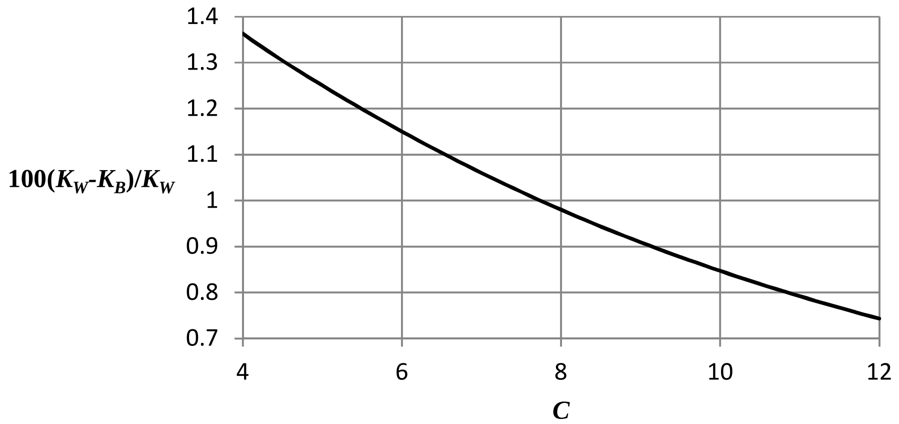
Chapter 10

10-1 From Eqs. (10-4) and (10-5)

$$K_W - K_B = \frac{4C-1}{4C-4} + \frac{0.615}{C} - \frac{4C+2}{4C-3}$$

Plot $100(K_W - K_B)/\,K_W$ vs. C for $4 \le C \le 12$ obtaining



We see the maximum and minimum occur at C = 4 and 12 respectively where

Maximum = 1.36 % *Ans.*, and Minimum = 0.743 % *Ans.*

10-2 $A = Sd^m$

$\dim(A_{\text{uscu}}) = [\dim(S)\dim(d^m)]_{\text{uscu}} = \text{kpsi}\cdot\text{in}^m$

$\dim(A_{\text{SI}}) = [\dim(S)\dim(d^m)]_{\text{SI}} = \text{MPa}\cdot\text{mm}^m$

$$A_{\text{SI}} = \frac{\text{MPa}}{\text{kpsi}} \cdot \frac{\text{mm}^m}{\text{in}^m} A_{\text{uscu}} = 6.894\,757\,(25.4)^m A_{\text{uscu}} \cong 6.895\,(25.4)^m A_{\text{uscu}} \quad \textit{Ans.}$$

For music wire, from Table 10-4:

$A_{\text{uscu}} = 201\ \text{kpsi}\cdot\text{in}^m$, $m = 0.145$; what is A_{SI}?

$A_{\text{SI}} = 6.895(25.4)^{0.145}\,(201) = 2215\ \text{MPa}\cdot\text{mm}^m$ *Ans.*

10-3 Given: Music wire, d = 2.5 mm, OD = 31 mm, plain ground ends, N_t = 14 coils.

(**a**) Table 10-1: $N_a = N_t - 1 = 14 - 1 = 13$ coils

$D = \text{OD} - d = 31 - 2.5 = 28.5$ mm

$C = D/d = 28.5/2.5 = 11.4$

Table 10-5: $d = 2.5/25.4 = 0.098$ in $\Rightarrow$ $G = 81.0(10^3)$ MPa

Eq. (10-9): $$k = \frac{d^4 G}{8D^3 N_a} = \frac{2.5^4(81)10^3}{8(28.5^3)13} = 1.314 \text{ N/mm} \qquad \textit{Ans.}$$

(**b**) Table 10-1: $L_s = d\,N_t = 2.5(14) = 35$ mm

Table 10-4: $m = 0.145$, $A = 2211$ MPa·mm^m

Eq. (10-14): $$S_{ut} = \frac{A}{d^m} = \frac{2211}{2.5^{0.145}} = 1936 \text{ MPa}$$

Table 10-6: $S_{sy} = 0.45(1936) = 871.2$ MPa

Eq. (10-5): $$K_B = \frac{4C+2}{4C-3} = \frac{4(11.4)+2}{4(11.4)-3} = 1.117$$

Eq. (10-7): $$F_s = \frac{\pi d^3 S_{sy}}{8K_B D} = \frac{\pi(2.5^3)871.2}{8(1.117)28.5} = 167.9 \text{ N} \qquad \textit{Ans.}$$

(**c**) $$L_0 = \frac{F_s}{k} + L_s = \frac{167.9}{1.314} + 35 = 162.8 \text{ mm} \qquad \textit{Ans.}$$

(**d**) $$(L_0)_{\text{cr}} = \frac{2.63(28.5)}{0.5} = 149.9 \text{ mm}$$ Spring needs to be supported. *Ans.*

10-4 Given: Design load, $F_1 = 130$ N.

Referring to Prob. 10-3 solution, $C = 11.4$, $N_a = 13$ coils, $S_{sy} = 871.2$ MPa, $F_s = 167.9$ N, $L_0 = 162.8$ mm and $(L_0)_{\text{cr}} = 149.9$ mm.
Eq. (10-18): $4 \le C \le 12$ $C = 11.4$ *O.K.*
Eq. (10-19): $3 \le N_a \le 15$ $N_a = 13$ *O.K.*
Eq. (10-17): $$\xi = \frac{F_s}{F_1} - 1 = \frac{167.9}{130} - 1 = 0.29$$

Eq. (10-20): $\xi \geq 0.15$, $\xi = 0.29$ *O.K.*
From Eq. (10-7) for static service

$$\tau_1 = K_B\left(\frac{8F_1D}{\pi d^3}\right) = 1.117\frac{8(130)(28.5)}{\pi(2.5)^3} = 674 \text{ MPa}$$

$$n = \frac{S_{sy}}{\tau_1} = \frac{871.2}{674} = 1.29$$

Eq. (10-21): $n_s \geq 1.2$, $n = 1.29$ *O.K.*

$$\tau_s = \tau_1\left(\frac{167.9}{130}\right) = 674\left(\frac{167.9}{130}\right) = 870.5 \text{ MPa}$$

$$S_{sy} \,/\, \tau_s = 871.2 \,/\, 870.5 \approx 1$$

$S_{sy}/\tau_s \geq (n_s)_d$: Not solid-safe (but was the basis of the design). *Not O.K.*

$L_0 \leq (L_0)_{cr}$: 162.8 ≥ 149.9 *Not O.K.*

Design is unsatisfactory. Operate over a rod? *Ans.*

10-5 Given: Oil-tempered wire, d = 0.2 in, D = 2 in, N_t = 12 coils, L_0 = 5 in, squared ends.

(**a**) Table 10-1: $L_s = d\,(N_t + 1) = 0.2(12 + 1) = 2.6$ in *Ans.*

(**b**) Table 10-1: $N_a = N_t - 2 = 12 - 2 = 10$ coils
Table 10-5: $G = 11.2$ Mpsi

Eq. (10-9):
$$k = \frac{d^4G}{8D^3N_a} = \frac{0.2^4\,(11.2)10^6}{8\left(2^3\right)10} = 28 \text{ lbf/in}$$

$$F_s = k\,y_s = k\,(L_0 - L_s) = 28(5 - 2.6) = 67.2 \text{ lbf} \qquad \textit{Ans.}$$

(**c**) Eq. (10-1): $C = D/d = 2/0.2 = 10$

Eq. (10-5):
$$K_B = \frac{4C+2}{4C-3} = \frac{4(10)+2}{4(10)-3} = 1.135$$

Eq. (10-7):
$$\tau_s = K_B\frac{8F_sD}{\pi d^3} = 1.135\frac{8(67.2)2}{\pi\left(0.2^3\right)} = 48.56\left(10^3\right) \text{ psi}$$

Table 10-4: $m = 0.187$, $A = 147$ kpsi·in^m

Eq. (10-14): $S_{ut} = \dfrac{A}{d^m} = \dfrac{147}{0.2^{0.187}} = 198.6 \text{ kpsi}$

Table 10-6: $S_{sy} = 0.50\, S_{ut} = 0.50(198.6) = 99.3$ kpsi

$$n_s = \frac{S_{sy}}{\tau_s} = \frac{99.3}{48.56} = 2.04 \qquad Ans.$$

10-6 Given: Oil-tempered wire, $d = 4$ mm, $C = 10$, plain ends, $L_0 = 80$ mm, and at $F = 50$ N, $y = 15$ mm.

(**a**) $k = F/y = 50/15 = 3.333$ N/mm *Ans.*

(**b**) $D = Cd = 10(4) = 40$ mm

OD $= D + d = 40 + 4 = 44$ mm *Ans.*

(**c**) From Table 10-5, $G = 77.2$ GPa

Eq. (10-9): $N_a = \dfrac{d^4 G}{8kD^3} = \dfrac{4^4(77.2)10^3}{8(3.333)40^3} = 11.6 \text{ coils}$

Table 10-1: $N_t = N_a = 11.6$ coils *Ans.*

(**d**) Table 10-1: $L_s = d\,(N_t + 1) = 4(11.6 + 1) = 50.4$ mm *Ans.*

(**e**) Table 10-4: $m = 0.187$, $A = 1855$ MPa·mm^m

Eq. (10-14): $S_{ut} = \dfrac{A}{d^m} = \dfrac{1855}{4^{0.187}} = 1431 \text{ MPa}$

Table 10-6: $S_{sy} = 0.50\, S_{ut} = 0.50(1431) = 715.5$ MPa

$y_s = L_0 - L_s = 80 - 50.4 = 29.6$ mm

$F_s = k\, y_s = 3.333(29.6) = 98.66$ N

Eq. (10-5): $K_B = \dfrac{4C+2}{4C-3} = \dfrac{4(10)+2}{4(10)-3} = 1.135$

Eq. (10-7): $\tau_s = K_B \dfrac{8F_s D}{\pi d^3} = 1.135 \dfrac{8(98.66)40}{\pi(4^3)} = 178.2 \text{ MPa}$

$$n_s = \frac{S_{sy}}{\tau_s} = \frac{715.5}{178.2} = 4.02 \qquad \textit{Ans.}$$

10-7 Static service spring with: HD steel wire, d = 0.080 in, OD = 0.880 in, N_t = 8 coils, plain and ground ends.

Preliminaries

Table 10-5: $A = 140 \text{ kpsi} \cdot \text{in}^m, \quad m = 0.190$

Eq. (10-14): $S_{ut} = \dfrac{A}{d^m} = \dfrac{140}{0.080^{0.190}} = 226.2 \text{ kpsi}$

Table 10-6: $S_{sy} = 0.45(226.2) = 101.8$ kpsi

Then,

$D = \text{OD} - d = 0.880 - 0.080 = 0.8$ in

Eq. (10-1): $C = D/d = 0.8/0.08 = 10$

Eq. (10-5): $K_B = \dfrac{4C+2}{4C-3} = \dfrac{4(10)+2}{4(10)-3} = 1.135$

Table 10-1: $N_a = N_t - 1 = 8 - 1 = 7$ coils

$L_s = dN_t = 0.08(8) = 0.64$ in

Eq. (10-7) For solid-safe, $n_s = 1.2$:

$$F_s = \frac{\pi d^3 S_{sy} / n_s}{8 K_B D} = \frac{\pi\left(0.08^3\right)\left[101.8\left(10^3\right)/1.2\right]}{8(1.135)(0.8)} = 18.78 \text{ lbf}$$

Eq. (10-9): $k = \dfrac{d^4 G}{8 D^3 N_a} = \dfrac{0.08^4 (11.5) 10^6}{8\left(0.8^3\right)7} = 16.43 \text{ lbf/in}$

$$y_s = \frac{F_s}{k} = \frac{18.78}{16.43} = 1.14 \text{ in}$$

(a) $L_0 = y_s + L_s = 1.14 + 0.64 = 1.78$ in *Ans.*

(b) Table 10-1: $p = \dfrac{L_0}{N_t} = \dfrac{1.78}{8} = 0.223$ in *Ans.*

(c) From above: F_s = 18.78 lbf *Ans.*

(d) From above: k = 16.43 lbf/in *Ans.*

(e) Table 10-2 and Eq. (10-13): $(L_0)_{\text{cr}} = \dfrac{2.63D}{\alpha} = \dfrac{2.63(0.8)}{0.5} = 4.21$ in

Since $L_0 < (L_0)_{\text{cr}}$, buckling is unlikely *Ans.*

10-8 Given: Design load, F_1 = 16.5 lbf.

Referring to Prob. 10-7 solution, C = 10, N_a = 7 coils, S_{sy} = 101.8 kpsi, F_s = 18.78 lbf, y_s = 1.14 in, L_0 = 1.78 in, and $(L_0)_{\text{cr}}$ = 4.21 in.

Eq. (10-18): $4 \le C \le 12 \qquad C = 10 \qquad$ *O.K.*

Eq. (10-19): $3 \le N_a \le 15$ $N_a = 7$ *O.K.*

Eq. (10-17): $\xi = \dfrac{F_s}{F_1} - 1 = \dfrac{18.78}{16.5} - 1 = 0.14$

Eq. (10-20): $\xi \ge 0.15$, $\xi = 0.14$ *not O.K.*, but probably acceptable.

