3 Equilibrium of a Particle, Free-Body Diagrams

Ref: Hibbeler § 3.3, Bedford & Fowler: Statics § 3.2-3.3

When a body is either not moving (zero velocity), or moving at a constant velocity (speed and direction), the sum of the external forces on the body is zero, and the body is said to be in *equilibrium*.

$$\sum \mathbf{F} = 0$$

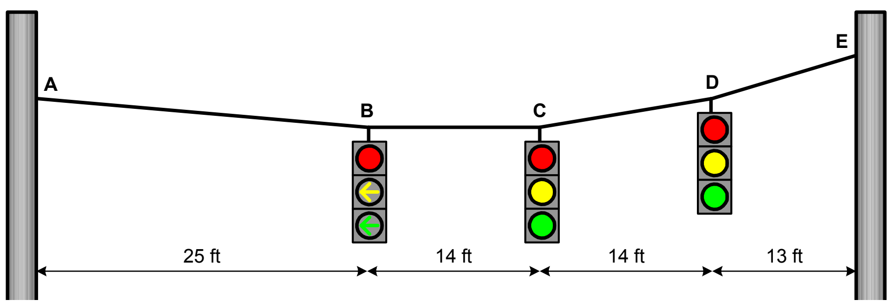
or, for two-dimensional equilibrium,

$$\sum F_x = 0$$
$$\sum F_y = 0$$

The picture showing the external forces acting on the object is called a *free-body diagram*. A free-body diagram is used to help solve problems both in statics and dynamics.

Example: Forces on Traffic Light Suspension Cables

Three traffic lights have been suspended between two poles at an intersection, as shown below.



Each of the three lights has a mass of 18 kg (approx. 40 lb). (Assume zero mass cables.) The tension in the cables has been adjusted such that the lights at points B and C are at the same height, and cable section AB is at an angle of 5° from horizontal.

a. Determine the downward force at points B, C, and D due to the mass of the lights.

b. Draw a free-body diagram at point B, and use it to find the vertical and horizontal components of force in cable AB.

c. Repeat part b. for the other lights, determining the force components in each cable section, and the angle of each cable section (measured from the +x direction).

Solution: Part a.

The mass of each traffic light is being acted on by gravity, so the force is calculated as

$$F_y = m\, g$$

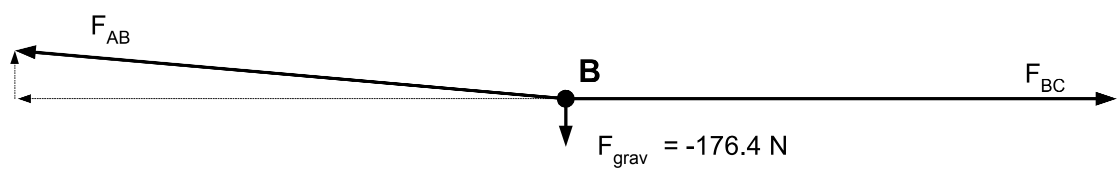
In MATLAB, this calculation looks like this:

```
» m = 18;                    %Mass of light (kg)
» g = 9.8;                   %Gravitational constant (m/s^2)
» F_grav = -m * g
F_grav =
 -176.4000
```

So the downward force exerted by each traffic light is 176.4 N. The minus sign has been included to show that the force is downward, i.e., in the –y direction.

If the lights are not moving up or down, the cable must apply an equal but oppositely directed force on the light, since the vertical force components must sum to zero if the body is in equilibrium.

Part b. – Free-Body Diagram for Light at B



Since cable section BC is horizontal, there is no vertical component of force in cable section BC. So, all of the weight of the traffic light at B must be carried by cable AB, more specifically, by the vertical component of force in cable AB.

```
» F_ABy = -F_grav
F_ABy =
  176.4000
```

The horizontal component of force in cable AB can be determined using the specified angle (cable AB is 5° from horizontal, or 175° from +x).

```
F_ABx = F_ABy / tan( 175 * pi/180)
F_ABx =
 -2016.3000
```

The horizontal component is 2016 N (about 450 lb_f) acting in the –x direction.

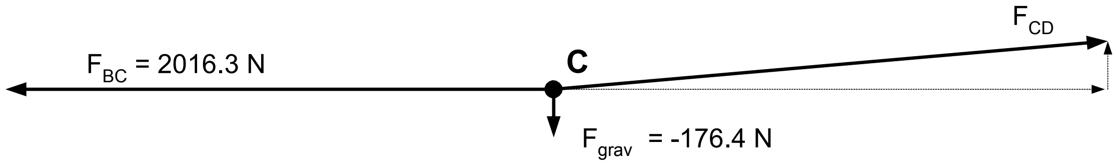
The force acting in the direction of the cable has a magnitude of 2024 N.

```
» F_AB  = sqrt( F_ABx ^ 2 + F_ABy ^ 2 )
F_AB =
  2024.0000
```

Finally, if the light at point B is not moving to the left or right, the sum of the horizontal forces acting on point B must be zero, so the horizontal component of force in section BC is +2016 N. Since there is no vertical component of force in section BC, this is also the total force on section BC at point B.

```
» F_BCx = -F_ABx;       % Since the sum of the x-components of force is zero
» F_BCy = 0;
» F_BC  = sqrt( F_BCx ^ 2 + F_BCy ^ 2 )
F_BC =
  2016.3000
```

Part b. – Free-Body Diagram for Light at C



The free body diagram for the light at point C is constructed using the following concepts:

