Chapter 17
Synchronous Motors

17.0 Introduction

The synchronous generators described in the previous chapter can operate either as generators or as motors. When operating as motors (by connecting them to a 3-phase source), they are called synchronous motors. As the name implies, synchronous motors run in synchronism with the revolving field. The speed of rotation is therefore tied to the frequency of the source. Because the frequency is fixed, the motor speed stays constant, irrespective of the load or voltage of the 3-phase line. However, synchronous motors are used not so much because they run at constant speed but because they possess other unique electrical properties. We will study these features in this chapter.

Most synchronous motors are rated between 150 kW (200 hp) and 15 MW (20,000 hp) and turn at speeds ranging from 150 to 1800 r/min. Consequently, these machines are mainly used in heavy industry.

Figure 17.1
Three-phase, unity power factor synchronous motor rated 3000 hp (2200 kW), 327 r/min, 4000 V, 60 Hz driving a compressor used in a pumping station on the Trans-Canada pipeline. Brushless excitation is provided by a 21 kW, 250 V alternator/rectifier, which is mounted on the shaft between the bearing pedestal and the main rotor. (Courtesy of General Electric)
(Fig. 17.1). At the other end of the power spectrum, we find tiny single-phase synchronous motors used in control devices and electric clocks. They are discussed in Chapter 18.

17.1 Construction

Synchronous motors are identical in construction to salient-pole ac generators. The \textit{stator} is composed of a slotted magnetic core, which carries a 3-phase lap winding. Consequently, the winding is also identical to that of a 3-phase induction motor.

The \textit{rotor} has a set of salient poles that are excited by a dc current (Fig. 17.2). The exciting coils are connected in series to two slip-rings, and the dc current is fed into the winding from an external exciter. Slots are also punched out along the circumference of the salient poles. They carry a squirrel-cage winding similar to that in a 3-phase induction motor. This \textit{damper winding} serves to start the motor.

Modern synchronous motors often employ brushless excitation, similar to that used in synchronous generators. Referring to Fig. 17.3, a relatively small 3-phase generator, called \textit{exciter}, and a 3-phase rectifier are mounted at one end of the motor shaft. The dc current \( I_e \) from the rectifier is fed directly into the salient-pole windings, without going through brushes and slip-rings. The current can be varied by controlling the small exciting current \( I_e \) that flows in the stationary field winding of the exciter. Fig. 17.4

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig17.2}
\caption{Rotor of a 50 Hz to 16 2/3 Hz frequency converter used to power an electric railway. The 4-pole rotor at the left is associated with a single-phase alternator rated 7000 kVA, 16 2/3 Hz, PF 85%. The rotor on the right is for a 6600 kVA, 50 Hz, 90% PF synchronous motor which drives the single-phase alternator. Both rotors are equipped with squirrel-cage windings. (Courtesy of ABB)}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig17.3}
\caption{Diagram showing the main components of a brushless exciter for a synchronous motor. It is similar to that of a synchronous generator.}
\end{figure}
shows how the exciter, rectifier, and salient poles are mounted in a 3000 kW synchronous motor.

The rotor and stator always have the same number of poles. As in the case of an induction motor, the number of poles determines the synchronous speed of the motor:

\[ n_s = \frac{120 \cdot f}{p} \quad (17.1) \]

where

- \( n_s \) = motor speed [r/min]
- \( f \) = frequency of the source [Hz]
- \( p \) = number of poles

Example 17-1

Calculate the number of salient poles on the rotor of the synchronous motor shown in Fig. 17.4a.

Solution

The motor operates at 60 Hz and runs at 200 r/min; consequently,

\[ n_s = \frac{120 \cdot f}{p} \]

\[ 200 = \frac{(120 \times 60)}{p} \]

\[ p = 36 \text{ poles} \]

The rotor possesses 18 north poles and 18 south poles.

