13 Curvilinear Motion: Rectangular Components

Ref: Hibbeler § 12.5, Bedford & Fowler: Dynamics § 2.3

A fixed x, y, z frame of reference is commonly used to describe the path of a particle in two situations:

1. The path "fits" the Cartesian coordinate system well (tends to follow straight lines, or simple paths.)
2. The path does not "fit" any commonly used coordinate system well.

The example used here falls into the latter category.

Example: Finding the Velocity and Acceleration of a Particle

The complex path of a particle for times between 0 and 10 seconds has been fit using multivariable regression to the following functions:

$$x(t) = 2\cos(t)$$

$$y(t) = 1.3 + 2.7\,t^2 - 0.031\,t^3$$

$$z(t) = 2.4 + 0.14\,t^3$$

The position of the particle at any point in time for which the functions are valid ($0 \le t \le 10$ sec.) can be determined as

$$\mathbf{r} = x\,\mathbf{i} + y\,\mathbf{j} + z\,\mathbf{k}$$

Given these functions for x, y, and z determine the position, and the magnitudes of the velocity and acceleration of the particle at t = 7 sec.

Solution

The first thing we might want to do is graph this particle path, just to see what it looks like. MATLAB can help here. First, declare the three functions of time that describe the particle's path:

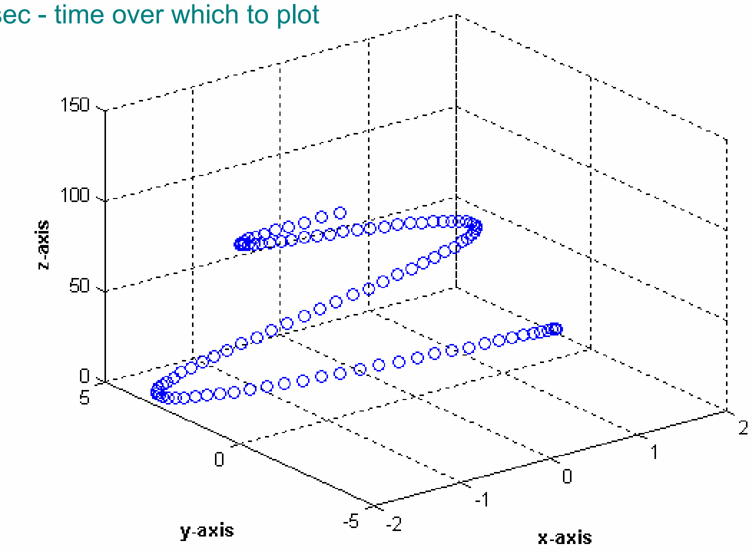
```
» x = inline('2 .* cos(t)', 't')
x =
    Inline function:
    x(t) = 2 .* cos(t)


» y = inline('1.3 + 0.27.*t.^2 - 0.031.*t.^3', 't')
y =
    Inline function:
    y(t) = 1.3 + 0.27.*t.^2 - 0.031.*t.^3


» z = inline('2.4 + 0.14.*t.^3', 't')
z =
    Inline function:
    z(t) = 2.4 + 0.14.*t.^3
```

Then ask MATLAB to plot the particle path by placing the three function names into the scatter3() function.

```
» t = 0 : 0.1: 10;            %sec - time over which to plot
» scatter3(x(t),y(t),z(t),'o')
» xlabel('x-axis')
» Ylabel('y-axis')
» zlabel('z-axis')
```



Solving for Position

Declare r(t) as a three-component matrix composed of the three functions. The position is described by function r(t), so finding the position at t = 7 sec. Simply requires evaluating r(t) at this time.

```
» r = inline('[ [2 .* cos(t)]   [1.3 + 0.27. *t.^2 - 0.031 .*t.^3]   [2.4 + 0.14. * t.^3] ]', 't')
r =
     Inline function:
     r(t) = [[2.*cos(t)] [1.3+0.27.*t.^2-0.031.*t.^3] [2.4+0.14.*t.^3]]
» r(7)
ans =
    1.5078    3.8970   50.4200
```

So, at t = 7 seconds, the particle is located at x = 1.5078, y = 3.8970, and z = 50.4200.

Solving for the Magnitude of the Velocity

For velocity we need to take the derivative of the x, y, and z functions. Here, the derivatives are obtained using MATLAB's symbolic math toolbox and cutting-and-pasting into three new inline functions: $v_x(t)$, $v_y(t)$, and $v_z(t)$.

