18 Equations of Motion: Normal and Tangential Components

Ref: Hibbeler § 13.5, Bedford & Fowler: Dynamics § 3.4

In an earlier example (#14) we looked at the motion of a skateboarder along a curved path. Now, we apply the equations of motion to that problem to investigate the significance of friction in the problem.

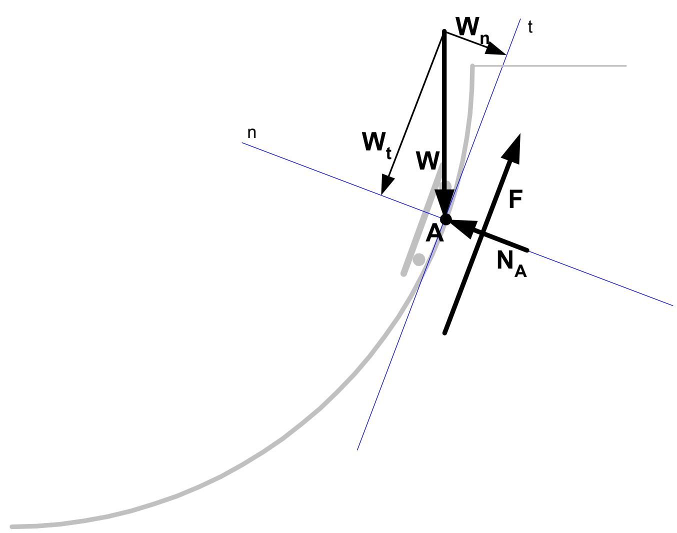
Example: A skateboarder in a quarter pipe.

A skateboarder with a mass of 55 kg has dropped into a 3 meter (radius) quarter pipe. When she's dropped a distance of 1 meter her speed is 3.2 m/s and is increasing by 7.8 m/s^2. At this location the surface is at an angle of 19.4° from vertical.

Draw a free body diagram of this system including a friction force. Neglect the size of the skateboarder and her board. Solve the equations of motion for the normal force, N_A, and the friction force, F. Then, calculate the coefficient of kinetic friction required to generate the observed friction force.

Solution

A free-body diagram for this system looks like the following:



First, information from the problem statement is entered into MATLAB.

```
» v = 3.2;                          %m / s
» a_t = 7.8;                        %m / s^2
» M = 55;                           %kg
» g = 9.807;                        %m / s^2
» alpha = 19.4 * pi /180;           %radians - angle from vertical
» rho = 3;                          %meters - circular path, constant rho
```

The tangential acceleration at this location was given in the problem statement, as a_t = 7.8 m/s^2. The normal acceleration can be calculated from the stated velocity (3.2 m/s) and the radius of curvature (3 m).

```
» a_n = v.^2 ./ rho                    %m / s^2

a_n =

   3.4133
```

The normal component of the equation of motion can be used to find N_A, but first we need the normal component of the skateboarder's weight.

```
» W = M .* g                           %Newtons

W =

  539.3850


» W_n = -W .* sin(alpha)               %Newtons - minus sign accounts for direction

W_n =

 -179.1627


» W_t =  W .* cos(alpha)               %Newtons

W_t =

  508.7602
```

Then find the normal force, N_A.

```
» % N_A + W_n = M * a_n        so...

» N_A = M .* a_n - W_n                 %Newtons

N_A =

  366.8961
```

Next, use the tangential component of the equation of motion to determine the friction force.

```
» % W_t + F = M * a_t          so...

» F = M .* a_t - W_t                   %Newtons

F =

  -79.7602
```

The friction force, F, and the normal force, NA, can be used together to calculate the apparent coefficient of kinetic friction for this problem.

```
» mu_k = abs(F) ./ N_A                 %need only the magnitude of F here

mu_k =

   0.2174
```

Annotated MATLAB Script Solution

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%               Friction Force on a Skateboarder                   %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%Set problem parameters as stated in the problem.
v = 3.2;                      %m/s
a_t = 7.8;                    %m/s^2
M = 55;                       %kg
g = 9.807;                    %m/s^2
alpha = 19.4 * pi /180;       %radians - angle from vertical
rho = 3;                      %meters - circular path, constant rho

%Solve for the normal acceleration.
a_n = v.^2 ./ rho;            %m/s^2
fprintf('\nNormal Acceleration = %1.1f M/s^2 \n', a_n)

%Find the normal and tangential components of the skateboarder’s weight
W = M .* g;                   %Newtons
W_n = -W .* sin(alpha);       %Newtons - minus sign accounts for direction
W_t =  W .* cos(alpha);%Newtons
fprintf('Skateboarder''s Weight = %1.0f N\n', W)
fprintf('\tNormal component     = %1.0f N\n', W_n)
fprintf('\tTangential component =  %1.0f N\n', W_t)

%EQ of MOTION: normal component
%N_A + W_n = M * a_n so...
N_A = M .* a_n - W_n;         %Newtons
fprintf('Normal Force = %1.2f N\n', N_A)

%EQ of MOTION: tangential component
%W_t + F = M * a_t so...
F = M .* a_t - W_t;           %Newtons
fprintf('Friction Force = %1.1f N\n', F)

                              %Calculate the coefficient of kinetic friction
mu_k = abs(F) ./ N_A;         %need only the magnitude of F here
fprintf('Coefficient of Kinetic Friction = %1.2f\n\n', mu_k)
```