

Dot Products

Ref: Hibbeler § 2.9, Bedford & Fowler: Statics § 2.5

Taking the *dot product* of an arbitrary vector with a *unit vector* oriented along a coordinate direction yields the *projection* of the arbitrary vector along the coordinate axis. When this scalar (magnitude) is multiplied by a unit vector in the coordinate direction, the result is the vector component in that coordinate direction. This is one common use of the dot product. The other is finding the angle between two vectors.

The dot product (or scalar product) can be calculated in two ways:

• In trigonometric terms, the equation for a dot product is written as

 $\mathbf{A} \bullet \mathbf{B} = \mathbf{A} \operatorname{B} \cos(\theta)$

Where θ is the angle between arbitrary vectors **A** and **B**.

• In matrix form, the equation is written

 $\mathbf{A} \bullet \mathbf{B} = \mathbf{A}_{\mathbf{x}} \mathbf{B}_{\mathbf{x}} + \mathbf{A}_{\mathbf{y}} \mathbf{B}_{\mathbf{y}} + \mathbf{A}_{\mathbf{z}} \mathbf{B}_{\mathbf{z}}$

MATLAB provides a dot product function, dot() to automatically perform the calculations required by the matrix form of the dot product. If you have two vectors written in matrix form, such as



Then **A**•**B** is the projection of **A** onto **B** (a magnitude, or scalar). Using MATLAB, the dot product is calculated like this (bold letters have not been used for matrix names in MATLAB):

» A = [1 2 3]; » B = [-1 -2 -1]; » dot(A,B) ans = -8 To verify this result, we can do the math term by term...

And we can use the trig. form of the dot product to find the angle between the two vectors. First, calculate the magnitude of the **A** and **B** vectors...

Then find the angle between the vectors using MATLAB's acos() function. The factor 180/pi has been included to convert the angle to degrees.

Finally, you can use a 3-d scatter plot in MATLAB to help you visualize these vectors, to see if your computed results make sense. To create the 3-d scatter plot, you must define the x, y, and z coordinates of the endpoints of each vector.

» $X = [0 \ 0; \ 1 \ -1];$ $Y = [0 \ 0; \ 2 \ -2];$ $Z = [0 \ 0; \ 3 \ -1];$

Here, the top-left elements form the starting coordinates for the **A** vector: (0, 0, 0). The lower-left elements for the end coordinates for the **A** vector: (1, 2, 3). Similarly, the right columns in each matrix represent the starting (0, 0, 0) and ending (-1, -2, -1) coordinates of the **B** vector.

To create the graph, issue the following statements to the MATLAB command window.



Note that the axes do not intercept at (0, 0, 0). Actually, the middle point on the plot is at (0, 0, 0). The '-o' option in the plot3 function is a line specification that determines line type, and marker symbol of the plotted lines. Here, the '-' represents a solid line and the 'o' is used for circular marker symbols.

The usefulness of the 3-d plot for visualization is the ability to rotate the graph to see how the plot looks from different angles. To rotate the plot, use the menu commands: Tools/Rotate 3D, then simply click inside the plot area, hold the left mouse button down, and drag the mouse pointer around the plot. The curve will respond to the location of the pointer.

In the graph below, the plot has been rotated until the plane of the vectors is approximately parallel with the page. In this view, the calculated angle of 150° looks to be about right for these vectors.



Annotated MATLAB Script Solution

```
% Define the vectors
A = [1 \ 2 \ 3];
B = [-1 - 2 - 1];
% Take the dot product
dot(A,B)
% Check MATLAB's dot product operator by calculating the dot product
% explicitly...
x = 1; y = 2;
                z = 3;
                                     % coordinate index definitions
A(x) * B(x) + A(y) * B(y) + A(z) * B(z)
% To calculate the angle between the vectors, first need the magnitude
% of each vector.
A mag = sqrt(A(x)^{2} + A(y)^{2} + A(z)^{2});
B_mag = sqrt(B(x)^2 + B(y)^2 + B(z)^2);
% Then calculate the angle
theta = 180/pi * acos( dot(A,B)/( A_mag * B_mag ) );
% To visualize the vectors, create matrices describing the starting and
% ending coordinates of each vector.
X = [0 0]
      1 -1];
Y = [0 0]
      2 -2];
Z = [0 0]
      3 -1];
% Then create a 3-d Scatter Plot
plot3(X, Y, Z, '-o') % Creates plot
                              % Draws gridlines
grid;
xlabel('\bf{x-axis}');
                             % Labels x-axis
ylabel('\bf{y-axis}');
zlabel('\bf{z-axis}');
                             % Labels y-axis
                             % Labels z-axis
view(29,20);
                             % 3-D graph viewpoint specification
```

Example: Finding the Components of a Force Vector

A force acts on a fixed pin with a magnitude of 270 N (about 60 lb_f) at an angle of 55° from horizontal.

a. Find the horizontal and vertical components of the force using dot products.



Solution

To find the magnitude of the x-component of **F**, calculate the dot product of **F** and a unit vector in the x direction, \mathbf{u}_x .

$$F_x = F \cos(\theta) = \frac{F \bullet u_x}{u_x}$$

Note: Most texts do not show u_x in the denominator of the last term of this series of equalities, since the magnitude of a unit vector is known to be one.

Similarly, the vertical component of **F** is calculated as:

Alternatively, the **F** and \mathbf{u}_{x} vectors can be written in vector form.

```
» F(1) = 270 * cos(55 * pi/180);
» F(2) = 270 * cos( (90-55) * pi/180 );
» F(3) = 0;
» u_x = [ 1   0   0 ];
```

And the dot product can be used to calculate the horizontal component of F.

Similarly, we can define a unit vector in the y-direction, and calculate the vertical component of the force.