7 Equilibrium of a Rigid Body

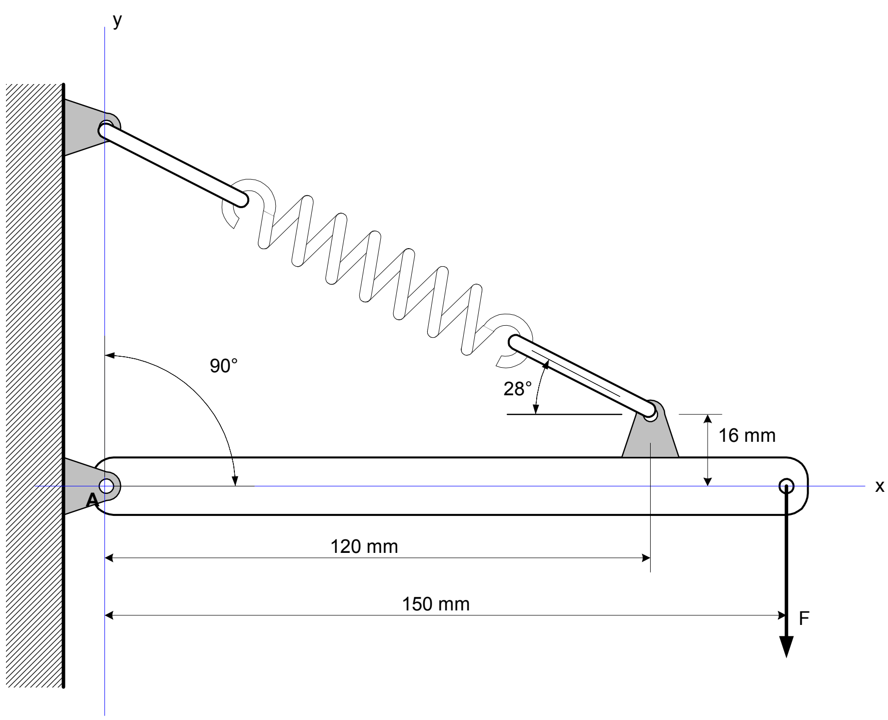
Ref: Hibbeler § 5.1-5.3, Bedford & Fowler: Statics § 5.1-5.2

When forces are applied to *rigid bodies*, the conditions of *equilibrium* can be used to determine any unknown forces, and the *reactions* at the supports.

Example 1: Spring Support

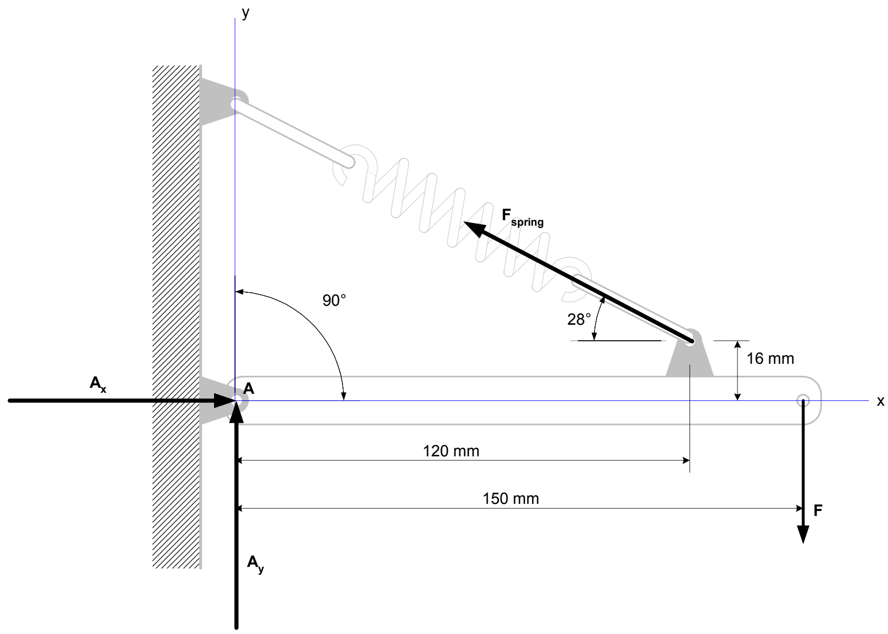
A force, F, is applied to the end of a bar to stretch a spring until the bar is horizontal. If the spring constant is k = 600 N/m and the unstretched length of the spring assembly (spring and connecting links) is 75 mm, and ignoring the mass of the bar, determine:

a) the magnitude of the applied force, F, and
b) the reactions at point A.