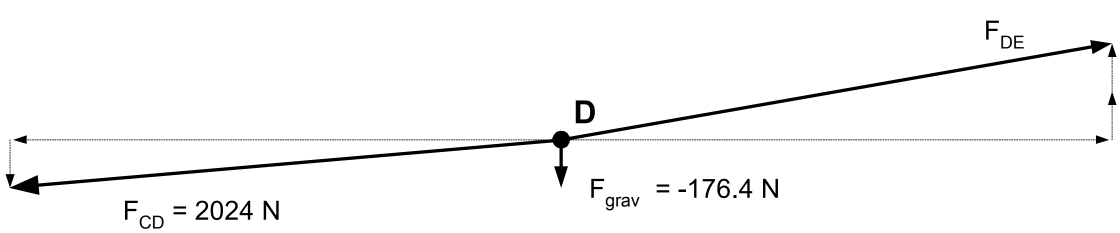
- If the light at point C is not moving left or right, then the horizontal component of force F_{CD} must be +2016 N.
- If the light at point C is not moving up or down, then the vertical component of force F_{CD} must be +176 N.

This is essentially the same as the free-body diagram at point B, just flipped left to right. So the magnitude of force F_{CD} is 2024 N, and acts at 5° from horizontal. This can be verified as follows:

```
» F_CDx = -F_BCx
F_CDx =
  2016.3000
» F_CDy = -F_grav
F_CDy =
  176.4000
» F_CD  = sqrt( F_CDx ^ 2 + F_CDy ^ 2 )
F_CD =
  2024.0000
» theta_CD = atan(F_CDy/F_CDx) * 180/pi
theta_CD =
    5.0000
```

Note: The value of F_{BC} used in this section is –2016.3 N, as shown in the free-body diagram for point C. See the Annotated MATLAB Script Solution to see how this sign change is handled for the complete problem solution.

Part b. – Free-Body Diagram for Light at D



The weight of the traffic light at point D exerts a downward force, F_{grav}, of 176.4 N. In addition, force F_{CD}, acting on point D, has a vertical component of 176.4 N, also acting downward. If the light at point D is not moving up or down, then these downward forces must be counterbalanced by the vertical component of force F_{DE}.

```
» F_DEy = -( F_grav + F_CDy )
F_DEy =
  352.8000
```

The horizontal component of force FDE must be equal to $–F_{CD\_x}$ if the light at point D is to be stationary.

```
F_DEx =
  2016.3000
```

Once the horizontal and vertical components are known, the angle (in degrees) of cable DE can be determined.

```
» theta_DE = atan(F_DEy/F_DEx) * 180/pi
theta_DE =
    9.9250
```

Annotated MATLAB Script Solution

```
%Traffic Light Suspension Cable
%
% Part a.
m = 18;                                   %Mass of light (kg)
g = 9.8;                                  %Gravitational constant (m/s^2)
F_grav = -m * g;                          %Force exerted by traffic light
fprintf('Part a.\n')
fprintf('m      = %8.1f kg\t\t g = %8.1f m/s^2\n',m,g);
fprintf('F_grav = %8.2f N\n\n',F_grav);


% Part b. - Light at Point B
F_ABy = -F_grav;                          %Vertical component of AB
F_ABx = F_ABy / tan( 175 * pi/180 );      %Horizontal component of AB
F_AB  = sqrt( F_ABx ^ 2 + F_ABy ^ 2 ); %Magnitude of AB
fprintf('Part b. - Light at Point B\n')
fprintf('F_ABx = %+8.1f N\t\t F_ABy = %+8.1f N\n',F_ABx,F_ABy)
fprintf('F_AB  = %+8.1f N\n\n',F_AB);

F_BCx = -F_ABx;                           %Horizontal component of BC
F_BCy = 0;                                %Vertical component of BC
F_BC  = sqrt( F_BCx ^ 2 + F_BCy ^ 2 ); %Magnitude of BC
fprintf('F_BCx = %+8.1f N\t\t F_BCy = %+8.1f N\n',F_ABx,F_ABy)
fprintf('F_BC  = %+8.1f N\n\n',F_BC);

% Part b. - Light at Point C
F_BCx = -F_BCx;          %F_BC acting on point C is in the -x direction
F_CDx = -F_BCx;                           %Horizontal component of CD
F_CDy = -F_grav;                          %Vertical component of CD
F_CD  = sqrt( F_CDx ^ 2 + F_CDy ^ 2 ); %Magnitude of CD
theta_CD = atan(F_CDy/F_CDx) * 180/pi; %Angle from horizontal force acts
fprintf('Part b. - Light at Point C\n')
fprintf('F_CDx = %+8.1f N\t\t F_CDy    = %+8.1f N\n',F_CDx,F_CDy)
fprintf('F_CD  = %+8.1f N\t\t theta_CD = %+8.3f deg\n\n',F_CD,theta_CD);

% Part b. - Light at Point D
F_CDx = -F_CDx;          %F_CD acting on point d is in the -x direction
F_CDy = -F_CDy;          %F_CD acting on point C is in the -y direction
F_DEy = -( F_grav + F_CDy );              %Vertical component of DE
F_DEx = -F_CDx;                           %Horizontal component of DE
F_DE  = sqrt( F_DEx ^ 2 + F_DEy ^ 2 ); %Magnitude of DE
theta_DE = atan(F_DEy/F_DEx) * 180/pi; %Angle from horizontal force acts
fprintf('Part b. - Light at Point D\n')
fprintf('F_DEx = %+8.1f N\t\t F_DEy    = %+8.1f N\n',F_DEx,F_DEy)
fprintf('F_DE  = %+8.1f N\t\t theta_DE = %+8.3f deg\n\n',F_DE,theta_DE);
```