Figure 17.4a

Synchronous motor rated 4000 hp (3000 kW), 200 r/min, 6.9 kV, 60 Hz, 80% power factor designed to drive an ore crusher. The brushless exciter (alternator/rectifier) is mounted on the overhung shaft and is rated 50 kW, 250 V. (Courtesy of General Electric)

Figure 17.4b

Close-up of the 50 kW exciter, showing the armature winding and 5 of the 6 diodes used to rectify the ac current. (Courtesy of General Electric)
17.2 Starting a synchronous motor

A synchronous motor cannot start by itself; consequently, the rotor is usually equipped with a squirrel-cage winding so that it can start up as an induction motor. When the stator is connected to the 3-phase line, the motor accelerates until it reaches a speed slightly below synchronous speed. The dc excitation is suppressed during this starting period.

While the rotor accelerates, the rotating flux created by the stator sweeps across the slower moving salient poles. Because the coils on the rotor possess a relatively large number of turns, a high voltage is induced in the rotor winding when it turns at low speeds. This voltage appears between the slip-rings and it decreases as the rotor accelerates, eventually becoming negligible when the rotor approaches synchronous speed. To limit the voltage, and to improve the starting torque, we either short-circuit the slip-rings or connect them to an auxiliary resistor during the starting period.

If the power capacity of the supply line is limited, we sometimes have to apply reduced voltage to the stator. As in the case of induction motors, we use either autotransformers or series reactors to limit the starting current (see Chapter 20). Very large synchronous motors (20 MW and more) are sometimes brought up to speed by an auxiliary motor, called a pony motor. Finally, in some big installations the motor may be brought up to speed by a variable-frequency electronic source.

17.3 Pull-in torque

As soon as the motor is running at close to synchronous speed, the rotor is excited with dc current. This produces alternate N and S poles around the circumference of the rotor (Fig. 17.5). If the poles at this moment happen to be facing poles of opposite polarity on the stator, a strong magnetic attraction is set up between them. The mutual attraction locks the rotor and stator poles together, and the rotor is literally yanked into step with the revolving field. The torque developed at this moment is appropriately called the pull-in torque.

![Figure 17.5](image)
The poles of the rotor are attracted to the opposite poles on the stator. At no-load the axes of the poles coincide.

The pull-in torque of a synchronous motor is powerful, but the dc current must be applied at the right moment. For example, if it should happen that the emerging N, S poles of the rotor are opposite the N, S poles of the stator, the resulting magnetic repulsion produces a violent mechanical shock. The motor will immediately slow down and the circuit breakers will trip. In practice, starters for synchronous motors are designed to detect the precise moment when excitation should be applied. The motor then pulls automatically and smoothly into step with the revolving field.

Once the motor turns at synchronous speed, no voltage is induced in the squirrel-cage winding and so it carries no current. Consequently, the behavior of a synchronous motor is entirely different from that of an induction motor. Basically, a synchronous motor rotates because of the magnetic attraction between the poles of the rotor and the opposite poles of the stator.

To reverse the direction of rotation, we simply interchange any two lines connected to the stator.

17.4 Motor under load—
general description

When a synchronous motor runs at no-load, the rotor poles are directly opposite the stator poles and their axes coincide (Fig. 17.5). However, if we apply a mechanical load, the rotor poles fall slightly
behind the stator poles, but the rotor continues to turn at synchronous speed. The mechanical angle $\alpha$ between the poles increases progressively as we increase the load (Fig. 17.6). Nevertheless, the magnetic attraction keeps the rotor locked to the revolving field, and the motor develops an ever more powerful torque as the angle increases.

But there is a limit. If the mechanical load exceeds the pull-out torque of the motor, the rotor poles suddenly pull away from the stator poles and the motor comes to a halt. A motor that pulls out of step creates a major disturbance on the line, and the circuit breakers immediately trip. This protects the motor because both the squirrel-cage and stator windings overheat rapidly when the machine ceases to run at synchronous speed.

The pull-out torque depends upon the magneto-motive force developed by the rotor and the stator poles. The mmf of the rotor poles depends upon the dc excitation $I_x$, while that of the stator depends upon the ac current flowing in the windings. The pull-out torque is usually 1.5 to 2.5 times the nominal full-load torque.

The mechanical angle $\alpha$ between the rotor and stator poles has a direct bearing on the stator current. As the angle increases, the current increases. This is to be expected because a larger angle corresponds to a bigger mechanical load, and the increased power can only come from the 3-phase ac source.