From Eq. (10-7) for static service

$$\tau_1 = K_B\left(\frac{8F_1D}{\pi d^3}\right) = 1.135\frac{8(16.5)(0.8)}{\pi(0.080)^3} = 74.5\left(10^3\right)\text{ psi} = 74.5\text{ kpsi}$$

$$n = \frac{S_{sy}}{\tau_1} = \frac{101.8}{74.5} = 1.37$$

Eq. (10-21): $n_s \ge 1.2$, $n = 1.37$ *O.K.*

$$\tau_s = \tau_1\left(\frac{18.78}{16.5}\right) = 74.5\left(\frac{18.78}{16.5}\right) = 84.8\text{ kpsi}$$

$$n_s = S_{sy} / \tau_s = 101.8 / 84.8 = 1.20$$

Eq. (10-21): $n_s \ge 1.2$, $n_s = 1.2$ It is solid-safe (basis of design). *O.K.*

Eq. (10-13) and Table 10-2: $L_0 \le (L_0)_{cr}$ 1.78 in $\le$ 4.21 in *O.K.*

10-9 Given: A228 music wire, squared and ground ends, $d = 0.007$ in, OD = 0.038 in, $L_0 = 0.58$ in, $N_t = 38$ coils.

$D = \text{OD} - d = 0.038 - 0.007 = 0.031$ in

Eq. (10-1): $C = D/d = 0.031/0.007 = 4.429$

Eq. (10-5): $K_B = \dfrac{4C+2}{4C-3} = \dfrac{4(4.429)+2}{4(4.429)-3} = 1.340$

Table 10-1: $N_a = N_t - 2 = 38 - 2 = 36$ coils (high)

Table 10-5: $G = 12.0$ Mpsi

Eq. (10-9): $k = \dfrac{d^4G}{8D^3N_a} = \dfrac{0.007^4(12.0)10^6}{8\left(0.031^3\right)36} = 3.358$ lbf/in

Table 10-1: $L_s = dN_t = 0.007(38) = 0.266$ in

$y_s = L_0 - L_s = 0.58 - 0.266 = 0.314$ in

$F_s = ky_s = 3.358(0.314) = 1.054$ lbf

Eq. (10-7): $\tau_s = K_B\dfrac{8F_sD}{\pi d^3} = 1.340\dfrac{8(1.054)0.031}{\pi\left(0.007^3\right)} = 325.1\left(10^3\right)$ psi (1)

Table 10-4: $A = 201$ kpsi·in^m, $m = 0.145$

Eq. (10-14): $S_{ut} = \dfrac{A}{d^m} = \dfrac{201}{0.007^{0.145}} = 412.7 \text{ kpsi}$

Table 10-6: $S_{sy} = 0.45\, S_{ut} = 0.45(412.7) = 185.7$ kpsi

$\tau_s > S_{sy}$, that is, 325.1 > 185.7 kpsi, the spring is not solid-safe. Return to Eq. (1) with $F_s = ky_s$ and $\tau_s = S_{sy}/n_s$, and solve for y_s, giving

$$y_s = \frac{\left(S_{sy}/n_s\right)\pi d^3}{8K_B kD} = \frac{\left[185.7\left(10^3\right)/1.2\right]\pi\left(0.007^3\right)}{8(1.340)3.358(0.031)} = 0.149 \text{ in}$$

The free length should be wound to

$$L_0 = L_s + y_s = 0.266 + 0.149 = 0.415 \text{ in} \qquad Ans.$$

This only addresses the solid-safe criteria. There are additional problems.

10-10 Given: B159 phosphor-bronze, squared and ground. ends, d = 0.014 in, OD = 0.128 in, L_0 = 0.50 in, N_t = 16 coils.

D = OD – d = 0.128 – 0.014 = 0.114 in

Eq. (10-1): $C = D/d = 0.114/0.014 = 8.143$

Eq. (10-5): $K_B = \dfrac{4C+2}{4C-3} = \dfrac{4(8.143)+2}{4(8.143)-3} = 1.169$

Table 10-1: $N_a = N_t - 2 = 16 - 2 = 14$ coils

Table 10-5: G = 6 Mpsi

Eq. (10-9): $k = \dfrac{d^4 G}{8D^3 N_a} = \dfrac{0.014^4(6)10^6}{8\left(0.114^3\right)14} = 1.389 \text{ lbf/in}$

Table 10-1: $L_s = dN_t = 0.014(16) = 0.224$ in

$y_s = L_0 - L_s = 0.50 - 0.224 = 0.276$ in

$F_s = ky_s = 1.389(0.276) = 0.3834$ lbf

Eq. (10-7): $\tau_s = K_B \dfrac{8F_s D}{\pi d^3} = 1.169 \dfrac{8(0.3834)0.114}{\pi\left(0.014^3\right)} = 47.42\left(10^3\right) \text{ psi} \qquad (1)$

Table 10-4: $A = 145 \text{ kpsi}\cdot\text{in}^m$, $m = 0$

Eq. (10-14): $S_{ut} = \dfrac{A}{d^m} = \dfrac{145}{0.014^0} = 145 \text{ kpsi}$

Table 10-6: $S_{sy} = 0.35\, S_{ut} = 0.35(135) = 47.25$ kpsi

$\tau_s > S_{sy}$, that is, 47.42 > 47.25 kpsi, the spring is not solid-safe. Return to Eq. (1) with $F_s = ky_s$ and $\tau_s = S_{sy}/n_s$, and solve for y_s, giving

$$y_s = \frac{\left(S_{sy}/n_s\right)\pi d^3}{8K_B kD} = \frac{\left[47.25\left(10^3\right)/1.2\right]\pi\left(0.014^3\right)}{8(1.169)1.389(0.114)} = 0.229 \text{ in}$$

The free length should be wound to

$L_0 = L_s + y_s = 0.224 + 0.229 = 0.453$ in *Ans*.

10-11 Given: A313 stainless steel, squared and ground ends, $d = 0.050$ in, OD = 0.250 in, $L_0 = 0.68$ in, $N_t = 11.2$ coils.

$D = \text{OD} - d = 0.250 - 0.050 = 0.200$ in

Eq. (10-1): $C = D/d = 0.200/0.050 = 4$

Eq. (10-5): $$K_B = \frac{4C+2}{4C-3} = \frac{4(4)+2}{4(4)-3} = 1.385$$

Table 10-1: $N_a = N_t - 2 = 11.2 - 2 = 9.2$ coils

Table 10-5: $G = 10$ Mpsi

Eq. (10-9): $$k = \frac{d^4 G}{8D^3 N_a} = \frac{0.050^4(10)10^6}{8\left(0.2^3\right)9.2} = 106.1 \text{ lbf/in}$$

Table 10-1: $L_s = dN_t = 0.050(11.2) = 0.56$ in

$y_s = L_0 - L_s = 0.68 - 0.56 = 0.12$ in

$F_s = ky_s = 106.1(0.12) = 12.73$ lbf

Eq. (10-7): $$\tau_s = K_B \frac{8F_s D}{\pi d^3} = 1.385 \frac{8(12.73)0.2}{\pi\left(0.050^3\right)} = 71.8\left(10^3\right) \text{ psi}$$

Table 10-4: $A = 169$ kpsi·in^m, $m = 0.146$

Eq. (10-14): $$S_{ut} = \frac{A}{d^m} = \frac{169}{0.050^{0.146}} = 261.7 \text{ kpsi}$$

Table 10-6: $S_{sy} = 0.35\, S_{ut} = 0.35(261.7) = 91.6$ kpsi

$$n_s = \frac{S_{sy}}{\tau_s} = \frac{91.6}{71.8} = 1.28$$ Spring is solid-safe ($n_s > 1.2$) *Ans*.

10-12 Given: A227 hard-drawn wire, squared and ground ends, $d = 0.148$ in, OD = 2.12 in, $L_0 = 2.5$ in, $N_t = 5.75$ coils.

$D = \text{OD} - d = 2.12 - 0.148 = 1.972$ in

Eq. (10-1): $C = D/d = 1.972/0.148 = 13.32$ (high)

Eq. (10-5): $$K_B = \frac{4C+2}{4C-3} = \frac{4(13.32)+2}{4(13.32)-3} = 1.099$$

Table 10-1: $N_a = N_t - 2 = 5.75 - 2 = 3.75$ coils

Table 10-5: $G = 11.4$ Mpsi

Eq. (10-9): $$k = \frac{d^4 G}{8D^3 N_a} = \frac{0.148^4(11.4)10^6}{8\left(1.972^3\right)3.75} = 23.77 \text{ lbf/in}$$

Table 10-1: $L_s = dN_t = 0.148(5.75) = 0.851$ in

$y_s = L_0 - L_s = 2.5 - 0.851 = 1.649$ in

$F_s = ky_s = 23.77(1.649) = 39.20$ lbf

Eq. (10-7): $\tau_s = K_B \dfrac{8F_s D}{\pi d^3} = 1.099 \dfrac{8(39.20)1.972}{\pi(0.148^3)} = 66.7(10^3)$ psi

Table 10-4: A = 140 kpsi·in^m, m = 0.190

Eq. (10-14): $S_{ut} = \dfrac{A}{d^m} = \dfrac{140}{0.148^{0.190}} = 201.3$ kpsi

Table 10-6: S_{sy} = 0.35 S_{ut} = 0.45(201.3) = 90.6 kpsi

$$n_s = \frac{S_{sy}}{\tau_s} = \frac{90.6}{66.7} = 1.36$$ Spring is solid-safe (n_s > 1.2) *Ans.*

10-13 Given: A229 OQ&T steel, squared and ground ends, d = 0.138 in, OD = 0.92 in, L_0 = 2.86 in, N_t = 12 coils.

D = OD – d = 0.92 – 0.138 = 0.782 in

Eq. (10-1): $C = D/d$ = 0.782/0.138 = 5.667

Eq. (10-5): $K_B = \dfrac{4C+2}{4C-3} = \dfrac{4(5.667)+2}{4(5.667)-3} = 1.254$

Table 10-1: $N_a = N_t - 2 = 12 - 2 = 10$ coils

A229 OQ&T steel is not given in Table 10-5. From Table A-5, for carbon steels, G = 11.5 Mpsi.

Eq. (10-9): $k = \dfrac{d^4 G}{8D^3 N_a} = \dfrac{0.138^4(11.5)10^6}{8(0.782^3)10} = 109.0$ lbf/in

Table 10-1: $L_s = dN_t$ = 0.138(12) = 1.656 in

$y_s = L_0 - L_s$ = 2.86 – 1.656 = 1.204 in

$F_s = ky_s$ = 109.0(1.204) = 131.2 lbf

Eq. (10-7): $\tau_s = K_B \dfrac{8F_s D}{\pi d^3} = 1.254 \dfrac{8(131.2)0.782}{\pi(0.138^3)} = 124.7(10^3)$ psi (1)

Table 10-4: A = 147 kpsi·in^m, m = 0.187

Eq. (10-14): $S_{ut} = \dfrac{A}{d^m} = \dfrac{147}{0.138^{0.187}} = 212.9$ kpsi

Table 10-6: S_{sy} = 0.50 S_{ut} = 0.50(212.9) = 106.5 kpsi

$\tau_s > S_{sy}$, that is, 124.7 > 106.5 kpsi, the spring is not solid-safe. Return to Eq. (1) with $F_s = ky_s$ and $\tau_s = S_{sy}/n_s$, and solve for y_s, giving

$$y_s = \frac{(S_{sy}/n_s)\pi d^3}{8K_B kD} = \frac{\left[106.5(10^3)/1.2\right]\pi(0.138^3)}{8(1.254)109.0(0.782)} = 0.857 \text{ in}$$

The free length should be wound to

$L_0 = L_s + y_s = 1.656 + 0.857 = 2.51$ in *Ans.*

10-14 Given: A232 chrome-vanadium steel, squared and ground ends, $d = 0.185$ in, OD = 2.75 in, $L_0 = 7.5$ in, $N_t = 8$ coils.

$D = \text{OD} - d = 2.75 - 0.185 = 2.565$ in

Eq. (10-1): $C = D/d = 2.565/0.185 = 13.86$ (high)

Eq. (10-5): $$K_B = \frac{4C+2}{4C-3} = \frac{4(13.86)+2}{4(13.86)-3} = 1.095$$

Table 10-1: $N_a = N_t - 2 = 8 - 2 = 6$ coils

Table 10-5: $G = 11.2$ Mpsi.

Eq. (10-9): $$k = \frac{d^4 G}{8D^3 N_a} = \frac{0.185^4(11.2)10^6}{8\left(2.565^3\right)6} = 16.20 \text{ lbf/in}$$

Table 10-1: $L_s = dN_t = 0.185(8) = 1.48$ in

$y_s = L_0 - L_s = 7.5 - 1.48 = 6.02$ in

$F_s = ky_s = 16.20(6.02) = 97.5$ lbf

Eq. (10-7): $$\tau_s = K_B\frac{8F_sD}{\pi d^3} = 1.095\frac{8(97.5)2.565}{\pi\left(0.185^3\right)} = 110.1\left(10^3\right) \text{ psi} \qquad (1)$$

Table 10-4: $A = 169$ kpsi·in^m, $m = 0.168$

Eq. (10-14): $$S_{ut} = \frac{A}{d^m} = \frac{169}{0.185^{0.168}} = 224.4 \text{ kpsi}$$

Table 10-6: $S_{sy} = 0.50\, S_{ut} = 0.50(224.4) = 112.2$ kpsi

$$n_s = \frac{S_{sy}}{\tau_s} = \frac{112.2}{110.1} = 1.02$$ Spring is not solid-safe ($n_s < 1.2$)

Return to Eq. (1) with $F_s = ky_s$ and $\tau_s = S_{sy}/n_s$, and solve for y_s, giving

$$y_s = \frac{\left(S_{sy}/n_s\right)\pi d^3}{8K_BkD} = \frac{\left[112.2\left(10^3\right)/1.2\right]\pi\left(0.185^3\right)}{8(1.095)16.20(2.565)} = 5.109 \text{ in}$$

The free length should be wound to

$L_0 = L_s + y_s = 1.48 + 5.109 = 6.59$ in *Ans.*

10-15 Given: A313 stainless steel, squared and ground ends, $d = 0.25$ mm, OD = 0.95 mm, $L_0 = 12.1$ mm, $N_t = 38$ coils.

