19 Principle of Work and Energy

Ref: Hibbeler § 14.3, Bedford & Fowler: Dynamics § 8.1-8.2

The *principle of work and energy* states that the work done by all of the external forces and couples as a rigid body moves between positions 1 and 2 is equal to the change in the body's potential energy. Hibbeler writes the resulting equation as

$$T_1 + \sum U_{1-2} = T_2$$ Hibbeler (14-7)

The same equation is written as

$$U_{12} = T_2 - T_1$$ Bedford and Fowler: Dynamics (8.4)

by Bedford and Fowler.

The difference between these two equations is simply nomenclature. Both $\sum U_{1-2}$ and U_{12} represent the sum of work done by all external forces and couples on the body. We can use the principle of work and energy to solve problems involving force, displacement, and velocity.

Example: Skid-to-Stop Braking Distance

The driver of a 1600 kg passenger car hits the brakes and skids to a stop to try to avoid hitting a deer. When the deer suddenly appeared in the headlights [at an estimated distance of 300 ft (91 m)] the car was moving at 75 mph (120 km/hr).

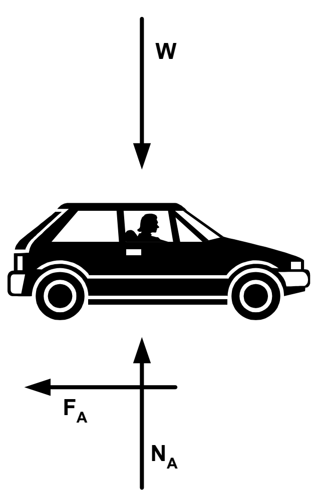
After the incident long skid marks were visible on the pavement, and the deer was seen running over a nearby hill. Did the driver get the car stopped in time, or did the deer manage to get out of the way?

Assumptions:

- Assume a 1.5 second reaction time – that is, it took 1.5 seconds from the time the driver saw the deer until he or she pressed the brake pedal.
- Assume a dry, flat road with a coefficient of friction, $\mu_k = 0.80$.

Solution

A free body diagram of this system shows the various forces acting on the car.



Because the road was assumed to be flat, the normal force, N_A, is equal to the vehicle's weight.

```
» M = 1600;                      %kg
» g = 9.807;                     %m / s^2
» W = M .* g;                    %Newtons
» N_A = W                        %Newtons
N_A =
  15691
```

The friction force during the skid (wheels locked) is calculated using the normal force and the coefficient of kinetic friction, μ_k.

```
» mu_k = 0.80;
» F_A = mu_k .* N_A              %Newtons
F_A =
  12553
```

The length of the skid can be determined using the principle of work and energy.

$$T_1 + \sum U_{1-2} = T_2$$

$$\tfrac{1}{2} m\, v_1^{\,2} + \sum U_{1-2} = \tfrac{1}{2} m\, v_2^{\,2}$$

The final velocity, v_2, is zero (skid-to-stop), and the only force acting on the car during the deceleration is the friction force, F_A, acting through the skid distance, s.

$$\tfrac{1}{2} m\, v_1^{\,2} + \left(-F_A\, s\right) = 0$$

This equation can be solved for the skid distance.

```
» v(1) = 75;                     %mi / hr
» v(1) = v(1) .* 1609 ./ 3600;   %m / s
» s = 0.5 .* M .* v(1).^2 / F_A  %meters
s =
  71.6100
```

That looks good; it is possible to stop in less than 91 meters, the initial distance between the car and the deer. But, we also need to account for the distance the car traveled in the 1.5 seconds between the time the driver saw the deer and the moment the brakes were applied.

```
» t_reaction = 1.5;              %sec
» s_reaction = v(1) .* t_reaction  %meters
s_reaction =
  50.2813
```

The total distance traveled between the moment the deer was spotted and the vehicle coming to a complete stop is 50.2813 + 71.6100 = 121.8912m.

```
» s_total = s + s_reaction       %meters
s_total =
  121.8912
```

The driver did not get stopped within 91 m. The deer had to get out of the way in order to escape injury.

Annotated MATLAB Script Solution

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%                      Skid-to-Stop Calculations                       %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%Calculate the magnitude of the normal force.
M = 1600;                                           %kg
g = 9.807;                                          %m/s^2
W = M .* g;                                         %Newtons
N_A = W                                             %Newtons
fprintf('Magnitude of the normal force = %1.0f N\n', N_A)

%Calculate the friction force.
mu_k = 0.80;
F_A = mu_k .* N_A;                                  %Newtons
fprintf('Friction force = %1.0f N\n', F_A)

%Calculate the skid distance
v(1) = 75;                                          %mi/hr
v(1) = v(1) .* 1609 ./ 3600;                        %m/s
s = 0.5 .* M .* v(1).^2 / F_A ;                     %meters
fprintf('Skid distance = %1.0f m\n', s)

%Calculate the distance traveled during the driver’s reaction time.
t_reaction = 1.5;                                   %sec
s_reaction = v(1) .* t_reaction;                    %meters
fprintf('Distance traveled during the drivers reaction ')
fprintf('time = %1.0f m\n', s_reaction)

%Calculate the total distance traveled after spotting the deer.
s_total = s + s_reaction;                           %meters
fprintf('Total distance traveled after spotting the ')
fprintf('deer %1.0f m\n', s_total)
```