$$\sigma_a = 2.28 \left| \frac{2.67 - (-10.67)}{2} \right| = 15.2 \text{ kpsi}$$

$$\sigma_m = 2.28 \frac{2.67 + (-10.67)}{2} = -9.12 \text{ kpsi}$$

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Since the midrange stress is negative,

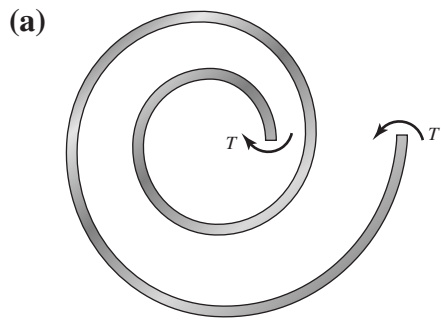
$$n_y = \left| \frac{S_y}{\sigma_{\min}} \right| = \left| \frac{54}{-10.67} \right| = 5.06$$

$$S_a = S_e = 24.4 \text{ kpsi}$$

$$n_f = \frac{S_a}{\sigma_a} = \frac{24.4}{15.2} = 1.61$$

Thus the design is controlled by the threat of fatigue equally at the fillet and the hole; the minimum factor of safety is $n_f = 1.61$. *Ans.*

6-24



Curved beam in pure bending where $M = -T$ throughout. The maximum stress will occur at the inner fiber where $r_c = 20 \text{ mm}$, but will be compressive. The maximum tensile stress will occur at the outer fiber where $r_c = 60 \text{ mm}$. Why?

Inner fiber where $r_c = 20 \text{ mm}$

$$r_n = \frac{h}{\ln(r_o/r_i)} = \frac{5}{\ln(22.5/17.5)} = 19.8954 \text{ mm}$$

$$e = 20 - 19.8954 = 0.1046 \text{ mm}$$

$$c_i = 19.8954 - 17.5 = 2.395 \text{ mm}$$

$$A = 25 \text{ mm}^2$$

$$\sigma_i = \frac{Mc_i}{Aer_i} = \frac{-T(2.395)10^{-3}}{25(10^{-6})0.1046(10^{-3})17.5(10^{-3})}(10^{-6}) = -52.34 T \quad (1)$$

where T is in $\text{N} \cdot \text{m}$, and σ_i is in MPa .

$$\sigma_m = \frac{1}{2}(-52.34T) = -26.17T, \quad \sigma_a = 26.17T$$

For the endurance limit, $S'_e = 0.5(770) = 385 \text{ MPa}$

$$k_a = 4.51(770)^{-0.265} = 0.775$$

$$d_e = 0.808[5(5)]^{1/2} = 4.04 \text{ mm}$$

$$k_b = (4.04/7.62)^{-0.107} = 1.07$$

$$S_e = 0.775(1.07)385 = 319.3 \text{ MPa}$$

For a compressive midrange component, $\sigma_a = S_e/n_f$. Thus,

$$26.17T = 319.3/3 \Rightarrow T = 4.07 \text{ N} \cdot \text{m}$$

Outer fiber where $r_c = 60 \text{ mm}$

$$r_n = \frac{5}{\ln(62.5/57.5)} = 59.96526 \text{ mm}$$

$$e = 60 - 59.96526 = 0.03474 \text{ mm}$$

$$c_o = 62.5 - 59.96526 = 2.535 \text{ mm}$$

$$\sigma_o = -\frac{Mc_i}{Aer_i} = -\frac{-T(2.535)10^{-3}}{25(10^{-6})0.03474(10^{-3})62.5(10^{-3})}(10^{-6}) = 46.7 T$$

Comparing this with Eq. (1), we see that it is less in magnitude, but the midrange component is *tension*.

$$\sigma_a = \sigma_m = \frac{1}{2}(46.7T) = 23.35T$$

Using Eq. (6-46), for modified Goodman, we have

$$\frac{23.35T}{319.3} + \frac{23.35T}{770} = \frac{1}{3} \Rightarrow T = 3.22 \text{ N} \cdot \text{m} \quad \text{Ans.}$$

(b) Gerber, Eq. (6-47), at the outer fiber,

$$\frac{3(23.35T)}{319.3} + \left[\frac{3(23.35T)}{770} \right]^2 = 1$$

$$\text{reduces to} \quad T^2 + 26.51T - 120.83 = 0$$

$$T = \frac{1}{2} \left(-26.51 + \sqrt{26.51^2 + 4(120.83)} \right) = 3.96 \text{ N} \cdot \text{m} \quad \text{Ans.}$$

(c) To guard against yield, use T of part (b) and the inner stress.

$$n_y = \frac{420}{52.34(3.96)} = 2.03 \quad \text{Ans.}$$

6-25 From Prob. 6-24, $S_e = 319.3 \text{ MPa}$, $S_y = 420 \text{ MPa}$, and $S_{ut} = 770 \text{ MPa}$

(a) Assuming the beam is straight,

$$\sigma_{\max} = \frac{6M}{bh^2} = \frac{6T}{5^3[(10^{-3})^3]} = 48(10^6)T$$

$$\text{Goodman:} \quad \frac{24T}{319.3} + \frac{24T}{770} = \frac{1}{3} \Rightarrow T = 3.13 \text{ N} \cdot \text{m} \quad \text{Ans.}$$

$$\text{(b) Gerber:} \quad \frac{3(24)T}{319.3} + \left[\frac{3(24)T}{770} \right]^2 = 1$$

$$T^2 + 25.79T - 114.37 = 1$$

$$T = \frac{1}{2} \left[-25.79 + \sqrt{25.79^2 + 4(114.37)} \right] = 3.86 \text{ N} \cdot \text{m} \quad \text{Ans.}$$