$D = \text{OD} - d = 0.95 - 0.25 = 0.7$ mm

Eq. (10-1): $C = D/d = 0.7/0.25 = 2.8$ (low)

Eq. (10-5): $$K_B = \frac{4C+2}{4C-3} = \frac{4(2.8)+2}{4(2.8)-3} = 1.610$$

Table 10-1: $N_a = N_t - 2 = 38 - 2 = 36$ coils (high)

Table 10-5: $G = 69.0(10^3)$ MPa.

Eq. (10-9): $$k = \frac{d^4 G}{8D^3 N_a} = \frac{0.25^4 (69.0)10^3}{8\left(0.7^3\right)36} = 2.728 \text{ N/mm}$$

Table 10-1: $L_s = dN_t = 0.25(38) = 9.5$ mm

$y_s = L_0 - L_s = 12.1 - 9.5 = 2.6$ mm

$F_s = ky_s = 2.728(2.6) = 7.093$ N

Eq. (10-7): $$\tau_s = K_B \frac{8F_s D}{\pi d^3} = 1.610 \frac{8(7.093)0.7}{\pi\left(0.25^3\right)} = 1303 \text{ MPa} \qquad (1)$$

Table 10-4 (dia. less than table): $A = 1867$ MPa·mm^m, $m = 0.146$

Eq. (10-14): $$S_{ut} = \frac{A}{d^m} = \frac{1867}{0.25^{0.146}} = 2286 \text{ MPa}$$

Table 10-6: $S_{sy} = 0.35\, S_{ut} = 0.35(2286) = 734$ MPa

$\tau_s > S_{sy}$, that is, 1303 > 734 MPa, the spring is not solid-safe. Return to Eq. (1) with $F_s = ky_s$ and $\tau_s = S_{sy}/n_s$, and solve for y_s, giving

$$y_s = \frac{\left(S_{sy}/n_s\right)\pi d^3}{8K_B kD} = \frac{(734/1.2)\pi\left(0.25^3\right)}{8(1.610)2.728(0.7)} = 1.22 \text{ mm}$$

The free length should be wound to

$L_0 = L_s + y_s = 9.5 + 1.22 = 10.72$ mm *Ans.*

This only addresses the solid-safe criteria. There are additional problems.

10-16 Given: A228 music wire, squared and ground ends, $d = 1.2$ mm, OD = 6.5 mm, $L_0 = 15.7$ mm, $N_t = 10.2$ coils.

$D = \text{OD} - d = 6.5 - 1.2 = 5.3$ mm

Eq. (10-1): $C = D/d = 5.3/1.2 = 4.417$

Eq. (10-5): $$K_B = \frac{4C+2}{4C-3} = \frac{4(4.417)+2}{4(4.417)-3} = 1.368$$

Table (10-1): $N_a = N_t - 2 = 10.2 - 2 = 8.2$ coils

Table 10-5 ($d = 1.2/25.4 = 0.0472$ in): $G = 81.7(10^3)$ MPa.

Eq. (10-9): $$k = \frac{d^4 G}{8D^3 N_a} = \frac{1.2^4 (81.7)10^3}{8\left(5.3^3\right)8.2} = 17.35 \text{ N/mm}$$

Table 10-1: $L_s = dN_t = 1.2(10.2) = 12.24$ mm

$y_s = L_0 - L_s = 15.7 - 12.24 = 3.46$ mm

$F_s = ky_s = 17.35(3.46) = 60.03$ N

Eq. (10-7): $\tau_s = K_B \dfrac{8F_sD}{\pi d^3} = 1.368\dfrac{8(60.03)5.3}{\pi(1.2^3)} = 641.4 \text{ MPa}$ (1)

Table 10-4: A = 2211 MPa·mm^m, m = 0.145

Eq. (10-14): $S_{ut} = \dfrac{A}{d^m} = \dfrac{2211}{1.2^{0.145}} = 2153 \text{ MPa}$

Table 10-6: S_{sy} = 0.45 S_{ut} = 0.45(2153) = 969 MPa

$$n_s = \frac{S_{sy}}{\tau_s} = \frac{969}{641.4} = 1.51$$ Spring is solid-safe (n_s > 1.2) *Ans.*

10-17 Given: A229 OQ&T steel, squared and ground ends, d = 3.5 mm, OD = 50.6 mm, L_0 = 75.5 mm, N_t = 5.5 coils.

D = OD − d = 50.6 − 3.5 = 47.1 mm

Eq. (10-1): $C = D/d$ = 47.1/3.5 = 13.46 (high)

Eq. (10-5): $K_B = \dfrac{4C+2}{4C-3} = \dfrac{4(13.46)+2}{4(13.46)-3} = 1.098$

Table 10-1: $N_a = N_t - 2$ = 5.5 − 2 = 3.5 coils

A229 OQ&T steel is not given in Table 10-5. From Table A-5, for carbon steels, $G = 79.3(10^3)$ MPa.

Eq. (10-9): $k = \dfrac{d^4G}{8D^3N_a} = \dfrac{3.5^4(79.3)10^3}{8(47.1^3)3.5} = 4.067 \text{ N/mm}$

Table 10-1: $L_s = dN_t$ = 3.5(5.5) = 19.25 mm

$y_s = L_0 - L_s$ = 75.5 − 19.25 = 56.25 mm

$F_s = ky_s$ = 4.067(56.25) = 228.8 N

Eq. (10-7): $\tau_s = K_B \dfrac{8F_sD}{\pi d^3} = 1.098\dfrac{8(228.8)47.1}{\pi(3.5^3)} = 702.8 \text{ MPa}$ (1)

Table 10-4: A = 1855 MPa·mm^m, m = 0.187

Eq. (10-14): $S_{ut} = \dfrac{A}{d^m} = \dfrac{1855}{3.5^{0.187}} = 1468 \text{ MPa}$

Table 10-6: S_{sy} = 0.50 S_{ut} = 0.50(1468) = 734 MPa

$$n_s = \frac{S_{sy}}{\tau_s} = \frac{734}{702.8} = 1.04$$ Spring is not solid-safe (n_s < 1.2)

Return to Eq. (1) with $F_s = ky_s$ and $\tau_s = S_{sy}/n_s$, and solve for y_s, giving

$$y_s = \frac{(S_{sy}/n_s)\pi d^3}{8K_BkD} = \frac{(734/1.2)\pi(3.5^3)}{8(1.098)4.067(47.1)} = 48.96 \text{ mm}$$

The free length should be wound to

$L_0 = L_s + y_s = 19.25 + 48.96 = 68.2$ mm *Ans.*

10-18 Given: B159 phosphor-bronze, squared and ground ends, d = 3.8 mm, OD = 31.4 mm, L_0 = 71.4 mm, N_t = 12.8 coils.

D = OD − d = 31.4 − 3.8 = 27.6 mm

Eq. (10-1): $C = D/d = 27.6/3.8 = 7.263$

Eq. (10-5): $$K_B = \frac{4C+2}{4C-3} = \frac{4(7.263)+2}{4(7.263)-3} = 1.192$$

Table 10-1: $N_a = N_t - 2 = 12.8 - 2 = 10.8$ coils

Table 10-5: $G = 41.4(10^3)$ MPa.

Eq. (10-9): $$k = \frac{d^4G}{8D^3N_a} = \frac{3.8^4(41.4)10^3}{8(27.6^3)10.8} = 4.752 \text{ N/mm}$$

Table 10-1: $L_s = dN_t = 3.8(12.8) = 48.64$ mm

$y_s = L_0 - L_s = 71.4 - 48.64 = 22.76$ mm

$F_s = ky_s = 4.752(22.76) = 108.2$ N

Eq. (10-7): $$\tau_s = K_B\frac{8F_sD}{\pi d^3} = 1.192\frac{8(108.2)27.6}{\pi(3.8^3)} = 165.2 \text{ MPa} \qquad (1)$$

Table 10-4 (d = 3.8/25.4 = 0.150 in): $A = 932$ MPa·mm^m, $m = 0.064$

Eq. (10-14): $$S_{ut} = \frac{A}{d^m} = \frac{932}{3.8^{0.064}} = 855.7 \text{ MPa}$$

Table 10-6: $S_{sy} = 0.35\, S_{ut} = 0.35(855.7) = 299.5$ MPa

$$n_s = \frac{S_{sy}}{\tau_s} = \frac{299.5}{165.2} = 1.81$$ Spring is solid-safe ($n_s > 1.2$) *Ans.*

10-19 Given: A232 chrome-vanadium steel, squared and ground ends, d = 4.5 mm, OD = 69.2 mm, L_0 = 215.6 mm, N_t = 8.2 coils.

D = OD − d = 69.2 − 4.5 = 64.7 mm

Eq. (10-1): $C = D/d = 64.7/4.5 = 14.38$ (high)

Eq. (10-5): $$K_B = \frac{4C+2}{4C-3} = \frac{4(14.38)+2}{4(14.38)-3} = 1.092$$

Table 10-1: $N_a = N_t - 2 = 8.2 - 2 = 6.2$ coils

Table 10-5: $G = 77.2(10^3)$ MPa.

Eq. (10-9): $$k = \frac{d^4G}{8D^3N_a} = \frac{4.5^4(77.2)10^3}{8(64.7^3)6.2} = 2.357 \text{ N/mm}$$

Table 10-1: $L_s = dN_t = 4.5(8.2) = 36.9$ mm

$y_s = L_0 - L_s = 215.6 - 36.9 = 178.7$ mm
$F_s = ky_s = 2.357(178.7) = 421.2$ N

Eq. (10-7): $$\tau_s = K_B \frac{8F_sD}{\pi d^3} = 1.092\frac{8(421.2)64.7}{\pi(4.5^3)} = 832 \text{ MPa} \qquad (1)$$

Table 10-4: $A = 2005$ MPa·mm^m, $m = 0.168$

Eq. (10-14): $$S_{ut} = \frac{A}{d^m} = \frac{2005}{4.5^{0.168}} = 1557 \text{ MPa}$$

Table 10-6: $S_{sy} = 0.50\, S_{ut} = 0.50(1557) = 779$ MPa

$\tau_s > S_{sy}$, that is, 832 > 779 MPa, the spring is not solid-safe. Return to Eq. (1) with $F_s = ky_s$ and $\tau_s = S_{sy}/n_s$, and solve for y_s, giving

$$y_s = \frac{(S_{sy}/n_s)\pi d^3}{8K_BkD} = \frac{(779/1.2)\pi(4.5^3)}{8(1.092)2.357(64.7)} = 139.5 \text{ mm}$$

The free length should be wound to

$$L_0 = L_s + y_s = 36.9 + 139.5 = 176.4 \text{ mm} \qquad \textit{Ans.}$$

This only addresses the solid-safe criteria. There are additional problems.

10-20 Given: A227 HD steel.
From the figure: $L_0 = 4.75$ in, OD = 2 in, and $d = 0.135$ in. Thus

$$D = \text{OD} - d = 2 - 0.135 = 1.865 \text{ in}$$

(a) By counting, $N_t = 12.5$ coils. Since the ends are squared along 1/4 turn on each end,

$$N_a = 12.5 - 0.5 = 12 \text{ turns} \qquad \textit{Ans.}$$
$$p = 4.75 / 12 = 0.396 \text{ in} \qquad \textit{Ans.}$$

The solid stack is 13 wire diameters

$$L_s = 13(0.135) = 1.755 \text{ in} \qquad \textit{Ans.}$$

(b) From Table 10-5, $G = 11.4$ Mpsi

$$k = \frac{d^4G}{8D^3N_a} = \frac{0.135^4(11.4)(10^6)}{8(1.865^3)(12)} = 6.08 \text{ lbf/in} \qquad \textit{Ans.}$$

(c) $F_s = k(L_0 - L_s) = 6.08(4.75 - 1.755) = 18.2$ lbf *Ans.*

(d) $C = D/d = 1.865/0.135 = 13.81$

$$K_B = \frac{4(13.81)+2}{4(13.81)-3} = 1.096$$

$$\tau_s = K_B \frac{8F_s D}{\pi d^3} = 1.096 \frac{8(18.2)(1.865)}{\pi\left(0.135^3\right)} = 38.5\left(10^3\right) \text{ psi} = 38.5 \text{ kpsi} \quad \textit{Ans.}$$

10-21 For the wire diameter analyzed, G = 11.75 Mpsi per Table 10-5. Use squared and ground ends. The following is a spread-sheet study using Fig. 10-3 for parts (*a*) and (*b*). For N_a, $k = F_{\max}/y = 20/2 = 10$ lbf/in. For τ_s, $F = F_s = 20(1 + \xi) = 20(1 + 0.15) = 23$ lbf.

(a) Spring over a Rod					(b) Spring in a Hole				
Source	Parameter	Values			Source	Parameter	Values		
	d	0.075	0.080	0.085		d	0.075	0.080	0.085
	ID	0.800	0.800	0.800		OD	0.950	0.950	0.950
	D	0.875	0.880	0.885		D	0.875	0.870	0.865
Eq. (10-1)	C	11.667	11.000	10.412	Eq. (10-1)	C	11.667	10.875	10.176
Eq. (10-9)	N_a	6.937	8.828	11.061	Eq. (10-9)	N_a	6.937	9.136	11.846
Table 10-1	N_t	8.937	10.828	13.061	Table 10-1	N_t	8.937	11.136	13.846
Table 10-1	L_s	0.670	0.866	1.110	Table 10-1	L_s	0.670	0.891	1.177
$1.15y + L_s$	L_0	2.970	3.166	3.410	$1.15y + L_s$	L_0	2.970	3.191	3.477
Eq. (10-13)	$(L_0)_{cr}$	4.603	4.629	4.655	Eq. (10-13)	$(L_0)_{cr}$	4.603	4.576	4.550
Table 10-4	A	201.000	201.000	201.000	Table 10-4	A	201.000	201.000	201.000
Table 10-4	m	0.145	0.145	0.145	Table 10-4	m	0.145	0.145	0.145
Eq. (10-14)	S_{ut}	292.626	289.900	287.363	Eq. (10-14)	S_{ut}	292.626	289.900	287.363
Table 10-6	S_{sy}	131.681	130.455	129.313	Table 10-6	S_{sy}	131.681	130.455	129.313
Eq. (10-5)	K_B	1.115	1.122	1.129	Eq. (10-5)	K_B	1.115	1.123	1.133
Eq. (10-7)	τ_s	135.335	112.948	95.293	Eq. (10-7)	τ_s	135.335	111.787	93.434
Eq. (10-3)	n_s	0.973	1.155	1.357	Eq. (10-3)	n_s	0.973	1.167	1.384
Eq. (10-22)	fom	–0.282	–0.391	–0.536	Eq. (10-22)	fom	–0.282	–0.398	–0.555

For $n_s \geq 1.2$, the optimal size is d = 0.085 in for both cases.

