

Chapter 5

5-1 $S_y = 350 \text{ MPa}$.

MSS: $\sigma_1 - \sigma_3 = S_y / n \Rightarrow n = \frac{S_y}{(\sigma_1 - \sigma_3)}$

DE: $\sigma' = (\sigma_A^2 - \sigma_A \sigma_B + \sigma_B^2)^{1/2} = (\sigma_x^2 - \sigma_x \sigma_y + \sigma_y^2 + 3\tau_{xy}^2)^{1/2}$

$$n = \frac{S_y}{\sigma'}$$

(a) MSS: $\sigma_1 = 100 \text{ MPa}, \sigma_2 = 100 \text{ MPa}, \sigma_3 = 0$

$$n = \frac{350}{100 - 0} = 3.5 \quad \text{Ans.}$$

DE: $\sigma' = (100^2 - 100(100) + 100^2)^{1/2} = 100 \text{ MPa}, \quad n = \frac{350}{100} = 3.5 \quad \text{Ans.}$

(b) MSS: $\sigma_1 = 100 \text{ MPa}, \sigma_2 = 50 \text{ MPa}, \sigma_3 = 0$

$$n = \frac{350}{100 - 0} = 3.5 \quad \text{Ans.}$$

DE: $\sigma' = (100^2 - 100(50) + 50^2)^{1/2} = 86.6 \text{ MPa}, \quad n = \frac{350}{86.6} = 4.04 \quad \text{Ans.}$

(c) $\sigma_A, \sigma_B = \frac{100}{2} \pm \sqrt{\left(\frac{100}{2}\right)^2 + (-75)^2} = 140, -40 \text{ MPa}$

$$\sigma_1 = 140, \sigma_2 = 0, \sigma_3 = -40 \text{ MPa}$$

MSS: $n = \frac{350}{140 - (-40)} = 1.94 \quad \text{Ans.}$

DE: $\sigma' = [100^2 + 3(-75^2)]^{1/2} = 164 \text{ MPa}, \quad n = \frac{350}{164} = 2.13 \quad \text{Ans.}$

(d) $\sigma_A, \sigma_B = \frac{-50 - 75}{2} \pm \sqrt{\left(\frac{-50 + 75}{2}\right)^2 + (-50)^2} = -11.0, -114.0 \text{ MPa}$

$$\sigma_1 = 0, \sigma_2 = -11.0, \sigma_3 = -114.0 \text{ MPa}$$

MSS: $n = \frac{350}{0 - (-114.0)} = 3.07 \quad \text{Ans.}$

DE: $\sigma' = [(-50)^2 - (-50)(-75) + (-75)^2 + 3(-50)^2]^{1/2} = 109.0 \text{ MPa}$

$$n = \frac{350}{109.0} = 3.21 \quad \text{Ans.}$$

(e) $\sigma_A, \sigma_B = \frac{100 + 20}{2} \pm \sqrt{\left(\frac{100 - 20}{2}\right)^2 + (-20)^2} = 104.7, 15.3 \text{ MPa}$

$$\sigma_1 = 104.7, \sigma_2 = 15.3, \sigma_3 = 0 \text{ MPa}$$

$$\text{MSS: } n = \frac{350}{104.7 - 0} = 3.34 \quad \text{Ans.}$$

$$\text{DE: } \sigma' = \left[100^2 - 100(20) + 20^2 + 3(-20)^2 \right]^{1/2} = 98.0 \text{ MPa}$$

$$n = \frac{350}{98.0} = 3.57 \quad \text{Ans.}$$

5-2 $S_y = 350 \text{ MPa}$.

$$\text{MSS: } \sigma_1 - \sigma_3 = S_y / n \Rightarrow n = \frac{S_y}{(\sigma_1 - \sigma_3)}$$

$$\text{DE: } \sigma' = (\sigma_A^2 - \sigma_A \sigma_B + \sigma_B^2)^{1/2} = \frac{S_y}{n} \Rightarrow n = \frac{S_y}{\sigma'}$$

$$\text{(a) MSS: } \sigma_1 = 100 \text{ MPa}, \sigma_3 = 0 \Rightarrow n = \frac{350}{100 - 0} = 3.5 \quad \text{Ans.}$$

$$\text{DE: } n = \frac{350}{[100^2 - (100)(100) + 100^2]^{1/2}} = 3.5 \quad \text{Ans.}$$

$$\text{(b) MSS: } \sigma_1 = 100, \sigma_3 = -100 \text{ MPa} \Rightarrow n = \frac{350}{100 - (-100)} = 1.75 \quad \text{Ans.}$$

$$\text{DE: } n = \frac{350}{[100^2 - (100)(-100) + (-100)^2]^{1/2}} = 2.02 \quad \text{Ans.}$$

$$\text{(c) MSS: } \sigma_1 = 100 \text{ MPa}, \sigma_3 = 0 \Rightarrow n = \frac{350}{100 - 0} = 3.5 \quad \text{Ans.}$$

$$\text{DE: } n = \frac{350}{[100^2 - (100)(50) + 50^2]^{1/2}} = 4.04 \quad \text{Ans.}$$

$$\text{(d) MSS: } \sigma_1 = 100, \sigma_3 = -50 \text{ MPa} \Rightarrow n = \frac{350}{100 - (-50)} = 2.33 \quad \text{Ans.}$$

$$\text{DE: } n = \frac{350}{[100^2 - (100)(-50) + (-50)^2]^{1/2}} = 2.65 \quad \text{Ans.}$$

$$\text{(e) MSS: } \sigma_1 = 0, \sigma_3 = -100 \text{ MPa} \Rightarrow n = \frac{350}{0 - (-100)} = 3.5 \quad \text{Ans.}$$

$$\text{DE: } n = \frac{350}{[(-50)^2 - (-50)(-100) + (-100)^2]^{1/2}} = 4.04 \quad \text{Ans.}$$

5-3 From Table A-20, $S_y = 37.5 \text{ kpsi}$

$$\text{MSS: } \sigma_1 - \sigma_3 = S_y / n \Rightarrow n = \frac{S_y}{(\sigma_1 - \sigma_3)}$$

$$\text{DE: } \sigma' = (\sigma_A^2 - \sigma_A \sigma_B + \sigma_B^2)^{1/2} = (\sigma_x^2 - \sigma_x \sigma_y + \sigma_y^2 + 3\tau_{xy}^2)^{1/2}$$

$$n = \frac{S_y}{\sigma'}$$

$$\text{(a) MSS: } \sigma_1 = 25 \text{ kpsi, } \sigma_3 = 0 \Rightarrow n = \frac{37.5}{25 - 0} = 1.5 \quad \text{Ans.}$$

$$\text{DE: } n = \frac{37.5}{[25^2 - (25)(15) + 15^2]^{1/2}} = 1.72 \quad \text{Ans.}$$

$$\text{(b) MSS: } \sigma_1 = 15 \text{ kpsi, } \sigma_3 = -15 \Rightarrow n = \frac{37.5}{15 - (-15)} = 1.25 \quad \text{Ans.}$$

$$\text{DE: } n = \frac{37.5}{[15^2 - (15)(-15) + (-15)^2]^{1/2}} = 1.44 \quad \text{Ans.}$$

$$\text{(c) } \sigma_A, \sigma_B = \frac{20}{2} \pm \sqrt{\left(\frac{20}{2}\right)^2 + (-10)^2} = 24.1, -4.1 \text{ kpsi}$$

$$\sigma_1 = 24.1, \sigma_2 = 0, \sigma_3 = -4.1 \text{ kpsi}$$

$$\text{MSS: } n = \frac{37.5}{24.1 - (-4.1)} = 1.33 \quad \text{Ans.}$$

$$\text{DE: } \sigma' = [20^2 + 3(-10)^2]^{1/2} = 26.5 \text{ kpsi} \Rightarrow n = \frac{37.5}{26.5} = 1.42 \quad \text{Ans.}$$

$$\text{(d) } \sigma_A, \sigma_B = \frac{-12 + 15}{2} \pm \sqrt{\left(\frac{-12 - 15}{2}\right)^2 + (-9)^2} = 17.7, -14.7 \text{ kpsi}$$

$$\sigma_1 = 17.7, \sigma_2 = 0, \sigma_3 = -14.7 \text{ kpsi}$$

$$\text{MSS: } n = \frac{37.5}{17.7 - (-14.7)} = 1.16 \quad \text{Ans.}$$

$$\text{DE: } \sigma' = [(-12)^2 - (-12)(15) + 15^2 + 3(-9)^2]^{1/2} = 28.1 \text{ kpsi}$$

$$n = \frac{37.5}{28.1} = 1.33 \quad \text{Ans.}$$

$$\text{(e) } \sigma_A, \sigma_B = \frac{-24 - 24}{2} \pm \sqrt{\left(\frac{-24 + 24}{2}\right)^2 + (-15)^2} = -9, -39 \text{ kpsi}$$

$$\sigma_1 = 0, \sigma_2 = -9, \sigma_3 = -39 \text{ kpsi}$$

$$\text{MSS: } n = \frac{37.5}{0 - (-39)} = 0.96 \quad \text{Ans.}$$

$$\text{DE: } \sigma' = \left[(-24)^2 - (-24)(-24) + (-24)^2 + 3(-15)^2 \right]^{1/2} = 35.4 \text{ kpsi}$$

$$n = \frac{37.5}{35.4} = 1.06 \quad \text{Ans.}$$

5-4 From Table A-20, $S_y = 47$ kpsi.

$$\text{MSS: } \sigma_1 - \sigma_3 = S_y / n \Rightarrow n = \frac{S_y}{(\sigma_1 - \sigma_3)}$$

$$\text{DE: } \sigma' = (\sigma_A^2 - \sigma_A \sigma_B + \sigma_B^2)^{1/2} = \frac{S_y}{n} \Rightarrow n = \frac{S_y}{\sigma'}$$

(a) MSS: $\sigma_1 = 30 \text{ kpsi}, \sigma_3 = 0 \Rightarrow n = \frac{47}{30 - 0} = 1.57 \quad \text{Ans.}$

DE: $n = \frac{47}{[30^2 - (30)(30) + 30^2]^{1/2}} = 1.57 \quad \text{Ans.}$

(b) MSS: $\sigma_1 = 30, \sigma_3 = -30 \text{ kpsi} \Rightarrow n = \frac{47}{30 - (-30)} = 0.78 \quad \text{Ans.}$

DE: $n = \frac{47}{[30^2 - (30)(-30) + (-30)^2]^{1/2}} = 0.90 \quad \text{Ans.}$

(c) MSS: $\sigma_1 = 30 \text{ kpsi}, \sigma_3 = 0 \Rightarrow n = \frac{47}{30 - 0} = 1.57 \quad \text{Ans.}$

DE: $n = \frac{47}{[30^2 - (30)(15) + 15^2]^{1/2}} = 1.81 \quad \text{Ans.}$

(d) MSS: $\sigma_1 = 0, \sigma_3 = -30 \text{ kpsi} \Rightarrow n = \frac{47}{0 - (-30)} = 1.57 \quad \text{Ans.}$

DE: $n = \frac{47}{[(-30)^2 - (-30)(-15) + (-15)^2]^{1/2}} = 1.81 \quad \text{Ans.}$

(e) MSS: $\sigma_1 = 10, \sigma_3 = -50 \text{ kpsi} \Rightarrow n = \frac{47}{10 - (-50)} = 0.78 \quad \text{Ans.}$

DE: $n = \frac{47}{[(-50)^2 - (-50)(10) + 10^2]^{1/2}} = 0.84 \quad \text{Ans.}$

5-5 Note: The drawing in this manual may not be to the scale of original drawing. The measurements were taken from the original drawing.

(a) MSS and DE:

$$n = \frac{OB}{OA} = \frac{4.95''}{1.41''} = 3.51 \text{ Ans.}$$

(b) MSS:

$$n = \frac{OD}{OC} = \frac{3.91''}{1.12''} = 3.49 \text{ Ans.}$$

DE:

$$n = \frac{OE}{OC} = \frac{4.51''}{1.12''} = 4.03 \text{ Ans.}$$

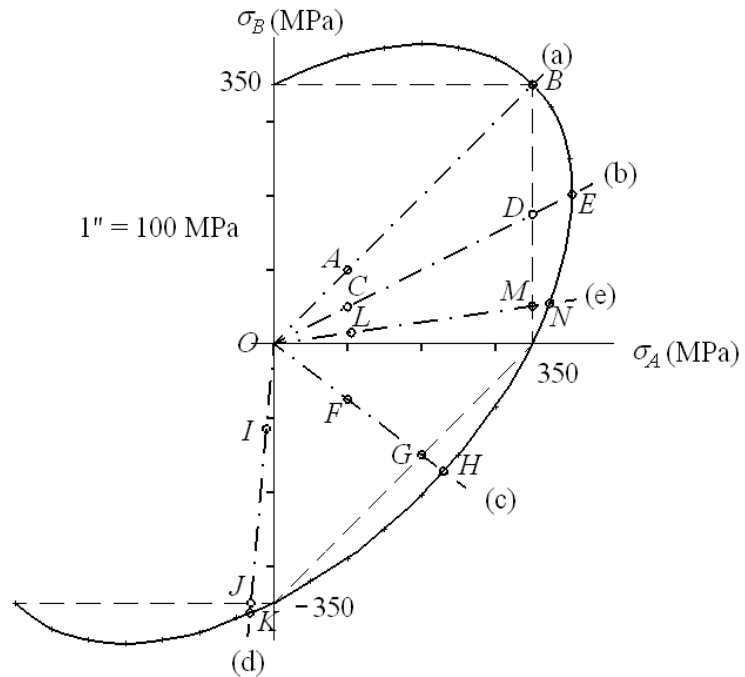
(c) MSS:

$$n = \frac{OG}{OF} = \frac{2.50''}{1.25''} = 2.00 \text{ Ans.}$$

$$\text{DE: } n = \frac{OH}{OF} = \frac{2.86''}{1.25''} = 2.29 \text{ Ans.}$$

$$\text{(d) MSS: } n = \frac{OJ}{OI} = \frac{3.51''}{1.15''} = 3.05 \text{ Ans., DE: } n = \frac{OK}{OI} = \frac{3.65''}{1.15''} = 3.17 \text{ Ans.}$$

$$\text{(e) MSS: } n = \frac{OM}{OL} = \frac{3.54''}{1.06''} = 3.34 \text{ Ans., DE: } n = \frac{ON}{OL} = \frac{3.77''}{1.06''} = 3.56 \text{ Ans.}$$



5-6 Note: The drawing in this manual may not be to the scale of original drawing. The measurements were taken from the original drawing.

(a) $\sigma_A = 25$ kpsi, $\sigma_B = 15$ kpsi

MSS:

$$n = \frac{OB}{OA} = \frac{4.37''}{2.92''} = 1.50 \quad \text{Ans.}$$

DE:

$$n = \frac{OC}{OA} = \frac{5.02''}{2.92''} = 1.72 \quad \text{Ans.}$$

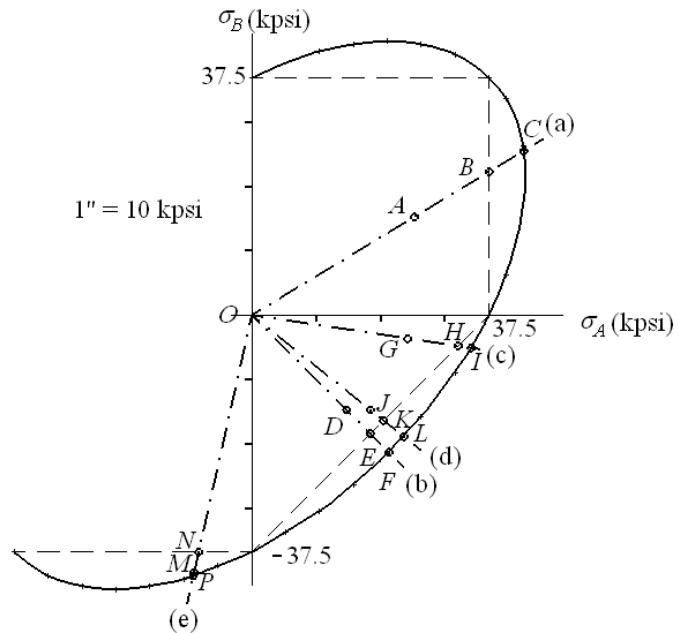
(b) $\sigma_A = 15 \text{ kpsi}$, $\sigma_B = -15 \text{ kpsi}$

MSS:

$$n = \frac{OE}{OD} = \frac{2.66''}{2.12''} = 1.25 \quad \text{Ans.}$$

DE:

$$n = \frac{OF}{OD} = \frac{3.05''}{2.12''} = 1.44 \quad \text{Ans.}$$



$$\text{(c) } \sigma_A, \sigma_B = \frac{20}{2} \pm \sqrt{\left(\frac{20}{2}\right)^2 + (-10)^2} = 24.1, -4.1 \text{ kpsi}$$

$$\text{MSS: } n = \frac{OH}{OG} = \frac{3.25''}{2.43''} = 1.34 \quad \text{Ans.} \quad \text{DE: } n = \frac{OI}{OG} = \frac{3.46''}{2.43''} = 1.42 \quad \text{Ans.}$$

$$\textbf{(d)} \quad \sigma_A, \sigma_B = \frac{-12+15}{2} \pm \sqrt{\left(\frac{-12-15}{2}\right)^2 + (-9)^2} = 17.7, -14.7 \text{ MPa}$$

$$\text{MSS: } n = \frac{OK}{OJ} = \frac{2.67''}{2.30''} = 1.16 \quad \text{Ans.} \quad \text{DE: } n = \frac{OL}{OJ} = \frac{3.06''}{2.30''} = 1.33 \quad \text{Ans.}$$

$$\text{(e) } \sigma_A, \sigma_B = \frac{-24 - 24}{2} \pm \sqrt{\left(\frac{-24 + 24}{2}\right)^2 + (-15)^2} = -9, -39 \text{ kpsi}$$