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(c) Using $\sigma_{\max} = 52.34(10^6)T$ from Prob. 6-24,

$$n_y = \frac{420}{52.34(3.86)} = 2.08 \quad \text{Ans.}$$

6-26

(a)
$$\tau_{\max} = \frac{16K_{fs}T_{\max}}{\pi d^3}$$

Fig. 6-21 for $H_B > 200$, $r = 3$ mm, $q_s \doteq 1$

$$K_{fs} = 1 + q_s(K_{ts} - 1)$$

$$K_{fs} = 1 + 1(1.6 - 1) = 1.6$$

$$T_{\max} = 2000(0.05) = 100 \text{ N} \cdot \text{m}, \quad T_{\min} = \frac{500}{2000}(100) = 25 \text{ N} \cdot \text{m}$$

$$\tau_{\max} = \frac{16(1.6)(100)(10^{-6})}{\pi(0.02)^3} = 101.9 \text{ MPa}$$

$$\tau_{\min} = \frac{500}{2000}(101.9) = 25.46 \text{ MPa}$$

$$\tau_m = \frac{1}{2}(101.9 + 25.46) = 63.68 \text{ MPa}$$

$$\tau_a = \frac{1}{2}(101.9 - 25.46) = 38.22 \text{ MPa}$$

$$S_{su} = 0.67S_{ut} = 0.67(320) = 214.4 \text{ MPa}$$

$$S_{sy} = 0.577S_y = 0.577(180) = 103.9 \text{ MPa}$$

$$S'_e = 0.5(320) = 160 \text{ MPa}$$

$$k_a = 57.7(320)^{-0.718} = 0.917$$

$$d_e = 0.370(20) = 7.4 \text{ mm}$$

$$k_b = \left(\frac{7.4}{7.62}\right)^{-0.107} = 1.003$$

$$k_c = 0.59$$

$$S_e = 0.917(1.003)(0.59)(160) = 86.8 \text{ MPa}$$

Modified Goodman, Table 6-6

$$n_f = \frac{1}{(\tau_a/S_e) + (\tau_m/S_{su})} = \frac{1}{(38.22/86.8) + (63.68/214.4)} = 1.36 \quad \text{Ans.}$$

(b) Gerber, Table 6-7

$$\begin{aligned} n_f &= \frac{1}{2} \left(\frac{S_{su}}{\tau_m} \right)^2 \frac{\tau_a}{S_e} \left[-1 + \sqrt{1 + \left(\frac{2\tau_m S_e}{S_{su} \tau_a} \right)^2} \right] \\ &= \frac{1}{2} \left(\frac{214.4}{63.68} \right)^2 \frac{38.22}{86.8} \left\{ -1 + \sqrt{1 + \left[\frac{2(63.68)(86.8)}{214.4(38.22)} \right]^2} \right\} = 1.70 \quad \text{Ans.} \end{aligned}$$

6-27 $S_y = 800 \text{ MPa}$, $S_{ut} = 1000 \text{ MPa}$

(a) From Fig. 6-20, for a notch radius of 3 mm and $S_{ut} = 1 \text{ GPa}$, $q \doteq 0.92$.

$$K_f = 1 + q(K_t - 1) = 1 + 0.92(3 - 1) = 2.84$$

$$\sigma_{\max} = -K_f \frac{4P}{\pi d^2} = -\frac{2.84(4)P}{\pi(0.030)^2} = -4018P$$

$$\sigma_m = -\sigma_a = \frac{1}{2}(-4018P) = -2009P$$

$$T = fP \left(\frac{D+d}{4} \right)$$

$$T_{\max} = 0.3P \left(\frac{0.150 + 0.03}{4} \right) = 0.0135P$$

From Fig. 6-21, $q_s \doteq 0.95$. Also, K_{ts} is given as 1.8. Thus,

$$K_{fs} = 1 + q_s(K_{ts} - 1) = 1 + 0.95(1.8 - 1) = 1.76$$

$$\tau_{\max} = \frac{16K_{fs}T}{\pi d^3} = \frac{16(1.76)(0.0135P)}{\pi(0.03)^3} = 4482P$$

$$\tau_a = \tau_m = \frac{1}{2}(4482P) = 2241P$$

Eqs. (6-55) and (6-56):

$$\sigma'_a = \sigma'_m = [(\sigma_a/0.85)^2 + 3\tau_a^2]^{1/2} = [(-2009P/0.85)^2 + 3(2241P)^2]^{1/2} = 4545P$$

$$S'_e = 0.5(1000) = 500 \text{ MPa}$$

$$k_a = 4.51(1000)^{-0.265} = 0.723$$

$$k_b = \left(\frac{30}{7.62} \right)^{-0.107} = 0.864$$

$$S_e = 0.723(0.864)(500) = 312.3 \text{ MPa}$$

Modified Goodman: $\frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}} = \frac{1}{n}$

$$\frac{4545P}{312.3(10^6)} + \frac{4545P}{1000(10^6)} = \frac{1}{3} \Rightarrow P = 17.5(10^3) \text{ N} = 17.5 \text{ kN} \quad \text{Ans.}$$

