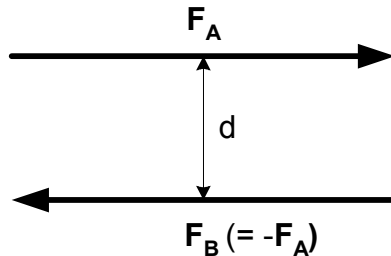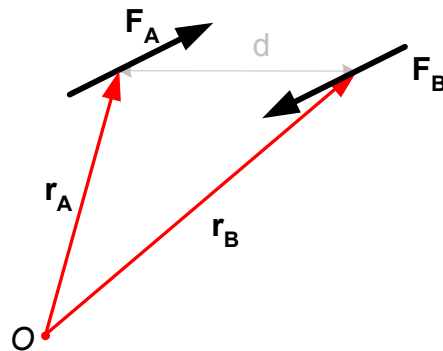# 5 | Moment of a Couple

Ref: Hibbeler § 4.6, Bedford & Fowler: Statics § 4.4

A *couple* is a pair of forces, equal in magnitude, oppositely directed, and displaced by perpendicular distance, d.

Since the forces are equal and oppositely directed, the resultant force is zero. But the displacement of the force couple (d) does create a *couple moment*.

The moment, **M**, about some arbitrary point *O* can be calculated.

$$\mathbf{M} = \mathbf{r}_A \times \mathbf{F}_A + \mathbf{r}_B \times \mathbf{F}_B$$
$$= \mathbf{r}_A \times \mathbf{F}_A + \mathbf{r}_B \times \left(-\mathbf{F}_A \right)$$

If point *O* is placed on the line of action of one of the forces, say **F**$_B$, then that force causes no rotation (or tendency toward rotation) and the calculation of the moment is simplified.
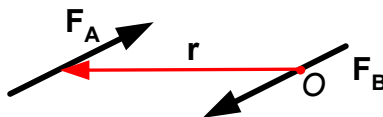

$$\mathbf{M} = \mathbf{r} \times \mathbf{F}_A$$

This is a significant result: The couple moment, **M**, depends only on the position vector **r** between forces **F**$_A$ and **F**$_B$. The couple moment does not have to be determined relative to the location of a point or an axis.

Example A: Moment from a Large Hand Wheel

The stem on a valve has two hand wheels: a small wheel (30 cm diameter) used to spin the valve quickly as it is opened and closed, and a large wheel (80 cm diameter) that may be used to free a stuck valve, or seat the valve tightly when it is fully closed.
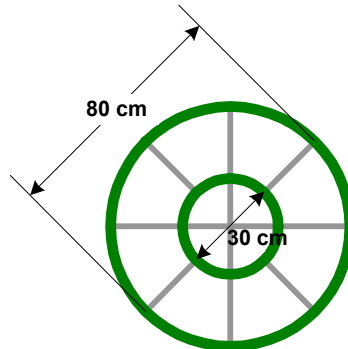
If the operator can impose a force of 150 N on each side of the large wheel (a force couple), what moment is imposed on the valve stem?
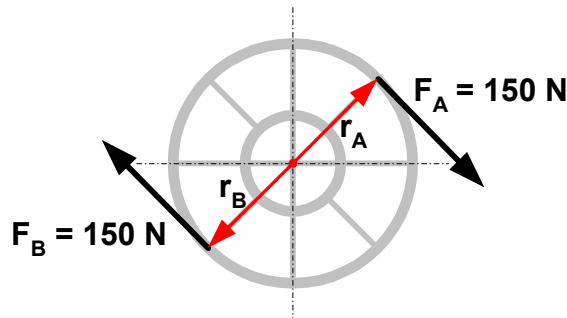
Solution 1

As drawn, both the force and position vectors have x- and y-components. The vectors may be defined as:

» F_A = [106.06 -106.06 0];	%Newtons
» r_A = [28.284 28.284 0];	%centimeters
» F_B = [-106.06 106.06 0]	%Newtons
» r_B = [-28.284 -28.284 0];	%centimeters

The moment of the couple can be calculated using the cross product operator on the Matrix toolbar.
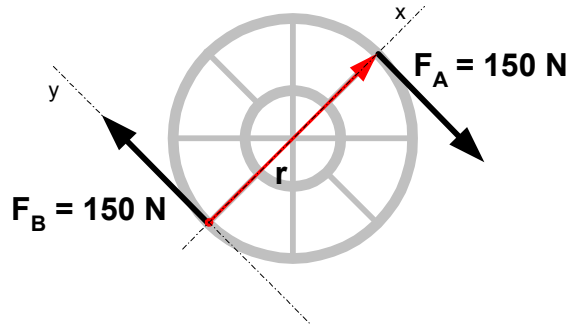
» M = cross(r_A,F_A) + cross(r_B,F_B);	%Newton centimeters
» M = M/100	%Newton meters

M =

      0   0   -120.0000

The result is a couple moment of 120 N·m directed in the –z direction (into the page).

