It has been suggested to heat a sidewalk with electric heating elements in order to melt ice and snow. The 100-mm thick concrete sidewalk has heating elements embedded at half the thickness, spaced $d$ apart.

Your task is to minimize both the cost of the sidewalk and the required temperature of the heating elements while providing a uniform temperature to the top surface of the sidewalk.

The cost of the sidewalk is proportional to the number of heating elements and the temperature $T$ at which they operate. Calculate the cost of the sidewalk per linear meter as

$$C = \frac{\$8}{d} + \$2.60 \left(1 + \frac{T}{200^\circ C}\right)$$

where $d$ is in meters and $T$ is in °C. The required temperature of the heating elements is such that the top surface of the sidewalk is never less than 5°C, and the spacing $d$ between the heating elements must be so that the temperature of the upper surface does not vary more than 3°C. The spacing $d$ cannot be less than 25 mm.

For the sidewalk, the thermal conductivity is 0.30 W/mK
For the air, the convective heat transfer coefficient is 52 W/m²K

Submit a report (due via email before 7 pm on December 13, 2016) that provides – in narrative - the details of your finite-difference model, the steps that you followed to make sure that it was producing correct results, all of the calculations you performed in order to arrive at your solution to the problem, and a complete description of your solution. Illustrate your report with appropriate graphs and figures. Provide an Excel file with calculations that support your conclusions.
Develop a general-purpose Excel application to solve linear programming problems using the simplex method.

Section 15.1.3 provides a great deal of guidance and a worked-out example of developing and maintaining a tableau to keep track of the various components of the simplex method during the solution process. This would be a good place to start. The resources listed in the course outline, especially the Cengage resource, has more detailed explanations, more worked-out examples and contains the changes that must be incorporated to solve minimization problems.

You must test your application by solving all four of the simplex method examples provided in class. Once this is done, you must demonstrate your application by formulating and solving both of these problems:

An automobile company has two versions of the same model car, a two-door coupe and the full-size four-door model. Profit earned is $14,500/car for the 2-door and $16,000 for the 4-door. The 2-door model takes 15 hours/car to build, the 4-door 20 hours/car; there are 8000 hours/year available for production. Facilities are available to store 400 2-door and 350 4-door cars. Consumer demand, based on 240,000 cars, has been determined to be 700/car for the 2-door model and 500/car for the 4-door. Determine how many of each car should be produced to maximize profit and what that profit is.

An aerospace company is developing a new fuel additive for commercial airliners. The additive is composed of three ingredients, $X$, $Y$ and $Z$. For peak performance, the total amount of additive must be at least 6 mL/L of fuel. For safety reasons, the sum of the highly flammable $X$ and $Y$ ingredients must not exceed 2.5 mL/L. In addition, the amount of the $X$ ingredient must always be equal or greater than the $Y$, and the $Z$ must be greater than half the $Y$. If the cost per mL for the ingredients $X$, $Y$ and $Z$ is $0.04$, $0.035$ and $0.15$, respectively, determine the minimum cost mixture for each liter of fuel.

For extra points, solve these 6 linear programming problems using the Solver within Microsoft Excel and describe your solution method.

Submit a report (due via email before 7 pm on December 13, 2016) that provides – in narrative - the details of your general purpose simplex method application, detailed solutions of the example problems supplied in class, and your formulations and solutions to the two problems above. Illustrate your report with appropriate graphs and figures. Provide an Excel file with your application and the input data and instructions for all of the examples and demonstration problems required.
Develop a general Excel application to solve up to six simultaneous ordinary differential equations using an adaptive 4th-order Runge-Kutta approach and use it in a numerical demonstration of modal analysis.

The undamped natural frequencies $\omega$ of the 2-DOF system below are given by the following:

$$m_1 m_2 \omega^4 - [m_1 k_2 + m_2 (k_1 + k_2)]\omega^2 + k_1 k_2 = 0$$

Model the undamped system with parameters

- $m_1 = 100$ kg
- $m_2 = 24$ kg
- $k_1 = 21$ kN/m
- $k_2 = 29$ kN/m

and apply an impulsive force of the form

$$F(t) = F_0 \sin \left( \frac{\pi t}{T} \right) \; ; \; 0 \leq t \leq T$$

to mass 2 when the system is at rest, where $T$ is the time of the impulse (typically 2-5 ms). After time $T$ the system will be vibrating freely. During the free vibration, record the response of the system, that is, the accelerations, velocities and positions of the two masses, for an appropriate amount of time. Performing an FFT on these signals (for example, the acceleration of mass 2) should directly reveal the natural frequencies of the system.

Repeat the analysis above with $c_1 = c_2 = 20$ N-s/m. Comment on the differences.

Repeat the problem with an additional mass $m_3 = 10$ kg below $m_2$, suspended via a spring with a rate of $k_3 = 20$ kN/m and damper with $c_3 = 5$ N-s/m. Comment on the differences.

Submit a report (due via email before 7 pm on December 13, 2016) that provides – in narrative - the details of your general purpose application, the detailed model of these problems, the solutions of the system responses, the FFT of the responses and your analyses of the entire solution process. Illustrate your report with appropriate graphs and figures. Provide the Excel files of your application, with all input data and analysis.