Yield (conservative): $n_y = \frac{S_y}{\sigma'_a + \sigma'_m}$

$$n_y = \frac{800(10^6)}{2(4545)(17.5)(10^3)} = 5.03 \quad \text{Ans.}$$

(actual): $\sigma'_{\max} = (\sigma_{\max}^2 + 3\tau_{\max}^2)^{1/2} = [(-4018P)^2 + 3(4482P)^2]^{1/2}$

$$= 8741P$$

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{800(10^6)}{8741(17.5)10^3} = 5.22$$

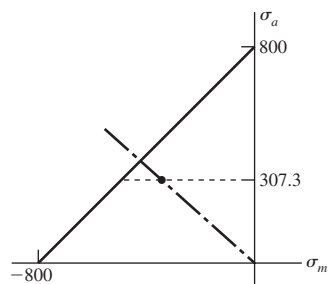
(b) If the shaft is not rotating, $\tau_m = \tau_a = 0$.

$$\sigma_m = -\sigma_a = -2009P$$

$$k_b = 1 \quad (\text{axial})$$

$$k_c = 0.85 \quad (\text{Since there is no tension, } k_c = 1 \text{ might be more appropriate.})$$

$$S_e = 0.723(1)(0.85)(500) = 307.3 \text{ MPa}$$



$$n_f = \frac{307.3(10^6)}{2009P} \Rightarrow P = \frac{307.3(10^6)}{3(2009)} = 51.0(10^3) \text{ N} \\ = 51.0 \text{ kN} \quad \text{Ans.}$$

$$\text{Yield:} \quad n_y = \frac{800(10^6)}{2(2009)(51.0)(10^3)} = 3.90 \quad \text{Ans.}$$

6-28 From Prob. 6-27, $K_f = 2.84$, $K_{fs} = 1.76$, $S_e = 312.3 \text{ MPa}$

$$\sigma_{\max} = -K_f \frac{4P_{\max}}{\pi d^2} = -2.84 \left[\frac{(4)(80)(10^{-3})}{\pi(0.030)^2} \right] = -321.4 \text{ MPa}$$

$$\sigma_{\min} = \frac{20}{80}(-321.4) = -80.4 \text{ MPa}$$

$$T_{\max} = f P_{\max} \left(\frac{D+d}{4} \right) = 0.3(80)(10^3) \left(\frac{0.150 + 0.03}{4} \right) = 1080 \text{ N} \cdot \text{m}$$

$$T_{\min} = \frac{20}{80}(1080) = 270 \text{ N} \cdot \text{m}$$

$$\tau_{\max} = K_{fs} \frac{16T_{\max}}{\pi d^3} = 1.76 \left[\frac{16(1080)}{\pi(0.030)^3} (10^{-6}) \right] = 358.5 \text{ MPa}$$

$$\tau_{\min} = \frac{20}{80}(358.5) = 89.6 \text{ MPa}$$

$$\sigma_a = \frac{321.4 - 80.4}{2} = 120.5 \text{ MPa}$$

$$\sigma_m = \frac{-321.4 - 80.4}{2} = -200.9 \text{ MPa}$$

$$\tau_a = \frac{358.5 - 89.6}{2} = 134.5 \text{ MPa}$$

$$\tau_m = \frac{358.5 + 89.6}{2} = 224.1 \text{ MPa}$$

Eqs. (6-55) and (6-56):

$$\sigma'_a = [(\sigma_a/0.85)^2 + 3\tau_a^2]^{1/2} = [(120.5/0.85)^2 + 3(134.5)^2]^{1/2} = 272.7 \text{ MPa}$$

$$\sigma'_m = [(-200.9/0.85)^2 + 3(224.1)^2]^{1/2} = 454.5 \text{ MPa}$$

Goodman:

$$(\sigma_a)_e = \frac{\sigma'_a}{1 - \sigma'_m/S_{ut}} = \frac{272.7}{1 - 454.5/1000} = 499.9 \text{ MPa}$$

Let $f = 0.9$

$$a = \frac{[0.9(1000)]^2}{312.3} = 2594 \text{ MPa}$$

$$b = -\frac{1}{3} \log \left[\frac{0.9(1000)}{312.3} \right] = -0.1532$$

$$N = \left[\frac{(\sigma_a)_e}{a} \right]^{1/b} = \left[\frac{499.9}{2594} \right]^{1/-0.1532} = 46\,520 \text{ cycles} \quad \text{Ans.}$$

6-29

$$S_y = 490 \text{ MPa}, \quad S_{ut} = 590 \text{ MPa}, \quad S_e = 200 \text{ MPa}$$

$$\sigma_m = \frac{420 + 140}{2} = 280 \text{ MPa}, \quad \sigma_a = \frac{420 - 140}{2} = 140 \text{ MPa}$$