### 17.5 Motor under load—simple calculations

We can get a better understanding of the operation of a synchronous motor by referring to the equivalent circuit shown in Fig. 17.7a. It represents one phase of a wye-connected motor. It is identical to the equivalent circuit of an ac generator, because both machines are built the same way. Thus, the flux $\Phi$ created by the rotor induces a voltage $E_o$ in the stator. This flux depends on the dc exciting current $I_x$. Consequently, $E_o$ varies with the excitation.

As already mentioned, the rotor and stator poles are lined up at no-load. Under these conditions, induced voltage $E_o$ is in phase with the fine-to-neutral voltage $E$ (Fig. 17.7b). It, in addition, we adjust the excitation so that $E_o = E$, the motor “floats” on the line and the line current $I$ is practically zero. In effect, the only current needed is to supply the small windage and friction losses in the motor, and so it is negligible.

What happens if we apply a mechanical load to the shaft? The motor will begin to slow down, causing the rotor poles to fall behind the stator poles by an angle $\alpha$. Due to this mechanical shift, $E_o$ reaches its maximum value a little later than before. Thus, referring to Fig. 17.7c, $E_o$ is now $\delta$ electrical degrees behind $E$. The mechanical displacement $\alpha$ produces an electrical phase shift $\delta$ between $E_o$ and $E$.

The phase shift produces a difference of potential $E_x$ across the synchronous reactance $X_s$ given by

$$E_x = E - E_o$$

Consequently, a current $I$ must flow in the circuit, given by

$$jX_s = E_x$$

from which

$$I = -jE_x/X_s = -j(E - E_o)/X_s$$
Example 17-2a
A 500 hp, 720 r/min synchronous motor connected to a 3980 V, 3-phase line generates an excitation voltage $E_o$ of 1790 V (line-to-neutral) when the dc exciting current is 25 A. The synchronous reactance is 22 $\Omega$ and the torque angle between $E_o$ and $E$ is 30°.

Calculate
a. The value of $E_x$
b. The ac line current
c. The power factor of the motor
d. The approximate horsepower developed by the motor
e. The approximate torque developed at the shaft

Solution
This problem can best be solved by using vector notation.

a. The voltage $E$ (line-to-neutral) applied to the motor has a value

$$E = E_l / \sqrt{3} = 3980 / \sqrt{3}$$
$$= 2300 \text{ V}$$

Let us select $E$ as the reference phasor, whose angle with respect to the horizontal axis is assumed to be zero. Thus,

$$E = 2300 \angle 0^\circ$$

It follows that $E_o$ is given by the phasor

$$E_o = 1790 \angle -30^\circ$$

The equivalent circuit per phase is given in Fig. 17.8a.

Moving clockwise around the circuit and applying Kirchhoff’s voltage law we can write

$$-E + E_x + E_o = 0$$

$$E_x = E - E_o$$

$$= 2300 \angle 0^\circ - 1790 \angle -30^\circ$$

$$= 2300 (\cos 0^\circ + j \sin 0^\circ) - 1790 (\cos -30^\circ + j \sin -30^\circ)$$

$$= 2300 - 1550 + j 895$$

$$= 750 + j 895$$

$$= 1168 \angle 50^\circ$$
Thus, phasor $E_x$ has a value of 1168 V and it leads phasor $E$ by $50^\circ$.

b. The line current $I$ is given by

$$I = \frac{j22 \times E_x}{22 \angle 90^\circ} = 53 \angle -40^\circ$$

Thus, phasor $I$ has a value of 53 A and it lags $40^\circ$ behind phasor $E$.

c. The power factor of the motor is given by the cosine of the angle between the line-to-neutral voltage $E$ across the motor terminals and the current $I$. Hence,

$$\text{power factor} = \cos \theta = \cos 40^\circ = 0.766, \text{ or } 76.6\%$$

The power factor is lagging because the current lags behind the voltage.

The complete phasor diagram is shown in Fig. 17.8b.

d. Total active power input to the stator:

$$P_i = 3 \times E_{LN}I_L \cos \theta = 3 \times 2300 \times 53 \times \cos 40^\circ = 280 \, 142 \, W = 280.1 \, kW$$

Neglecting the $I^2R$ losses and iron losses in the stator, the electrical power transmitted across the airgap to the rotor is 280.1 kW.