10-22 In Prob. 10-21, there is an advantage of first selecting d as one can select from the available sizes (Table A-28). Selecting C first requires a calculation of d where then a size must be selected from Table A-28.

Consider part (a) of the problem. It is required that

$$\text{ID} = D - d = 0.800 \text{ in.} \qquad (1)$$

From Eq. (10-1), $D = Cd$. Substituting this into the first equation yields

$$d = \frac{0.800}{C-1} \qquad (2)$$

Starting with C = 10, from Eq. (2) we find that d = 0.089 in. From Table A-28, the closest diameter is d = 0.090 in. Substituting this back into Eq. (1) gives D = 0.890 in, with C = 0.890/0.090 = 9.889, which are acceptable. From this point the solution is the same as Prob. 10-21. For part (b), use

$$\text{OD} = D + d = 0.950 \text{ in.} \qquad (3)$$

and,
$$d = \frac{0.800}{C-1} \qquad (4)$$

(a) Spring over a rod				(b) Spring in a Hole		
Source		Parameter	Values	Source	Parameter	Values
	C	10.000	10.5		C	10.000
Eq. (2)	d	0.089	0.084	Eq. (4)	d	0.086
Table A-28	d	0.090	0.085	Table A-28	d	0.085
Eq. (1)	D	0.890	0.885	Eq. (3)	D	0.865
Eq. (10-1)	C	9.889	10.412	Eq. (10-1)	C	10.176
Eq. (10-9)	N_a	13.669	11.061	Eq. (10-9)	N_a	11.846
Table 10-1	N_t	15.669	13.061	Table 10-1	N_t	13.846
Table 10-1	L_s	1.410	1.110	Table 10-1	L_s	1.177
$1.15y + L_s$	L_0	3.710	3.410	$1.15y + L_s$	L_0	3.477
Eq. (10-13)	$(L_0)_{cr}$	4.681	4.655	Eq. (10-13)	$(L_0)_{cr}$	4.550
Table 10-4	A	201.000	201.000	Table 10-4	A	201.000
Table 10-4	m	0.145	0.145	Table 10-4	m	0.145
Eq. (10-14)	S_{ut}	284.991	287.363	Eq. (10-14)	S_{ut}	287.363
Table 10-6	S_{sy}	128.246	129.313	Table 10-6	S_{sy}	129.313
Eq. (10-5)	K_B	1.135	1.128	Eq. (10-5)	K_B	1.135
Eq. (10-7)	τ_s	81.167	95.223	Eq. (10-7)	τ_s	93.643
$n_s = S_{sy}/\tau_s$	n_s	1.580	1.358	$n_s = S_{sy}/\tau_s$	n_s	1.381
Eq. (10-22)	fom	-0.725	-0.536	Eq. (10-22)	fom	-0.555

Again, for $n_s \geq 1.2$, the optimal size is = 0.085 in.

Although this approach used less iterations than in Prob. 10-21, this was due to the initial values picked and not the approach.

10-23 One approach is to select A227 HD steel for its low cost. Try $L_0 = 48$ mm, then for $y = 48 - 37.5 = 10.5$ mm when $F = 45$ N. The spring rate is $k = F/y = 45/10.5 = 4.286$ N/mm.

For a clearance of 1.25 mm with screw, ID = 10 + 1.25 = 11.25 mm. Starting with $d = 2$ mm,

$$D = \text{ID} + d = 11.25 + 2 = 13.25 \text{ mm}$$

$$C = D/d = 13.25/2 = 6.625 \quad \text{(acceptable)}$$

Table 10-5 ($d = 2/25.4 = 0.0787$ in): $G = 79.3$ GPa

Eq. (10-9): $$N_a = \frac{d^4 G}{8kD^3} = \frac{2^4(79.3)10^3}{8(4.286)13.25^3} = 15.9 \text{ coils}$$

Assume squared and closed.

Table 10-1: $N_t = N_a + 2 = 15.9 + 2 = 17.9$ coils

$L_s = dN_t = 2(17.9) = 35.8$ mm

$y_s = L_0 - L_s = 48 - 35.8 = 12.2$ mm

$F_s = ky_s = 4.286(12.2) = 52.29$ N

Eq. (10-5): $$K_B = \frac{4C+2}{4C-3} = \frac{4(6.625)+2}{4(6.625)-3} = 1.213$$

Eq. (10-7): $$\tau_s = K_B \frac{8F_sD}{\pi d^3} = 1.213\left[\frac{8(52.29)13.25}{\pi(2^3)}\right] = 267.5 \text{ MPa}$$

Table 10-4: $A = 1783 \text{ MPa} \cdot \text{mm}^m$, $m = 0.190$

Eq. (10-14): $$S_{ut} = \frac{A}{d^m} = \frac{1783}{2^{0.190}} = 1563 \text{ MPa}$$

Table 10-6: $S_{sy} = 0.45S_{ut} = 0.45(1563) = 703.3$ MPa

$$n_s = \frac{S_{sy}}{\tau_s} = \frac{703.3}{267.5} = 2.63 > 1.2 \qquad O.K.$$

No other diameters in the given range work. So specify

A227-47 HD steel, $d = 2$ mm, $D = 13.25$ mm, ID = 11.25 mm, OD = 15.25 mm, squared and closed, $N_t = 17.9$ coils, $N_a = 15.9$ coils, $k = 4.286$ N/mm, $L_s = 35.8$ mm, and $L_0 = 48$ mm. *Ans.*

10-24 Select A227 HD steel for its low cost. Try $L_0 = 48$ mm, then for $y = 48 - 37.5 = 10.5$ mm when $F = 45$ N. The spring rate is $k = F/y = 45/10.5 = 4.286$ N/mm.

For a clearance of 1.25 mm with screw, ID = 10 + 1.25 = 11.25 mm.

$$D - d = 11.25 \qquad (1)$$

and, $$D = Cd \qquad (2)$$

Starting with $C = 8$, gives $D = 8d$. Substitute into Eq. (1) resulting in $d = 1.607$ mm. Selecting the nearest diameter in the given range, $d = 1.6$ mm. From this point, the calculations are shown in the third column of the spreadsheet output shown. We see that for $d = 1.6$ mm, the spring is not solid safe. Iterating on C we find that $C = 6.5$ provides acceptable results with the specifications

A227-47 HD steel, $d = 2$ mm, $D = 13.25$ mm, ID = 11.25 mm, OD = 15.25 mm, squared and closed, $N_t = 17.9$ coils, $N_a = 15.9$ coils, $k = 4.286$ N/mm, $L_s = 35.8$ mm, and $L_0 = 48$ mm. *Ans.*

Source		Parameter Values		
	C	8.000	7	6.500
Eq. (2)	d	1.607	1.875	2.045
Table A-28	d	1.600	1.800	2.000
Eq. (1)	D	12.850	13.050	13.250
Eq. (10-1)	C	8.031	7.250	6.625
Eq. (10-9)	N_a	7.206	10.924	15.908
Table 10-1	N_t	9.206	12.924	17.908
Table 10-1	L_s	14.730	23.264	35.815
$L_0 - L_s$	y_s	33.270	24.736	12.185
$F_s = ky_s$	F_s	142.594	106.020	52.224
Table 10-4	A	1783.000	1783.000	1783.000
Table 10-4	m	0.190	0.190	0.190
Eq. (10-14)	S_{ut}	1630.679	1594.592	1562.988
Table 10-6	S_{sy}	733.806	717.566	703.345
Eq. (10-5)	K_B	1.172	1.200	1.217
Eq. (10-7)	τ_s	1335.568	724.943	268.145
$n_s = S_{sy}/\tau_s$	n_s	0.549	0.990	2.623

The only difference between selecting C first rather than d as was done in Prob. 10-23, is that once d is calculated, the closest wire size must be selected. Iterating on d uses available wire sizes from the beginning.

10-25 A stock spring catalog may have over two hundred pages of compression springs with up to 80 springs per page listed.

- Students should be made aware that such catalogs exist.
- Many springs are selected from catalogs rather than designed.
- The wire size you want may not be listed.
- Catalogs may also be available on disk or the web through search routines. For example, disks are available from Century Spring at

  1 - (800) - 237 - 5225

  www.centuryspring.com
- It is better to familiarize yourself with vendor resources rather than invent them yourself.
- Sample catalog pages can be given to students for study.

10-26 Given: ID = 0.6 in, C = 10, L_0 = 5 in, L_s = 5 − 3 = 2 in, sq. & grd ends, unpeened, HD A227 wire.

(**a**) With ID = $D - d$ = 0.6 in and $C = D/d = 10 \Rightarrow 10\,d - d = 0.6 \Rightarrow d = 0.0667$ in *Ans*., and D = 0.667 in.

(**b**) Table 10-1: $L_s = dN_t = 2$ in $\Rightarrow N_t = 2/0.0667 = 30$ coils *Ans.*

(**c**) Table 10-1: $N_a = N_t - 2 = 30 - 2 = 28$ coils

Table 10-5: $G = 11.5$ Mpsi

Eq. (10-9): $$k = \frac{d^4G}{8D^3N_a} = \frac{0.0667^4(11.5)10^6}{8(0.667^3)28} = 3.424 \text{ lbf/in} \qquad \textit{Ans.}$$

(**d**) Table 10-4: $A = 140$ kpsi·in^m, $m = 0.190$

Eq. (10-14): $$S_{ut} = \frac{A}{d^m} = \frac{140}{0.0667^{0.190}} = 234.2 \text{ kpsi}$$

Table 10-6: $S_{sy} = 0.45\, S_{ut} = 0.45\,(234.2) = 105.4$ kpsi

$$F_s = ky_s = 3.424(3) = 10.27 \text{ lbf}$$

Eq. (10-5): $$K_B = \frac{4C+2}{4C-3} = \frac{4(10)+2}{4(10)-3} = 1.135$$

Eq. (10-7):

$$\tau_s = K_B\frac{8F_sD}{\pi d^3} = 1.135\frac{8(10.27)0.667}{\pi(0.0667^3)}$$

$$= 66.72(10^3) \text{ psi} = 66.72 \text{ kpsi}$$

$$n_s = \frac{S_{sy}}{\tau_s} = \frac{105.4}{66.72} = 1.58 \qquad \textit{Ans.}$$

(**e**) $\tau_a = \tau_m = 0.5\tau_s = 0.5(66.72) = 33.36$ kpsi, $r = \tau_a / \tau_m = 1$. Using the Gerber fatigue failure criterion with Zimmerli data,

Eq. (10-30): $S_{su} = 0.67\, S_{ut} = 0.67(234.2) = 156.9$ kpsi

The Gerber ordinate intercept for the Zimmerli data is

$$S_{se} = \frac{S_{sa}}{1-(S_{sm}/S_{su})^2} = \frac{35}{1-(55/156.9)^2} = 39.9 \text{ kpsi}$$

Table 6-7, p. 315,

$$S_{sa} = \frac{r^2S_{su}^2}{2S_{se}}\left[-1+\sqrt{1+\left(\frac{2S_{se}}{rS_{su}}\right)^2}\right]$$

$$= \frac{1^2(156.9^2)}{2(39.9)}\left\{-1+\sqrt{1+\left[\frac{2(39.9)}{1(156.9)}\right]^2}\right\} = 37.61 \text{ kpsi}$$

$$n_f = \frac{S_{sa}}{\tau_a} = \frac{37.61}{33.36} = 1.13 \qquad \textit{Ans.}$$

10-27 Given: OD ≤ 0.9 in, $C = 8$, $L_0 = 3$ in, $L_s = 1$ in, $y_s = 3 - 1 = 2$ in, sq. ends, unpeened, music wire.