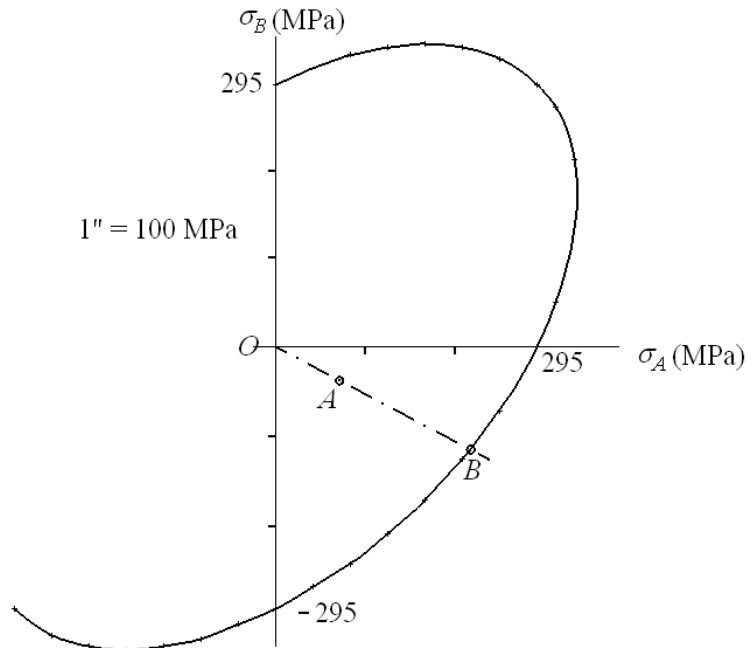
$$\text{MSS: } n = \frac{ON}{OM} = \frac{3.85''}{4.00''} = 0.96 \quad \text{Ans.} \quad \text{DE: } n = \frac{OP}{OM} = \frac{4.23''}{4.00''} = 1.06 \quad \text{Ans.}$$

5-7 $S_y = 295 \text{ MPa}$, $\sigma_A = 75 \text{ MPa}$, $\sigma_B = -35 \text{ MPa}$,

$$(a) \quad n = \frac{S_y}{(\sigma_A^2 - \sigma_A \sigma_B + \sigma_B^2)^{1/2}} = \frac{295}{[75^2 - 75(-35) + (-35)^2]^{1/2}} = 3.03 \quad \text{Ans.}$$

(b) Note: The drawing in this manual may not be to the scale of original drawing. The measurements were taken from the original drawing.

$$n = \frac{OB}{OA} = \frac{2.50''}{0.83''} = 3.01 \quad \text{Ans.}$$

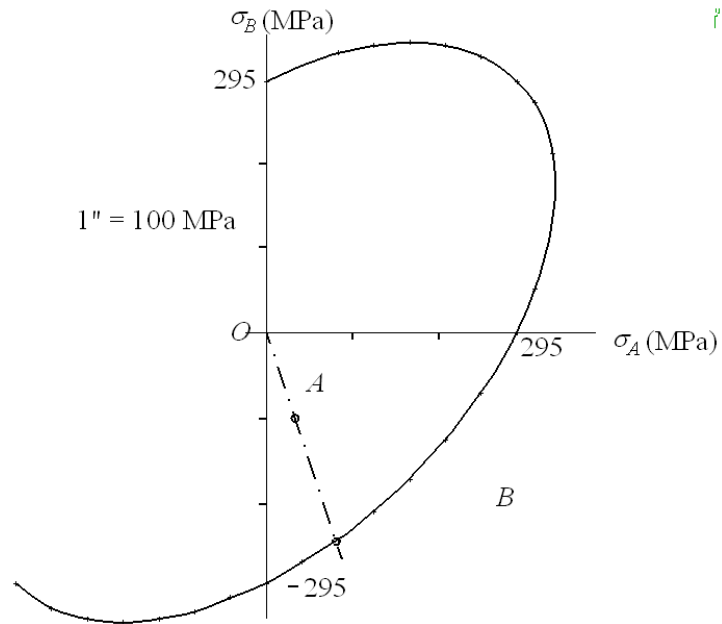


5-8 $S_y = 295 \text{ MPa}$, $\sigma_A = 30 \text{ MPa}$, $\sigma_B = -100 \text{ MPa}$,

$$(a) \quad n = \frac{S_y}{(\sigma_A^2 - \sigma_A \sigma_B + \sigma_B^2)^{1/2}} = \frac{295}{[30^2 - 30(-100) + (-100)^2]^{1/2}} = 2.50 \quad \text{Ans.}$$

(b) Note: The drawing in this manual may not be to the scale of original drawing. The measurements were taken from the original drawing.

$$n = \frac{OB}{OA} = \frac{2.50''}{0.83''} = 3.01 \quad \text{Ans.}$$

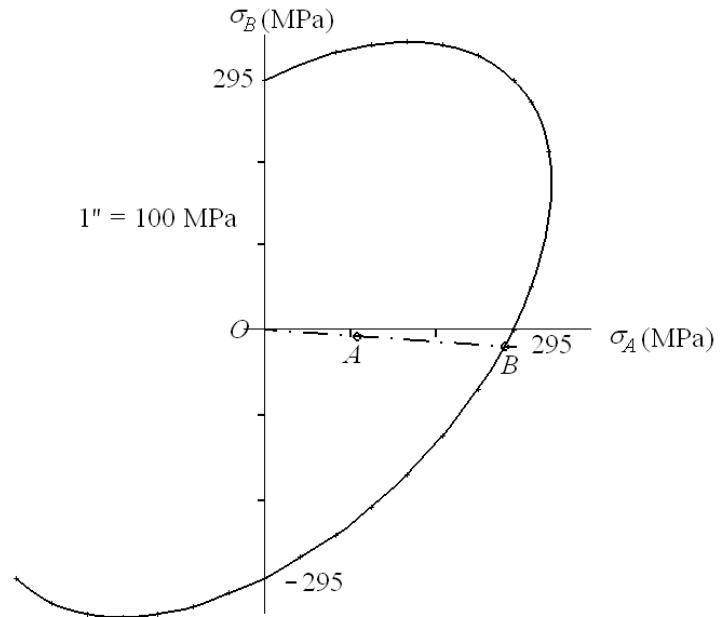


5-9 $S_y = 295 \text{ MPa}$, σ_A , $\sigma_B = \frac{100}{2} \pm \sqrt{\left(\frac{100}{2}\right)^2 + (-25)^2} = 105.9, -5.9 \text{ MPa}$

(a) $n = \frac{S_y}{(\sigma_A^2 - \sigma_A \sigma_B + \sigma_B^2)^{1/2}} = \frac{295}{[105.9^2 - 105.9(-5.9) + (-5.9)^2]^{1/2}} = 2.71 \text{ Ans.}$

(b) Note: The drawing in this manual may not be to the scale of original drawing. The measurements were taken from the original drawing.

$n = \frac{OB}{OA} = \frac{2.87''}{1.06''} = 2.71 \text{ Ans.}$

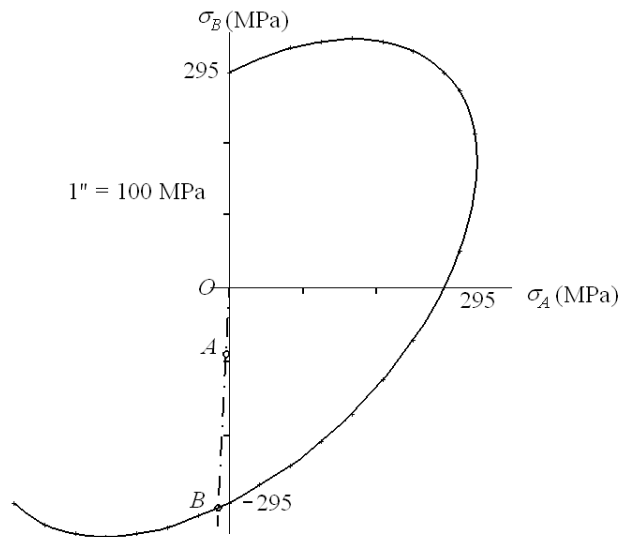


5-10 $S_y = 295 \text{ MPa}$, $\sigma_A, \sigma_B = \frac{-30-65}{2} \pm \sqrt{\left(\frac{-30+65}{2}\right)^2 + 40^2} = -3.8, -91.2 \text{ MPa}$

(a) $n = \frac{S_y}{(\sigma_A^2 - \sigma_A \sigma_B + \sigma_B^2)^{1/2}} = \frac{295}{[(-3.8)^2 - (-3.8)(-91.2) + (-91.2)^2]^{1/2}} = 3.30 \text{ Ans.}$

(b) Note: The drawing in this manual may not be to the scale of original drawing. The measurements were taken from the original drawing.

$n = \frac{OB}{OA} = \frac{3.00''}{0.90''} = 3.33 \text{ Ans.}$

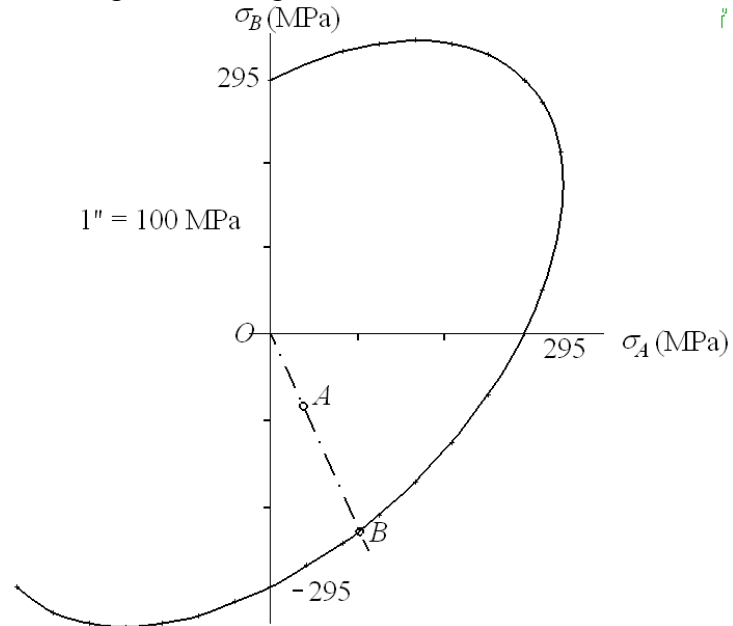


5-11 $S_y = 295 \text{ MPa}$, $\sigma_A, \sigma_B = \frac{-80+30}{2} \pm \sqrt{\left(\frac{-80-30}{2}\right)^2 + (-10)^2} = 30.9, -80.9 \text{ MPa}$

(a) $n = \frac{S_y}{(\sigma_A^2 - \sigma_A \sigma_B + \sigma_B^2)^{1/2}} = \frac{295}{[30.9^2 - 30.9(-80.9) + (-80.9)^2]^{1/2}} = 2.95 \quad \text{Ans.}$

(b) Note: The drawing in this manual may not be to the scale of original drawing. The measurements were taken from the original drawing.

$n = \frac{OB}{OA} = \frac{2.55''}{0.87''} = 2.93 \quad \text{Ans.}$



5-12 $S_{yt} = 60 \text{ kpsi}$, $S_{yc} = 75 \text{ kpsi}$. Eq. (5-26) for yield is

$$n = \left(\frac{\sigma_1}{S_{yt}} - \frac{\sigma_3}{S_{yc}} \right)^{-1}$$

(a) $\sigma_1 = 25 \text{ kpsi}$, $\sigma_3 = 0 \Rightarrow n = \left(\frac{25}{60} - \frac{0}{75} \right)^{-1} = 2.40 \quad \text{Ans.}$

(b) $\sigma_1 = 15$, $\sigma_3 = -15 \text{ kpsi} \Rightarrow n = \left(\frac{15}{60} - \frac{-15}{75} \right)^{-1} = 2.22 \quad \text{Ans.}$

$$(c) \sigma_A, \sigma_B = \frac{20}{2} \pm \sqrt{\left(\frac{20}{2}\right)^2 + (-10)^2} = 24.1, -4.1 \text{ kpsi},$$

$$\sigma_1 = 24.1, \sigma_2 = 0, \sigma_3 = -4.1 \text{ kpsi} \Rightarrow n = \left(\frac{24.1}{60} - \frac{-4.1}{75} \right)^{-1} = 2.19 \quad \text{Ans.}$$

$$(d) \sigma_A, \sigma_B = \frac{-12+15}{2} \pm \sqrt{\left(\frac{-12-15}{2}\right)^2 + (-9)^2} = 17.7, -14.7 \text{ kpsi}$$

$$\sigma_1 = 17.7, \sigma_2 = 0, \sigma_3 = -14.7 \text{ kpsi} \Rightarrow n = \left(\frac{17.7}{60} - \frac{-14.7}{75} \right)^{-1} = 2.04 \quad \text{Ans.}$$

$$(e) \sigma_A, \sigma_B = \frac{-24-24}{2} \pm \sqrt{\left(\frac{-24+24}{2}\right)^2 + (-15)^2} = -9, -39 \text{ kpsi}$$

$$\sigma_1 = 0, \sigma_2 = -9, \sigma_3 = -39 \text{ kpsi} \Rightarrow n = \left(\frac{0}{60} - \frac{-39}{75} \right)^{-1} = 1.92 \quad \text{Ans.}$$

5-13 Note: The drawing in this manual may not be to the scale of original drawing. The measurements were taken from the original drawing.

$$(a) \sigma_A = 25, \sigma_B = 15 \text{ kpsi}$$

$$n = \frac{OB}{OA} = \frac{3.49''}{1.46''} = 2.39 \quad \text{Ans.}$$

$$(b) \sigma_A = 15, \sigma_B = -15 \text{ kpsi}$$

$$n = \frac{OD}{OC} = \frac{2.36''}{1.06''} = 2.23 \quad \text{Ans.}$$

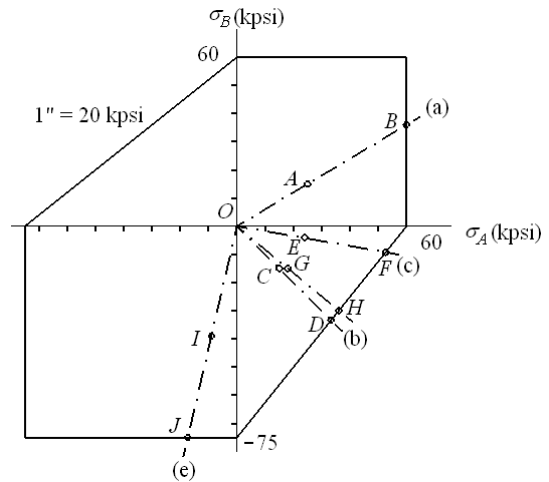
(c)

$$\sigma_A, \sigma_B = \frac{20}{2} \pm \sqrt{\left(\frac{20}{2}\right)^2 + (-10)^2} = 24.1, -4.1 \text{ kpsi}$$

$$n = \frac{OF}{OE} = \frac{2.67''}{1.22''} = 2.19 \quad \text{Ans.}$$

(d)

$$\sigma_A, \sigma_B = \frac{-12+15}{2} \pm \sqrt{\left(\frac{-12-15}{2}\right)^2 + (-9)^2} = 17.7, -14.7 \text{ kpsi}$$



(e)

$$\sigma_A, \sigma_B = \frac{-24 - 24}{2} \pm \sqrt{\left(\frac{-24 + 24}{2}\right)^2 + (-15)^2} = -9, -39 \text{ kpsi}$$

$$n = \frac{OJ}{OI} = \frac{3.85''}{2.00''} = 1.93 \text{ Ans.}$$

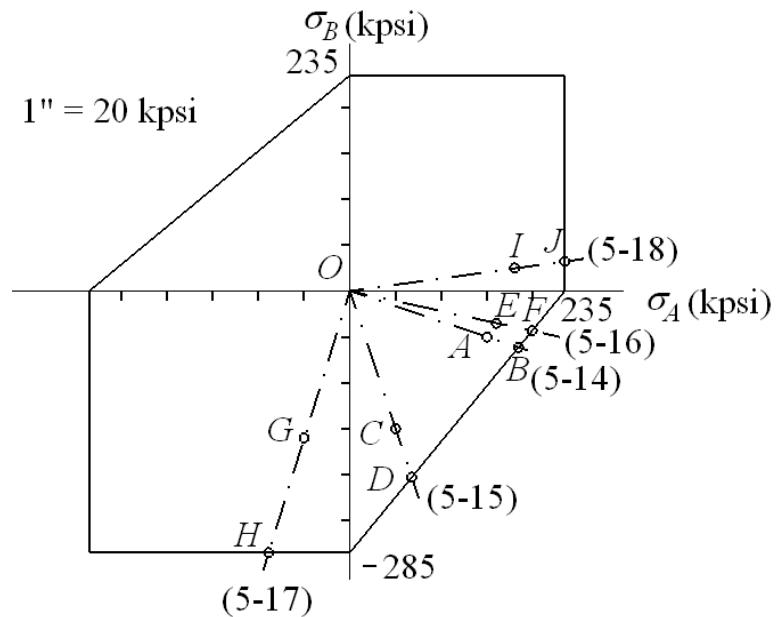
5-14 Since $\varepsilon_f > 0.05$, and $S_{yt} \neq S_{yc}$, the Coulomb-Mohr theory for ductile materials will be used.