Goodman:

$$(\sigma_a)_e = \frac{\sigma_a}{1 - \sigma_m/S_{ut}} = \frac{140}{1 - (280/590)} = 266.5 \text{ MPa} > S_e \therefore \text{finite life}$$

$$a = \frac{[0.9(590)]^2}{200} = 1409.8 \text{ MPa}$$

$$b = -\frac{1}{3} \log \frac{0.9(590)}{200} = -0.141\,355$$

$$N = \left(\frac{266.5}{1409.8} \right)^{-1/0.143\,55} = 131\,200 \text{ cycles}$$

$$N_{\text{remaining}} = 131\,200 - 50\,000 = 81\,200 \text{ cycles}$$

Second loading: $(\sigma_m)_2 = \frac{350 + (-200)}{2} = 75 \text{ MPa}$

$$(\sigma_a)_2 = \frac{350 - (-200)}{2} = 275 \text{ MPa}$$

$$(\sigma_a)_{e2} = \frac{275}{1 - (75/590)} = 315.0 \text{ MPa}$$

(a) Miner's method

$$N_2 = \left(\frac{315}{1409.8} \right)^{-1/0.141355} = 40\,200 \text{ cycles}$$

$$\frac{n_1}{N_1} + \frac{n_2}{N_2} = 1 \Rightarrow \frac{50\,000}{131\,200} + \frac{n_2}{40\,200} = 1$$

$$n_2 = 24\,880 \text{ cycles } \textit{Ans.}$$

(b) Manson's method

Two data points: 0.9(590 MPa), 10^3 cycles
266.5 MPa, 81 200 cycles

$$\frac{0.9(590)}{266.5} = \frac{a_2(10^3)^{b_2}}{a_2(81\,200)^{b_2}}$$

$$1.9925 = (0.012\,315)^{b_2}$$

$$b_2 = \frac{\log 1.9925}{\log 0.012\,315} = -0.156\,789$$

$$a_2 = \frac{266.5}{(81\,200)^{-0.156\,789}} = 1568.4 \text{ MPa}$$

$$n_2 = \left(\frac{315}{1568.4} \right)^{1/-0.156\,789} = 27\,950 \text{ cycles } \textit{Ans.}$$

6-30 (a) Miner's method

$$a = \frac{[0.9(76)]^2}{30} = 155.95 \text{ kpsi}$$

$$b = -\frac{1}{3} \log \frac{0.9(76)}{30} = -0.119\,31$$

$$\sigma_1 = 48 \text{ kpsi}, \quad N_1 = \left(\frac{48}{155.95} \right)^{1/-0.119\,31} = 19\,460 \text{ cycles}$$

$$\sigma_2 = 38 \text{ kpsi}, \quad N_2 = \left(\frac{38}{155.95} \right)^{1/-0.119\,31} = 137\,880 \text{ cycles}$$

$$\sigma_3 = 32 \text{ kpsi}, \quad N_3 = \left(\frac{32}{155.95} \right)^{1/-0.119\,31} = 582\,150 \text{ cycles}$$

$$\frac{n_1}{N_1} + \frac{n_2}{N_2} + \frac{n_3}{N_3} = 1$$

$$\frac{4000}{19\,460} + \frac{60\,000}{137\,880} + \frac{n_3}{582\,150} = 1 \Rightarrow n_3 = 209\,160 \text{ cycles } \textit{Ans.}$$

(b) Manson's method

The life remaining after the first cycle is $N_{R_1} = 19\,460 - 4000 = 15\,460$ cycles. The two data points required to define $S'_{e,1}$ are $[0.9(76), 10^3]$ and $(48, 15\,460)$.

$$\frac{0.9(76)}{48} = \frac{a_2(10^3)^{b_2}}{a_2(15\,460)^{b_2}} \Rightarrow 1.425 = (0.064\,683)^{b_2}$$

$$\begin{aligned}
 b_2 &= \frac{\log(1.425)}{\log(0.064\,683)} = -0.129\,342 \\
 a_2 &= \frac{48}{(15\,460)^{-0.129\,342}} = 167.14 \text{ kpsi} \\
 N_2 &= \left(\frac{38}{167.14} \right)^{-1/0.129\,342} = 94\,110 \text{ cycles} \\
 N_{R_2} &= 94\,110 - 60\,000 = 34\,110 \text{ cycles} \\
 \frac{0.9(76)}{38} &= \frac{a_3(10^3)^{b_3}}{a_3(34\,110)^{b_3}} \Rightarrow 1.8 = (0.029\,317)^{b_3} \\
 b_3 &= \frac{\log 1.8}{\log(0.029\,317)} = -0.166\,531, \quad a_3 = \frac{38}{(34\,110)^{-0.166\,531}} = 216.10 \text{ kpsi} \\
 N_3 &= \left(\frac{32}{216.1} \right)^{-1/0.166\,531} = 95\,740 \text{ cycles} \quad \text{Ans.}
 \end{aligned}$$

6-31 Using Miner's method

$$\begin{aligned}
 a &= \frac{[0.9(100)]^2}{50} = 162 \text{ kpsi} \\
 b &= -\frac{1}{3} \log \frac{0.9(100)}{50} = -0.085\,091 \\
 \sigma_1 &= 70 \text{ kpsi}, \quad N_1 = \left(\frac{70}{162} \right)^{1/-0.085\,091} = 19\,170 \text{ cycles} \\
 \sigma_2 &= 55 \text{ kpsi}, \quad N_2 = \left(\frac{55}{162} \right)^{1/-0.085\,091} = 326\,250 \text{ cycles} \\
 \sigma_3 &= 40 \text{ kpsi}, \quad N_3 \rightarrow \infty \\
 \frac{0.2N}{19\,170} + \frac{0.5N}{326\,250} + \frac{0.3N}{\infty} &= 1 \\
 N &= 83\,570 \text{ cycles} \quad \text{Ans.}
 \end{aligned}$$