Note: Capital letters are used to represent symbolic variables while lower case letters are assigned a value or to a function.

```
» syms T                                %Define a symbolic variable T
» X = 2 .* cos(T);                      %Define X
» diff(X)                               %Differentiate on X
ans =
 -2*sin(T)
```

```
» v_x = inline('-2 .* sin(t)','t');                  %Assign v_x(t)

» Y = 1.3 + 0.27.*T.^2 - 0.031.*T.^3;                 %Define Y
» diff(Y)                                             %Differentiate onY
ans =
 27/50*T-93/1000*T^2
» v_y = inline('27/50 - 93/1000 .*t.^2', 't');        %Assign v_y(t)

» Z = 2.4 + 0.14.*T.^3;                               %Define Z
» diff(Z)                                             %Differentiate onZ
ans =
 21/50*T^2
» v_z = inline('21/50 .* t.^2', 't');                 %Assign v_z(t)
```

The magnitude of the velocity at t = 7 seconds can now be determined.

```
» v = sqrt(v_x(7).^2 + v_y(7).^2 + v_z(7).^2)
v =
  21.0095
```

Solving for the Magnitude of the Acceleration

To find the acceleration, we need to differentiate the v_x, v_y, and v_z functions with respect to time. The derivatives obtained using MATLAB's symbolic math toolbox and cutting-and-pasting into three new inline functions: $a_x(t)$, $a_y(t)$, and $a_z(t)$.

```
» diff(-2*sin(T), T)                                  %Differentiate v_x
ans =
 -2*cos(T)
» a_x = inline('-2.*cos(t)','t')                      %Assign a_x(t)
 a_x =
     Inline function:
     a_x(t) = -2.*cos(t)

» diff(27/50 .* T - 93/1000 .* T.^2, T)               %Differentiate v_y
ans =
 27/50-93/500*T
» a_y = inline('0.54 - 0.186.*t','t')                 %Assign a_y(t)
a_y =
     Inline function:
     a_y(t) = 0.54 - 0.186.*t
```

```
» diff(21/50 .* T.^2, T)                                  %Differentiate v_z
ans =
 21/25*T
» a_z = inline('0.84.*t','t')                             %Assign a_z(t)
a_z =
    Inline function:
    a_z(t) = 0.84.*t
```

The magnitude of the acceleration at t = 7 seconds is

```
» a = sqrt( a_x(7).^2 + a_y(7).^2 + a_z(7).^2 )
a =
 6.1179
```

Annotated MATLAB Script Solution

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%          Curvilinear Motion:  Rectangular Components           %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%Declare the time functions for the particle's path.
x = inline('2 .* cos(t)', 't');
y = inline('1.3 + 0.27.*t.^2 - 0.031.*t.^3', 't');
z = inline('2.4 + 0.14.*t.^3', 't');

%Create a plot to see the particle path
t = 0 : 0.1: 10;%sec - time over which to plot
scatter3(x(t),y(t),z(t),'o')
xlabel('{\bf x-axis}')
Ylabel('{\bf y-axis}')
zlabel('{\bf z-axis}')

%Declare the r(t) function.
r = inline('[[2.*cos(t)] [1.3+0.27.*t.^2-0.031.*t.^3]
[2.4+0.14.*t.^3]]','t');

%Evaluate r(t) at t = 7 to find the position.
position = r(7);
fprintf('\nr(7) = [ %1.3f, %1.3f, %1.3f ]\n', r(7))

%Take the time derivatives of x, y, and z functions.
syms T                                       %Define a symbolic variable T
X = 2 .* cos(T);                             %Define X
diff(X)                                      %Differentiate on X
v_x = inline('-2 .* sin(t)','t');            %Assign v_x(t)

Y = 1.3 + 0.27.*T.^2 - 0.031.*T.^3;          %Define Y
diff(Y)                                      %Differentiate on Y
```

```
v_y = inline('27/50 - 93/1000 .*t.^2', 't'); %Assign v_y(t)

Z = 2.4 + 0.14.*T.^3;                         %Define Z
diff(Z)                                       %Differentiate on Z
v_z = inline('21/50 .* t.^2', 't');           %Assign v_z(t)

%Calculate the magnitude of the velocity at 7 seconds.
v = sqrt(v_x(7).^2 + v_y(7).^2 + v_z(7).^2);
fprintf('Magnitude of the velocity at 7 seconds = %1.3f \n', v)

%Take the time derivatives of the velocity functions.
diff(-2*sin(T), T)                            %Differentiate on v_x
a_x = inline('-2.*cos(t)','t');               %Assign a_x(t)

diff(27/50 .* T - 93/1000 .* T.^2, T);        %Differentiate on v_y
a_y = inline('0.54 - 0.186.*t','t');          %Assign a_y(t)

diff(21/50 .* T.^2, T)                        %Differentiate on v_z
a_z = inline('0.84.*t','t');                  %Assign a_z(t)

%Calculate the magnitude of the acceleration at 7 seconds.
a = sqrt(a_x(7).^2 + a_y(7).^2 + a_z(7).^2);
fprintf('Magnitude of the acceleration at 7 seconds = %1.3f \n\n', a)
```