(a) From Eq. (5-26),

$$n = \left(\frac{\sigma_1}{S_{yt}} - \frac{\sigma_3}{S_{yc}} \right)^{-1} = \left(\frac{150}{235} - \frac{-50}{285} \right)^{-1} = 1.23 \quad \text{Ans.}$$

(b) Plots for Problems 5-14 to 5-18 are found here. Note: The drawing in this manual may not be to the scale of original drawing. The measurements were taken from the original drawing.

$$n = \frac{OB}{OA} = \frac{1.94''}{1.58''} = 1.23 \text{ Ans.} \quad 1'' = 20 \text{ kpsi}$$



5-15 (a) From Eq. (5-26),

$$n = \left(\frac{\sigma_1}{S_{yt}} - \frac{\sigma_3}{S_{yc}} \right)^{-1} = \left(\frac{50}{235} - \frac{-150}{285} \right)^{-1} = 1.35 \quad \text{Ans.}$$

(b) The plot for this problem is found on the page for Prob. 5-14. Note: The drawing in this manual may not be to the scale of original drawing. The measurements were taken from the original drawing.

$$n = \frac{OD}{OC} = \frac{2.14''}{1.58''} = 1.35 \text{ Ans.}$$

5-16 $\sigma_A, \sigma_B = \frac{125}{2} \pm \sqrt{\left(\frac{125}{2}\right)^2 + (-75)^2} = 160, -35 \text{ kpsi}$

(a) From Eq. (5-26),

$$n = \left(\frac{\sigma_1}{S_{yt}} - \frac{\sigma_3}{S_{yc}} \right)^{-1} = \left(\frac{160}{235} - \frac{-35}{285} \right)^{-1} = 1.24 \text{ Ans.}$$

(b) The plot for this problem is found on the page for Prob. 5-14. Note: The drawing in this manual may not be to the scale of original drawing. The measurements were taken from the original drawing.

$$n = \frac{OF}{OE} = \frac{2.04''}{1.64''} = 1.24 \text{ Ans.}$$

5-17 $\sigma_A, \sigma_B = \frac{-80-125}{2} \pm \sqrt{\left(\frac{-80+125}{2}\right)^2 + 50^2} = -47.7, -157.3 \text{ kpsi}$

(a) From Eq. (5-26),

$$n = \left(\frac{\sigma_1}{S_{yt}} - \frac{\sigma_3}{S_{yc}} \right)^{-1} = \left(\frac{0}{235} - \frac{-157.3}{285} \right)^{-1} = 1.81 \text{ Ans.}$$

(b) The plot for this problem is found on the page for Prob. 5-14. Note: The drawing in this manual may not be to the scale of original drawing. The measurements were taken from the original drawing.

$$n = \frac{OH}{OG} = \frac{2.99''}{1.64''} = 1.82 \text{ Ans.}$$

5-18 $\sigma_A, \sigma_B = \frac{125+80}{2} \pm \sqrt{\left(\frac{125-80}{2}\right)^2 + (-75)^2} = 180.8, 24.2 \text{ kpsi}$

(a) From Eq. (5-26),

$$n = \left(\frac{\sigma_1}{S_{yt}} - \frac{\sigma_3}{S_{yc}} \right)^{-1} = \left(\frac{180.8}{235} - \frac{0}{285} \right)^{-1} = 1.30 \quad \text{Ans.}$$

(b) The plot for this problem is found on the page for Prob. 5-14. Note: The drawing in this manual may not be to the scale of original drawing. The measurements were taken from the original drawing.

$$n = \frac{OJ}{OI} = \frac{2.37''}{1.83''} = 1.30 \quad \text{Ans.}$$

5-19 $S_{ut} = 30$ kpsi, $S_{uc} = 90$ kpsi

BCM: Eqs. (5-31), p. 236

MM: see Eqs. (5-32), p. 236

(a) $\sigma_A = 25$ kpsi, $\sigma_B = 15$ kpsi

$$\text{BCM: Eq. (5-31a), } n = \frac{S_{ut}}{\sigma_A} = \frac{30}{25} = 1.2 \quad \text{Ans.}$$

$$\text{MM: Eq. (5-32a), } n = \frac{S_{ut}}{\sigma_A} = \frac{30}{25} = 1.2 \quad \text{Ans.}$$

(b) $\sigma_A = 15$ kpsi, $\sigma_B = -15$ kpsi,

$$\text{BCM: Eq. (5-31a), } n = \left(\frac{15}{30} - \frac{-15}{90} \right)^{-1} = 1.5 \quad \text{Ans.}$$

$$\text{MM: } \sigma_A \geq 0 \geq \sigma_B, \text{ and } |\sigma_B / \sigma_A| \leq 1, \text{ Eq. (5-32a), } n = \frac{S_{ut}}{\sigma_A} = \frac{30}{15} = 2.0 \quad \text{Ans.}$$

$$(c) \sigma_A, \sigma_B = \frac{20}{2} \pm \sqrt{\left(\frac{20}{2} \right)^2 + (-10)^2} = 24.14, -4.14 \text{ kpsi}$$

$$\text{BCM: Eq. (5-31b), } n = \left(\frac{24.14}{30} - \frac{-4.14}{90} \right)^{-1} = 1.18 \quad \text{Ans.}$$

$$\text{MM: } \sigma_A \geq 0 \geq \sigma_B, \text{ and } |\sigma_B / \sigma_A| \leq 1, \text{ Eq. (5-32a), } n = \frac{S_{ut}}{\sigma_A} = \frac{30}{24.14} = 1.24 \quad \text{Ans.}$$

$$(d) \sigma_A, \sigma_B = \frac{-15+10}{2} \pm \sqrt{\left(\frac{-15-10}{2} \right)^2 + (-15)^2} = 17.03, -22.03 \text{ kpsi}$$

$$\text{BCM: Eq. (5-31b), } n = \left(\frac{17.03}{30} - \frac{-22.03}{90} \right)^{-1} = 1.23 \text{ Ans.}$$

$$\text{MM: } \sigma_A \geq 0 \geq \sigma_B, \text{ and } |\sigma_B / \sigma_A| \geq 1, \text{ Eq. (5-32b),}$$

$$n = \left[\frac{(S_{uc} - S_{ut})\sigma_A}{S_{uc}S_{ut}} - \frac{\sigma_B}{S_{uc}} \right]^{-1} = \left[\frac{(90 - 30)17.03}{90(30)} - \frac{-22.03}{90} \right]^{-1} = 1.60 \text{ Ans.}$$

$$\text{(e) } \sigma_A, \sigma_B = \frac{-20 - 20}{2} \pm \sqrt{\left(\frac{-20 + 20}{2} \right)^2 + (-15)^2} = -5, -35 \text{ kpsi}$$

$$\text{BCM: Eq. (5-31c), } n = -\frac{S_{uc}}{\sigma_B} = -\frac{90}{-35} = 2.57 \text{ Ans.}$$

$$\text{MM: Eq. (5-32c), } n = -\frac{S_{uc}}{\sigma_B} = -\frac{90}{-35} = 2.57 \text{ Ans.}$$

5-20 Note: The drawing in this manual may not be to the scale of original drawing. The measurements were taken from the original drawing.

$$\text{(a) } \sigma_A = 25, \sigma_B = 15 \text{ kpsi}$$

BCM & MM:

$$n = \frac{OB}{OA} = \frac{1.74''}{1.46''} = 1.19 \text{ Ans.}$$

$$\text{(b) } \sigma_A = 15, \sigma_B = -15 \text{ kpsi}$$

$$\text{BCM: } n = \frac{OC}{OD} = \frac{1.59''}{1.06''} = 1.5 \text{ Ans.}$$

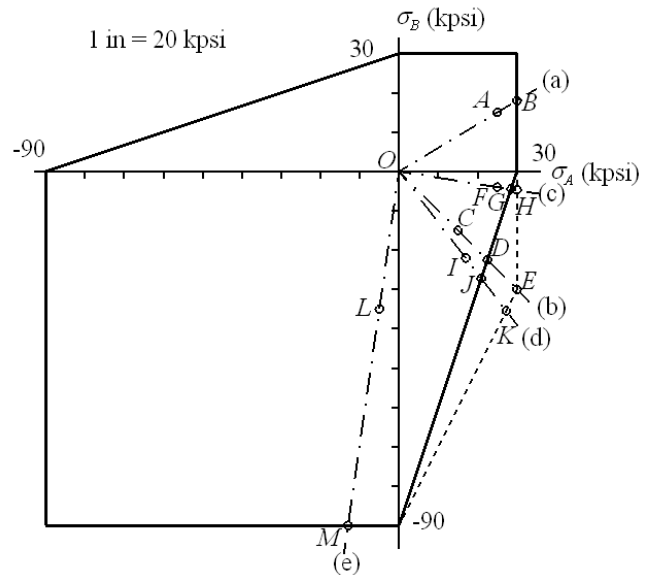
$$\text{MM: } n = \frac{OE}{OC} = \frac{2.12''}{1.06''} = 2.0 \text{ Ans.}$$

$$\begin{aligned} \text{(c) } \sigma_A, \sigma_B &= \frac{20}{2} \pm \sqrt{\left(\frac{20}{2} \right)^2 + (-10)^2} \\ &= 24.14, -4.14 \text{ kpsi} \end{aligned}$$

$$\text{BCM: } n = \frac{OG}{OF} = \frac{1.44''}{1.22''} = 1.18 \text{ Ans.}$$

$$\text{MM: } n = \frac{OH}{OF} = \frac{1.52''}{1.22''} = 1.25 \text{ Ans.}$$

$$\text{(d) } \sigma_A, \sigma_B = \frac{-15 + 10}{2} \pm \sqrt{\left(\frac{-15 - 10}{2} \right)^2 + (-15)^2} = 17.03, -22.03 \text{ kpsi}$$



$$\text{BCM: Eq. (5-31b), } n = \left(\frac{15}{31} - \frac{-50}{109} \right)^{-1} = 1.06 \text{ Ans.}$$

$$\text{MM: Eq. (3-32b), } n = \left[\frac{(S_{uc} - S_{ut})\sigma_A}{S_{uc}S_{ut}} - \frac{\sigma_B}{S_{uc}} \right]^{-1} = \left[\frac{(109 - 31)15}{109(31)} - \frac{-50}{109} \right]^{-1} = 1.24 \text{ Ans.}$$

(b), (c) The plot is shown in the solution to Prob. 5-21.

$$\text{BCM: } n = \frac{OD}{OC} = \frac{2.78''}{2.61''} = 1.07 \text{ Ans.}$$

$$\text{MM: } n = \frac{OE}{OC} = \frac{3.25''}{2.61''} = 1.25 \text{ Ans.}$$

5-23 From Table A-24, $S_{ut} = 31$ kpsi, $S_{uc} = 109$ kpsi

BCM: Eq. (5-31), MM: Eqs. (5-32)

$$\sigma_A, \sigma_B = \frac{15}{2} \pm \sqrt{\left(\frac{15}{2}\right)^2 + (-10)^2} = 20, -5 \text{ kpsi}$$

$$\text{(a) BCM: Eq. (5-32b), } n = \left(\frac{20}{31} - \frac{-5}{109} \right)^{-1} = 1.45 \text{ Ans.}$$

$$\text{MM: Eq. (5-32a), } n = \frac{S_{ut}}{\sigma_A} = \frac{31}{20} = 1.55 \text{ Ans.}$$

(b), (c) The plot is shown in the solution to Prob. 5-21.

$$\text{BCM: } n = \frac{OG}{OF} = \frac{1.48''}{1.03''} = 1.44 \text{ Ans.}$$

$$\text{MM: } n = \frac{OH}{OF} = \frac{1.60''}{1.03''} = 1.55 \text{ Ans.}$$

5-24 From Table A-24, $S_{ut} = 31$ kpsi, $S_{uc} = 109$ kpsi

BCM: Eq. (5-31), MM: Eqs. (5-32)

$$\sigma_A, \sigma_B = \frac{-10 - 25}{2} \pm \sqrt{\left(\frac{-10 + 25}{2}\right)^2 + (-10)^2} = -5, -30 \text{ kpsi}$$

$$\text{(a) BCM: Eq. (5-31c), } n = -\frac{S_{uc}}{\sigma_B} - \frac{109}{-30} = 3.63 \text{ Ans.}$$

$$\text{MM: Eq. (5-32c), } n = -\frac{S_{uc}}{\sigma_B} = -\frac{109}{-30} = 3.63 \text{ Ans.}$$

(b), (c) The plot is shown in the solution to Prob. 5-21.

$$\text{BCM and MM: } n = \frac{OJ}{OI} = \frac{5.53''}{1.52''} = 3.64 \text{ Ans.}$$

5-25 From Table A-24, $S_{ut} = 31$ kpsi, $S_{uc} = 109$ kpsi

BCM: Eq. (5-31), MM: Eqs. (5-32)

$$\sigma_A, \sigma_B = \frac{-35+13}{2} \pm \sqrt{\left(\frac{-35-13}{2}\right)^2 + (-10)^2} = 15, -37 \text{ kpsi}$$

$$\text{(a) BCM: Eq. (5-31b), } n = \left(\frac{15}{31} - \frac{-37}{109}\right)^{-1} = 1.21 \text{ Ans.}$$

$$\text{MM: Eq. (5-32b), } n = \left[\frac{(S_{uc} - S_{ut})\sigma_A}{S_{uc}S_{ut}} - \frac{\sigma_B}{S_{uc}}\right]^{-1} = \left[\frac{(109-31)15}{109(31)} - \frac{-37}{109}\right]^{-1} = 1.46 \text{ Ans.}$$

(b), (c) The plot is shown in the solution to Prob. 5-21.

$$\text{BCM: } n = \frac{OL}{OK} = \frac{2.42''}{2.00''} = 1.21 \text{ Ans.}$$

$$\text{MM: } n = \frac{OM}{OK} = \frac{2.91''}{2.00''} = 1.46 \text{ Ans.}$$

5-26 $S_{ut} = 36$ kpsi, $S_{uc} = 35$ kpsi

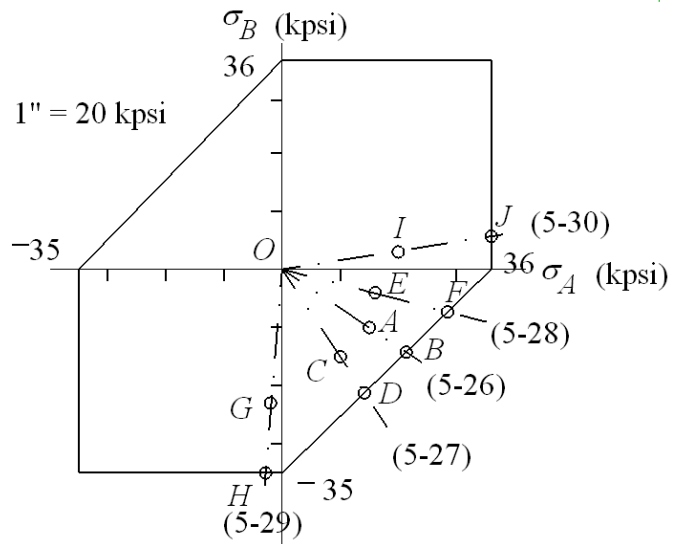
BCM: Eq. (5-31),

(a) $\sigma_A = 15$, $\sigma_B = -10$ kpsi.

$$\text{BCM: Eq. (5-31b), } n = \left(\frac{15}{36} - \frac{-10}{35}\right)^{-1} = 1.42 \text{ Ans.}$$

(b) The plot is shown below is for Probs. 5-26 to 5-30. Note: The drawing in this manual may not be to the scale of original drawing. The measurements were taken from the original drawing.

$$n = \frac{OB}{OA} = \frac{1.28''}{0.90''} = 1.42 \quad \text{Ans.}$$



5-27 $S_{ut} = 36 \text{ kpsi}$, $S_{uc} = 35 \text{ kpsi}$

BCM: Eq. (5-31),

(a) $\sigma_A = 15$, $\sigma_B = -15 \text{ kpsi}$.

BCM: Eq. (5-31b), $n = \left(\frac{10}{36} - \frac{-15}{35} \right)^{-1} = 1.42 \quad \text{Ans.}$

(b) The plot is shown in the solution to Prob. 5-26.

$$n = \frac{OD}{OC} = \frac{1.28''}{0.90''} = 1.42 \quad \text{Ans.}$$

5-28 $S_{ut} = 36 \text{ kpsi}$, $S_{uc} = 35 \text{ kpsi}$

BCM: Eq. (5-31),

(a) $\sigma_A, \sigma_B = \frac{12}{2} \pm \sqrt{\left(\frac{12}{2} \right)^2 + (-8)^2} = 16, -4 \text{ kpsi}$

BCM: Eq. (5-31b), $n = \left(\frac{16}{36} - \frac{-4}{35} \right)^{-1} = 1.79 \quad \text{Ans.}$

(b) The plot is shown in the solution to Prob. 5-26.

$$n = \frac{OF}{OE} = \frac{1.47''}{0.82''} = 1.79 \text{ Ans.}$$

5-29 $S_{ut} = 36 \text{ kpsi}, S_{uc} = 35 \text{ kpsi}$

BCM: Eq. (5-31),

$$(a) \sigma_A, \sigma_B = \frac{-10-15}{2} \pm \sqrt{\left(\frac{-10+15}{2}\right)^2 + 10^2} = -2.2, -22.8 \text{ kpsi}$$

$$\text{BCM: Eq. (5-31c), } n = -\frac{35}{-22.8} = 1.54 \text{ Ans.}$$

(b) The plot is shown in the solution to Prob. 5-26.

$$n = \frac{OH}{OG} = \frac{1.76''}{1.15''} = 1.53 \text{ Ans.}$$

5-30 $S_{ut} = 36 \text{ kpsi}, S_{uc} = 35 \text{ kpsi}$

BCM: Eq. (5-31),

$$(a) \sigma_A, \sigma_B = \frac{15+8}{2} \pm \sqrt{\left(\frac{15-8}{2}\right)^2 + (-8)^2} = 20.2, 2.8 \text{ kpsi}$$

$$\text{BCM: Eq. (5-31a), } n = \frac{36}{20.2} = 1.78 \text{ Ans.}$$

(b) The plot is shown in the solution to Prob. 5-26.