6-32 Given $S_{ut} = 245\text{LN}(1, 0.0508)$ kpsiFrom Table 7-13: $a = 1.34, b = -0.086, C = 0.12$

$$\begin{aligned}
 k_a &= 1.34\bar{S}_{ut}^{-0.086}\text{LN}(1, 0.120) \\
 &= 1.34(245)^{-0.086}\text{LN}(1, 0.12) \\
 &= 0.835\text{LN}(1, 0.12)
 \end{aligned}$$

$$k_b = 1.02 \quad (\text{as in Prob. 6-1})$$

$$\text{Eq. (6-70)} \quad S_e = 0.835(1.02)\text{LN}(1, 0.12)[107\text{LN}(1, 0.139)]$$

$$\bar{S}_e = 0.835(1.02)(107) = 91.1 \text{ kpsi}$$

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Now

$$C_{Se} \doteq (0.12^2 + 0.139^2)^{1/2} = 0.184$$

$$S_e = 91.1 \text{LN}(1, 0.184) \text{ kpsi} \quad \text{Ans.}$$

6-33 A Priori Decisions:

- Material and condition: 1018 CD, $S_{ut} = 440 \text{LN}(1, 0.03)$, and $S_y = 370 \text{LN}(1, 0.061)$ MPa
- Reliability goal: $R = 0.999$ ($z = -3.09$)
- Function:

Critical location—hole

- Variabilities:

$$C_{ka} = 0.058$$

$$C_{kc} = 0.125$$

$$C_\phi = 0.138$$

$$C_{Se} = (C_{ka}^2 + C_{kc}^2 + C_\phi^2)^{1/2} = (0.058^2 + 0.125^2 + 0.138^2)^{1/2} = 0.195$$

$$C_{kc} = 0.10$$

$$C_{Fa} = 0.20$$

$$C_{\sigma a} = (0.10^2 + 0.20^2)^{1/2} = 0.234$$

$$C_n = \sqrt{\frac{C_{Se}^2 + C_{\sigma a}^2}{1 + C_{\sigma a}^2}} = \sqrt{\frac{0.195^2 + 0.234^2}{1 + 0.234^2}} = 0.297$$

Resulting in a design factor n_f of,

$$\text{Eq. (6-88): } n_f = \exp[-(-3.09)\sqrt{\ln(1 + 0.297^2)} + \ln \sqrt{1 + 0.297^2}] = 2.56$$

- Decision: Set $n_f = 2.56$

Now proceed deterministically using the mean values:

$$\text{Table 6-10: } \bar{k}_a = 4.45(440)^{-0.265} = 0.887$$

$$k_b = 1$$

$$\text{Table 6-11: } \bar{k}_c = 1.43(440)^{-0.0778} = 0.891$$

$$\text{Eq. (6-70): } \bar{S}'_e = 0.506(440) = 222.6 \text{ MPa}$$

$$\text{Eq. (6-71): } \bar{S}_e = 0.887(1)0.891(222.6) = 175.9 \text{ MPa}$$

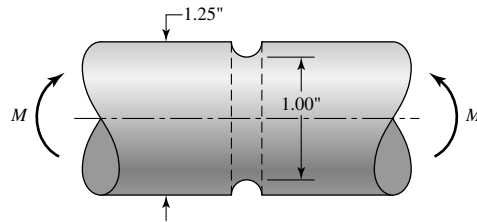
From Prob. 6-10, $K_f = 2.23$. Thus,

$$\bar{\sigma}_a = \bar{K}_f \frac{\bar{F}_a}{A} = \bar{K}_f \frac{\bar{F}_a}{t(60 - 12)} = \frac{\bar{S}_e}{\bar{n}_f}$$

$$\text{and, } t = \frac{\bar{n}_f \bar{K}_f \bar{F}_a}{48 \bar{S}_e} = \frac{2.56(2.23)15(10^3)}{48(175.9)} = 10.14 \text{ mm}$$

Decision: Depending on availability, (1) select $t = 10$ mm, recalculate n_f and R , and determine whether the reduced reliability is acceptable, or, (2) select $t = 11$ mm or larger, and determine whether the increase in cost and weight is acceptable. *Ans.*

6-34



Rotation is presumed. M and S_{ut} are given as deterministic, but notice that σ is not; therefore, a reliability estimation can be made.

From Eq. (6-70):

$$\begin{aligned} S'_e &= 0.506(110)\text{LN}(1, 0.138) \\ &= 55.7\text{LN}(1, 0.138) \text{ kpsi} \end{aligned}$$

Table 6-10:

$$\begin{aligned} k_a &= 2.67(110)^{-0.265}\text{LN}(1, 0.058) \\ &= 0.768\text{LN}(1, 0.058) \end{aligned}$$

Based on $d = 1$ in, Eq. (6-20) gives

$$k_b = \left(\frac{1}{0.30}\right)^{-0.107} = 0.879$$

Conservatism is not necessary

$$\begin{aligned} S_e &= 0.768[\text{LN}(1, 0.058)](0.879)(55.7)[\text{LN}(1, 0.138)] \\ \bar{S}_e &= 37.6 \text{ kpsi} \\ C_{Se} &= (0.058^2 + 0.138^2)^{1/2} = 0.150 \\ S_e &= 37.6\text{LN}(1, 0.150) \end{aligned}$$