(**a**) Try OD = $D + d = 0.9$ in, $C = D/d = 8 \Rightarrow D = 8d \Rightarrow 9d = 0.9 \Rightarrow d = 0.1$ *Ans.*

$D = 8(0.1) = 0.8$ in

(**b**) Table 10-1: $L_s = d\,(N_t + 1) \;\Rightarrow\; N_t = L_s / d - 1 = 1/0.1 - 1 = 9$ coils *Ans.*

Table 10-1: $N_a = N_t - 2 = 9 - 2 = 7$ coils

(**c**) Table 10-5: $G = 11.75$ Mpsi

Eq. (10-9):
$$k = \frac{d^4 G}{8D^3 N_a} = \frac{0.1^4(11.75)10^6}{8\left(0.8^3\right)7} = 40.98 \text{ lbf/in} \qquad \textit{Ans.}$$

(**d**) $F_s = k y_s = 40.98(2) = 81.96$ lbf

Eq. (10-5):
$$K_B = \frac{4C+2}{4C-3} = \frac{4(8)+2}{4(8)-3} = 1.172$$

Eq. (10-7):
$$\tau_s = K_B \frac{8F_s D}{\pi d^3} = 1.172\frac{8(81.96)0.8}{\pi\left(0.1^3\right)} = 195.7\left(10^3\right) \text{ psi} = 195.7 \text{ kpsi}$$

Table 10-4: $A = 201$ kpsi·in^m, $m = 0.145$

Eq. (10-14):
$$S_{ut} = \frac{A}{d^m} = \frac{201}{0.1^{0.145}} = 280.7 \text{ kpsi}$$

Table 10-6: $S_{sy} = 0.45\,S_{ut} = 0.45(280.7) = 126.3$ kpsi

$$n_s = \frac{S_{sy}}{\tau_s} = \frac{126.3}{195.7} = 0.645 \qquad \textit{Ans.}$$

(**e**) $\tau_a = \tau_m = \tau_s/2 = 195.7/2 = 97.85$ kpsi. Using the Gerber fatigue failure criterion with Zimmerli data,

Eq. (10-30): $S_{su} = 0.67\,S_{ut} = 0.67(280.7) = 188.1$ kpsi

The Gerber ordinate intercept for the Zimmerli data is
$$S_{se} = \frac{S_{sa}}{1-\left(S_{sm}/S_{su}\right)^2} = \frac{35}{1-\left(55/188.1\right)^2} = 38.3 \text{ kpsi}$$

Table 6-7, p. 315,
$$S_{sa} = \frac{r^2 S_{su}^2}{2S_{se}}\left[-1+\sqrt{1+\left(\frac{2S_{se}}{rS_{su}}\right)^2}\,\right]$$
$$= \frac{1^2\left(188.1^2\right)}{2(38.3)}\left\{-1+\sqrt{1+\left[\frac{2(38.3)}{1(188.1)}\right]^2}\,\right\} = 36.83 \text{ kpsi}$$

$$n_f = \frac{S_{sa}}{\tau_a} = \frac{36.83}{97.85} = 0.376 \quad Ans.$$

Obviously, the spring is severely under designed and will fail statically and in fatigue. Increasing C would improve matters. Try C = 12. This yields n_s = 1.83 and n_f = 1.00.

10-28 Given: $F_{\max}$ = 300 lbf, $F_{\min}$ = 150 lbf, Δy = 1 in, OD = 2.1 – 0.2 = 1.9 in, C = 7, unpeened, squared & ground, oil-tempered wire.

(**a**) D = OD – d = 1.9 – d (1)

$C = D/d = 7 \Rightarrow D = 7d$ (2)

Substitute Eq. (2) into (1)

$7d = 1.9 - d \Rightarrow d = 1.9/8 = 0.2375$ in *Ans.*

(**b**) From Eq. (2): $D = 7d = 7(0.2375) = 1.663$ in *Ans.*

(**c**) $$k = \frac{\Delta F}{\Delta y} = \frac{300-150}{1} = 150 \text{ lbf/in} \quad Ans.$$

(**d**) Table 10-5: G = 11.6 Mpsi

Eq. (10-9): $$N_a = \frac{d^4 G}{8D^3 k} = \frac{0.2375^4(11.6)10^6}{8(1.663^3)150} = 6.69 \text{ coils}$$

Table 10-1: $N_t = N_a + 2 = 8.69$ coils *Ans.*

(**e**) Table 10-4: A = 147 kpsi·in^m, m = 0.187

Eq. (10-14): $$S_{ut} = \frac{A}{d^m} = \frac{147}{0.2375^{0.187}} = 192.3 \text{ kpsi}$$

Table 10-6: $S_{sy} = 0.5\, S_{ut} = 0.5(192.3) = 96.15$ kpsi

Eq. (10-5): $$K_B = \frac{4C+2}{4C-3} = \frac{4(7)+2}{4(7)-3} = 1.2$$

Eq. (10-7): $$\tau_s = K_B \frac{8F_s D}{\pi d^3} = S_{sy}$$

$$F_s = \frac{\pi d^3 S_{sy}}{8K_B D} = \frac{\pi\left(0.2375^3\right)96.15\left(10^3\right)}{8\left(1.2\right)1.663} = 253.5 \text{ lbf}$$

$y_s = F_s / k = 253.5/150 = 1.69$ in

Table 10-1: $L_s = N_t\, d = 8.46(0.2375) = 2.01$ in

$L_0 = L_s + y_s = 2.01 + 1.69 = 3.70$ in *Ans.*

10-29 For a coil radius given by:

$$R = R_1 + \frac{R_2 - R_1}{2\pi N}\theta$$

The torsion of a section is $T = PR$ where $dL = R\, d\theta$

$$\begin{aligned}
\delta_P &= \frac{\partial U}{\partial P} = \frac{1}{GJ}\int T\frac{\partial T}{\partial P}dL = \frac{1}{GJ}\int_0^{2\pi N} PR^3\, d\theta \\
&= \frac{P}{GJ}\int_0^{2\pi N}\left(R_1 + \frac{R_2 - R_1}{2\pi N}\theta\right)^3 d\theta \\
&= \frac{P}{GJ}\left(\frac{1}{4}\right)\left(\frac{2\pi N}{R_2 - R_1}\right)\left[\left(R_1 + \frac{R_2 - R_1}{2\pi N}\theta\right)^4\right]_0^{2\pi N} \\
&= \frac{\pi PN}{2GJ(R_2 - R_1)}\left(R_2^4 - R_1^4\right) = \frac{\pi PN}{2GJ}(R_1 + R_2)\left(R_1^2 + R_2^2\right)
\end{aligned}$$

$$J = \frac{\pi}{32}d^4 \quad \therefore\ \delta_p = \frac{16PN}{Gd^4}(R_1 + R_2)\left(R_1^2 + R_2^2\right)$$

$$k = \frac{P}{\delta_P} = \frac{d^4 G}{16N(R_1 + R_2)\left(R_1^2 + R_2^2\right)} \qquad \textit{Ans.}$$

10-30 Given: $F_{\min} = 4$ lbf, $F_{\max} = 18$ lbf, $k = 9.5$ lbf/in, OD ≤ 2.5 in, $n_f = 1.5$.

For a food service machinery application select A313 Stainless wire.
Table 10-5: $G = 10(10^6)$ psi
Note that for $0.013 \le d \le 0.10$ in $A = 169$, $m = 0.146$
$0.10 < d \le 0.20$ in $A = 128$, $m = 0.263$

$$F_a = \frac{18 - 4}{2} = 7 \text{ lbf}, \quad F_m = \frac{18 + 4}{2} = 11 \text{ lbf}, \quad r = 7/11$$

Try, $d = 0.080$ in, $S_{ut} = \dfrac{169}{(0.08)^{0.146}} = 244.4$ kpsi

$S_{su} = 0.67S_{ut} = 163.7$ kpsi, $S_{sy} = 0.35S_{ut} = 85.5$ kpsi

Try unpeened using Zimmerli's endurance data: $S_{sa} = 35$ kpsi, $S_{sm} = 55$ kpsi

Gerber: $$S_{se} = \frac{S_{sa}}{1-(S_{sm}/S_{su})^2} = \frac{35}{1-(55/163.7)^2} = 39.5 \text{ kpsi}$$

$$S_{sa} = \frac{(7/11)^2(163.7)^2}{2(39.5)}\left\{-1+\sqrt{1+\left[\frac{2(39.5)}{(7/11)(163.7)}\right]^2}\right\} = 35.0 \text{ kpsi}$$

$$\alpha = S_{sa}/n_f = 35.0/1.5 = 23.3 \text{ kpsi}$$

$$\beta = \frac{8F_a}{\pi d^2}(10^{-3}) = \left[\frac{8(7)}{\pi(0.08^2)}\right](10^{-3}) = 2.785 \text{ kpsi}$$

$$C = \frac{2(23.3)-2.785}{4(2.785)} + \sqrt{\left[\frac{2(23.3)-2.785}{4(2.785)}\right]^2 - \frac{3(23.3)}{4(2.785)}} = 6.97$$

$$D = Cd = 6.97(0.08) = 0.558 \text{ in}$$

$$K_B = \frac{4C+2}{4C-3} = \frac{4(6.97)+2}{4(6.97)-3} = 1.201$$

$$\tau_a = K_B\left(\frac{8F_aD}{\pi d^3}\right) = 1.201\left[\frac{8(7)(0.558)}{\pi(0.08^3)}(10^{-3})\right] = 23.3 \text{ kpsi}$$

$$n_f = 35/23.3 = 1.50 \quad \text{checks}$$

$$N_a = \frac{Gd^4}{8kD^3} = \frac{10(10^6)(0.08)^4}{8(9.5)(0.558)^3} = 31.02 \text{ coils}$$

$$N_t = 31.02 + 2 = 33 \text{ coils}, \quad L_s = dN_t = 0.08(33) = 2.64 \text{ in}$$

$$y_{\max} = F_{\max}/k = 18/9.5 = 1.895 \text{ in}$$

$$y_s = (1+\xi)y_{\max} = (1+0.15)(1.895) = 2.179 \text{ in}$$

$$L_0 = 2.64 + 2.179 = 4.819 \text{ in}$$

$$(L_0)_{cr} = 2.63\frac{D}{\alpha} = \frac{2.63(0.558)}{0.5} = 2.935 \text{ in}$$

$$\tau_s = 1.15(18/7)\tau_a = 1.15(18/7)(23.3) = 68.9 \text{ kpsi}$$

$$n_s = S_{sy}/\tau_s = 85.5/68.9 = 1.24$$

$$f = \sqrt{\frac{kg}{\pi^2 d^2 D N_a \gamma}} = \sqrt{\frac{9.5(386)}{\pi^2(0.08^2)(0.558)(31.02)(0.283)}} = 109 \text{ Hz}$$

These steps are easily implemented on a spreadsheet, as shown below, for different diameters.

	d_1	d_2	d_3	d_4
d	0.080	0.0915	0.1055	0.1205
m	0.146	0.146	0.263	0.263
A	169.000	169.000	128	128
S_{ut}	244.363	239.618	231.257	223.311
S_{su}	163.723	160.544	154.942	149.618
S_{sy}	85.527	83.866	80.940	78.159
S_{se}	39.452	39.654	40.046	40.469
S_{sa}	35.000	35.000	35.000	35.000
α	23.333	23.333	23.333	23.333
β	2.785	2.129	1.602	1.228
C	6.977	9.603	13.244	17.702
D	0.558	0.879	1.397	2.133
K_B	1.201	1.141	1.100	1.074
τ_a	23.333	23.333	23.333	23.333
n_f	1.500	1.500	1.500	1.500
N_a	30.993	13.594	5.975	2.858
N_t	32.993	15.594	7.975	4.858
L_S	2.639	1.427	0.841	0.585
y_s	2.179	2.179	2.179	2.179
L_0	4.818	3.606	3.020	2.764
$(L_0)_{\text{cr}}$	2.936	4.622	7.350	11.220
τ_s	69.000	69.000	69.000	69.000
n_s	1.240	1.215	1.173	1.133
f,(Hz)	108.895	114.578	118.863	121.775

The shaded areas depict conditions outside the recommended design conditions. Thus, one spring is satisfactory. The specifications are: A313 stainless wire, unpeened, squared and ground, d = 0.0915 in, OD = 0.879 + 0.092 = 0.971 in, L_0 = 3.606 in, and N_t = 15.59 turns *Ans*.

10-31 The steps are the same as in Prob. 10-30 except that the Gerber-Zimmerli criterion is replaced with Goodman-Zimmerli:

$$S_{se} = \frac{S_{sa}}{1 - \left(S_{sm}/S_{su}\right)}$$

The problem then proceeds as in Prob. 10-30. The results for the wire sizes are shown below (see solution to Prob. 10-30 for additional details).

Iteration of d for the first trial									
	d_1	d_2	d_3	d_4		d_1	d_2	d_3	d_4
d	0.080	0.0915	0.1055	0.1205	d	0.080	0.0915	0.1055	0.1205
m	0.146	0.146	0.263	0.263	K_B	1.151	1.108	1.078	1.058
A	169.000	169.000	128.000	128.000	τ_a	29.008	29.040	29.090	29.127
S_{ut}	244.363	239.618	231.257	223.311	n_f	1.500	1.500	1.500	1.500
S_{su}	163.723	160.544	154.942	149.618	N_a	14.191	6.456	2.899	1.404
S_{sy}	85.527	83.866	80.940	78.159	N_t	16.191	8.456	4.899	3.404
S_{se}	52.706	53.239	54.261	55.345	L_s	1.295	0.774	0.517	0.410
S_{sa}	43.513	43.560	43.634	43.691	y_{max}	2.875	2.875	2.875	2.875
α	29.008	29.040	29.090	29.127	L_0	4.170	3.649	3.392	3.285
β	2.785	2.129	1.602	1.228	$(L_0)_{\text{cr}}$	3.809	5.924	9.354	14.219
C	9.052	12.309	16.856	22.433	τ_s	85.782	85.876	86.022	86.133
D	0.724	1.126	1.778	2.703	n_s	0.997	0.977	0.941	0.907
					f (Hz)	140.040	145.559	149.938	152.966

Without checking all of the design conditions, it is obvious that none of the wire sizes satisfy $n_s \geq 1.2$. Also, the Gerber line is closer to the yield line than the Goodman. Setting $n_f = 1.5$ for Goodman makes it impossible to reach the yield line ($n_s < 1$) . The table below uses $n_f = 2$.