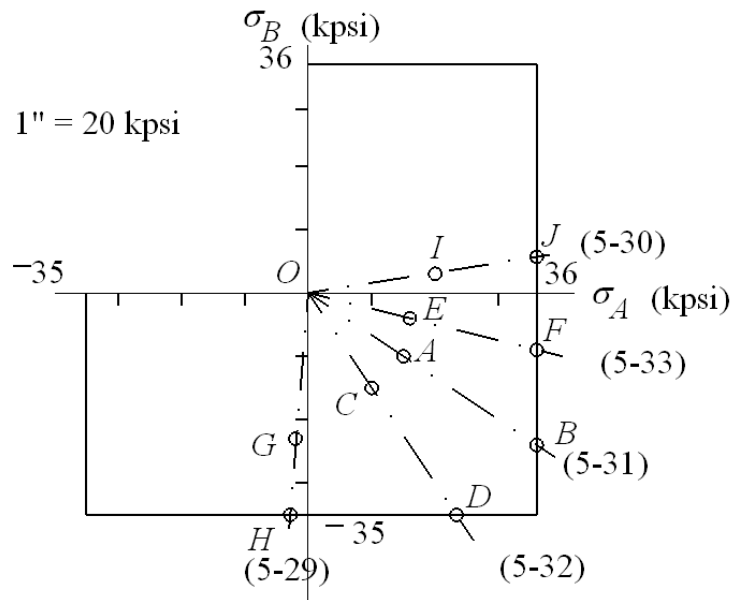
$$n = \frac{OJ}{OI} = \frac{1.82''}{1.02''} = 1.78 \text{ Ans.}$$

5-31 $S_{ut} = 36 \text{ kpsi}, S_{uc} = 35 \text{ kpsi}$. MM: Use Eq. (5-32). For this problem, MM reduces to the MNS theory.

$$(a) \sigma_A = 15, \sigma_B = -10 \text{ kpsi. Eq. (5-32a), } n = \frac{S_{ut}}{\sigma_A} = \frac{36}{15} = 2.4 \text{ Ans.}$$

(b) The plot on the next page is for Probs. 5-31 to 5-35. Note: The drawing in this manual may not be to the scale of original drawing. The measurements were taken from the original drawing.

$$n = \frac{OB}{OA} = \frac{2.16''}{0.90''} = 2.43 \quad \text{Ans.}$$



5-32 $S_{ut} = 36$ kpsi, $S_{uc} = 35$ kpsi. MM: Use Eq. (5-32). For this problem, MM reduces to the MNS theory.

(a) $\sigma_A = 10$, $\sigma_B = -15$ kpsi. Eq. (3-32b) is not valid and must use Eq. (3-32c),

$$n = -\frac{S_{uc}}{\sigma_B} = -\frac{35}{-15} = 2.33 \quad \text{Ans.}$$

(b) The plot is shown in the solution to Prob. 5-31.

$$n = \frac{OD}{OC} = \frac{2.10''}{0.90''} = 2.33 \quad \text{Ans.}$$

5-33 $S_{ut} = 36$ kpsi, $S_{uc} = 35$ kpsi. MM: Use Eq. (5-32). For this problem, MM reduces to the MNS theory.

$$(a) \quad \sigma_A, \sigma_B = \frac{12}{2} \pm \sqrt{\left(\frac{12}{2}\right)^2 + (-8)^2} = 16, -4 \text{ kpsi}$$

$$n = \frac{S_{ut}}{\sigma_A} = \frac{36}{16} = 2.25 \quad \text{Ans.}$$

(b) The plot is shown in the solution to Prob. 5-31.

$$n = \frac{OF}{OE} = \frac{1.86''}{0.82''} = 2.27 \quad \text{Ans.}$$

- 5-34** $S_{ut} = 36$ kpsi, $S_{uc} = 35$ kpsi. MM: Use Eq. (5-32). For this problem, MM reduces to the MNS theory.

$$(a) \sigma_A, \sigma_B = \frac{-10-15}{2} \pm \sqrt{\left(\frac{-10+15}{2}\right)^2 + 10^2} = -2.2, -22.8 \text{ kpsi}$$

$$n = -\frac{S_{uc}}{\sigma_B} = -\frac{35}{-22.8} = 1.54 \text{ Ans.}$$

- (b) The plot is shown in the solution to Prob. 5-31.

$$n = \frac{OH}{OG} = \frac{1.76''}{1.15''} = 1.53 \text{ Ans.}$$

- 5-35** $S_{ut} = 36$ kpsi, $S_{uc} = 35$ kpsi. MM: Use Eq. (5-32). For this problem, MM reduces to the MNS theory.

$$(a) \sigma_A, \sigma_B = \frac{15+8}{2} \pm \sqrt{\left(\frac{15-8}{2}\right)^2 + (-8)^2} = 20.2, 2.8 \text{ kpsi}$$

$$n = \frac{S_{ut}}{\sigma_A} = \frac{36}{20.2} = 1.78 \text{ Ans.}$$

- (b) The plot is shown in the solution to Prob. 5-31.

$$n = \frac{OJ}{OI} = \frac{1.82''}{1.02''} = 1.78 \text{ Ans.}$$

- 5-36** Given: AISI 1006 CD steel, $F = 0.55$ kN, $P = 4.0$ kN, and $T = 25$ N·m. From Table A-20, $S_y = 280$ MPa. Apply the DE theory to stress elements A and B

$$A: \sigma_x = \frac{4P}{\pi d^2} = \frac{4(4)10^3}{\pi(0.015^2)} = 22.6(10^6) \text{ Pa} = 22.6 \text{ MPa}$$

$$\tau_{xy} = \frac{16T}{\pi d^3} + \frac{4V}{3A} = \frac{16(25)}{\pi(0.015^3)} + \frac{4}{3} \left[\frac{0.55(10^3)}{(\pi/4)0.015^2} \right] = 41.9(10^6) \text{ Pa} = 41.9 \text{ MPa}$$

$$\sigma' = \left[22.6^2 + 3(41.9^2) \right]^{1/2} = 76.0 \text{ MPa}$$

$$n = \frac{280}{76.0} = 3.68 \text{ Ans.}$$

$$B: \sigma_x = \frac{32Fl}{\pi d^3} + \frac{4P}{\pi d^2} = \frac{32(0.55)10^3(0.1)}{\pi(0.015^3)} + \frac{4(4)10^3}{\pi(0.015^2)} = 189(10^6) \text{ Pa} = 189 \text{ MPa}$$

$$\tau_{xy} = \frac{16T}{\pi d^3} = \frac{16(25)}{\pi(0.015^3)} = 37.7(10^6) \text{ Pa} = 37.7 \text{ MPa}$$

$$\sigma' = (\sigma_x^2 + 3\tau_{xy}^2)^{1/2} = [189^2 + 3(37.7^2)]^{1/2} = 200 \text{ MPa}$$

$$n = \frac{S_y}{\sigma'} = \frac{280}{200} = 1.4 \quad \text{Ans.}$$

5-37 From Prob. 3-44, the critical location is at the top of the beam at $x = 27$ in from the left end, where there is only a bending stress of $\sigma = -7\,456$ psi. Thus, $\sigma' = 7\,456$ psi and

$$(S_y)_{\min} = n\sigma' = 2(7\,456) = 14\,912 \text{ psi}$$

$$\text{Choose } (S_y)_{\min} = 15 \text{ kpsi} \quad \text{Ans.}$$

5-38 From Table A-20 for 1020 CD steel, $S_y = 57$ kpsi. From Eq. (3-42), p.102,

$$T = \frac{63\,025H}{n} \quad (1)$$

where n is the shaft speed in rev/min. From Eq. (5-3), for the MSS theory,

$$\tau_{\max} = \frac{S_y}{2n_d} = \frac{16T}{\pi d^3} \quad (2)$$

where n_d is the design factor. Substituting Eq. (1) into Eq. (2) and solving for d gives

$$d = \left[\frac{32(63\,025)Hn_d}{n\pi S_y} \right]^{1/3} \quad (3)$$

Substituting $H = 20$ hp, $n_d = 3$, $n = 1750$ rev/min, and $S_y = 57(10^3)$ psi results in

$$d_{\min} = \left[\frac{32(63\,025)20(3)}{1750\pi(57)10^3} \right]^{1/3} = 0.728 \text{ in} \quad \text{Ans.}$$

5-39 From Table A-20, $S_y = 54$ kpsi. From the solution of Prob. 3-68, in the plane of analysis

$$\sigma_1 = 16.5 \text{ kpsi}, \sigma_2 = -1.19 \text{ kpsi}, \text{ and } \tau_{\max} = 8.84 \text{ kpsi}$$

The state of stress is *plane stress*. Thus, the three-dimensional principal stresses are

$$\sigma_1 = 16.5 \text{ kpsi}, \sigma_2 = 0, \text{ and } \sigma_3 = -1.19 \text{ kpsi}$$

$$\text{MSS: From Eq. (5-3), } n = \frac{S_y}{\sigma_1 - \sigma_3} = \frac{54}{16.5 - (-1.19)} = 3.05 \quad \text{Ans.}$$

Note: Whenever the two principal stresses of a plane stress state are of opposite sign, the maximum shear stress found in the analysis is the *true* maximum shear stress. Thus, the factor of safety could have been found from

$$n = \frac{S_y}{2\tau_{\max}} = \frac{54}{2(8.84)} = 3.05 \quad \text{Ans.}$$

DE: The von Mises stress can be found from the principal stresses or from the stresses found in part (d) of Prob. 3-68. That is,

Eqs. (5-13) and (5-19)

$$n = \frac{S_y}{\sigma'} = \frac{S_y}{(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2)^{1/2}} = \frac{54}{[16.5^2 - 16.5(-1.19) + (-1.19)^2]^{1/2}} \\ = 3.15 \quad \text{Ans.}$$

or, Eqs. (5-15) and (5-19) using the results of part (d) of Prob. 3-68

$$n = \frac{S_y}{\sigma'} = \frac{S_y}{(\sigma^2 + 3\tau^2)^{1/2}} = \frac{54}{[15.3^2 + 3(4.43^2)]^{1/2}} \\ = 3.15 \quad \text{Ans.}$$

5-40 From Table A-20, $S_y = 370 \text{ MPa}$. From the solution of Prob. 3-69, in the plane of analysis

$$\sigma_1 = 275 \text{ MPa}, \sigma_2 = -12.1 \text{ MPa}, \text{ and } \tau_{\max} = 144 \text{ MPa}$$

The state of stress is *plane stress*. Thus, the three-dimensional principal stresses are

$$\sigma_1 = 275 \text{ MPa}, \sigma_2 = 0, \text{ and } \sigma_3 = -12.1 \text{ MPa}$$

$$\text{MSS: From Eq. (5-3), } n = \frac{S_y}{\sigma_1 - \sigma_3} = \frac{370}{275 - (-12.1)} = 1.29 \quad \text{Ans.}$$

DE: From Eqs. (5-13) and (5-19)

$$n = \frac{S_y}{\sigma'} = \frac{S_y}{\left(\sigma_A^2 - \sigma_A \sigma_B + \sigma_B^2\right)^{1/2}} = \frac{370}{\left[275^2 - 275(-12.1) + (-12.1)^2\right]^{1/2}}$$

= 1.32 *Ans.*

5-41 From Table A-20, $S_y = 54$ kpsi. From the solution of Prob. 3-70, in the plane of analysis

$$\sigma_1 = 22.6 \text{ kpsi}, \sigma_2 = -1.14 \text{ kpsi}, \text{ and } \tau_{\max} = 11.9 \text{ kpsi}$$

The state of stress is *plane stress*. Thus, the three-dimensional principal stresses are

$$\sigma_1 = 22.6 \text{ kpsi}, \sigma_2 = 0, \text{ and } \sigma_3 = -1.14 \text{ kpsi}$$

MSS: From Eq. (5-3), $n = \frac{S_y}{\sigma_1 - \sigma_3} = \frac{54}{22.6 - (-1.14)} = 2.27$ *Ans.*

DE: From Eqs. (5-13) and (5-19)

$$n = \frac{S_y}{\sigma'} = \frac{S_y}{\left(\sigma_A^2 - \sigma_A \sigma_B + \sigma_B^2\right)^{1/2}} = \frac{54}{\left[22.6^2 - 22.6(-1.14) + (-1.14)^2\right]^{1/2}} \\ = 2.33 \quad \text{Ans.}$$

5-42 From Table A-20, $S_y = 370$ MPa. From the solution of Prob. 3-71, in the plane of analysis

$$\sigma_1 = 78.2 \text{ MPa}, \sigma_2 = -5.27 \text{ MPa}, \text{ and } \tau_{\max} = 41.7 \text{ MPa}$$

The state of stress is *plane stress*. Thus, the three-dimensional principal stresses are

$$\sigma_1 = 78.2 \text{ MPa}, \sigma_2 = 0, \text{ and } \sigma_3 = -5.27 \text{ MPa}$$

MSS: From Eq. (5-3), $n = \frac{S_y}{\sigma_1 - \sigma_3} = \frac{370}{78.2 - (-5.27)} = 4.43$ *Ans.*

DE: From Eqs. (5-13) and (5-19)

$$n = \frac{S_y}{\sigma'} = \frac{S_y}{\left(\sigma_A^2 - \sigma_A \sigma_B + \sigma_B^2\right)^{1/2}} = \frac{370}{\left[78.2^2 - 78.2(-5.27) + (-5.27)^2\right]^{1/2}} \\ = 4.57 \quad \text{Ans.}$$

5-43 From Table A-20, $S_y = 54$ kpsi. From the solution of Prob. 3-72, in the plane of analysis

$$\sigma_1 = 36.7 \text{ kpsi}, \sigma_2 = -1.47 \text{ kpsi}, \text{ and } \tau_{\max} = 19.1 \text{ kpsi}$$

The state of stress is *plane stress*. Thus, the three-dimensional principal stresses are

$$\sigma_1 = 36.7 \text{ kpsi}, \sigma_2 = 0, \text{ and } \sigma_3 = -1.47 \text{ kpsi}$$

MSS: From Eq. (5-3), $n = \frac{S_y}{\sigma_1 - \sigma_3} = \frac{54}{36.7 - (-1.47)} = 1.41$ *Ans.*

DE: From Eqs. (5-13) and (5-19)

$$n = \frac{S_y}{\sigma'} = \frac{S_y}{(\sigma_A^2 - \sigma_A \sigma_B + \sigma_B^2)^{1/2}} = \frac{54}{[36.7^2 - 36.7(-1.47) + (-1.47)^2]^{1/2}}$$

= 1.44 *Ans.*

5-44 From Table A-20, $S_y = 370$ MPa. From the solution of Prob. 3-73, in the plane of analysis

$$\sigma_1 = 376 \text{ MPa}, \sigma_2 = -42.4 \text{ MPa}, \text{ and } \tau_{\max} = 209 \text{ MPa}$$

The state of stress is *plane stress*. Thus, the three-dimensional principal stresses are

$$\sigma_1 = 376 \text{ MPa}, \sigma_2 = 0, \text{ and } \sigma_3 = -42.4 \text{ MPa}$$

MSS: From Eq. (5-3), $n = \frac{S_y}{\sigma_1 - \sigma_3} = \frac{370}{376 - (-42.4)} = 0.88$ *Ans.*

DE: From Eqs. (5-13) and (5-19)

$$n = \frac{S_y}{\sigma'} = \frac{S_y}{(\sigma_A^2 - \sigma_A \sigma_B + \sigma_B^2)^{1/2}} = \frac{370}{[376^2 - 376(-42.4) + (-42.4)^2]^{1/2}}$$

= 0.93 *Ans.*

5-45 From Table A-20, $S_y = 54$ kpsi. From the solution of Prob. 3-74, in the plane of analysis

$$\sigma_1 = 7.19 \text{ kpsi}, \sigma_2 = -17.0 \text{ kpsi}, \text{ and } \tau_{\max} = 12.1 \text{ kpsi}$$

The state of stress is *plane stress*. Thus, the three-dimensional principal stresses are

$$\sigma_1 = 7.19 \text{ kpsi}, \sigma_2 = 0, \text{ and } \sigma_3 = -17.0 \text{ kpsi}$$

MSS: From Eq. (5-3), $n = \frac{S_y}{\sigma_1 - \sigma_3} = \frac{54}{7.19 - (-17.0)} = 2.23$ *Ans.*

DE: From Eqs. (5-13) and (5-19)

$$n = \frac{S_y}{\sigma'} = \frac{S_y}{(\sigma_A^2 - \sigma_A \sigma_B + \sigma_B^2)^{1/2}} = \frac{54}{[7.19^2 - 7.19(-17.0) + (-17.0)^2]^{1/2}}$$

= 2.51 *Ans.*

5-46 From Table A-20, $S_y = 54$ kpsi. From the solution of Prob. 3-76, in the plane of analysis

$$\sigma_1 = 1.72 \text{ kpsi}, \sigma_2 = -35.9 \text{ kpsi}, \text{ and } \tau_{\max} = 18.8 \text{ kpsi}$$

The state of stress is *plane stress*. Thus, the three-dimensional principal stresses are

$$\sigma_1 = 1.72 \text{ kpsi}, \sigma_2 = 0, \text{ and } \sigma_3 = -35.9 \text{ kpsi}$$

MSS: From Eq. (5-3), $n = \frac{S_y}{\sigma_1 - \sigma_3} = \frac{54}{1.72 - (-35.9)} = 1.44$ *Ans.*

DE: From Eqs. (5-13) and (5-19)

$$n = \frac{S_y}{\sigma'} = \frac{S_y}{\left(\sigma_A^2 - \sigma_A \sigma_B + \sigma_B^2\right)^{1/2}} = \frac{54}{\left[1.72^2 - 1.72(-35.9) + (-35.9)^2\right]^{1/2}} \\ = 1.47 \quad \text{Ans.}$$

5-47 From Table A-20, $S_y = 370$ MPa. From the solution of Prob. 3-77,
Bending: $\sigma_B = 68.6$ MPa, Torsion: $\tau_B = 37.7$ MPa

For a plane stress analysis it was found that $\tau_{\max} = 51.0$ MPa. With combined bending and torsion, the plane stress analysis yields the true $\tau_{\max}$.