Fig. A-15-14: $D/d = 1.25$, $r/d = 0.125$. Thus $K_t = 1.70$ and Eqs. (6-78), (6-79) and Table 6-15 give

$$\begin{aligned} K_f &= \frac{1.70\text{LN}(1, 0.15)}{1 + (2/\sqrt{0.125})[(1.70 - 1)/(1.70)](3/110)} \\ &= 1.598\text{LN}(1, 0.15) \\ \sigma &= K_f \frac{32M}{\pi d^3} = 1.598[\text{LN}(1 - 0.15)] \left[\frac{32(1400)}{\pi(1)^3} \right] \\ &= 22.8\text{LN}(1, 0.15) \text{ kpsi} \end{aligned}$$

From Eq. (5-43), p. 242:

$$z = -\frac{\ln \left[(37.6/22.8) \sqrt{(1 + 0.15^2)/(1 + 0.15^2)} \right]}{\sqrt{\ln[(1 + 0.15^2)(1 + 0.15^2)]}} = -2.37$$

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From Table A-10, $p_f = 0.00889$

$$\therefore R = 1 - 0.00889 = 0.991 \quad \text{Ans.}$$

Note: The correlation method uses only the mean of S_{ut} ; its variability is already included in the 0.138. When a deterministic load, in this case M , is used in a reliability estimate, engineers state, "For a *Design Load* of M , the reliability is 0.991." They are in fact referring to a *Deterministic Design Load*.

- 6-35** For completely reversed torsion, k_a and k_b of Prob. 6-34 apply, but k_c must also be considered.

$$\begin{aligned} \text{Eq. 6-74:} \quad k_c &= 0.328(110)^{0.125} \text{LN}(1, 0.125) \\ &= 0.590 \text{LN}(1, 0.125) \end{aligned}$$

Note 0.590 is close to 0.577.

$$\begin{aligned} S_{Se} &= k_a k_b k_c S'_e \\ &= 0.768[\text{LN}(1, 0.058)](0.878)[0.590 \text{LN}(1, 0.125)][55.7 \text{LN}(1, 0.138)] \\ \bar{S}_{Se} &= 0.768(0.878)(0.590)(55.7) = 22.2 \text{ kpsi} \\ C_{Se} &= (0.058^2 + 0.125^2 + 0.138^2)^{1/2} = 0.195 \\ S_{Se} &= 22.2 \text{LN}(1, 0.195) \text{ kpsi} \end{aligned}$$

Fig. A-15-15: $D/d = 1.25$, $r/d = 0.125$, then $K_{ts} = 1.40$. From Eqs. (6-78), (6-79) and Table 6-15

$$\begin{aligned} K_{ts} &= \frac{1.40 \text{LN}(1, 0.15)}{1 + (2/\sqrt{0.125}) [(1.4 - 1)/1.4](3/110)} = 1.34 \text{LN}(1, 0.15) \\ \tau &= K_{ts} \frac{16T}{\pi d^3} \\ \tau &= 1.34[\text{LN}(1, 0.15)] \left[\frac{16(1.4)}{\pi(1)^3} \right] \\ &= 9.55 \text{LN}(1, 0.15) \text{ kpsi} \end{aligned}$$

From Eq. (5-43), p. 242:

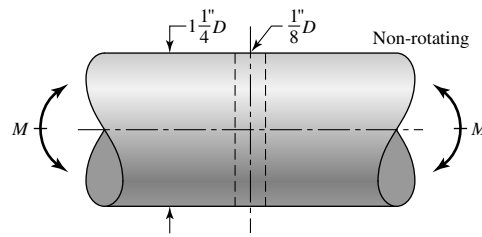
$$z = -\frac{\ln \left[(22.2/9.55) \sqrt{(1 + 0.15^2)/(1 + 0.195^2)} \right]}{\sqrt{\ln [(1 + 0.195^2)(1 + 0.15^2)]}} = -3.43$$

From Table A-10, $p_f = 0.0003$

$$R = 1 - p_f = 1 - 0.0003 = 0.9997 \quad \text{Ans.}$$

For a design with completely-reversed torsion of 1400 lbf · in, the reliability is 0.9997. The improvement comes from a smaller stress-concentration factor in torsion. See the note at the end of the solution of Prob. 6-34 for the reason for the phraseology.

6-36



$$S_{ut} = 58 \text{ kpsi}$$

$$S'_e = 0.506(58)\text{LN}(1, 0.138)$$

$$= 29.3\text{LN}(1, 0.138) \text{ kpsi}$$

Table 6-10:

$$k_a = 14.5(58)^{-0.719}\text{LN}(1, 0.11)$$

$$= 0.782\text{LN}(1, 0.11)$$

Eq. (6-24):

$$d_e = 0.37(1.25) = 0.463 \text{ in}$$

$$k_b = \left(\frac{0.463}{0.30} \right)^{-0.107} = 0.955$$

$$S_e = 0.782[\text{LN}(1, 0.11)](0.955)[29.3\text{LN}(1, 0.138)]$$

$$\bar{S}_e = 0.782(0.955)(29.3) = 21.9 \text{ kpsi}$$

$$C_{Se} = (0.11^2 + 0.138^2)^{1/2} = 0.150$$

Table A-16: $d/D = 0$, $a/D = 0.1$, $A = 0.83 \therefore K_t = 2.27$.

