Resolving forces refers to the process of finding two or more forces which, when combined, will produce a force with the same magnitude and direction as the original. The most common use of the process is finding the components of the original force in the Cartesian coordinate directions: x, y, and z.

A resultant force is the force (magnitude and direction) obtained when two or more forces are combined (i.e., added as vectors).

Breaking down a force into its Cartesian coordinate components (e.g., \( F_x, F_y \)) and using Cartesian components to determine the force and direction of a resultant force are common tasks when solving statics problems. These will be demonstrated here using a two-dimensional problem involving co-planar forces.

Example: Co-Planar Forces

Two boys are playing by pulling on ropes connected to a hook in a rafter. The bigger one pulls on the rope with a force of 270 N (about 60 lbf) at an angle of 55° from horizontal. The smaller boy pulls with a force of 180 N (about 40 lbf) at an angle of 110° from horizontal.

a. Which boy is exerting the greatest vertical force (downward) on the hook?

b. What is the net force (magnitude and direction) on the hook – that is, calculate the resultant force.

Solution

First, consider the 270 N force acting at 55° from horizontal. The x- and y-components of force are indicated schematically, as
The x- and y-components of the first force (270 N) can be calculated using a little trigonometry involving the included angle, 55°:

\[ \cos(55°) = \frac{F_{x1}}{270 \text{ N}}, \quad \text{or} \quad F_{x1} = (270 \text{ N})\cos(55°) \]

and

\[ \sin(55°) = \frac{F_{y1}}{270 \text{ N}}, \quad \text{or} \quad F_{y1} = (270 \text{ N})\sin(55°). \]

MATLAB can be used to solve for \( F_{x1} \) and \( F_{y1} \) using its built-in \( \sin() \) and \( \cos() \) functions, but these functions assume that the angle will be expressed as radians, not degrees. The factor \( \pi/180 \) is used to convert the angle from degrees to radians. Note that \( \pi \) is a predefined variable in MATLAB.

```matlab
» F_x1 = 270 * cos( 55 * pi/180 )
F_x1 =
154.8656
» F_x1 = 270 * sin( 55 * pi/180 )
F_x1 =
221.1711
```

**Your Turn**

Show that the x- and y-components of the second force (180 N acting at 110° from the x-axis) are 61.5 N (-x direction) and 169 N (-y direction), respectively. Note that trigonometry relationships are based on the included angle of the triangle (20°, as shown at the right), not the coordinate angle (-110° from the x-axis).

**Answer, part a)**

The larger boy exerts the greatest vertical force (221 N) on the hook. The vertical force exerted by the smaller boy is only 169 N.
**Solution, continued**

To determine the combined force on the hook, \( F_R \), first add the two y-components calculated above, to determine the combined y-directed force, \( F_{Ry} \), on the hook:

\[
F_{Ry} = F_{y1} + F_{y2}
\]

\[
F_{Ry} = 390.3157
\]

The y-component of the resultant force is 390 N (directed down, or in the \(-y\) direction). Note that the direction has not been accounted for in this calculation.

Then add the two x-components to determine the combined x-directed force, \( F_{Rx} \), on the hook. Note that the two x-component forces are acting in opposite directions, so the combined x-directed force, \( F_{Rx} \), is smaller than either of the components, and directed in the \(+x\) direction.
\[ \text{F}_{\text{Rx}} = \text{F}_{\text{x1}} + ( -\text{F}_{\text{x2}} ) \]

\[ \text{F}_{\text{Rx}} = 93.3020 \]

The minus sign was included before \( \text{F}_{\text{x2}} \) because it is directed in the \(-x\) direction. The result is an \( x\)-component of the resultant force of 93 N in the \(+x\) direction.