MSS: From Eq. (5-3), $n = \frac{S_y}{2\tau_{\max}} = \frac{370}{2(51.0)} = 3.63$ *Ans.*

DE: From Eqs. (5-15) and (5-19)

$$n = \frac{S_y}{\sigma'} = \frac{S_y}{\left(\sigma_B^2 + 3\tau_B^2\right)^{1/2}} = \frac{370}{\left[68.6^2 + 3(37.7^2)\right]^{1/2}} \\ = 3.91 \quad \text{Ans.}$$

5-48 From Table A-20, $S_y = 54$ kpsi. From the solution of Prob. 3-79,
Bending: $\sigma_C = 3460$ psi, Torsion: $\tau_C = 882$ kpsi

For a plane stress analysis it was found that $\tau_{\max} = 1940$ psi. With combined bending and torsion, the plane stress analysis yields the true $\tau_{\max}$.

MSS: From Eq. (5-3), $n = \frac{S_y}{2\tau_{\max}} = \frac{54(10^3)}{2(1940)} = 13.9$ *Ans.*

DE: From Eqs. (5-15) and (5-19)

$$n = \frac{S_y}{\sigma'} = \frac{S_y}{(\sigma_C^2 + 3\tau_C^2)^{1/2}} = \frac{54(10^3)}{[3460^2 + 3(882^2)]^{1/2}} \\ = 14.3 \quad \text{Ans.}$$

5-49 From Table A-20, $S_y = 54$ kpsi. From the solution of Prob. 3-80, in the plane of analysis

$$\sigma_1 = 17.8 \text{ kpsi}, \sigma_2 = -1.46 \text{ kpsi}, \text{ and } \tau_{\max} = 9.61 \text{ kpsi}$$

The state of stress is *plane stress*. Thus, the three-dimensional principal stresses are

$$\sigma_1 = 17.8 \text{ kpsi}, \sigma_2 = 0, \text{ and } \sigma_3 = -1.46 \text{ kpsi}$$

$$\text{MSS: From Eq. (5-3), } n = \frac{S_y}{\sigma_1 - \sigma_3} = \frac{54}{17.8 - (-1.46)} = 2.80 \quad \text{Ans.}$$

DE: From Eqs. (5-13) and (5-19)

$$n = \frac{S_y}{\sigma'} = \frac{S_y}{(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2)^{1/2}} = \frac{54}{[17.8^2 - 17.8(-1.46) + (-1.46)^2]^{1/2}} \\ = 2.91 \quad \text{Ans.}$$

5-50 From Table A-20, $S_y = 54$ kpsi. From the solution of Prob. 3-81, in the plane of analysis

$$\sigma_1 = 17.5 \text{ kpsi}, \sigma_2 = -1.13 \text{ kpsi}, \text{ and } \tau_{\max} = 9.33 \text{ kpsi}$$

The state of stress is *plane stress*. Thus, the three-dimensional principal stresses are

$$\sigma_1 = 17.5 \text{ kpsi}, \sigma_2 = 0, \text{ and } \sigma_3 = -1.13 \text{ kpsi}$$

$$\text{MSS: From Eq. (5-3), } n = \frac{S_y}{\sigma_1 - \sigma_3} = \frac{54}{17.5 - (-1.13)} = 2.90 \quad \text{Ans.}$$

DE: From Eqs. (5-13) and (5-19)

$$n = \frac{S_y}{\sigma'} = \frac{S_y}{(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2)^{1/2}} = \frac{54}{[17.5^2 - 17.5(-1.13) + (-1.13)^2]^{1/2}} \\ = 2.98 \quad \text{Ans.}$$

5-51 From Table A-20, $S_y = 54$ kpsi. From the solution of Prob. 3-82, in the plane of analysis

$$\sigma_1 = 21.5 \text{ kpsi}, \sigma_2 = -1.20 \text{ kpsi}, \text{ and } \tau_{\max} = 11.4 \text{ kpsi}$$

The state of stress is *plane stress*. Thus, the three-dimensional principal stresses are

$$\sigma_1 = 21.5 \text{ kpsi}, \sigma_2 = 0, \text{ and } \sigma_3 = -1.20 \text{ kpsi}$$

MSS: From Eq. (5-3), $n = \frac{S_y}{\sigma_1 - \sigma_3} = \frac{54}{21.5 - (-1.20)} = 2.38$ *Ans.*

DE: From Eqs. (5-13) and (5-19)

$$n = \frac{S_y}{\sigma'} = \frac{S_y}{\left(\sigma_A^2 - \sigma_A \sigma_B + \sigma_B^2\right)^{1/2}} = \frac{54}{\left[21.5^2 - 21.5(-1.20) + (-1.20)^2\right]^{1/2}} \\ = 2.44 \quad \text{Ans.}$$

5-52 From Table A-20, $S_y = 54$ kpsi. From the solution of Prob. 3-83, the concern was failure due to twisting of the flat bar where it was found that $\tau_{\max} = 14.3$ kpsi in the middle of the longest side of the rectangular cross section. The bar is also in bending, but the bending stress is zero where $\tau_{\max}$ exists.

MSS: From Eq. (5-3), $n = \frac{S_y}{2\tau_{\max}} = \frac{54}{2(14.3)} = 1.89$ *Ans.*

DE: From Eqs. (5-15) and (5-19)

$$n = \frac{S_y}{\sigma'} = \frac{S_y}{\left(3\tau_{\max}^2\right)^{1/2}} = \frac{54}{\left[3(14.3^2)\right]^{1/2}} = 2.18 \quad \text{Ans.}$$

5-53 From Table A-20, $S_y = 54$ kpsi. From the solution of Prob. 3-84, in the plane of analysis

$$\sigma_1 = 34.7 \text{ kpsi}, \sigma_2 = -6.7 \text{ kpsi}, \text{ and } \tau_{\max} = 20.7 \text{ kpsi}$$

The state of stress is *plane stress*. Thus, the three-dimensional principal stresses are

$$\sigma_1 = 34.7 \text{ kpsi}, \sigma_2 = 0, \text{ and } \sigma_3 = -6.7 \text{ kpsi}$$

MSS: From Eq. (5-3), $n = \frac{S_y}{\sigma_1 - \sigma_3} = \frac{54}{34.7 - (-6.7)} = 1.30$ *Ans.*

DE: From Eqs. (5-13) and (5-19)

$$n = \frac{S_y}{\sigma'} = \frac{S_y}{(\sigma_A^2 - \sigma_A \sigma_B + \sigma_B^2)^{1/2}} = \frac{54}{[34.7^2 - 34.7(-6.7) + (-6.7)^2]^{1/2}} \\ = 1.40 \quad \text{Ans.}$$

5-54 From Table A-20, $S_y = 54$ kpsi. From the solution of Prob. 3-85, in the plane of analysis

$$\sigma_1 = 51.1 \text{ kpsi}, \sigma_2 = -4.58 \text{ kpsi}, \text{ and } \tau_{\max} = 27.8 \text{ kpsi}$$

The state of stress is *plane stress*. Thus, the three-dimensional principal stresses are

$$\sigma_1 = 51.1 \text{ kpsi}, \sigma_2 = 0, \text{ and } \sigma_3 = -4.58 \text{ kpsi}$$

MSS: From Eq. (5-3), $n = \frac{S_y}{\sigma_1 - \sigma_3} = \frac{54}{51.1 - (-4.58)} = 0.97 \quad \text{Ans.}$

DE: From Eqs. (5-13) and (5-19)

$$n = \frac{S_y}{\sigma'} = \frac{S_y}{(\sigma_A^2 - \sigma_A \sigma_B + \sigma_B^2)^{1/2}} = \frac{54}{[51.1^2 - 51.1(-4.58) + (-4.58)^2]^{1/2}} \\ = 1.01 \quad \text{Ans.}$$

5-55 From Table A-20, $S_y = 54$ kpsi. From the solution of Prob. 3-86, in the plane of analysis

$$\sigma_1 = 59.7 \text{ kpsi}, \sigma_2 = -3.92 \text{ kpsi}, \text{ and } \tau_{\max} = 31.8 \text{ kpsi}$$

The state of stress is *plane stress*. Thus, the three-dimensional principal stresses are

$$\sigma_1 = 59.7 \text{ kpsi}, \sigma_2 = 0, \text{ and } \sigma_3 = -3.92 \text{ kpsi}$$

MSS: From Eq. (5-3), $n = \frac{S_y}{\sigma_1 - \sigma_3} = \frac{54}{59.7 - (-3.92)} = 0.85 \quad \text{Ans.}$

DE: From Eqs. (5-13) and (5-19)

$$n = \frac{S_y}{\sigma'} = \frac{S_y}{(\sigma_A^2 - \sigma_A \sigma_B + \sigma_B^2)^{1/2}} = \frac{54}{[59.7^2 - 59.7(-3.92) + (-3.92)^2]^{1/2}} \\ = 0.87 \quad \text{Ans.}$$

5-56 For Prob. 3-84, from Prob. 5-53 solution, with 1018 CD, DE theory yields, $n = 1.40$.

From Table A-21, for 4140 Q&T @400°F, $S_y = 238$ kpsi. From Prob. (3-87) solution which considered stress concentrations for Prob. 3-84

$$\sigma_1 = 53.0 \text{ kpsi}, \sigma_2 = -8.48 \text{ kpsi}, \text{ and } \tau_{\max} = 30.7 \text{ kpsi}$$

DE: From Eqs. (5-13) and (5-19)

$$n = \frac{S_y}{\sigma'} = \frac{S_y}{\left(\sigma_A^2 - \sigma_A \sigma_B + \sigma_B^2\right)^{1/2}} = \frac{238}{\left[53.0^2 - 53.0(-8.48) + (-8.48)^2\right]^{1/2}}$$
$$= 4.12 \quad \text{Ans.}$$

Using the 4140 versus the 1018 CD, the factor of safety increases by a factor of $4.12/1.40 = 2.94$. *Ans.*

5-57 Design Decisions Required:

- Material and condition
- Design factor
- Failure model
- Diameter of pin

Using $F = 416$ lbf from Ex. 5-3,

$$\sigma_{\max} = \frac{32M}{\pi d^3}$$
$$d = \left(\frac{32M}{\pi \sigma_{\max}}\right)^{1/3}$$

Decision 1: Select the same material and condition of Ex. 5-3 (AISI 1035 steel, $S_y = 81$ kpsi)

Decision 2: Since we prefer the pin to yield, set n_d a little larger than 1. Further explanation will follow.

Decision 3: Use the Distortion Energy static failure theory.

Decision 4: Initially set $n_d = 1$

$$\sigma_{\max} = \frac{S_y}{n_d} = \frac{S_y}{1} = 81\,000 \text{ psi}$$

$$d = \left(\frac{32(416)(15)}{\pi(81\,000)}\right)^{1/3} = 0.922 \text{ in}$$

Choose preferred size of $d = 1.000$ in

$$F = \frac{\pi(1)^3(81\,000)}{32(15)} = 530 \text{ lbf}$$

$$n = \frac{530}{416} = 1.274$$

Set design factor to $n_d = 1.274$

Adequacy Assessment:

$$\sigma_{\max} = \frac{S_y}{n_d} = \frac{81\,000}{1.274} = 63\,580 \text{ psi}$$

$$d = \left(\frac{32(416)(15)}{\pi(63\,580)} \right)^{\frac{1}{3}} = 1.000 \text{ in (OK)}$$

$$F = \frac{\pi(1)^3(81\,000)}{32(15)} = 530 \text{ lbf}$$

$$n = \frac{530}{416} = 1.274 \text{ (OK)}$$

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- 5-58** From Table A-20, for a thin walled cylinder made of AISI 1020 CD steel, $S_{yt} = 57$ kpsi, $S_{ut} = 68$ kpsi.

Since $r/t = 7.5/0.0625 = 120 > 10$, the shell can be considered thin-wall. From the solution of Prob. 3-93 the principal stresses are

$$\sigma_1 = \sigma_2 = \frac{pd}{4t} = \frac{p(15)}{4(0.0625)} = 60p, \quad \sigma_3 = -p$$

From Eq. (5-12)

$$\begin{aligned} \sigma' &= \frac{1}{\sqrt{2}} \left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \right]^{1/2} \\ &= \frac{1}{\sqrt{2}} \left[(60p - 60p)^2 + (60p + p)^2 + (-p - 60p)^2 \right]^{1/2} = 61p \end{aligned}$$

For yield, $\sigma' = S_y \Rightarrow 61p = 57(10^3) \Rightarrow p = 934 \text{ psi}$ *Ans.*

For rupture, $61p = 68 \Rightarrow p = 1.11 \text{ kpsi}$ *Ans.*

5-59 For AISI 1020 HR steel, from Tables A-5 and A-20, $w = 0.282 \text{ lbf/in}^3$, $S_y = 30 \text{ kpsi}$, and $\nu = 0.292$. Then, $\rho = w/g = 0.282/386 \text{ lbf}\cdot\text{s}^2/\text{in}$. For the problem, $r_i = 3 \text{ in}$, and $r_o = 5 \text{ in}$. Substituting into Eqs. (3-55), p. 115, gives

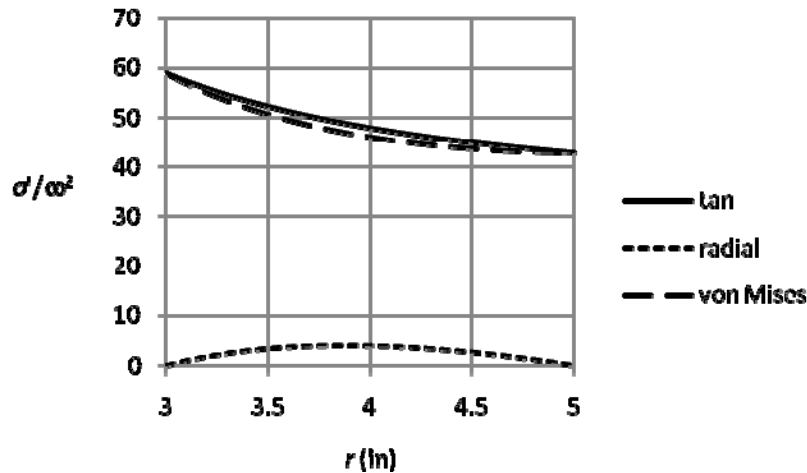
$$\begin{aligned}\sigma_t &= \frac{0.282}{386} \omega^2 \left(\frac{3+0.292}{8} \right) \left[9+25+\frac{9(25)}{r^2}-\frac{1+3(0.292)}{3+0.292} r^2 \right] \\ &= 3.006(10^{-4}) \omega^2 \left(34+\frac{225}{r^2}-0.5699r^2 \right) = F(r) \omega^2 \quad (1)\end{aligned}$$

$$\sigma_r = 3.006(10^{-4}) \omega^2 \left(34-\frac{225}{r^2}-r^2 \right) = G(r) \omega^2 \quad (2)$$

For the distortion-energy theory, the von Mises stress will be

$$\sigma' = \left(\sigma_t^2 - \sigma_t \sigma_r + \sigma_r^2 \right)^{1/2} = \omega^2 \left[F^2(r) - F(r)G(r) + G^2(r) \right]^{1/2} \quad (3)$$

Although it was noted that the maximum radial stress occurs at $r = (r_o r_i)^{1/2}$ we are more interested as to where the von Mises stress is a maximum. One could take the derivative of Eq. (3) and set it to zero to find where the maximum occurs. However, it is much easier to plot σ'/ω^2 for $3 \leq r \leq 5 \text{ in}$. Plotting Eqs. (1) through (3) results



in

It can be seen that there is no maxima, and the greatest value of the von Mises stress is the tangential stress at $r = r_i$. Substituting $r = 3$ in into Eq. (1) and setting $\sigma' = S_y$ gives

$$\omega = \left[\frac{30(10^3)}{3.006(10^{-4}) \left(34 + \frac{225}{3^2} - 0.5699(3^2) \right)} \right]^{1/2} = 1361 \text{ rad/s}$$

$$n = \frac{60\omega}{2\pi} = \frac{60(1361)}{2\pi} = 13\,000 \text{ rev/min} \quad \text{Ans.}$$