Solution 2

Perhaps a more reasonable positioning of the axes for this problem might look like this:

In this case, the vector definitions become:

> » F_A = [0 -150 0];   %Newtons

> » r   = [80   0 0];   %centimeters

Since the position vector **r** originates from the line of action of force F_B, F_B does not contribute to the moment. The moment is then calculated as

> » M = cross(r,F_A);   %Newton centimeters

> » M = M/100   %Newton meters

> M =

>   0   0   -120

The result is, of course, the same no matter how the axes are situated.

Solution 3: Using Scalars

Finally, the problem can also be solved using a scalar formulation. The perpendicular distance between the forces is 80 cm. With axes established as in Solution 2, the moment can be calculated as

> » F = 150;   %Newtons

> » d = 80;   %centimeters

> » M= d * F;   %Newton centimeters

> » M = M/100   %Newton meters

> M =

>   120

The direction must be determined using the right-hand rule.

Annotated MATLAB Script Solution

```
%Solution 1
%
%Define the vectors
F_A = [ 106.06 -106.06 0];            %Newtons
r_A = [ 28.284  28.284 0];            %centimeters
F_B = [-106.06  106.06 0];            %Newtons
r_B = [-28.284 -28.284 0];            %centimeters
%Compute the moment using MATLAB's cross product function
M = cross(r_A,F_A) + cross(r_B,F_B);   %Newton centimeters
%Convert to Newton meters
M = M/100;                            %Newton meters
fprintf('The resulting couple moment is = [ %1.2f  %1.2f  %1.2f ] (Nm)\n', M)
```

```
%Solution 2
%
%Define the vectors
F_A = [   0 -150 0];                  %Newtons
r   = [ 80    0 0];                   %centimeters
%Compute the moment
M = cross(r,F_A);                     %Newton centimeters
%Convert to Newton meters
M = M/100;                            %Newton meters
fprintf('The resulting couple moment is = [ %1.2f  %1.2f  %1.2f ] (Nm)\n', M)
```

```
%Solution 3
%
%Define the applied force and the perpendicular distance between the forces
F = 150;                              %Newtons
d = 80;                               %centimeters
%Compute the moment
M= d * F;                             %Newton centimeters
%Convert to Newton meters
M = M/100;                            %Newton meters
fprintf('The resulting couple moment is = %1.2f (Nm)\n', M)
fprintf('  The direction must be determined using the right-hand rule.\n\n')
```

Example B: Moment from the Small Hand Wheel

The moment resulting from applying the same forces to the smaller hand wheel can be determined using any of the three solution procedures outlined above. Solution 2 is shown here.
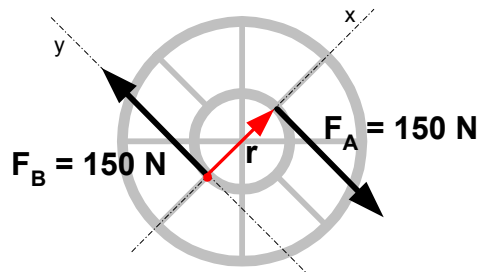
```matlab
%Example B: Moment from the Small Hand Wheel
%  The moment resulting from applying the same forces to the
%  smaller hand wheel can be determined using any of the three
%  solution procedures above.  Solution 2 is shown here.
%
%Define the vectors
F_A = [ 0 -150 0];                      %Newtons
r   = [ 30   0 0];                      %centimeters
%Compute the moments
M = cross(r,F_A);                       %Newton centimeters
%Convert to Newton meters
M = M/100;                              %Newton meters
fprintf('The resulting couple moment for the smaller wheel is = [ %1.2f …
         %1.2f %1.2f ] (Nm)\n', M)
```