Solution

First, a free-body diagram is drawn.



We have assumed that the reactions at A will include both x and y components, and will calculate the actual values of A_x and A_y as part of the solution process.

First, a little trigonometry is required to obtain the extended length of the spring assembly.

```
» deg2rad = pi /180;                        %Conversion factor from degrees to radians
» L_ext = 120 / cos( 28 * deg2rad )         %mm
L_ext =
  135.9084
```

Then, we can calculate the actual extension of the spring…

```
» L_unstretched = 75;                       %mm
» L_stretch = L_ext - L_unstretched         %mm
L_stretch =
   60.9084
```

…and the spring force, F_{spring}.

```
» k_spring = 600;                           %N/m
» F_spring = k_spring * L_stretch / 1000    %Newtons
F_spring =
   36.5450
```

Next, we calculate the x and y components of the spring force, using the angle from the positive x axis (not just 28°) so that the direction of the force is accounted for.

```
» alpha = (180 - 28) * deg2rad;             %angle from +x axis
» Fsp_x = F_spring * cos(alpha)             %Newtons
Fsp_x =
  -32.2674
```

```
» Fsp_y = F_spring * sin(alpha)                  %Newtons

Fsp_y =

  17.1569
```

At equilibrium the sum of the moments at A must be zero. We can use this to solve for the applied force, F.

```
» F = -(abs(Fsp_x) * 16 + Fsp_y * 120) / 150     %Newtons

F =

 -17.1673
```

Note: The absolute value function was used on F_{sp_x} since the direction of rotation was accounted for as the equilibrium equation was written. Here, counter-clockwise rotation was assumed positive.

The equilibrium relationships for the x and y components of force can be used to determine the reactions at A. First, x-component equilibrium requires that the sum of the x components of force be zero. This relationship is used to solve for A_x.

```
» A_x = -Fsp_x                                   %Newtons

A_x =

  32.2674
```

Finally, the equilibrium relationship for the sum of the y components of force is used to calculate A_y.

```
» A_y = -Fsp_y – F                               %Newtons

A_y =

   0.0105
```

Annotated MATLAB Script Solution

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%                     Spring Support Problem                      %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%Create conversion factor from degrees to radians
deg2rad = pi /180;

%Find the total extended lngth of the spring assembly.
L_ext = 120 / cos( 28 * deg2rad );             %mm
fprintf('\nTotal extended length = %1.0f mm\n', L_ext)

%Calculate the spring's extension.
L_unstretched = 75;                            %mm
L_stretch = L_ext - L_unstretched;             %mm
fprintf('Spring''s extension = %1.0f mm\n\n', L_stretch)

%Calculate spring force.
k_spring = 600;                                %N/m
F_spring = k_spring * L_stretch  / 1000;       %Newtons
fprintf('Spring force = %1.1f N\n\n', F_spring)

%Find x and y components of spring force.
alpha = (180 - 28) * deg2rad;                  %angle from +x axis
Fsp_x = F_spring * cos(alpha);                 %Newtons
Fsp_y = F_spring * sin(alpha);                 %Newtons
fprintf('x-component of spring force = %1.1f N', Fsp_x)
fprintf('\t<= acting in -x direction\n')
fprintf('y-component of spring force = %1.1f N\n\n', Fsp_y)

%Use the sum of the moments about A to find applied force F
%We know => 16 |Fsp_x| + 120 Fsp_y+ 150 F = 0
%Solving for F
F = -(abs(Fsp_x) * 16 + Fsp_y * 120) / 150;  %Newtons
fprintf('Applied force = %1.1f N', F)
fprintf('\t<= acting to cause clockwise rotation\n\n')


%Use x and y component force balences to solve for the reactions at A
%We know => A_x + Fsp_x = 0
%Solving for A_x
A_x = -Fsp_x;                                  %Newtons
fprintf('A_x =  %1.1f N', A_x)
fprintf('\t<= acting in +x direction\n')

%We know => A_y + Fsp_y +F  = 0
%Solving for A_y
A_y = -Fsp_y - F;                              %Newtons
fprintf('A_y =  %1.1f N\n', A_y)
```