5-60 Since $r/t = 1.75/0.065 = 26.9 > 10$, we can use thin-walled equations. From Eqs. (3-53) and (3-54), p. 114,

$$d_i = 3.5 - 2(0.065) = 3.37 \text{ in}$$

$$\sigma_t = \frac{p(d_i + t)}{2t}$$

$$\sigma_t = \frac{500(3.37 + 0.065)}{2(0.065)} = 13\,212 \text{ psi} = 13.2 \text{ kpsi}$$

$$\sigma_l = \frac{pd_i}{4t} = \frac{500(3.37)}{4(0.065)} = 6481 \text{ psi} = 6.48 \text{ kpsi}$$

$$\sigma_r = -p_i = -500 \text{ psi} = -0.5 \text{ kpsi}$$

These are all principal stresses, thus, from Eq. (5-12),

$$\sigma' = \frac{1}{\sqrt{2}} \left\{ (13.2 - 6.48)^2 + [6.48 - (-0.5)]^2 + (-0.5 - 13.2)^2 \right\}^{1/2}$$

$$= 11.87 \text{ kpsi}$$

$$n = \frac{S_y}{\sigma'} = \frac{46}{11.87}$$

$$n = 3.88 \quad \text{Ans.}$$

5-61 From Table A-20, $S_y = 320 \text{ MPa}$

With $p_i = 0$, Eqs. (3-49), p. 113, are

$$\sigma_t = -\frac{r_o^2 p_o}{r_o^2 - r_i^2} \left(1 + \frac{r_i^2}{r^2} \right) = c \left(1 + \frac{b^2}{r^2} \right) \quad (1)$$

$$\sigma_r = -\frac{r_o^2 p_o}{r_o^2 - r_i^2} \left(1 - \frac{r_i^2}{r^2} \right) = c \left(1 - \frac{b^2}{r^2} \right)$$

For the distortion-energy theory, the von Mises stress is

$$\sigma' = \left(\sigma_t^2 - \sigma_t \sigma_r + \sigma_r^2 \right)^{1/2} = c \left[\left(1 + \frac{b^2}{r^2} \right)^2 - \left(1 + \frac{b^2}{r^2} \right) \left(1 - \frac{b^2}{r^2} \right) + \left(1 - \frac{b^2}{r^2} \right)^2 \right]^{1/2}$$

$$= c \left(1 + 3 \frac{b^4}{r^4} \right)^{1/2}$$

We see that the maximum von Mises stress occurs where r is a minimum at $r = r_i$. Here, $\sigma_r = 0$ and thus $\sigma' = -\sigma_t$. Setting $-\sigma_t = S_y = 320$ MPa at $r = 0.1$ m in Eq. (1) results in

$$-\sigma_t|_{r=r_i} = \frac{2r_o^2 p_o}{r_o^2 - r_i^2} = \frac{2(0.15^2) p_o}{0.15^2 - 0.1^2} = 3.6 p_o = 320 \Rightarrow p_o = 88.9 \text{ MPa} \quad \text{Ans.}$$

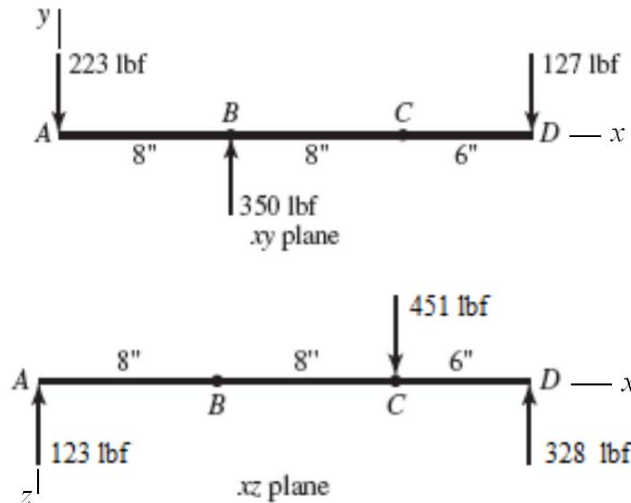
- 5-62** From Table A-24, $S_{ut} = 31$ kpsi for grade 30 cast iron. From Table A-5, $\nu = 0.211$ and $w = 0.260$ lbf/in³. In Prob. 5-59, it was determined that the maximum stress was the tangential stress at the inner radius, where the radial stress is zero. Thus at the inner radius, Eq. (3-55), p. 115, gives

$$\begin{aligned} \sigma_t &= \rho \omega^2 \left(\frac{3+\nu}{8} \right) \left(2r_o^2 + r_i^2 - \frac{1+3\nu}{3+\nu} r_i^2 \right) = \frac{0.260}{386} \omega^2 \left(\frac{3.211}{8} \right) \left[2(5^2) + 3^2 - \frac{1+3(0.211)}{3.211} 3^2 \right] \\ &= 0.01471 \omega^2 = 31(10^3) \quad \Rightarrow \quad 1452 \text{ rad/sec} \end{aligned}$$

$$n = 60(1452)/(2\pi) = 13\,870 \text{ rev/min} \quad \text{Ans.}$$

- 5-63** From Table A-20, for AISI 1035 CD, $S_y = 67$ kpsi.

From force and bending-moment equations, the ground reaction forces are found in two planes as shown.



The maximum bending moment will be at B or C. Check which is larger. In the xy plane,

$$M_B = 223(8) = 1784 \text{ lbf} \cdot \text{in} \text{ and } M_C = 127(6) = 762 \text{ lbf} \cdot \text{in}.$$

$$\text{In the } xz \text{ plane, } M_B = 123(8) = 984 \text{ lbf} \cdot \text{in} \text{ and } M_C = 328(6) = 1968 \text{ lbf} \cdot \text{in}.$$

$$M_B = [(1784)^2 + (984)^2]^{\frac{1}{2}} = 2037 \text{ lbf} \cdot \text{in}$$

$$M_C = [(762)^2 + (1968)^2]^{\frac{1}{2}} = 2110 \text{ lbf} \cdot \text{in}$$

So point *C* governs. The torque transmitted between *B* and *C* is $T = (300 - 50)(4) = 1000$ lbf·in. The stresses are

$$\tau_{xz} = \frac{16T}{\pi d^3} = \frac{16(1000)}{\pi d^3} = \frac{5093}{d^3} \text{ psi}$$

$$\sigma_x = \frac{32M_C}{\pi d^3} = \frac{32(2110)}{\pi d^3} = \frac{21492}{d^3} \text{ psi}$$

For combined bending and torsion, the maximum shear stress is found from

$$\tau_{\max} = \left[\left(\frac{\sigma_x}{2} \right)^2 + \tau_{xz}^2 \right]^{1/2} = \left[\left(\frac{21.49}{d^3} \right)^2 + \left(\frac{5.09}{d^3} \right)^2 \right]^{1/2} = \frac{11.89}{d^3} \text{ kpsi}$$

Max Shear Stress theory is chosen as a conservative failure theory. From Eq. (5-3)

$$\tau_{\max} = \frac{S_y}{2n} = \frac{11.89}{d^3} = \frac{67}{2(2)} \quad \Rightarrow \quad d = 0.892 \text{ in} \quad \text{Ans.}$$

5-64 As in Prob. 5-63, we will assume this to be a statics problem. Since the proportions are unchanged, the bearing reactions will be the same as in Prob. 5-63 and the bending moment will still be maximum at point *C*. Thus

$$\text{xy plane: } M_C = 127(3) = 381 \text{ lbf} \cdot \text{in}$$

$$\text{xz plane: } M_C = 328(3) = 984 \text{ lbf} \cdot \text{in}$$

So

$$M_{\max} = [(381)^2 + (984)^2]^{1/2} = 1055 \text{ lbf} \cdot \text{in}$$

$$\sigma_x = \frac{32M_C}{\pi d^3} = \frac{32(1055)}{\pi d^3} = \frac{10746}{d^3} \text{ psi} = \frac{10.75}{d^3} \text{ kpsi}$$

Since the torsional stress is unchanged,

$$\tau_{xz} = \frac{5.09}{d^3} \text{ kpsi}$$

For combined bending and torsion, the maximum shear stress is found from

$$\tau_{\max} = \left[\left(\frac{\sigma_x}{2} \right)^2 + \tau_{xz}^2 \right]^{1/2} = \left[\left(\frac{10.75}{2d^3} \right)^2 + \left(\frac{5.09}{d^3} \right)^2 \right]^{1/2} = \frac{7.40}{d^3} \text{ kpsi}$$

Using the MSS theory, as was used in Prob. 5-63, gives

$$\tau_{\max} = \frac{S_y}{2n} = \frac{7.40}{d^3} = \frac{67}{2(2)} \quad \Rightarrow \quad d = 0.762 \text{ in} \quad \text{Ans.}$$

- 5-65** For AISI 1018 HR, Table A-20 gives $S_y = 32$ kpsi. Transverse shear stress is maximum at the neutral axis, and zero at the outer radius. Bending stress is maximum at the outer radius, and zero at the neutral axis.

Model (c): From Prob. 3-40, at outer radius,

$$\sigma' = \sigma = 17.8 \text{ kpsi}$$

$$n = \frac{S_y}{\sigma'} = \frac{32}{17.8} = 1.80$$

At neutral axis,

$$\sigma' = \sqrt{3\tau^2} = \sqrt{3(3.4)^2} = 5.89 \text{ kpsi}$$

$$n = \frac{S_y}{\sigma'} = \frac{32}{5.89} = 5.43$$

The bending stress at the outer radius dominates. $n = 1.80$ Ans.

Model (d): Assume the bending stress at the outer radius will dominate, as in model (c). From Prob. 3-40,

$$\sigma' = \sigma = 25.5 \text{ kpsi}$$

$$n = \frac{S_y}{\sigma'} = \frac{32}{25.5} = 1.25 \quad \text{Ans.}$$

Model (e): From Prob. 3-40,

$$\sigma' = \sigma = 17.8 \text{ kpsi}$$

$$n = \frac{S_y}{\sigma'} = \frac{32}{17.8} = 1.80 \quad \text{Ans.}$$

Model (d) is the most conservative, thus safest, and requires the least modeling time.

Model (c) is probably the most accurate, but model (e) yields the same results with less modeling effort.

- 5-66** For AISI 1018 HR, from Table A-20, $S_y = 32$ kpsi. Model (d) yields the largest bending moment, so designing to it is the most conservative approach. The bending moment is $M = 312.5$ lbf-in. For this case, the principal stresses are

$$\sigma_1 = \frac{32M}{\pi d^3}, \quad \sigma_2 = \sigma_3 = 0$$

Using a conservative yielding failure theory use the MSS theory and Eq. (5-3)

$$\sigma_1 - \sigma_3 = \frac{S_y}{n} \quad \Rightarrow \quad \frac{32M}{\pi d^3} = \frac{S_y}{n} \quad \Rightarrow \quad d = \left(\frac{32Mn}{\pi S_y} \right)^{1/3}$$

$$\text{Thus, } d = \left[\frac{32(312.5)2.5}{\pi(32)10^3} \right]^{1/3} = 0.629 \text{ in } \therefore \text{ Use } d = \frac{11}{16} \text{ in } \quad \text{Ans.}$$

5-67 When the ring is set, the hoop tension in the ring is equal to the screw tension.

$$\sigma_t = \frac{r_i^2 p_i}{r_o^2 - r_i^2} \left(1 + \frac{r_o^2}{r^2} \right)$$

We have the hoop tension at any radius. The differential hoop tension dF is

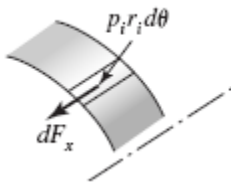
$$dF = w\sigma_t dr$$

$$F = \int_{r_i}^{r_o} w\sigma_t dr = \frac{wr_i^2 p_i}{r_o^2 - r_i^2} \int_{r_i}^{r_o} \left(1 + \frac{r_o^2}{r^2} \right) dr = wr_i p_i \quad (1)$$

The screw equation is

$$F_i = \frac{T}{0.2d} \quad (2)$$

From Eqs. (1) and (2)



$$p_i = \frac{F}{wr_i} = \frac{T}{0.2dwr_i}$$

$$dF_x = fp_i r_i d\theta$$

$$\begin{aligned} F_x &= \int_0^{2\pi} fp_i wr_i d\theta = \frac{fTw}{0.2dwr_i} r_i \int_0^{2\pi} d\theta \\ &= \frac{2\pi fT}{0.2d} \quad \text{Ans.} \end{aligned}$$

5-68 $T = 20 \text{ N}\cdot\text{m}$, $S_y = 450 \text{ MPa}$

(a) From Prob. 5-67, $T = 0.2 F_i d$

$$F_i = \frac{T}{0.2d} = \frac{20}{0.2[6(10^{-3})]} = 16.7(10^3) \text{ N} = 16.7 \text{ kN} \quad \text{Ans.}$$

(b) From Prob. 5-67, $F = wr_i p_i$

$$p_i = \frac{F}{wr_i} = \frac{F_i}{wr_i} = \frac{16.7(10^3)}{[12(10^{-3})][(25/2)(10^{-3})]} = 111.3(10^6) \text{ Pa} = 111.3 \text{ MPa} \quad \text{Ans.}$$

$$(c) \quad \sigma_t = \frac{r_i^2 p_i}{r_o^2 - r_i^2} \left(1 + \frac{r_o^2}{r} \right)_{r=r_i} = \frac{p_i (r_i^2 + r_o^2)}{r_o^2 - r_i^2}$$

$$= \frac{111.3(0.0125^2 + 0.025^2)}{0.025^2 - 0.0125^2} = 185.5 \text{ MPa} \quad \text{Ans.}$$

$$\sigma_r = -p_i = -111.3 \text{ MPa}$$

(d)

$$\tau_{\max} = \frac{\sigma_1 - \sigma_3}{2} = \frac{\sigma_t - \sigma_r}{2}$$

$$= \frac{185.5 - (-111.3)}{2} = 148.4 \text{ MPa} \quad \text{Ans.}$$

$$\sigma' = (\sigma_A^2 - \sigma_A \sigma_B + \sigma_B^2)^{1/2}$$

$$= [185.5^2 - (185.5)(-111.3) + (-111.3)^2]^{1/2}$$

$$= 259.7 \text{ MPa} \quad \text{Ans.}$$

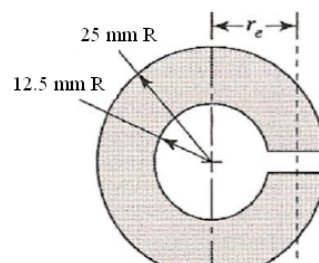
(e) Maximum Shear Stress Theory

$$n = \frac{S_y}{2\tau_{\max}} = \frac{450}{2(148.4)} = 1.52 \quad \text{Ans.}$$

Distortion Energy theory

$$n = \frac{S_y}{\sigma'} = \frac{450}{259.7} = 1.73 \quad \text{Ans.}$$

5-69 The moment about the center caused by the force F is $F r_e$ where r_e is the effective



radius. This is balanced by the moment about the center caused by the tangential (hoop) stress.

$$\begin{aligned}
 Fr_e &= \int_{r_i}^{r_o} r \sigma_t w dr \\
 &= \frac{w p_i r_i^2}{r_o^2 - r_i^2} \int_{r_i}^{r_o} \left(r + \frac{r_o^2}{r} \right) dr \\
 r_e &= \frac{w p_i r_i^2}{F (r_o^2 - r_i^2)} \left(\frac{r_o^2 - r_i^2}{2} + r_o^2 \ln \frac{r_o}{r_i} \right)
 \end{aligned}$$

From Prob. 5-67, $F = w r_i p_i$. Therefore,

$$r_e = \frac{r_i}{r_o^2 - r_i^2} \left(\frac{r_o^2 - r_i^2}{2} + r_o^2 \ln \frac{r_o}{r_i} \right)$$

For the conditions of Prob. 5-67, $r_i = 12.5$ mm and $r_o = 25$ mm

$$r_e = \frac{12.5}{25^2 - 12.5^2} \left(\frac{25^2 - 12.5^2}{2} + 25^2 \ln \frac{25}{12.5} \right) = 17.8 \text{ mm} \quad \text{Ans.}$$

5-70 (a) The nominal radial interference is $\delta_{\text{nom}} = (2.002 - 2.001) / 2 = 0.0005$ in.

