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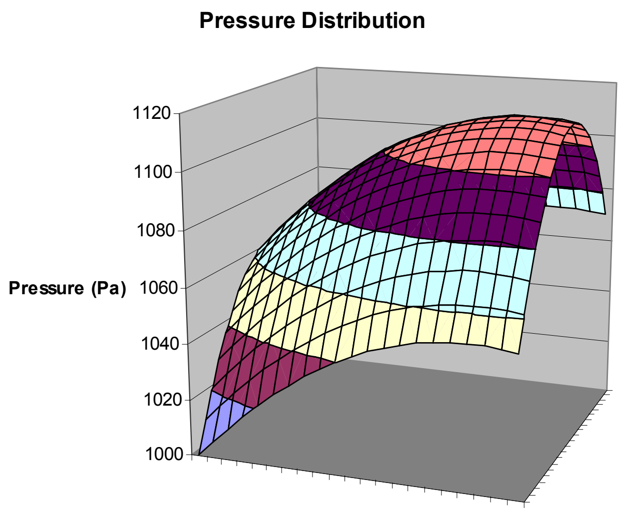
Resultant of a Generalized Distributed Loading

Ref: Hibbeler § 9.5, Bedford & Fowler: Statics § 7.4

When a load is continuously distributed across an area it is possible to determine the equivalent resultant force, and the position at which the resultant force acts. This is termed the *resultant of a generalized distributed loading*. This is the generalized case of the simple distributed loading considered earlier (#6).

Example: Load Distribution Expressed as a Function of x and y

The pressure on a surface is distributed as illustrated in the following surface plot:



This plot is described by the following function:

$$p(x,y) = 1000 + 230\,x - 210\,x^2 + 120\,y - 70\,y^2$$

The x and y values range from 0 to 1 meter. The pressure is expressed in Pa.

Determine the magnitude and location of the resultant force.

Solution

The magnitude of the resultant force is obtained by integration.

$$F_R = \int_A p(x,y)\,dA = \int_y \int_x p(x,y)\,dx\,dy$$

To perform this integration we first need to create an m-file for the pressure function, p(x,y), and save it in the MATLAB search path. We then use the MATLAB dblquad() function to carry out the integration on the new function.

```
function p = PressureFunction(x, y)
%Saved as PressureFunction.m in the MATLAB search path.

%Pressure Function (Pa)
p = 1000 + 230 .*x - 210 .*x.^2 + 120 .* y - 70 .* y.^2;
```

```
» F_R = dblquad('PressureFunction', 0, 1, 0, 1)                    %Newtons

F_R =

  1081.7
```

The location at which the resultant force acts is found by calculating the centroid of the volume defined by the distributed loading diagram. We again need to create a new function to calculate the 1^{st} moment of p(x, y). The centroid is then found by dividing the x and y 1^{st} moments by magnitude of the resultant force.

```
function FM = FirstMomentX(x, y)
%Saved as FirstMomentX.m in the MATLAB search path.

%Pressure Function (Pa)
FM = x .* (1000 + 230 .*x - 210 .*x.^2 + 120 .* y - 70 .* y.^2);
```

```
» x_loc = dblquad('FirstMomentX',0,1,0,1) ./ F_R          %Meters

x_loc =

   0.5015
```

```
function FM = FirstMomentY(x, y)
%Saved as FirstMomentY.m in the MATLAB search path.

%Pressure Function (Pa)
FM = y .* (1000 + 230 .*x - 210 .*x.^2 + 120 .* y - 70 .* y.^2);
```

```
» y_loc = dblquad('FirstMomentY',0,1,0,1) ./ F_R          %Meters

y_loc =

   0.5039
```

Annotated MATLAB Script Solution

```
function p = PressureFunction(x, y)
%Saved as PressureFunction.m in the MATLAB search path.

%Pressure Function (Pa)
p = 1000 + 230 .*x - 210 .*x.^2 + 120 .* y - 70 .* y.^2;
```

```
function FM = FirstMomentX(x, y)
%Saved as FirstMomentX.m in the MATLAB search path.

%Pressure Function (Pa)
FM = x .* (1000 + 230 .*x - 210 .*x.^2 + 120 .* y - 70 .* y.^2);
```

```
function FM = FirstMomentY(x, y)
%Saved as FirstMomentY.m in the MATLAB search path.

%Pressure Function (Pa)
FM = y .* (1000 + 230 .*x - 210 .*x.^2 + 120 .* y - 70 .* y.^2);
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%          Resultant of a Generalized Distributed Loading               %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%The magnitude of the resultant force is calculated.
F_R = dblquad('PressureFunction', 0, 1, 0, 1);%Newtons
fprintf('\nThe magnitude of the resultant force is = %1.0f N\n', F_R)

%The location of the resultant force is calculated.
x_loc = dblquad('FirstMomentX', 0, 1, 0, 1) ./ F_R;%meters
y_loc = dblquad('FirstMomentY', 0, 1, 0, 1) ./ F_R;%meters
fprintf('The location of the resultant force is = (%1.3f, %1.3f)m\n', x_loc, y_loc)
```