From Eqs. (6-78) and (6-79) and Table 6-15

$$\mathbf{K}_f = \frac{2.27\text{LN}(1, 0.10)}{1 + (2/\sqrt{0.125})[(2.27 - 1)/2.27](5/58)} = 1.783\text{LN}(1, 0.10)$$

Table A-16:

$$Z = \frac{\pi A D^3}{32} = \frac{\pi(0.83)(1.25^3)}{32} = 0.159 \text{ in}^3$$

$$\sigma = \mathbf{K}_f \frac{M}{Z} = 1.783\text{LN}(1, 0.10) \left(\frac{1.6}{0.159} \right)$$

$$= 17.95\text{LN}(1, 0.10) \text{ kpsi}$$

$$\bar{\sigma} = 17.95 \text{ kpsi}$$

$$C_\sigma = 0.10$$

Eq. (5-43), p. 242:

$$z = -\frac{\ln[(21.9/17.95)\sqrt{(1 + 0.10^2)/(1 + 0.15^2)}]}{\sqrt{\ln[(1 + 0.15^2)(1 + 0.10^2)]}} = -1.07$$

Table A-10:

$$p_f = 0.1423$$

$$R = 1 - p_f = 1 - 0.1423 = 0.858 \text{ Ans.}$$

For a completely-reversed design load M_a of 1400 lbf · in, the reliability estimate is 0.858.

174 Solutions Manual • Instructor's Solution Manual to Accompany Mechanical Engineering Design**6-37** For a non-rotating bar subjected to completely reversed torsion of $T_a = 2400 \text{ lbf} \cdot \text{in}$

From Prob. 6-36:

$$S'_e = 29.3 \text{LN}(1, 0.138) \text{ kpsi}$$

$$\mathbf{k}_a = 0.782 \text{LN}(1, 0.11)$$

$$k_b = 0.955$$

For $\mathbf{k}_c$ use Eq. (6-74):

$$\mathbf{k}_c = 0.328(58)^{0.125} \text{LN}(1, 0.125)$$

$$= 0.545 \text{LN}(1, 0.125)$$

$$\mathbf{S}_{Se} = 0.782[\text{LN}(1, 0.11)](0.955)[0.545 \text{LN}(1, 0.125)][29.3 \text{LN}(1, 0.138)]$$

$$\bar{S}_{Se} = 0.782(0.955)(0.545)(29.3) = 11.9 \text{ kpsi}$$

$$C_{Se} = (0.11^2 + 0.125^2 + 0.138^2)^{1/2} = 0.216$$

Table A-16: $d/D = 0$, $a/D = 0.1$, $A = 0.92$, $K_{ts} = 1.68$

From Eqs. (6-78), (6-79), Table 6-15

$$\begin{aligned} \mathbf{K}_{fs} &= \frac{1.68 \text{LN}(1, 0.10)}{1 + (2/\sqrt{0.125})[(1.68 - 1)/1.68](5/58)} \\ &= 1.403 \text{LN}(1, 0.10) \end{aligned}$$

Table A-16:

$$J_{\text{net}} = \frac{\pi A D^4}{32} = \frac{\pi(0.92)(1.25^4)}{32} = 0.2201$$

$$\begin{aligned} \tau_a &= \mathbf{K}_{fs} \frac{T_a c}{J_{\text{net}}} \\ &= 1.403[\text{LN}(1, 0.10)] \left[\frac{2.4(1.25/2)}{0.2201} \right] \\ &= 9.56 \text{LN}(1, 0.10) \text{ kpsi} \end{aligned}$$

From Eq. (5-43), p. 242:

$$z = -\frac{\ln \left[(11.9/9.56) \sqrt{(1 + 0.10^2)/(1 + 0.216^2)} \right]}{\sqrt{\ln[(1 + 0.10^2)(1 + 0.216^2)]}} = -0.85$$

Table A-10, $p_f = 0.1977$

$$R = 1 - p_f = 1 - 0.1977 = 0.80 \quad \text{Ans.}$$

6-38 This is a very important task for the student to attempt before starting Part 3. It illustrates the drawback of the deterministic factor of safety method. It also identifies the a priori decisions and their consequences.

The range of force fluctuation in Prob. 6-23 is -16 to $+4$ kip, or 20 kip. Repeatedly-applied F_a is 10 kip. The stochastic properties of this heat of AISI 1018 CD are given.

Function	Consequences
Axial	$F_a = 10 \text{ kip}$
Fatigue load	$C_{Fa} = 0$
	$C_{kc} = 0.125$
Overall reliability $R \geq 0.998$; with twin fillets	$z = -3.09$
$R \geq \sqrt{0.998} \geq 0.999$	$C_{Kf} = 0.11$
Cold rolled or machined surfaces	$C_{ka} = 0.058$
Ambient temperature	$C_{kd} = 0$
Use correlation method	$C_{\phi} = 0.138$
Stress amplitude	$C_{Kf} = 0.11$
	$C_{\sigma a} = 0.11$
Significant strength S_e	$C_{Se} = (0.058^2 + 0.125^2 + 0.138^2)^{1/2}$ $= 0.195$

Choose the mean design factor which will meet the reliability goal

$$C_n = \sqrt{\frac{0.195^2 + 0.11^2}{1 + 0.11^2}} = 0.223$$

$$\bar{n} = \exp[-(-3.09)\sqrt{\ln(1 + 0.223^2)} + \ln \sqrt{1 + 0.223^2}]$$

$$\bar{n} = 2.02$$

Review the number and quantitative consequences of the designer's a priori decisions to accomplish this. The operative equation is the definition of the design factor

$$\sigma_a = \frac{S_e}{n}$$

$$\bar{\sigma}_a = \frac{\bar{S}_e}{\bar{n}} \Rightarrow \frac{\bar{K}_f F_a}{w_2 h} = \frac{\bar{S}_e}{\bar{n}}$$