From Eq. (3-57), p. 116,

$$\begin{aligned}
 p &= \frac{E \delta}{2 R^3} \left[\frac{(r_o^2 - R^2)(R^2 - r_i^2)}{r_o^2 - r_i^2} \right] \\
 &= \frac{30(10^6) 0.0005}{2(1^3)} \left[\frac{(1.5^2 - 1^2)(1^2 - 0.625^2)}{1.5^2 - 0.625^2} \right] = 3072 \text{ psi} \quad \text{Ans.}
 \end{aligned}$$

Inner member: $p_i = 0$, $p_o = p = 3072$ psi. At fit surface $r = R = 1$ in,

$$\text{Eq. (3-49), p. 113,} \quad \sigma_t = -p \frac{R^2 + r_i^2}{R^2 - r_i^2} = -3072 \left(\frac{1^2 + 0.625^2}{1^2 - 0.625^2} \right) = -7010 \text{ psi}$$

$$\sigma_r = -p = -3072 \text{ psi}$$

Eq. (5-13)

$$\begin{aligned}\sigma' &= (\sigma_A^2 - \sigma_A \sigma_B + \sigma_B^2)^{1/2} \\ &= [(-7010)^2 - (-7010)(-3072) + (-3072)^2]^{1/2} = 6086 \text{ psi} \quad \text{Ans.}\end{aligned}$$

Outer member: $p_i = p = 3072 \text{ psi}$, $p_o = 0$. At fit surface $r = R = 1 \text{ in}$,

Eq. (3-49), p. 113,
$$\sigma_t = p \frac{r_o^2 + R^2}{r_o^2 - R^2} = 3072 \left(\frac{1.5^2 + 1^2}{1.5^2 - 1^2} \right) = 7987 \text{ psi}$$

$$\sigma_r = -p = -3072 \text{ psi}$$

Eq. (5-13)

$$\begin{aligned}\sigma' &= (\sigma_A^2 - \sigma_A \sigma_B + \sigma_B^2)^{1/2} \\ &= [7987^2 - 7987(-3072) + (-3072)^2]^{1/2} = 9888 \text{ psi} \quad \text{Ans.}\end{aligned}$$

(b) For a solid inner tube,

$$p = \frac{30(10^6)0.0005}{2(1^3)} \left[\frac{(1.5^2 - 1^2)(1^2)}{1.5^2} \right] = 4167 \text{ psi} \quad \text{Ans.}$$

Inner member: $\sigma_t = \sigma_r = -p = -4167 \text{ psi}$

$$\sigma' = [(-4167)^2 - (-4167)(-4167) + (-4167)^2]^{1/2} = 4167 \text{ psi} \quad \text{Ans.}$$

Outer member: $p_i = p = 4167 \text{ psi}$, $p_o = 0$. At fit surface $r = R = 1 \text{ in}$,

Eq. (3-49), p. 113,
$$\sigma_t = p \frac{r_o^2 + R^2}{r_o^2 - R^2} = 4167 \left(\frac{1.5^2 + 1^2}{1.5^2 - 1^2} \right) = 10834 \text{ psi}$$

$$\sigma_r = -p = -4167 \text{ psi}$$

Eq. (5-13)

$$\begin{aligned}\sigma' &= (\sigma_A^2 - \sigma_A \sigma_B + \sigma_B^2)^{1/2} \\ &= [10834^2 - 10834(-4167) + (-4167)^2]^{1/2} = 13410 \text{ psi} \quad \text{Ans.}\end{aligned}$$

- 5-71** From Table A-5, $E = 207 (10^3) \text{ MPa}$. The nominal radial interference is $\delta_{\text{nom}} = (40 - 39.98) / 2 = 0.01 \text{ mm}$.

From Eq. (3-57), p. 116,

$$p = \frac{E\delta}{2R^3} \left[\frac{(r_o^2 - R^2)(R^2 - r_i^2)}{r_o^2 - r_i^2} \right]$$

$$= \frac{207(10^3)0.01}{2(20^3)} \left[\frac{(32.5^2 - 20^2)(20^2 - 10^2)}{32.5^2 - 10^2} \right] = 26.64 \text{ MPa} \quad \text{Ans.}$$

Inner member: $p_i = 0$, $p_o = p = 26.64 \text{ MPa}$. At fit surface $r = R = 20 \text{ mm}$,

Eq. (3-49), p. 113, $\sigma_t = -p \frac{R^2 + r_i^2}{R^2 - r_i^2} = -26.64 \left(\frac{20^2 + 10^2}{20^2 - 10^2} \right) = -44.40 \text{ MPa}$

$$\sigma_r = -p = -26.64 \text{ MPa}$$

Eq. (5-13)

$$\sigma' = (\sigma_A^2 - \sigma_A \sigma_B + \sigma_B^2)^{1/2}$$

$$= [(-44.40)^2 - (-44.40)(-26.64) + (-26.64)^2]^{1/2} = 38.71 \text{ MPa} \quad \text{Ans.}$$

Outer member: $p_i = p = 26.64 \text{ MPa}$, $p_o = 0$. At fit surface $r = R = 20 \text{ mm}$,

Eq. (3-49), p. 113, $\sigma_t = p \frac{r_o^2 + R^2}{r_o^2 - R^2} = 26.64 \left(\frac{32.5^2 + 20^2}{32.5^2 - 20^2} \right) = 59.12 \text{ MPa}$

$$\sigma_r = -p = -26.64 \text{ MPa}$$

Eq. (5-13)

$$\sigma' = (\sigma_A^2 - \sigma_A \sigma_B + \sigma_B^2)^{1/2}$$

$$= [59.12^2 - 59.12(-26.64) + (-26.64)^2]^{1/2} = 76.03 \text{ MPa} \quad \text{Ans.}$$

- 5-72** From Table A-5, $E = 207 (10^3) \text{ MPa}$. The nominal radial interference is $\delta_{\text{nom}} = (40.008 - 39.972) / 2 = 0.018 \text{ mm}$.

From Eq. (3-57), p. 116,

$$p = \frac{E\delta}{2R^3} \left[\frac{(r_o^2 - R^2)(R^2 - r_i^2)}{r_o^2 - r_i^2} \right]$$

$$= \frac{207(10^3)0.018}{2(20^3)} \left[\frac{(32.5^2 - 20^2)(20^2 - 10^2)}{32.5^2 - 10^2} \right] = 47.94 \text{ MPa} \quad \text{Ans.}$$

Inner member: $p_i = 0$, $p_o = p = 47.94 \text{ MPa}$. At fit surface $r = R = 20 \text{ mm}$,

Eq. (3-49), p. 113, $\sigma_t = -p \frac{R^2 + r_i^2}{R^2 - r_i^2} = -47.94 \left(\frac{20^2 + 10^2}{20^2 - 10^2} \right) = -79.90 \text{ MPa}$

$$\sigma_r = -p = -47.94 \text{ MPa}$$

Eq. (5-13)

$$\sigma' = \left(\sigma_A^2 - \sigma_A \sigma_B + \sigma_B^2 \right)^{1/2}$$

$$= \left[(-79.90)^2 - (-79.90)(-47.94) + (-47.94)^2 \right]^{1/2} = 69.66 \text{ MPa} \quad \text{Ans.}$$

Outer member: $p_i = p = 47.94 \text{ MPa}$, $p_o = 0$. At fit surface $r = R = 20 \text{ mm}$,

Eq. (3-49), p. 113, $\sigma_t = p \frac{r_o^2 + R^2}{r_o^2 - R^2} = 47.94 \left(\frac{32.5^2 + 20^2}{32.5^2 - 20^2} \right) = 106.4 \text{ MPa}$

$$\sigma_r = -p = -47.94 \text{ MPa}$$

Eq. (5-13)

$$\sigma' = \left(\sigma_A^2 - \sigma_A \sigma_B + \sigma_B^2 \right)^{1/2}$$

$$= \left[106.4^2 - 106.4(-47.94) + (-47.94)^2 \right]^{1/2} = 136.8 \text{ MPa} \quad \text{Ans.}$$

5-73 From Table A-5, for carbon steel, $E_s = 30 \text{ kpsi}$, and $\nu_s = 0.292$. While for $E_{ci} = 14.5 \text{ Mpsi}$, and $\nu_{ci} = 0.211$. For ASTM grade 20 cast iron, from Table A-24, $S_{ut} = 22 \text{ kpsi}$.

For midrange values, $\delta = (2.001 - 2.0002)/2 = 0.0004 \text{ in.}$

Eq. (3-50), p. 116,

$$p = \frac{\delta}{R \left[\frac{1}{E_o} \left(\frac{r_o^2 + R^2}{r_o^2 - R^2} + \nu_o \right) + \frac{1}{E_i} \left(\frac{R^2 + r_i^2}{R^2 - r_i^2} - \nu_i \right) \right]}$$

$$= \frac{0.0004}{1 \left[\frac{1}{14.5(10^6)} \left(\frac{2^2 + 1^2}{2^2 - 1^2} + 0.211 \right) + \frac{1}{30(10^6)} \left(\frac{1^2}{1^2} - 0.292 \right) \right]} = 2613 \text{ psi}$$

At fit surface, with $p_i = p = 2613 \text{ psi}$, and $p_o = 0$, from Eq. (3-50), p. 113

$$\sigma_t = p \frac{r_o^2 + R^2}{r_o^2 - R^2} = 2613 \left(\frac{2^2 + 1^2}{2^2 - 1^2} \right) = 4355 \text{ psi}$$

$$\sigma_r = -p = -2613 \text{ psi}$$

From Modified-Mohr theory, Eq. (5-32a), since $\sigma_A > 0 > \sigma_B$ and $|\sigma_B / \sigma_A| < 1$,

$$n = \frac{S_{ut}}{\sigma_A} = \frac{22}{4.355} = 5.05 \quad \text{Ans.}$$

5-74 $E = 207 \text{ GPa}$

Eq. (3-57), p. 116, can be written in terms of diameters,

$$p = \frac{E \delta_d}{2D^3} \left[\frac{(d_o^2 - D^2)(D^2 - d_i^2)}{(d_o^2 - d_i^2)} \right] = \frac{207(10^3)(0.062)}{2(45)^3} \left[\frac{(50^2 - 45^2)(45^2 - 40^2)}{(50^2 - 40^2)} \right]$$

$$p = 15.80 \text{ MPa}$$

Outer member: From Eq. (3-50),

$$\text{Outer radius: } (\sigma_t)_o = \frac{45^2(15.80)}{50^2 - 45^2}(2) = 134.7 \text{ MPa}$$

$$(\sigma_r)_o = 0$$

$$\text{Inner radius: } (\sigma_t)_i = \frac{45^2(15.80)}{50^2 - 45^2} \left(1 + \frac{50^2}{45^2} \right) = 150.5 \text{ MPa}$$

$$(\sigma_r)_i = -15.80 \text{ MPa}$$

$$\text{Bending (no slipping): } I = (\pi/64)(50^4 - 40^4) = 181.1 (10^3) \text{ mm}^4$$

$$\text{At } r_o : \quad (\sigma_x)_o = \pm \frac{Mc}{I} = \pm \frac{675(0.05/2)}{181.1 (10^{-9})} = \pm 93.2(10^6) \text{ Pa} = \pm 93.2 \text{ MPa}$$

$$\text{At } r_i : \quad (\sigma_x)_i = \pm \frac{675(0.045/2)}{181.1 (10^{-9})} = \pm 83.9(10^6) \text{ Pa} = \pm 83.9 \text{ MPa}$$

$$\text{Torsion: } J = 2I = 362.2 (10^3) \text{ mm}^4$$

$$\text{At } r_o : \quad (\tau_{xy})_o = \frac{Tc}{J} = \frac{900(0.05/2)}{362.2 (10^{-9})} = 62.1(10^6) \text{ Pa} = 62.1 \text{ MPa}$$

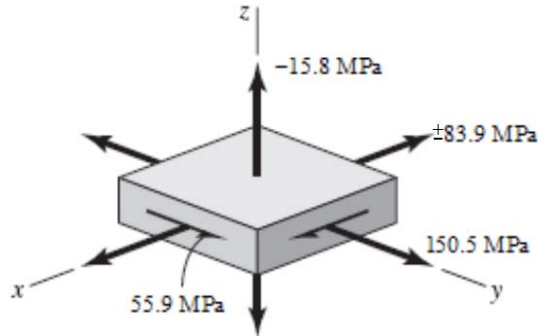
$$\text{At } r_i : \quad (\tau_{xy})_i = \frac{900(0.045/2)}{362.2 (10^{-9})} = 55.9(10^6) \text{ Pa} = 55.9 \text{ MPa}$$

Outer radius, is plane stress. Since the tangential stress is positive the von Mises stress will be maximum with a negative bending stress. That is,

$$\sigma_x = -93.2 \text{ MPa}, \sigma_y = 134.7 \text{ MPa}, \tau_{xy} = 62.1 \text{ MPa}$$

$$\begin{aligned} \text{Eq. (5-15)} \quad \sigma' &= (\sigma_x^2 - \sigma_x \sigma_y + \sigma_y^2 + 3\tau_{xy}^2)^{1/2} \\ &= [(-93.2)^2 - (-93.2)(134.7) + 134.7^2 + 3(62.1)^2]^{1/2} = 226 \text{ MPa} \\ n_o &= \frac{S_y}{\sigma'} = \frac{415}{226} = 1.84 \quad \text{Ans.} \end{aligned}$$

Inner radius, 3D state of stress



From Eq. (5-14) with $\tau_{yz} = \tau_{zx} = 0$ and $\sigma_x = + 83.9 \text{ MPa}$

$$\sigma' = \frac{1}{\sqrt{2}} [(83.9 - 150.5)^2 + (150.5 + 15.8)^2 + (-15.8 - 83.9)^2 + 6(55.9)^2]^{1/2} = 174 \text{ MPa}$$

With $\sigma_x = - 83.9 \text{ MPa}$

$$\sigma' = \frac{1}{\sqrt{2}} [(-83.9 - 150.5)^2 + (150.5 + 15.8)^2 + (-15.8 + 83.9)^2 + 6(55.9)^2]^{1/2} = 230 \text{ MPa}$$

$$n_i = \frac{S_y}{\sigma'} = \frac{415}{230} = 1.80 \quad \text{Ans.}$$

5-75 From the solution of Prob. 5-74, $p = 15.80 \text{ MPa}$

Inner member: From Eq. (3-50),

$$\text{Outer radius: } (\sigma_t)_o = -\frac{r_o^2 + r_i^2}{r_o^2 - r_i^2} p_o = -\frac{45^2 + 40^2}{45^2 - 40^2} (15.80) = -134.8 \text{ MPa}$$

$$(\sigma_r)_o = -p = -15.80 \text{ MPa}$$

$$\text{Inner radius: } (\sigma_t)_i = -\frac{2r_o^2}{r_o^2 - r_i^2} p_o = -\frac{2(45^2)}{45^2 - 40^2} (15.80) = -150.6 \text{ MPa}$$

$$(\sigma_r)_i = 0$$

$$\text{Bending (no slipping): } I = (\pi/64)(50^4 - 40^4) = 181.1 (10^3) \text{ mm}^4$$

$$\text{At } r_o: \quad (\sigma_x)_o = \pm \frac{Mc}{I} = \pm \frac{675(0.045/2)}{181.1 (10^{-9})} = \pm 83.9 (10^6) \text{ Pa} = \pm 83.9 \text{ MPa}$$

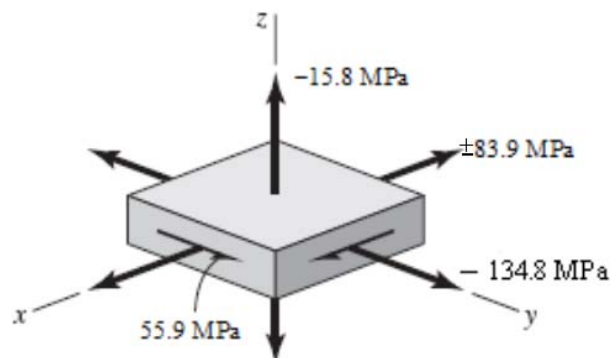
$$\text{At } r_i: \quad (\sigma_x)_i = \pm \frac{675(0.040/2)}{181.1 (10^{-9})} = \pm 74.5 (10^6) \text{ Pa} = \pm 74.5 \text{ MPa}$$

$$\text{Torsion: } J = 2I = 362.2 (10^3) \text{ mm}^4$$

$$\text{At } r_o: \quad (\tau_{xy})_o = \frac{Tc}{J} = \frac{900(0.045/2)}{362.2 (10^{-9})} = 55.9 (10^6) \text{ Pa} = 55.9 \text{ MPa}$$

$$\text{At } r_i: \quad (\tau_{xy})_i = \frac{900(0.040/2)}{362.2 (10^{-9})} = 49.7 (10^6) \text{ Pa} = 49.7 \text{ MPa}$$

Outer radius, 3D state of stress



From Eq. (5-14) with $\tau_{yz} = \tau_{zx} = 0$ and $\sigma_x = + 83.9$ MPa

$$\sigma' = \frac{1}{\sqrt{2}} \left[(83.9 + 134.8)^2 + (-134.8 + 15.8)^2 + (-15.8 - 83.9)^2 + 6(55.9)^2 \right]^{1/2} = 213 \text{ MPa}$$

With $\sigma_x = - 83.9$ MPa

$$\sigma' = \frac{1}{\sqrt{2}} \left[(-83.9 + 134.8)^2 + (-134.8 + 15.8)^2 + (-15.8 + 83.9)^2 + 6(55.9)^2 \right]^{1/2} = 142 \text{ MPa}$$

$$n_o = \frac{S_y}{\sigma'} = \frac{415}{213} = 1.95 \quad \text{Ans.}$$

Inner radius, plane stress. Worst case is when σ_x is positive

$$\sigma_x = 74.5 \text{ MPa}, \sigma_y = -150.6 \text{ MPa}, \tau_{xy} = 49.7 \text{ MPa}$$

Eq. (5-15)

$$\begin{aligned} \sigma' &= \left(\sigma_x^2 - \sigma_x \sigma_y + \sigma_y^2 + 3\tau_{xy}^2 \right)^{1/2} \\ &= \left[74.5^2 - 74.5(-150.6) + (-150.6)^2 + 3(49.7)^2 \right]^{1/2} = 216 \text{ MPa} \\ n_i &= \frac{S_y}{\sigma'} = \frac{415}{216} = 1.92 \quad \text{Ans.} \end{aligned}$$

5-76 For AISI 1040 HR, from Table A-20, $S_y = 290$ MPa.

From Prob. 3-110, $p_{\max} = 65.2$ MPa. From Eq. (3-50) at the inner radius R of the outer member,

$$\sigma_t = p \frac{r_o^2 + R^2}{r_o^2 - R^2} = 65.2 \frac{50^2 + 25^2}{50^2 - 25^2} = 108.7 \text{ MPa}$$

$$\sigma_r = -p = -65.2 \text{ MPa}$$

These are principal stresses. From Eq. (5-13)

$$\sigma'_o = \left(\sigma_t^2 - \sigma_t \sigma_r + \sigma_r^2 \right)^{1/2} = \left[108.7^2 - 108.7(-65.2) + (-65.2)^2 \right]^{1/2} = 152.2 \text{ MPa}$$

$$n = \frac{S_y}{\sigma'_o} = \frac{290}{152.2} = 1.91 \quad \text{Ans.}$$

5-77 For AISI 1040 HR, from Table A-20, $S_y = 42$ kpsi.

