11 Calculating Moments of Inertia

Ref: Hibbeler § 10.1-10.2, Bedford & Fowler: Statics § 8.1-8.2

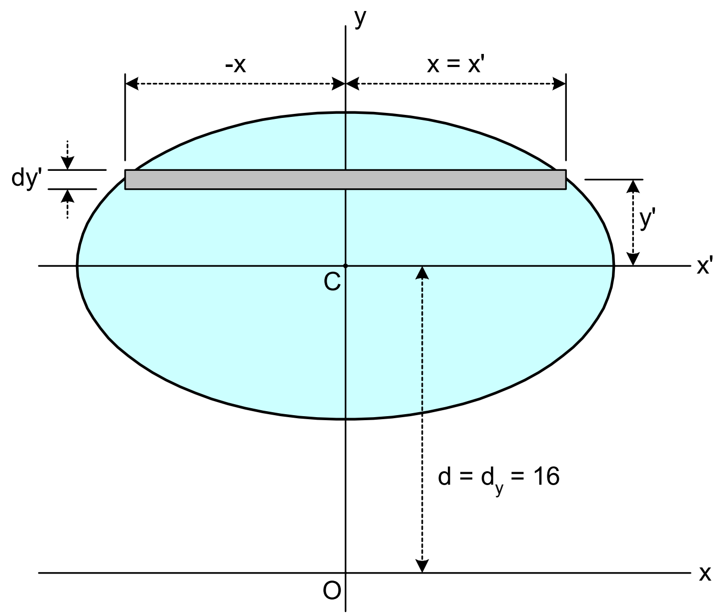
Calculating moments of inertia requires evaluating integrals. This can be accomplished either symbolically, or using numerical approximations. Mathcad's ability to integrate functions to generate numerical results is illustrated here.

Example: Moment of Inertia of an Elliptical Surface

Determine the moment of inertia of the ellipse illustrated below with respect to a) the centroidal x' axis, and b) the x axis.

The equation of the ellipse relative to centroidal axes is

$$\frac{x'^2}{8^2} + \frac{y'^2}{14^2} = 1$$



In this problem, x and y have units of cm.

Solution

The moment of inertia about the centroidal x axis is defined by the equation

$$I_{x'} = \int_A y'^2 \, dA$$

where dA is the area of the differential element indicated in the figure above.

$$dA = 2\, x\, dy'$$

So, the integral for moment of inertia becomes

$$I_{x'} = \int_A y'^2 \; 2\,x\,dy'$$

Furthermore, x (or x') can be related to y' using the equation of the ellipse.

Note: Because of the location of the axes, x = x' in this example.

$$x = x' = \sqrt{8^2\left(1 - \frac{y'^2}{14^2}\right)}$$

The equation for the moment of inertia becomes:

$$I_{x'} = \int_{-8}^{8} y'^2 \; 2\sqrt{8^2\left(1 - \frac{y'^2}{14^2}\right)}\,dy'$$

To perform this integration we need to place the integrand in an m-file function and call MATLAB's quad() function on the m-file.

```
function Ix_integrand = Moment_Of_Inertia_Integrand(y_prime)
%Saved as Moment_Of_Inertia_Integrand.m in the MATLAB
%search path.

x = sqrt(8.^2 .* (1 - y_prime.^2./14^2));
Ix_integrand = y_prime.^2 .* 2 .* x;
```

```
» Ix_prime = quad('Moment_Of_Inertia_Integrand',-8,8)          %cm^4

Ix_prime =

  4890
```

The moment of inertia relative to the original x axis can be found using the *parallel-axis theorem*.

$$I_x = I_{x'} + A\,d_y{}^2$$

Where A is the area of the ellipse, and d_y is the displacement of the centroidal y axis from the original y axis.

The required area can be calculated by integration in the same fashion as before.

```
function A = Area_Integrand(y_prime)
%Saved as Area_Integrand.m in the MATLAB search path.

x = sqrt(8.^2 .* (1 - y_prime.^2 ./ 14^2));
A = 2 .* x;
```

```
» A = quad('Area_Integrand',-8,8) %cm^2

A =

  241.2861
```

Then the moment of inertia about x can be determined.

```
» dy = 16;                                                      %cm
```

» I_x = Ix_prime + A .* dy.^2 %cm^4

I_x =

66660

Annotated MATLAB Script Solution

```
function Ix_integrand = Moment_Of_Inertia_Integrand(y_prime)
%Saved as Moment_Of_Inertia_Integrand.m in the MATLAB search path.

x = sqrt(8.^2 .* (1 - y_prime.^2./14^2));
Ix_integrand = y_prime.^2 .* 2 .* x;
```

```
function A = Area_Integrand(y_prime)
%Saved as Area_Integrand.m in the MATLAB search path.

x = sqrt(8.^2 .* (1 - y_prime.^2 ./ 14^2));
A = 2 .* x;
```

```
%Calculate the moment of inertia relative to the x' axis.
Ix_prime = quad('Moment_Of_Inertia_Integrand',-8,8);%cm^4
fprintf('\nThe moment of inertia relative to the x'' axis is %1.0f cm^4\n',
Ix_prime)

%Calculate the area of the ellipse.
A = quad('Area_Integrand',-8,8);%cm^2
fprintf('The area of the ellipse is %1.2f cm^2\n', A)

%Use the parallel-axis theorm to calculate the moment of inertia relative to
the x axis
dy = 16;%cm
I_x = Ix_prime + A .* dy.^2;%cm^4
fprintf('The moment of inertia relative to the x axis is = %1.0f cm^4\n', I_x)
```