Iteration of d for the second trial									
	d_1	d_2	d_3	d_4		d_1	d_2	d_3	d_4
d	0.080	0.0915	0.1055	0.1205	d	0.080	0.0915	0.1055	0.1205
m	0.146	0.146	0.263	0.263	K_B	1.221	1.154	1.108	1.079
A	169.000	169.000	128.000	128.000	τ_a	21.756	21.780	21.817	21.845
S_{ut}	244.363	239.618	231.257	223.311	n_f	2.000	2.000	2.000	2.000
S_{su}	163.723	160.544	154.942	149.618	N_a	40.243	17.286	7.475	3.539
S_{sy}	85.527	83.866	80.940	78.159	N_t	42.243	19.286	9.475	5.539
S_{se}	52.706	53.239	54.261	55.345	L_s	3.379	1.765	1.000	0.667
S_{sa}	43.513	43.560	43.634	43.691	y_{max}	2.875	2.875	2.875	2.875
α	21.756	21.780	21.817	21.845	L_0	6.254	4.640	3.875	3.542
β	2.785	2.129	1.602	1.228	$(L_0)_{\text{cr}}$	2.691	4.266	6.821	10.449
C	6.395	8.864	12.292	16.485	τ_s	64.336	64.407	64.517	64.600
D	0.512	0.811	1.297	1.986	n_s	1.329	1.302	1.255	1.210
					f (Hz)	98.936	104.827	109.340	112.409

The satisfactory spring has design specifications of: A313 stainless wire, unpeened, squared and ground, d = 0.0915 in, OD = 0.811 + 0.092 = 0.903 in, L_0 = 4.640 in, and .N_t = 19.3 turns. *Ans.*

10-32 This is the same as Prob. 10-30 since S_{sa} = 35 kpsi. Therefore, the specifications are: A313 stainless wire, unpeened, squared and ground, d = 0.0915 in, OD = 0.879 + 0.092 =

0.971 in, L_0 = 3.606 in, and N_t = 15.59 turns *Ans*.

10-33 For the Gerber-Zimmerli fatigue-failure criterion, $S_{su} = 0.67S_{ut}$,

$$S_{se} = \frac{S_{sa}}{1-(S_{sm}/S_{su})^2}\ , \qquad S_{sa} = \frac{r^2S_{su}^2}{2S_{se}}\left[-1+\sqrt{1+\left(\frac{2S_{se}}{rS_{su}}\right)^2}\,\right]$$

The equation for S_{sa} is the basic difference. The last 2 columns of diameters of Ex. 10-5 are presented below with additional calculations.

d	0.105	0.112	d	0.105	0.112
S_{ut}	278.691	276.096	N_a	8.915	6.190
S_{su}	186.723	184.984	L_s	1.146	0.917
S_{se}	38.325	38.394	L_0	3.446	3.217
S_{sy}	125.411	124.243	$(L_0)_{cr}$	6.630	8.160
S_{sa}	34.658	34.652	K_B	1.111	1.095
α	23.105	23.101	τ_a	23.105	23.101
β	1.732	1.523	n_f	1.500	1.500
C	12.004	13.851	τ_s	70.855	70.844
D	1.260	1.551	n_s	1.770	1.754
ID	1.155	1.439	f_n	105.433	106.922
OD	1.365	1.663	fom	–0.973	–1.022

There are only slight changes in the results.

10-34 As in Prob. 10-34, the basic change is S_{sa}.

For Goodman, $$S_{se} = \frac{S_{sa}}{1-(S_{sm}/S_{su})}$$

Recalculate S_{sa} with

$$S_{sa} = \frac{rS_{se}S_{su}}{rS_{su}+S_{se}}$$

Calculations for the last 2 diameters of Ex. 10-5 are given below.

d	0.105	0.112	d	0.105	0.112
S_{ut}	278.691	276.096	N_a	9.153	6.353
S_{su}	186.723	184.984	L_s	1.171	0.936
S_{se}	49.614	49.810	L_0	3.471	3.236
S_{sy}	125.411	124.243	$(L_0)_{cr}$	6.572	8.090
S_{sa}	34.386	34.380	K_B	1.112	1.096
α	22.924	22.920	τ_a	22.924	22.920
β	1.732	1.523	n_f	1.500	1.500
C	11.899	13.732	τ_s	70.301	70.289
D	1.249	1.538	n_s	1.784	1.768
ID	1.144	1.426	f_n	104.509	106.000
OD	1.354	1.650	fom	–0.986	–1.034

There are only slight differences in the results.

10-35 Use: E = 28.6 Mpsi, G = 11.5 Mpsi, $A = 140 \text{ kpsi} \cdot \text{in}^m$, m = 0.190, rel cost = 1.

Try $\quad d = 0.067\ in, \quad S_{ut} = \dfrac{140}{(0.067)^{0.190}} = 234.0 \text{ kpsi}$

Table 10-6: $\quad S_{sy} = 0.45S_{ut} = 105.3$ kpsi

Table 10-7: $\quad S_y = 0.75S_{ut} = 175.5$ kpsi

Eq. (10-34) with $D/d = C$ and $C_1 = C$

$$\sigma_A = \frac{F_{\max}}{\pi d^2}[(K)_A(16C) + 4] = \frac{S_y}{n_y}$$

$$\frac{4C^2 - C - 1}{4C(C-1)}(16C) + 4 = \frac{\pi d^2 S_y}{n_y F_{\max}}$$

$$4C^2 - C - 1 = (C-1)\left(\frac{\pi d^2 S_y}{4n_y F_{\max}} - 1\right)$$

$$C^2 - \frac{1}{4}\left(1 + \frac{\pi d^2 S_y}{4n_y F_{\max}} - 1\right)C + \frac{1}{4}\left(\frac{\pi d^2 S_y}{4n_y F_{\max}} - 2\right) = 0$$

$$C = \frac{1}{2}\left[\frac{\pi d^2 S_y}{16 n_y F_{\max}} \pm \sqrt{\left(\frac{\pi d^2 S_y}{16 n_y F_{\max}}\right)^2 - \frac{\pi d^2 S_y}{4 n_y F_{\max}} + 2}\right] \quad \text{take positive root}$$

$$= \frac{1}{2}\left\{\frac{\pi(0.067^2)(175.5)(10^3)}{16(1.5)(18)} + \sqrt{\left[\frac{\pi(0.067)^2(175.5)(10^3)}{16(1.5)(18)}\right]^2 - \frac{\pi(0.067)^2(175.5)(10^3)}{4(1.5)(18)} + 2}\right\} = 4.590$$

$$D = Cd = 4.59\left(0.067\right) = 0.3075 \text{ in}$$

$$F_i = \frac{\pi d^3 \tau_i}{8D} = \frac{\pi d^3}{8D}\left[\frac{33\,500}{\exp(0.105C)} \pm 1000\left(4 - \frac{C-3}{6.5}\right)\right]$$

Use the lowest F_i in the preferred range. This results in the best fom.

$$F_i = \frac{\pi(0.067)^3}{8(0.3075)}\left\{\frac{33\,500}{\exp[0.105(4.590)]} - 1000\left(4 - \frac{4.590-3}{6.5}\right)\right\} = 6.505 \text{ lbf}$$

For simplicity, we will round up to the next integer or half integer. Therefore, use $F_i = 7$ lbf

$$k = \frac{18-7}{0.5} = 22 \text{ lbf/in}$$

$$N_a = \frac{d^4 G}{8kD^3} = \frac{(0.067)^4(11.5)(10^6)}{8(22)(0.3075)^3} = 45.28 \text{ turns}$$

$$N_b = N_a - \frac{G}{E} = 45.28 - \frac{11.5}{28.6} = 44.88 \text{ turns}$$

$$L_0 = (2C - 1 + N_b)d = [2(4.590) - 1 + 44.88](0.067) = 3.555 \text{ in}$$

$$L_{18\text{ lbf}} = 3.555 + 0.5 = 4.055 \text{ in}$$

Body: $K_B = \dfrac{4C+2}{4C-3} = \dfrac{4(4.590)+2}{4(4.590)-3} = 1.326$

$$\tau_{\max} = \frac{8K_B F_{\max} D}{\pi d^3} = \frac{8(1.326)(18)(0.3075)}{\pi(0.067)^3}(10^{-3}) = 62.1 \text{ kpsi}$$

$$(n_y)_{\text{body}} = \frac{S_{sy}}{\tau_{\max}} = \frac{105.3}{62.1} = 1.70$$

$$r_2 = 2d = 2(0.067) = 0.134 \text{ in}, \quad C_2 = \frac{2r_2}{d} = \frac{2(0.134)}{0.067} = 4$$

$$(K)_B = \frac{4C_2 - 1}{4C_2 - 4} = \frac{4(4)-1}{4(4)-4} = 1.25$$

$$\tau_B = (K)_B\left[\frac{8F_{\max}D}{\pi d^3}\right] = 1.25\left[\frac{8(18)(0.3075)}{\pi(0.067)^3}\right](10^{-3}) = 58.58 \text{ kpsi}$$

$$(n_y)_B = \frac{S_{sy}}{\tau_B} = \frac{105.3}{58.58} = 1.80$$

$$\text{fom} = -(1)\frac{\pi^2 d^2 (N_b + 2)D}{4} = -\frac{\pi^2(0.067)^2(44.88+2)(0.3075)}{4} = -0.160$$

Several diameters, evaluated using a spreadsheet, are shown below.

d	0.067	0.072	0.076	0.081	0.085	0.09	0.095	0.104
S_{ut}	233.977	230.799	228.441	225.692	223.634	221.219	218.958	215.224
S_{sy}	105.290	103.860	102.798	101.561	100.635	99.548	98.531	96.851
S_y	175.483	173.100	171.331	169.269	167.726	165.914	164.218	161.418
C	4.589	5.412	6.099	6.993	7.738	8.708	9.721	11.650
D	0.307	0.390	0.463	0.566	0.658	0.784	0.923	1.212
F_i (calc)	6.505	5.773	5.257	4.675	4.251	3.764	3.320	2.621
F_i (rd)	7.0	6.0	5.5	5.0	4.5	4.0	3.5	3.0
k	22.000	24.000	25.000	26.000	27.000	28.000	29.000	30.000
N_a	45.29	27.20	19.27	13.10	9.77	7.00	5.13	3.15
N_b	44.89	26.80	18.86	12.69	9.36	6.59	4.72	2.75
L_0	3.556	2.637	2.285	2.080	2.026	2.071	2.201	2.605
$L_{18\text{ lbf}}$	4.056	3.137	2.785	2.580	2.526	2.571	2.701	3.105
K_B	1.326	1.268	1.234	1.200	1.179	1.157	1.139	1.115
τ_{max}	62.118	60.686	59.707	58.636	57.875	57.019	56.249	55.031
$(n_y)_{body}$	1.695	1.711	1.722	1.732	1.739	1.746	1.752	1.760
τ_B	58.576	59.820	60.495	61.067	61.367	61.598	61.712	61.712
$(n_y)_B$	1.797	1.736	1.699	1.663	1.640	1.616	1.597	1.569
$(n_y)_A$	1.500	1.500	1.500	1.500	1.500	1.500	1.500	1.500
fom	–0.160	–0.144	-0.138	–0.135	–0.133	–0.135	–0.138	–0.154

Except for the 0.067 in wire, all springs satisfy the requirements of length and number of coils. The 0.085 in wire has the highest fom.

10-36 Given: N_b = 84 coils, F_i = 16 lbf, OQ&T steel, OD = 1.5 in, d = 0.162 in.

$$D = \text{OD} - d = 1.5 - 0.162 = 1.338 \text{ in}$$

(a) Eq. (10-39):

$$L_0 = 2(D - d) + (N_b + 1)d$$
$$= 2(1.338 - 0.162) + (84 + 1)(0.162) = 16.12 \text{ in} \quad \textit{Ans.}$$

or $$2d + L_0 = 2(0.162) + 16.12 = 16.45 \text{ in overall}$$

(b) $$C = \frac{D}{d} = \frac{1.338}{0.162} = 8.26$$

$$K_B = \frac{4C+2}{4C-3} = \frac{4(8.26)+2}{4(8.26)-3} = 1.166$$

$$\tau_i = K_B\left[\frac{8F_iD}{\pi d^3}\right] = 1.166\frac{8(16)(1.338)}{\pi(0.162)^3} = 14\,950 \text{ psi} \quad \textit{Ans.}$$

(c) From Table 10-5 use: $G = 11.4(10^6)$ psi and $E = 28.5(10^6)$ psi

$$N_a = N_b + \frac{G}{E} = 84 + \frac{11.4}{28.5} = 84.4 \text{ turns}$$

$$k = \frac{d^4G}{8D^3N_a} = \frac{(0.162)^4(11.4)(10^6)}{8(1.338)^3(84.4)} = 4.855 \text{ lbf/in} \quad \textit{Ans.}$$

(d) Table 10-4: $A = 147 \text{ psi} \cdot \text{in}^m, \quad m = 0.187$

$$S_{ut} = \frac{147}{(0.162)^{0.187}} = 207.1 \text{ kpsi}$$
$$S_y = 0.75(207.1) = 155.3 \text{ kpsi}$$
$$S_{sy} = 0.50(207.1) = 103.5 \text{ kpsi}$$

Body

$$F = \frac{\pi d^3 S_{sy}}{\pi K_B D}$$
$$= \frac{\pi(0.162)^3(103.5)(10^3)}{8(1.166)(1.338)} = 110.8 \text{ lbf}$$

Torsional stress on hook point B

$$C_2 = \frac{2r_2}{d} = \frac{2(0.25 + 0.162 / 2)}{0.162} = 4.086$$
$$(K)_B = \frac{4C_2 - 1}{4C_2 - 4} = \frac{4(4.086) - 1}{4(4.086) - 4} = 1.243$$
$$F = \frac{\pi(0.162)^3(103.5)(10^3)}{8(1.243)(1.338)} = 103.9 \text{ lbf}$$