From Prob. 3-111, $p_{\max} = 9$ kpsi. From Eq. (3-50) at the inner radius R of the outer member,

$$\sigma_t = p \frac{r_o^2 + R^2}{r_o^2 - R^2} = 9 \frac{2^2 + 1^2}{2^2 - 1^2} = 15 \text{ kpsi}$$

$$\sigma_r = -p = -9 \text{ kpsi}$$

These are principal stresses. From Eq. (5-13)

$$\sigma'_o = (\sigma_t^2 - \sigma_t \sigma_r + \sigma_r^2)^{1/2} = [15^2 - 15(-9) + (-9)^2]^{1/2} = 21 \text{ kpsi}$$

$$n = \frac{S_y}{\sigma'_o} = \frac{42}{21} = 2 \quad \text{Ans.}$$

5-78 For AISI 1040 HR, from Table A-20, $S_y = 290 \text{ MPa}$.

From Prob. 3-111, $p_{\max} = 91.6 \text{ MPa}$. From Eq. (3-50) at the inner radius R of the outer member,

$$\sigma_t = p \frac{r_o^2 + R^2}{r_o^2 - R^2} = 91.6 \frac{50^2 + 25^2}{50^2 - 25^2} = 152.7 \text{ MPa}$$

$$\sigma_r = -p = -91.6 \text{ MPa}$$

These are principal stresses. From Eq. (5-13)

$$\sigma'_o = (\sigma_t^2 - \sigma_t \sigma_r + \sigma_r^2)^{1/2} = [152.7^2 - 152.7(-91.6) + (-91.6)^2]^{1/2} = 213.8 \text{ MPa}$$

$$n = \frac{S_y}{\sigma'_o} = \frac{290}{213.8} = 1.36 \quad \text{Ans.}$$

5-79 For AISI 1040 HR, from Table A-20, $S_y = 42 \text{ kpsi}$.

From Prob. 3-111, $p_{\max} = 12.94 \text{ kpsi}$. From Eq. (3-50) at the inner radius R of the outer member,

$$\sigma_t = p \frac{r_o^2 + R^2}{r_o^2 - R^2} = 12.94 \frac{2^2 + 1^2}{2^2 - 1^2} = 21.57 \text{ kpsi}$$

$$\sigma_r = -p = -12.94 \text{ kpsi}$$

These are principal stresses. From Eq. (5-13)

$$\sigma'_o = (\sigma_t^2 - \sigma_t \sigma_r + \sigma_r^2)^{1/2} = [21.57^2 - 21.57(-12.94) + (-12.94)^2]^{1/2} = 30.20 \text{ kpsi}$$

$$n = \frac{S_y}{\sigma'_o} = \frac{42}{30.2} = 1.39 \quad \text{Ans.}$$

5-80 For AISI 1040 HR, from Table A-20, $S_y = 290$ MPa.

From Prob. 3-111, $p_{\max} = 134$ MPa. From Eq. (3-50) at the inner radius R of the outer member,

$$\sigma_t = p \frac{r_o^2 + R^2}{r_o^2 - R^2} = 134 \frac{50^2 + 25^2}{50^2 - 25^2} = 223.3 \text{ MPa}$$

$$\sigma_r = -p = -134 \text{ MPa}$$

These are principal stresses. From Eq. (5-13)

$$\sigma'_o = (\sigma_t^2 - \sigma_t \sigma_r + \sigma_r^2)^{1/2} = [223.3^2 - 223.3(-134) + (-134)^2]^{1/2} = 312.6 \text{ MPa}$$

$$n = \frac{S_y}{\sigma'_o} = \frac{290}{312.6} = 0.93 \quad \text{Ans.}$$

5-81 For AISI 1040 HR, from Table A-20, $S_y = 42$ kpsi.

From Prob. 3-111, $p_{\max} = 19.13$ kpsi. From Eq. (3-50) at the inner radius R of the outer member,

$$\sigma_t = p \frac{r_o^2 + R^2}{r_o^2 - R^2} = 19.13 \frac{2^2 + 1^2}{2^2 - 1^2} = 31.88 \text{ kpsi}$$

$$\sigma_r = -p = -19.13 \text{ kpsi}$$

These are principal stresses. From Eq. (5-13)

$$\sigma'_o = (\sigma_t^2 - \sigma_t \sigma_r + \sigma_r^2)^{1/2} = [31.88^2 - 31.88(-19.13) + (-19.13)^2]^{1/2} = 44.63 \text{ kpsi}$$

$$n = \frac{S_y}{\sigma'_o} = \frac{42}{44.63} = 0.94 \quad \text{Ans.}$$

5-82

$$\sigma_p = \frac{1}{2} \left(2 \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \right) \pm \left[\left(\frac{K_I}{\sqrt{2\pi r}} \sin \frac{\theta}{2} \cos \frac{\theta}{2} \sin \frac{3\theta}{2} \right)^2 + \left(\frac{K_I}{\sqrt{2\pi r}} \sin \frac{\theta}{2} \cos \frac{\theta}{2} \cos \frac{3\theta}{2} \right)^2 \right]^{1/2}$$

$$= \frac{K_I}{\sqrt{2\pi r}} \left[\cos \frac{\theta}{2} \pm \left(\sin^2 \frac{\theta}{2} \cos^2 \frac{\theta}{2} \sin^2 \frac{3\theta}{2} + \sin^2 \frac{\theta}{2} \cos^2 \frac{\theta}{2} \cos^2 \frac{3\theta}{2} \right)^{1/2} \right]$$

$$= \frac{K_I}{\sqrt{2\pi r}} \left(\cos \frac{\theta}{2} \pm \sin \frac{\theta}{2} \cos \frac{\theta}{2} \right) = \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left(1 \pm \sin \frac{\theta}{2} \right)$$

Plane stress: The third principal stress is zero and

$$\sigma_1 = \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left(1 + \sin \frac{\theta}{2} \right), \sigma_2 = \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left(1 - \sin \frac{\theta}{2} \right), \sigma_3 = 0 \quad \text{Ans.}$$

Plane strain: Equations for σ_1 and σ_2 are still valid,. However,

$$\sigma_3 = \nu(\sigma_1 + \sigma_2) = 2\nu \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \quad \text{Ans.}$$

5-83 For $\theta = 0$ and plane strain, the principal stress equations of Prob. 5-82 give

$$\sigma_1 = \sigma_2 = \frac{K_I}{\sqrt{2\pi r}}, \quad \sigma_3 = 2\nu \frac{K_I}{\sqrt{2\pi r}} = 2\nu\sigma_1$$

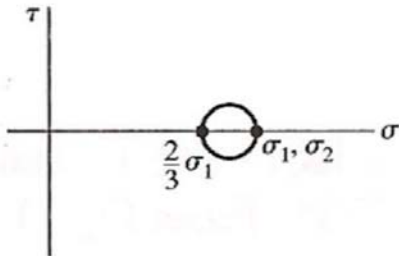
$$\text{(a) DE: Eq. (5-18)} \quad \frac{1}{\sqrt{2}} \left[(\sigma_1 - \sigma_1)^2 + (\sigma_1 - 2\nu\sigma_1)^2 + (2\nu\sigma_1 - \sigma_1)^2 \right]^{1/2} = S_y$$

$$\text{or, } \sigma_1 - 2\nu\sigma_1 = S_y$$

$$\text{For } \nu = \frac{1}{3}, \quad \left[1 - 2\left(\frac{1}{3}\right) \right] \sigma_1 = S_y \quad \Rightarrow \quad \sigma_1 = 3S_y \quad \text{Ans.}$$

$$\text{(a) MSS: Eq. (5-3), with } n=1 \quad \sigma_1 - \sigma_3 = S_y \quad \Rightarrow \quad \sigma_1 - 2\nu\sigma_1 = S_y$$

$$\nu = \frac{1}{3} \quad \Rightarrow \quad \sigma_1 = 3S_y \quad \text{Ans.}$$



$$\sigma_3 = \sigma_1 - S_y = 2S_y \quad \Rightarrow \quad \sigma_3 = \frac{2}{3}\sigma_1$$

Radius of largest circle

$$R = \frac{1}{2} \left(\sigma_1 - \frac{2}{3}\sigma_1 \right) = \frac{\sigma_1}{6}$$

5-84 Given: $a = 16 \text{ mm}$, $K_{Ic} = 80 \text{ MPa} \cdot \sqrt{\text{m}}$ and $S_y = 950 \text{ MPa}$

(a) Ignoring stress concentration

$$F = S_y A = 950(100 - 16)(12) = 958(10^3) \text{ N} = 958 \text{ kN} \quad \text{Ans.}$$

(b) From Fig. 5-26: $h/b = 1$, $a/b = 16/100 = 0.16$, $\beta = 1.3$

Eq. (5-37) $K_t = \beta \sigma \sqrt{\pi a}$

$$80 = 1.3 \frac{F}{100(12)} \sqrt{\pi(16)10^{-3}}$$

$$F = 329.4(10^3) \text{ N} = 329.4 \text{ kN} \quad \text{Ans.}$$

5-85 Given: $a = 0.5 \text{ in}$, $K_{Ic} = 72 \text{ ksi} \cdot \sqrt{\text{in}}$ and $S_y = 170 \text{ ksi}$, $S_{ut} = 192 \text{ ksi}$

$$r_o = 14/2 = 7 \text{ in}, \quad r_i = (14 - 2)/2 = 6 \text{ in}$$

$$\frac{a}{r_o - r_i} = \frac{0.5}{7 - 6} = 0.5, \quad \frac{r_i}{r_o} = \frac{6}{7} = 0.857$$

Fig. 5-30: $\beta \doteq 2.4$

Eq. (5-37): $K_{Ic} = \beta \sigma \sqrt{\pi a} \Rightarrow 72 = 2.4 \sigma \sqrt{\pi(0.5)} \Rightarrow \sigma = 23.9 \text{ ksi}$

Eq. (3-50), p. 113, at $r = r_o = 7 \text{ in}$:

$$\sigma_t = \frac{r_i^2 p_i}{r_o^2 - r_i^2} (2) \Rightarrow 23.9 = \frac{6^2 p_i}{7^2 - 6^2} (2) \Rightarrow p_i = 4.315 \text{ ksi} \quad \text{Ans.}$$

5-86

(a) First convert the data to radial dimensions to agree with the formulations of Fig. 3-33, p. 116. Thus,

$$r_o = 0.5625 \pm 0.001 \text{ in}$$

$$r_i = 0.1875 \pm 0.001 \text{ in}$$

$$R_o = 0.375 \pm 0.0002 \text{ in}$$

$$R_i = 0.376 \pm 0.0002 \text{ in}$$

The stochastic nature of the dimensions affects the $\delta = R_i - R_o$ relation in Eq. (3-57), p. 116, but not the others. Set $R = (1/2)(R_i + R_o) = 0.3755$. From Eq. (3-57)

$$\mathbf{p} = \frac{E\delta}{R} \left[\frac{(r_o^2 - R^2)(R^2 - r_i^2)}{2R^2(r_o^2 - r_i^2)} \right]$$

Substituting and solving with $E = 30$ Mpsi gives

$$\begin{aligned} \mathbf{p} &= \frac{30(10^6)\delta}{0.3755} \left[\frac{(0.5625^2 - 0.3755^2)(0.3755^2 - 0.1875^2)}{2(0.3755^2)(0.5625^2 - 0.1875^2)} \right] \\ &= 18.70(10^6)\delta \end{aligned}$$

Since $\delta = \mathbf{R}_i - \mathbf{R}_o$

$$\bar{\delta} = \bar{R}_i - \bar{R}_o = 0.376 - 0.375 = 0.001 \text{ in}$$

and
$$\hat{\sigma}_{\delta} = \left[\left(\frac{0.0002}{4} \right)^2 + \left(\frac{0.0002}{4} \right)^2 \right]^{1/2} = 0.0000707 \text{ in}$$

Then
$$C_{\delta} = \frac{\hat{\sigma}_{\delta}}{\bar{\delta}} = \frac{0.0000707}{0.001} = 0.0707$$

The tangential inner-cylinder stress at the shrink-fit surface is given by

$$\begin{aligned} \sigma_{it} &= -\mathbf{p} \frac{\bar{R}^2 + \bar{r}_i^2}{\bar{R}^2 - \bar{r}_i^2} = -18.70(10^6)\delta \left(\frac{0.3755^2 + 0.1875^2}{0.3755^2 - 0.1875^2} \right) \\ &= -31.1(10^6)\delta \end{aligned}$$

$$\bar{\sigma}_{it} = -31.1(10^6)\bar{\delta} = -31.1(10^6)(0.001) = -31.1(10^3) \text{ psi} = -31.1 \text{ kpsi}$$

Also
$$\hat{\sigma}_{\sigma_{it}} = |C_{\delta}\bar{\sigma}_{it}| = |0.0707(-31.1)(10^3)| = 2899 \text{ psi} = 2.899 \text{ kpsi}$$

$$\sigma_{it} = \mathbf{N}(-31.1, 2.899) \text{ kpsi} \quad \text{Ans.}$$

(b) The tangential stress for the outer cylinder at the shrink-fit surface is given by

$$\begin{aligned} \sigma_{it} &= \mathbf{p} \left(\frac{\bar{r}_o^2 + \bar{R}^2}{\bar{r}_o^2 - \bar{R}^2} \right) \\ &= 18.70(10^6)\delta \left(\frac{0.5625^2 + 0.3755^2}{0.5625^2 - 0.3755^2} \right) \\ &= 48.76(10^6)\delta \text{ psi} = 48.76(10^3)\delta \text{ kpsi} \end{aligned}$$

$$\bar{\sigma}_{ot} = 48.76(10^3)(0.001) = 48.76 \text{ kpsi}$$

$$\hat{\sigma}_{\sigma_{ot}} = C_{\delta}\bar{\sigma}_{ot} = 0.0707(48.76) = 3.445 \text{ kpsi}$$

$$\therefore \sigma_{ot} = \mathbf{N}(48.76, 3.445) \text{ kpsi} \quad \text{Ans.}$$

5-87 From Prob. 5-86, at the fit surface $\sigma_{ot} = \mathbf{N}(48.76, 3.445)$ kpsi. The radial stress is the fit pressure which was found to be

$$\mathbf{p} = 18.70(10^6) \delta$$

$$\bar{p} = 18.70(10^6)(0.001) = 18.70(10^3) \text{ psi}$$

$$\hat{\sigma}_p = C_\delta \bar{p} = 0.707(18.70)(10^3) = 1322 \text{ psi}$$

and so $\mathbf{p} = \mathbf{N}(18.7, 1.32)$ kpsi
and $\sigma_{or} = -\mathbf{N}(18.7, 1.32)$ kpsi

These represent the principal stresses. The von Mises stress is next assessed.

$$\bar{\sigma}_A = 48.8 \text{ kpsi}, \bar{\sigma}_B = -18.7 \text{ kpsi}$$

$$\begin{aligned} \bar{\sigma}' &= (\bar{\sigma}_A^2 - \bar{\sigma}_A \bar{\sigma}_B + \bar{\sigma}_B^2)^{1/2} = [48.8^2 - 48.8(-18.7) + (-18.7)^2]^{1/2} \\ &= 60.4 \text{ kpsi} \end{aligned}$$

$$\hat{\sigma}' = C_p \bar{\sigma}' = 0.707(60.4) = 4.27 \text{ kpsi}$$

Using the interference equation, Eq. (5-40),

$$z = -\frac{\bar{S} - \bar{\sigma}'}{(\hat{\sigma}_S^2 + \hat{\sigma}_{\sigma'}^2)^{1/2}} = -\frac{95.5 - 60.4}{(6.59^2 + 4.27^2)^{1/2}} = -4.5$$

From Table A-10, $p_f = \alpha = 0.000\,003\,40$

or 3.4 chances in a million. Ans.

5-88 Note to the Instructor. In the first printing, the pressure was incorrectly given to be $\mathbf{p} = \mathbf{N}(40, 2)$ MPa. This will be changed to $\mathbf{p} = \mathbf{N}(20, 1)$ MPa in subsequent printings. We are sorry for any inconvenience.

$$\sigma_t = \frac{\mathbf{p}d_i}{2t} = \frac{20\mathbf{N}(1, 0.05)(69)}{2(3)} = 230\mathbf{N}(1, 0.05) \text{ MPa}$$

$$\sigma_l = \frac{\sigma_t}{2} = 115\mathbf{N}(1, 0.05) \text{ MPa}$$

$$\sigma_r = -\mathbf{p} = -20\mathbf{N}(1, 0.05) \text{ MPa}$$

These three stresses are principal stresses whose variability is due to the loading. From Eq. (5-12), we find that the von Mises stress to be

$$\bar{\sigma}' = \left\{ \frac{(230 - 115)^2 + [115 - (-20)]^2 + (-20 - 230)^2}{2} \right\}^{1/2} = 217 \text{ MPa}$$

$$\hat{\sigma}_{\sigma'} = C_p \bar{\sigma}' = 0.05(217) = 10.9 \text{ MPa}$$

Using the interference equation, Eq. (5-40),

$$z = -\frac{\bar{S} - \bar{\sigma}'}{(\hat{\sigma}_s^2 + \hat{\sigma}_{\sigma'}^2)^{1/2}} = -\frac{350 - 217}{(29^2 + 10.9^2)^{1/2}} = -4.29$$

From Table A-10, the reliability is very high at

$$R = 1 - \Phi(4.29) \doteq 1 - 0.00000854 \doteq 1 \quad \text{Ans.}$$