Solve for thickness h . To do so we need

$$\bar{k}_a = 2.67 \bar{S}_{ut}^{-0.265} = 2.67(64)^{-0.265} = 0.887$$

$$k_b = 1$$

$$\bar{k}_c = 1.23 \bar{S}_{ut}^{-0.078} = 1.23(64)^{-0.078} = 0.889$$

$$\bar{k}_d = \bar{k}_e = 1$$

$$\bar{S}_e = 0.887(1)(0.889)(1)(1)(0.506)(64) = 25.5 \text{ kpsi}$$

Fig. A-15-5: $D = 3.75 \text{ in}$, $d = 2.5 \text{ in}$, $D/d = 3.75/2.5 = 1.5$, $r/d = 0.25/2.5 = 0.10$

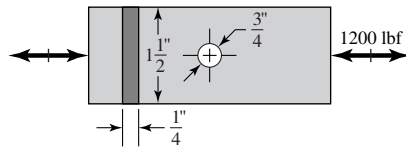
$$\therefore K_t = 2.1$$

$$\bar{K}_f = \frac{2.1}{1 + (2/\sqrt{0.25})[(2.1 - 1)/(2.1)](4/64)} = 1.857$$

$$h = \frac{\bar{K}_f \bar{n} F_a}{w_2 \bar{S}_e} = \frac{1.857(2.02)(10)}{2.5(25.5)} = 0.667 \text{ Ans.}$$

This thickness separates $\bar{S}_e$ and $\bar{\sigma}_a$ so as to realize the reliability goal of 0.999 at each shoulder. The design decision is to make t the next available thickness of 1018 CD steel strap from the same heat. This eliminates machining to the desired thickness and the extra cost of thicker work stock will be less than machining the fares. Ask your steel supplier what is available *in this heat*.

6-39



$$F_a = 1200 \text{ lbf}$$

$$S_{ut} = 80 \text{ kpsi}$$

(a) Strength

$$\begin{aligned} k_a &= 2.67(80)^{-0.265} \text{LN}(1, 0.058) \\ &= 0.836 \text{LN}(1, 0.058) \end{aligned}$$

$$k_b = 1$$

$$\begin{aligned} k_c &= 1.23(80)^{-0.078} \text{LN}(1, 0.125) \\ &= 0.874 \text{LN}(1, 0.125) \end{aligned}$$

$$\begin{aligned} S'_a &= 0.506(80) \text{LN}(1, 0.138) \\ &= 40.5 \text{LN}(1, 0.138) \text{ kpsi} \end{aligned}$$

$$\bar{S}_e = 0.836[\text{LN}(1, 0.058)](1)[0.874 \text{LN}(1, 0.125)][40.5 \text{LN}(1, 0.138)]$$

$$\bar{S}_e = 0.836(1)(0.874)(40.5) = 29.6 \text{ kpsi}$$

$$C_{Se} = (0.058^2 + 0.125^2 + 0.138^2)^{1/2} = 0.195$$

Stress: Fig. A-15-1; $d/w = 0.75/1.5 = 0.5$, $K_t = 2.17$. From Eqs. (6-78), (6-79) and Table 6-15

$$\begin{aligned} \mathbf{K}_f &= \frac{2.17 \text{LN}(1, 0.10)}{1 + (2/\sqrt{0.375})[(2.17 - 1)/2.17](5/80)} \\ &= 1.95 \text{LN}(1, 0.10) \end{aligned}$$

$$\sigma_a = \frac{\mathbf{K}_f F_a}{(w - d)t}, \quad C_\sigma = 0.10$$

$$\bar{\sigma}_a = \frac{\bar{K}_f F_a}{(w - d)t} = \frac{1.95(1.2)}{(1.5 - 0.75)(0.25)} = 12.48 \text{ kpsi}$$

$$\bar{S}_a = \bar{S}_e = 29.6 \text{ kpsi}$$

$$z = - \frac{\ln(\bar{S}_a/\bar{\sigma}_a) \sqrt{(1+C_\sigma^2)/(1+C_S^2)}}{\sqrt{\ln(1+C_\sigma^2)(1+C_S^2)}} \\ = - \frac{\ln[(29.6/12.48) \sqrt{(1+0.10^2)/(1+0.195^2)}]}{\sqrt{\ln(1+0.10^2)(1+0.195^2)}} = -3.9$$

From Table A-20

$$p_f = 4.481(10^{-5})$$

$$R = 1 - 4.481(10^{-5}) = 0.999\,955 \quad \text{Ans.}$$

(b) All computer programs will differ in detail.

6-40 Each computer program will differ in detail. When the programs are working, the experience should reinforce that the decision regarding $\bar{n}_f$ is independent of mean values of strength, stress or associated geometry. The reliability goal can be realized by noting the impact of all those a priori decisions.

6-41 Such subprograms allow a simple call when the information is needed. The calling program is often named an executive routine (executives tend to delegate chores to others and only want the answers).

6-42 This task is similar to Prob. 6-41.

6-43 Again, a similar task.

6-44 The results of Probs. 6-41 to 6-44 will be the basis of a class computer aid for fatigue problems. The codes should be made available to the class through the library of the computer network or main frame available to your students.

6-45 Peterson's notch sensitivity q has very little statistical basis. This subroutine can be used to show the variation in q , which is not apparent to those who embrace a deterministic q .

6-46 An additional program which is useful.