Normal stress on hook point A

$$C_1 = \frac{2r_1}{d} = \frac{1.338}{0.162} = 8.26$$
$$(K)_A = \frac{4C_1^2 - C_1 - 1}{4C_1(C_1 - 1)} = \frac{4(8.26)^2 - 8.26 - 1}{4(8.26)(8.26 - 1)} = 1.099$$
$$S_{yt} = \sigma = F\left[\frac{16(K)_A D}{\pi d^3} + \frac{4}{\pi d^2}\right]$$
$$F = \frac{155.3(10^3)}{\left[16(1.099)(1.338)\right] / \left[\pi(0.162)^3\right] + \left\{4 / \left[\pi(0.162)^2\right]\right\}} = 85.8 \text{ lbf}$$
$$= \min(110.8,\ 103.9,\ 85.8) = 85.8 \text{ lbf} \quad \textit{Ans.}$$

(e) Eq. (10-48):

$$y = \frac{F - F_i}{k} = \frac{85.8 - 16}{4.855} = 14.4 \text{ in} \quad \textit{Ans.}$$

10-37 $F_{\text{min}} = 9$ lbf, $F_{\text{max}} = 18$ lbf

$$F_a = \frac{18 - 9}{2} = 4.5 \text{ lbf}, \quad F_m = \frac{18 + 9}{2} = 13.5 \text{ lbf}$$

A313 stainless: $0.013 \le d \le 0.1$ $A = 169$ kpsi · in^m, $m = 0.146$
$0.1 \le d \le 0.2$ $A = 128$ kpsi · in^m, $m = 0.263$
$E = 28$ Mpsi, $G = 10$ Gpsi

Try $d = 0.081$ in and refer to the discussion following Ex. 10-7

$$S_{ut} = \frac{169}{(0.081)^{0.146}} = 243.9 \text{ kpsi}$$
$$S_{su} = 0.67S_{ut} = 163.4 \text{ kpsi}$$
$$S_{sy} = 0.35S_{ut} = 85.4 \text{ kpsi}$$
$$S_y = 0.55S_{ut} = 134.2 \text{ kpsi}$$

Table 10-8: $S_r = 0.45S_{ut} = 109.8$ kpsi

$$S_e = \frac{S_r / 2}{1 - [S_r / (2S_{ut})]^2} = \frac{109.8 / 2}{1 - [(109.8 / 2) / 243.9]^2} = 57.8 \text{ kpsi}$$
$$r = F_a / F_m = 4.5 / 13.5 = 0.333$$

Table 6-7: $$S_a = \frac{r^2 S_{ut}^2}{2S_e}\left[-1 + \sqrt{1 + \left(\frac{2S_e}{rS_{ut}}\right)^2}\right]$$

$$S_a = \frac{(0.333)^2(243.9^2)}{2(57.8)}\left[-1 + \sqrt{1 + \left[\frac{2(57.8)}{0.333(243.9)}\right]^2}\right] = 42.2 \text{ kpsi}$$

Hook bending

$$(\sigma_a)_A = F_a\left[(K)_A \frac{16C}{\pi d^2} + \frac{4}{\pi d^2}\right] = \frac{S_a}{(n_f)_A} = \frac{S_a}{2}$$
$$\frac{4.5}{\pi d^2}\left[\frac{(4C^2 - C - 1)16C}{4C(C-1)} + 4\right] = \frac{S_a}{2}$$

This equation reduces to a quadratic in C (see Prob. 10-35). The useable root for C is

$$C = 0.5\left[\frac{\pi d^2 S_a}{144} + \sqrt{\left(\frac{\pi d^2 S_a}{144}\right)^2 - \frac{\pi d^2 S_a}{36} + 2}\right]$$
$$= 0.5\left\{\frac{\pi(0.081)^2(42.2)(10^3)}{144} + \sqrt{\left[\frac{\pi(0.081)^2(42.2)(10^3)}{144}\right]^2 - \frac{\pi(0.081)^2(42.2)(10^3)}{36} + 2}\right\}$$
$$= 4.91$$

$$D = Cd = 0.398 \text{ in}$$
$$F_i = \frac{\pi d^3 \tau_i}{8D} = \frac{\pi d^3}{8D}\left[\frac{33\,500}{\exp(0.105C)} \pm 1000\left(4 - \frac{C-3}{6.5}\right)\right]$$

Use the lowest F_i in the preferred range.

$$F_i = \frac{\pi(0.081)^3}{8(0.398)}\left[\frac{33\,500}{\exp[0.105(4.91)]} - 1000\left(4 - \frac{4.91-3}{6.5}\right)\right]$$
$$= 8.55 \text{ lbf}$$

For simplicity we will round up to next 1/4 integer.

$$F_i = 8.75 \text{ lbf}$$
$$k = \frac{18-9}{0.25} = 36 \text{ lbf/in}$$
$$N_a = \frac{d^4 G}{8kD^3} = \frac{(0.081)^4(10)(10^6)}{8(36)(0.398)^3} = 23.7 \text{ turns}$$
$$N_b = N_a - \frac{G}{E} = 23.7 - \frac{10}{28} = 23.3 \text{ turns}$$
$$L_0 = (2C - 1 + N_b)d = [2(4.91) - 1 + 23.3](0.081) = 2.602 \text{ in}$$
$$L_{\max} = L_0 + (F_{\max} - F_i)/k = 2.602 + (18 - 8.75)/36 = 2.859 \text{ in}$$
$$(\sigma_a)_A = \frac{4.5(4)}{\pi d^2}\left(\frac{4C^2 - C - 1}{C-1} + 1\right)$$
$$= \frac{18(10^{-3})}{\pi(0.081^2)}\left[\frac{4(4.91^2) - 4.91 - 1}{4.91 - 1} + 1\right] = 21.1 \text{ kpsi}$$
$$(n_f)_A = \frac{S_a}{(\sigma_a)_A} = \frac{42.2}{21.1} = 2 \text{ checks}$$

Body: $$K_B = \frac{4C+2}{4C-3} = \frac{4(4.91)+2}{4(4.91)-3} = 1.300$$
$$\tau_a = \frac{8(1.300)(4.5)(0.398)}{\pi(0.081)^3}(10^{-3}) = 11.16 \text{ kpsi}$$
$$\tau_m = \frac{F_m}{F_a}\tau_a = \frac{13.5}{4.5}(11.16) = 33.47 \text{ kpsi}$$

The repeating allowable stress from Table 10-8 is

$$S_{sr} = 0.30 S_{ut} = 0.30(243.9) = 73.17 \text{ kpsi}$$

The Gerber intercept is given by Eq. (10-42) as

$$S_{se} = \frac{73.17/2}{1 - [(73.17/2)/163.4]^2} = 38.5 \text{ kpsi}$$

From Table 6-7,

$$(n_f)_{\text{body}} = \frac{1}{2}\left(\frac{163.4}{33.47}\right)^2\left(\frac{11.16}{38.5}\right)\left\{-1+\sqrt{1+\left[\frac{2(33.47)(38.5)}{163.4(11.16)}\right]^2}\right\} = 2.53$$

Let $r_2 = 2d = 2(0.081) = 0.162$

$$C_2 = \frac{2r_2}{d} = 4, \qquad (K)_B = \frac{4(4)-1}{4(4)-4} = 1.25$$

$$(\tau_a)_B = \frac{(K)_B}{K_B}\tau_a = \frac{1.25}{1.30}(11.16) = 10.73 \text{ kpsi}$$

$$(\tau_m)_B = \frac{(K)_B}{K_B}\tau_m = \frac{1.25}{1.30}(33.47) = 32.18 \text{ kpsi}$$

Table 10-8: $(S_{sr})_B = 0.28S_{ut} = 0.28(243.9) = 68.3$ kpsi

$$(S_{se})_B = \frac{68.3/2}{1-[(68.3/2)/163.4]^2} = 35.7 \text{ kpsi}$$

$$(n_f)_B = \frac{1}{2}\left(\frac{163.4}{32.18}\right)^2\left(\frac{10.73}{35.7}\right)\left\{-1+\sqrt{1+\left[\frac{2(32.18)(35.7)}{163.4(10.73)}\right]^2}\right\} = 2.51$$

Yield

Bending:

$$(\sigma_A)_{\max} = \frac{4F_{\max}}{\pi d^2}\left[\frac{(4C^2-C-1)}{C-1}+1\right]$$

$$= \frac{4(18)}{\pi(0.081^2)}\left[\frac{4(4.91)^2-4.91-1}{4.91-1}+1\right](10^{-3}) = 84.4 \text{ kpsi}$$

$$(n_y)_A = \frac{134.2}{84.4} = 1.59$$

Body:

$$\tau_i = (F_i/F_a)\tau_a = (8.75/4.5)(11.16) = 21.7 \text{ kpsi}$$

$$r = \tau_a/(\tau_m-\tau_i) = 11.16/(33.47-21.7) = 0.948$$

$$(S_{sa})_y = \frac{r}{r+1}(S_{sy}-\tau_i) = \frac{0.948}{0.948+1}(85.4-21.7) = 31.0 \text{ kpsi}$$

$$(n_y)_{\text{body}} = \frac{(S_{sa})_y}{\tau_a} = \frac{31.0}{11.16} = 2.78$$

Hook shear:

$$S_{sy} = 0.3S_{ut} = 0.3(243.9) = 73.2 \text{ kpsi}$$

$$\tau_{\max} = (\tau_a)_B + (\tau_m)_B = 10.73 + 32.18 = 42.9 \text{ kpsi}$$

$$(n_y)_B = \frac{73.2}{42.9} = 1.71$$

$$\text{fom} = -\frac{7.6\pi^2 d^2(N_b+2)D}{4} = -\frac{7.6\pi^2(0.081)^2(23.3+2)(0.398)}{4} = -1.239$$

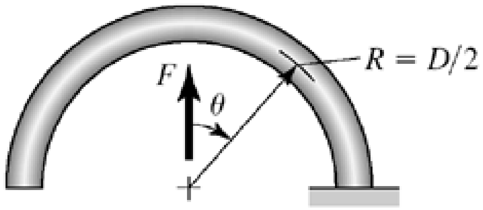
A tabulation of several wire sizes follow

d	0.081	0.085	0.092	0.098	0.105	0.12
S_{ut}	243.920	242.210	239.427	237.229	234.851	230.317
S_{su}	163.427	162.281	160.416	158.943	157.350	154.312
S_r	109.764	108.994	107.742	106.753	105.683	103.643
S_e	57.809	57.403	56.744	56.223	55.659	54.585
S_a	42.136	41.841	41.360	40.980	40.570	39.786
C	4.903	5.484	6.547	7.510	8.693	11.451
D	0.397	0.466	0.602	0.736	0.913	1.374
OD	0.478	0.551	0.694	0.834	1.018	1.494
F_i (calc)	8.572	7.874	6.798	5.987	5.141	3.637
F_i (rd)	8.75	9.75	10.75	11.75	12.75	13.75
k	36.000	36.000	36.000	36.000	36.000	36.000
N_a	23.86	17.90	11.38	8.03	5.55	2.77
N_b	23.50	17.54	11.02	7.68	5.19	2.42
L_0	2.617	2.338	2.127	2.126	2.266	2.918
$L_{18\text{ lbf}}$	2.874	2.567	2.328	2.300	2.412	3.036
$(\sigma_a)_A$	21.068	20.920	20.680	20.490	20.285	19.893
$(n_f)_A$	2.000	2.000	2.000	2.000	2.000	2.000
K_B	1.301	1.264	1.216	1.185	1.157	1.117
$(\tau_a)_{\text{body}}$	11.141	10.994	10.775	10.617	10.457	10.177
$(\tau_m)_{\text{body}}$	33.424	32.982	32.326	31.852	31.372	30.532
S_{sr}	73.176	72.663	71.828	71.169	70.455	69.095
S_{se}	38.519	38.249	37.809	37.462	37.087	36.371
$(n_f)_{\text{body}}$	2.531	2.547	2.569	2.583	2.596	2.616
$(K)_B$	1.250	1.250	1.250	1.250	1.250	1.250
$(\tau_a)_B$	10.705	10.872	11.080	11.200	11.294	11.391
$(\tau_m)_B$	32.114	32.615	33.240	33.601	33.883	34.173
$(S_{sr})_B$	68.298	67.819	67.040	66.424	65.758	64.489
$(S_{se})_B$	35.708	35.458	35.050	34.728	34.380	33.717
$(n_f)_B$	2.519	2.463	2.388	2.341	2.298	2.235
S_y	134.156	133.215	131.685	130.476	129.168	126.674
$(\sigma_A)_{\max}$	84.273	83.682	82.720	81.961	81.139	79.573
$(n_y)_A$	1.592	1.592	1.592	1.592	1.592	1.592
τ_i	21.663	23.820	25.741	27.723	29.629	31.097
r	0.945	1.157	1.444	1.942	2.906	4.703
$(S_{sy})_{\text{body}}$	85.372	84.773	83.800	83.030	82.198	80.611
$(S_{sa})_y$	30.958	32.688	34.302	36.507	39.109	40.832
$(n_y)_{\text{body}}$	2.779	2.973	3.183	3.438	3.740	4.012
$(S_{sy})_B$	73.176	72.663	71.828	71.169	70.455	69.095
$(\tau_B)_{\max}$	42.819	43.486	44.321	44.801	45.177	45.564
$(n_y)_B$	1.709	1.671	1.621	1.589	1.560	1.516
fom	−1.246	−1.234	−1.245	−1.283	−1.357	−1.639

optimal fom

The shaded areas show the conditions not satisfied.

