17 Kinetics of a Particle: Force and Acceleration

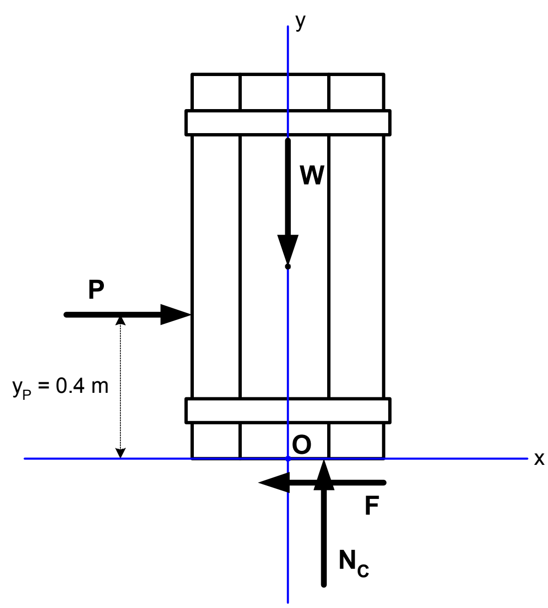
Ref: Hibbeler § 13.4, Bedford & Fowler: Dynamics § 3.1-3.4

In an earlier example (#8) we looked at "pushing a crate" and investigated the effect of changing the height of the push and the magnitude of the push on the rigid body. Specifically, we tested to see if the crate would tip or slip. In this example, we will determine the initial acceleration of the crate and the velocity after a short time at that acceleration.

Example: Pushing a Crate

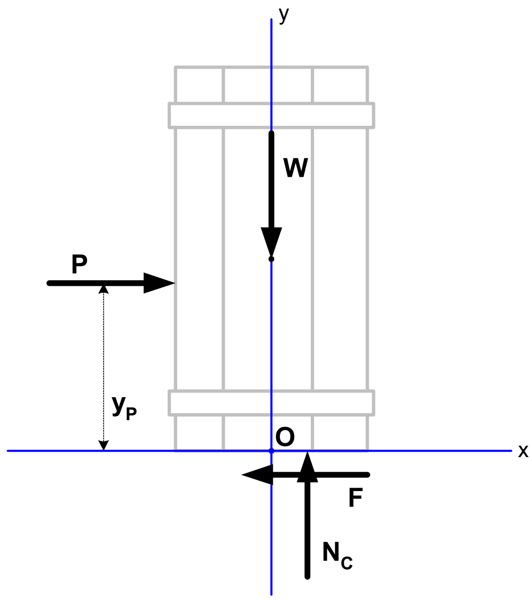
A crate weighs 35 kg, and is 0.5 m wide and 1.0 m in height. A force, P, is applied to the side of the crate at height y_P = 0.4 m from the floor. From the earlier example (#8) we know that this force is sufficient to overcome friction, and when applied at this height will not tip the crate. The coefficient of kinetic friction is 0.24.

Determine the initial acceleration and, assuming the acceleration remains constant, the expected velocity of the crate after 3 seconds.



Solution

First, a free-body diagram is drawn.



We start the solution by assigning values given in the problem statement to variables:

```
» M = 35;                    %kg
» P = 120;                   %Newtons
» y_p = 0.4;                 %meters
» g = 9.807;                 %meters/s^2
» mu_k = 0.24;
```

Then we calculate the weight of the crate.

```
» W = -M .* g                %Newtons
W =
 -343.2450
```

We can use the equation of motion for the y-components of force to determine the resultant normal force, N_C (but the result is not much of a surprise.)

```
» %EQ of MOTION: y components of force.
» % W + N_c = 0          so...
» N_c = -W                   %Newtons - acts in the +x direction
N_c =
  343.2450
```

Knowing N_C, the friction force can be determined.

```
» F= mu_k .* N_c;     %Newtons - this equation does not account for direction
» F = -F              %Newtons - sign changed since F acts in -x direction
F =
  -82.3788
```

Next, we use the equations of motions with the information on the free-body diagram to solve for the acceleration, a_x.

```
» %EQ of MOTION: x components of force.
» % P + F = M * a_x      so...
» a_x = (P + F) ./ M                                %m / s^2
a_x =
   1.0749
```

Now the velocity after 3 seconds can be determined.

```
» %Calculate the velocity after three seconds.
» v_o = 0;                                          %m / s
» delta_t = 3;                                      %s
» a = a_x;                                          %m / s^2
» v = v_o + a .* delta_t                            %m / s
v =
   3.2247
```

Annotated MATLAB Script Solution

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%            Pushing a Crate - Initial Acceleration             %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%Data from the problem statement...
M = 35;                                                   %kg
P = 120;                                                  %Newtons
y_p = 0.4;                                                %meters
g = 9.807;                                                %meters/s^2
mu_k = 0.24;

%Calculate the weight of the crate.
W = -M .* g;                                              %Newtons
fprintf('\nW = %1.1f N\t\t', W)

%EQ of MOTION: y components of force.
%W + N_c = 0 so...
N_c = -W;                                                 %Newtons
fprintf('N_c = %1.1f N\n', N_c)

%Calculate teh friction force, F.
F= mu_k .* N_c;   %Newtons - this equation does not account for direction
F = -F;           %Newtons - Sign changed since F acts in -x direction
fprintf('F = %1.1f N\t\t', F)

%EQ of MOTION: x components of force.
%P + F = M * a_x so...
a_x = (P + F) ./ M;                                       %m/s^2
fprintf('a_x = %1.3f m/s^2\n\n', a_x)
```

```
%Calculate the velocity after three seconds.
v_o = 0;                                                  %m / s
delta_t = 3;                                              %s
a = a_x;                                                  %m/s^2
v = v_o + a .* delta_t;                                   %m/s
fprintf('v = %1.3f m/s\n\n', v)
```