Once the \( x\)- and \( y\)-components of the resultant force have been determined, the magnitude can be calculated using

\[ F_{\text{R}} = \sqrt{F_{\text{Rx}}^2 + F_{\text{Ry}}^2} \]

The MATLAB calculation uses the built-in square-root function \text{sqrt}().

\[ \text{F}_{\text{R}} = \text{sqrt}( \text{F}_{\text{Rx}}^2 + \text{F}_{\text{Ry}}^2 ) \]

\[ \text{F}_{\text{R}} = 401.3124 \]
The angle of the resultant force can be calculated using any of three functions in MATLAB:

<table>
<thead>
<tr>
<th>Function</th>
<th>Argument(s)</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>atan(abs(Fx / Fy))</td>
<td>one argument:</td>
<td>Returns the included angle</td>
</tr>
<tr>
<td></td>
<td>abs(Fx / Fy)</td>
<td></td>
</tr>
<tr>
<td>atan2(Fy, Fx)</td>
<td>two arguments:</td>
<td>Returns the coordinate direction angle</td>
</tr>
<tr>
<td></td>
<td>Fx and Fy</td>
<td>Angle value is always between 0 and π radians (0 and 180°)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>A negative sign on the angle indicates a result in one of the lower</td>
</tr>
<tr>
<td></td>
<td></td>
<td>quadrants of the Cartesian coordinate system</td>
</tr>
<tr>
<td>cart2pol (Fx, Fy)</td>
<td>two arguments:</td>
<td>Returns the positive angle from the positive x-axis to the vector</td>
</tr>
<tr>
<td></td>
<td>Fx and Fy</td>
<td>Angle value always between 0 and 2π radians (0 and 360°)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>An angle value greater than 180° (π radians) indicates a result in</td>
</tr>
<tr>
<td></td>
<td></td>
<td>one of the lower quadrants of the Cartesian coordinate system</td>
</tr>
</tbody>
</table>

The `atan2()` function is used here, and \( F_{Ry} \) is negative because it is acting in the –y direction.

\[
\text{» F}_{\text{Rx}} = 93.302; \\
\text{» F}_{\text{Ry}} = -390.316; \\
\text{» theta } = \frac{180}{\pi} \cdot \text{atan2}( F_{\text{Ry}}, F_{\text{Rx}} )
\]

\[
\theta = -76.5562
\]

**Answer, part b**

The net force (magnitude and direction) on the hook is now known:

\[
F_R = 401 \text{ N (about 90 lbf)} \text{ acting at an angle 76.6° below the x-axis.}
\]
% Determine the x- and y-components of the two forces
% (270 N at -55°, and 180 N at -110°)
%
% Note: These trig. Calculations use the included angles
% (55° & 20°), with minus signs added to both y-component
% equations to indicate the forces act in the -y direction,
% and the F_x2 equation to show that this force acts in
% the -x direction.

% Calculate the x- and y- components of the first force (270 N)
F_x1 = 270 * cos( 55 * pi/180 );
F_y1 = -270 * sin( 55 * pi/180 );
fprintf( 'F_x1 = %8.3f N	 F_y1 = %+9.3f N\n',F_x1,F_y1);

% Calculate the x- and y- components of the first force (180 N)
F_x2 = -180 * sin( 20 * pi/180 );
F_y2 = -180 * cos( 20 * pi/180 );
fprintf( 'F_x2 = %7.3f N	 F_y2 = %9.3f N\n',F_x2,F_y2);

% Sum the y-components of the two forces to determine the
% y-component of the resultant force
F_Ry = F_y1 + F_y2;

% Sum the x-components of the two forces to determine the
% x-component of the resultant force
F_Rx = F_x1 + F_x2;
fprintf( 'F_Rx = %7.3f N	 F_Ry = %9.3f N\n',F_Rx,F_Ry);

% Calculate the magnitude of the resultant force
F_R = sqrt( F_Rx^2 + F_Ry^2 );
fprintf( 'F_R = %8.3f N\n',F_R);

% Calculate the angle of the resultant force
% (in degrees from the x-axis)
theta = atan2( F_Ry, F_Rx ) * 180/pi;
fprintf( 'theta = %7.3f N\n',theta);