10-38 For the hook,



$$M = FR\sin\theta, \quad \partial M/\partial F = R\sin\theta$$

$$\delta_F = \frac{1}{EI}\int_0^{\pi/2} F\left(R\sin\theta\right)^2 R\,d\theta = \frac{\pi}{2}\frac{FR^3}{EI}$$

The total deflection of the body and the two hooks

$$\delta = \frac{8FD^3N_b}{d^4G} + 2\left(\frac{\pi}{2}\frac{FR^3}{EI}\right) = \frac{8FD^3N_b}{d^4G} + \frac{\pi F(D/2)^3}{E(\pi/64)(d^4)}$$

$$= \frac{8FD^3}{d^4G}\left(N_b + \frac{G}{E}\right) = \frac{8FD^3N_a}{d^4G}$$

$$\therefore\ N_a = N_b + \frac{G}{E} \qquad \text{Q.E.D.}$$

10-39 Table 10-5 (d = 4 mm = 0.1575 in): E = 196.5 GPa

Table 10-4 for A227:

$$A = 1783\ \text{MPa}\cdot\text{mm}^m, \qquad m = 0.190$$

Eq. (10-14): $$S_{ut} = \frac{A}{d^m} = \frac{1783}{4^{0.190}} = 1370\ \text{MPa}$$

Eq. (10-57): $$S_y = \sigma_{\text{all}} = 0.78\,S_{ut} = 0.78(1370) = 1069\ \text{MPa}$$

$$D = \text{OD} - d = 32 - 4 = 28\ \text{mm}$$

$$C = D/d = 28/4 = 7$$

Eq. (10-43): $$K_i = \frac{4C^2 - C - 1}{4C(C-1)} = \frac{4\left(7^2\right) - 7 - 1}{4(7)(7-1)} = 1.119$$

Eq. (10-44): $$\sigma = K_i\frac{32Fr}{\pi d^3}$$

At yield, $Fr = M_y$, $\sigma = S_y$. Thus,

$$M_y = \frac{\pi d^3 S_y}{32K_i} = \frac{\pi\left(4^3\right)1069\left(10^{-3}\right)}{32(1.119)} = 6.00\ \text{N}\cdot\text{m}$$

Count the turns when $M = 0$

$$N = 2.5 - \frac{M_y}{k}$$

where from Eq. (10-51): $$k = \frac{d^4 E}{10.8DN}$$

Thus,

$$N = 2.5 - \frac{M_y}{d^4 E \ / \ (10.8DN)}$$

Solving for N gives

$$N = \frac{2.5}{1 + [10.8DM_y \ / \ (d^4 E)]}$$

$$= \frac{2.5}{1 + \left\{ \left[10.8(28)(6.00)\right] / \left[4^4(196.5)\right] \right\}} = 2.413 \text{ turns}$$

This means (2.5 – 2.413)(360°) or 31.3° from closed. *Ans*.

Treating the hand force as in the middle of the grip,

$$r = 112.5 - 87.5 + \frac{87.5}{2} = 68.75 \text{ mm}$$

$$F_{\max} = \frac{M_y}{r} = \frac{6.00\left(10^3\right)}{68.75} = 87.3 \text{ N} \quad \textit{Ans.}$$

10-40 The spring material and condition are unknown. Given $d = 0.081$ in and OD = 0.500,

(a) $D = 0.500 - 0.081 = 0.419$ in

Using $E = 28.6$ Mpsi for an estimate

$$k' = \frac{d^4 E}{10.8DN} = \frac{(0.081)^4(28.6)(10^6)}{10.8(0.419)(11)} = 24.7 \text{ lbf} \cdot \text{in/turn}$$

for each spring. The moment corresponding to a force of 8 lbf

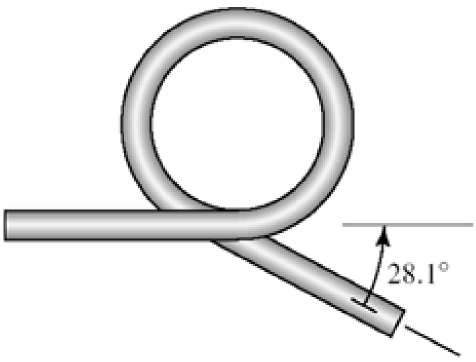
$$Fr = (8/2)(3.3125) = 13.25 \text{ lbf} \cdot \text{in/spring}$$

The fraction windup turn is

$$n = \frac{Fr}{k'} = \frac{13.25}{24.7} = 0.536 \text{ turns}$$

The arm swings through an arc of slightly less than 180°, say 165°. This uses up 165/360 or 0.458 turns. So $n = 0.536 - 0.458 = 0.078$ turns are left (or

$0.078(360°) = 28.1°$). The original configuration of the spring was



Ans.

(b)

$$C = \frac{D}{d} = \frac{0.419}{0.081} = 5.17$$

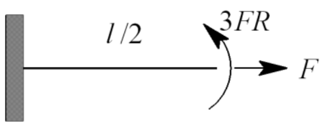
$$K_i = \frac{4C^2 - C - 1}{4C(C-1)} = \frac{4(5.17)^2 - 5.17 - 1}{4(5.17)(5.17-1)} = 1.168$$

$$\sigma = K_i \frac{32M}{\pi d^3} = 1.168\left[\frac{32(13.25)}{\pi(0.081)^3}\right] = 297(10^3)\ \text{psi} = 297\ \text{kpsi} \qquad \textit{Ans.}$$

To achieve this stress level, the spring had to have set removed.

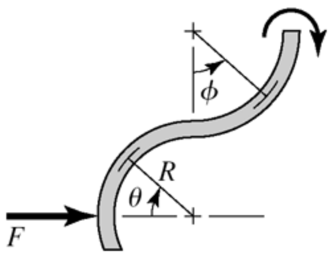
10-41 (**a**) Consider half and double results

Straight section:



$M = 3FR, \quad \dfrac{\partial M}{\partial F} = 3R$

Upper 180° section:



$$M = F[R + R(1 - \cos\phi)]$$
$$= FR(2 - \cos\phi), \quad \frac{\partial M}{\partial F} = R(2 - \cos\phi)$$

Lower section: $M = FR \sin\theta, \quad \dfrac{\partial M}{\partial F} = R\sin\theta$

Considering bending only:

$$\delta = \frac{\partial U}{\partial F} = \frac{2}{EI}\left[\int_0^{l/2} 9FR^2\, dx + \int_0^{\pi} FR^2(2-\cos\phi)^2 R\, d\phi + \int_0^{\pi/2} F(R\sin\theta)^2 R\, d\theta\right]$$
$$= \frac{2F}{EI}\left[\frac{9}{2}R^2 l + R^3\left(4\pi - 4\sin\phi\Big|_0^{\pi} + \frac{\pi}{2}\right) + R^3\left(\frac{\pi}{4}\right)\right]$$
$$= \frac{2FR^2}{EI}\left(\frac{19\pi}{4}R + \frac{9}{2}l\right) = \frac{FR^2}{2EI}(19\pi R + 18l)$$

The spring rate is

$$k = \frac{F}{\delta} = \frac{2EI}{R^2(19\pi R + 18l)} \qquad \textit{Ans.}$$

(**b**) Given: A227 HD wire, d = 2 mm, R = 6 mm, and l = 25 mm.

Table 10-5 (d = 2 mm = 0.0787 in): E = 197.2 GPa

$$k = \frac{2(197.2)10^9\pi\left(0.002^4\right)/(64)}{0.006^2\left[19\pi(0.006)+18(0.025)\right]} = 10.65\left(10^3\right)\text{ N/m} = 10.65\text{ N/mm} \qquad \textit{Ans.}$$

(**c**) The maximum stress will occur at the bottom of the top hook where the bending-moment is $3FR$ and the axial fore is F. Using curved beam theory for bending,

Eq. (3-65), p. 133: $$\sigma_i = \frac{Mc_i}{Aer_i} = \frac{3FRc_i}{\left(\pi d^2/4\right)e\left(R-d/2\right)}$$

Axial: $$\sigma_a = \frac{F}{A} = \frac{F}{\pi d^2/4}$$

Combining, $$\sigma_{\max} = \sigma_i + \sigma_a = \frac{4F}{\pi d^2}\left[\frac{3Rc_i}{e\left(R-d/2\right)} + 1\right] = S_y$$

$$F = \frac{\pi d^2 S_y}{4\left[\dfrac{3Rc_i}{e\left(R-d/2\right)} + 1\right]} \qquad (1) \qquad \textit{Ans.}$$

For the clip in part (b),

Eq. (10-14) and Table 10-4: $S_{ut} = A/d^m = 1783/2^{0.190} = 1563$ MPa

Eq. (10-57): $S_y = 0.78\, S_{ut} = 0.78(1563) = 1219$ MPa

Table 3-4, p. 135:

$$r_n = \frac{1^2}{2\left(6-\sqrt{6^2-1^2}\right)} = 5.95804 \text{ mm}$$

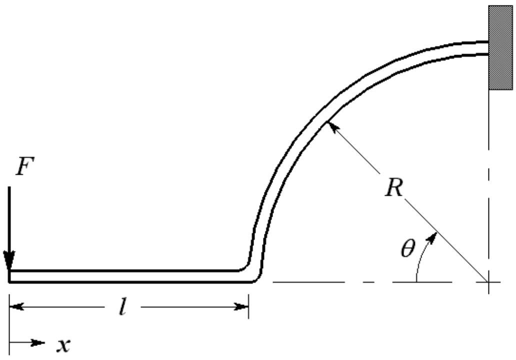
$$e = r_c - r_n = 6 - 5.95804 = 0.04196 \text{ mm}$$

$$c_i = r_n - (R - d/2) = 5.95804 - (6 - 2/2) = 0.95804 \text{ mm}$$

Eq. (1):

$$F = \frac{\pi\left(0.002^2\right)1219\left(10^6\right)}{4\left[\dfrac{3(6)0.95804}{0.04196(6-1)}+1\right]} = 46.0 \text{ N} \qquad \textit{Ans.}$$

10-42 (**a**)



$$M = -Fx, \quad \frac{\partial M}{\partial F} = -x \quad 0 \le x \le l$$

$$M = Fl + FR(1-\cos\theta), \quad \frac{\partial M}{\partial F} = l + R(1-\cos\theta) \quad 0 \le \theta \le \pi/2$$

$$\delta_F = \frac{1}{EI}\left\{\int_0^l -Fx(-x)dx + \int_0^{\pi/2} F\left[l + R(1-\cos\theta)\right]^2 R d\theta\right\}$$

$$= \frac{F}{12EI}\left\{4l^3 + 3R\left[2\pi l^2 + 4(\pi-2)lR + (3\pi-8)R^2\right]\right\}$$

The spring rate is

$$k = \frac{F}{\delta_F} = \frac{12EI}{4l^3 + 3R\left[2\pi l^2 + 4(\pi-2)lR + (3\pi-8)R^2\right]} \qquad \textit{Ans.}$$

(**b**) Given: A313 stainless wire, d = 0.063 in, R = 0.625 in, and l = 0.5 in.

Table 10-5: E = 28 Mpsi

$$I = \frac{\pi}{64} d^4 = \frac{\pi}{64}\left(0.063^4\right) = 7.733\left(10^{-7}\right) \text{ in}^4$$

$$k = \frac{12(28)10^6(7.733)10^{-7}}{4\left(0.5^3\right)+3(0.625)\left[2\pi\left(0.5^2\right)+4(\pi-2)0.5(0.625)+(3\pi-8)\left(0.625^2\right)\right]}$$

$$= 36.3 \text{ lbf/in} \qquad \textit{Ans.}$$

(**c**) Table 10-4: A = 169 kpsi·in^m, m = 0.146

Eq. (10-14): $S_{ut} = A/d^{\,m} = 169/0.063^{0.146} = 253.0$ kpsi

Eq. (10-57): $S_y = 0.61\,S_{ut} = 0.61(253.0) = 154.4$ kpsi

One can use curved beam theory as in the solution for Prob. 10-41. However, the equations developed in Sec. 10-12 are equally valid.

$$C = D/d = 2(0.625 + 0.063/2)/0.063 = 20.8$$

Eq. (10-43): $$K_i = \frac{4C^2 - C - 1}{4C(C-1)} = \frac{4\left(20.8^2\right) - 20.8 - 1}{4(20.8)(20.8-1)} = 1.037$$

Eq. (10-44), setting $\sigma = S_y$:

$$K_i \frac{32Fr}{\pi d^3} = S_y \quad \Rightarrow \quad 1.037 \frac{32F(0.5+0.625)}{\pi\left(0.063^3\right)} = 154.4\left(10^3\right)$$

Solving for F yields F = 3.25 lbf *Ans.*

Try solving part (c) of this problem using curved beam theory. You should obtain the same answer.

10-43 (**a**) $M = -Fx$

$$\sigma = \left|\frac{M}{I/c}\right| = \frac{Fx}{I/c} = \frac{Fx}{bh^2/6}$$

Constant stress,

$$\frac{bh^2}{6} = \frac{Fx}{\sigma} \quad \Rightarrow \quad h = \sqrt{\frac{6Fx}{b\sigma}} \qquad (1) \qquad \textit{Ans.}$$

At $x = l$,

$$h_o = \sqrt{\frac{6Fl}{b\sigma}} \qquad \Rightarrow \qquad h = h_o\sqrt{x/l} \qquad \textit{Ans.}$$

(**b**) $\quad M = -Fx, \quad \partial M/\partial F = -x$

$$y = \int_0^l \frac{M\left(\partial M/\partial F\right)}{EI}dx = \frac{1}{E}\int_0^l \frac{-Fx(-x)}{\frac{1}{12}bh_o^3\left(x/l\right)^{3/2}}dx = \frac{12Fl^{3/2}}{bh_o^3E}\int_0^l x^{1/2}dx$$

$$= \frac{2}{3}\frac{12Fl^{3/2}}{bh_o^3E}l^{3/2} = \frac{8Fl^3}{bh_o^3E}$$

$$k = \frac{F}{y} = \frac{bh_o^3E}{8l^3} \qquad \textit{Ans.}$$

10-44 Computer programs will vary.

10-45 Computer programs will vary.