Approximate horsepower developed:

$$P = 280.1 \times 10^3/746 = 375 \, \text{hp}$$

Figure 17.8b

See Example 17-2.

e. Approximate torque:

$$T = \frac{9.55 \times P}{n} = \frac{9.55 \times 280.1 \times 10^3}{720} = 3715 \, \text{N} \cdot \text{m}$$

Example 17-2b

The motor in Example 17-2a has a stator resistance of 0.64 $\Omega$ per phase and possesses the following losses:

$R^2$ losses in the rotor: 3.2 kW
Stator core loss: 3.3 kW
Windage and friction loss: 1.5 kW

Calculate

a. The actual horsepower developed
b. The actual torque developed at the shaft
c. The efficiency of the motor

Solution

a. Power input to the stator is 280.1 kW

Stator $I^2R$ losses = $3 \times 53^2 \times 0.64 \, \Omega = 5.4 \, kW$

Total stator losses = 5.4 + 3.3 = 8.7 kW

Power transmitted to the rotor = 280.1 – 8.7 = 271.4 kW

The power at the shaft is the power to the rotor minus the windage and friction losses. The rotor
$\dot{F}R$ losses are supplied by an external dc source and so they do not affect the mechanical power.

Power available at the shaft:

$$P_o = 271.4 - 1.5 = 269.9 \text{ kW}$$
$$= \frac{269.9 \times 10^3}{746} = 361.8 \text{ hp}$$

This power is very close to the approximate value calculated in Example 17.2a.

b. The corresponding torque is:

$$T = \frac{9.55 \times P}{n} = \frac{9.55 \times 269.9 \times 10^3}{720}$$
$$= 3580 \text{ N} \cdot \text{m}$$

c. Total losses = 5.4 + 3.3 + 3.2 + 1.5 = 13.4 kW

Total power input = 280.1 + 3.2 = 283.3 kW

Total power output = 269.9 kW

Efficiency = $\frac{269.9}{283.3} = 0.9527 = 95.3\%$

Note that the stator resistance of 0.64 $\Omega$ is very small compared to the reactance of 22 $\Omega$. Consequently, the true phasor diagram is very close to the phasor diagram of Fig. 17.8b.

17.6 Power and torque

When a synchronous motor operates under load, it draws active power from the line. The power is given by the same equation we previously used for the synchronous generator in Chapter 16:

$$P = (E_oE/X_s) \sin \delta \quad (16.5)$$

As in the case of a generator, the active power absorbed by the motor depends upon the supply voltage $E$, the excitation voltage $E_o$, and the phase angle $\delta$ between them. If we neglect the relatively small $\dot{F}R$ and iron losses in the stator, all the power is transmitted across the air gap to the rotor. This is analogous to the power $P_r$ transmitted across the air gap of an induction motor (Section 13.13). However, in a synchronous motor, the rotor $\dot{F}R$ losses are entirely supplied by the dc source. Consequently, all the power transmitted across the air gap is available in the form of mechanical power. The mechanical power developed by a synchronous motor is therefore expressed by the equation

$$P = \frac{E_oE}{X_s} \sin \delta \quad (17.2)$$

where

- $P$ = mechanical power of the motor, per phase [W]
- $E_o$ = line-to-neutral voltage induced by $I_x$ [V]
- $E$ = line-to-neutral voltage of the source [V]
- $X_s$ = synchronous reactance per phase [$\Omega$]
- $\delta$ = torque angle between $E_o$ and $E$ [electrical degrees]

This equation shows that the mechanical power increases with the torque angle, and its maximum value is reached when $\delta = 90^\circ$. The poles of the rotor are then midway between the N and S poles of the stator. The peak power $P_{\text{max}}$ (per-phase) is given by

$$P_{\text{max}} = \frac{E_oE}{X_s} \quad (17.3)$$

As far as torque is concerned, it is directly proportional to the mechanical power because the rotor speed is fixed. The torque is derived from Eq. 3.5:

$$T = \frac{9.55 \times P}{n_s} \quad (17.4)$$

where

- $T$ = torque, per phase [N m]
- $P$ = mechanical power, per phase [W]
- $n_s$ = synchronous speed [r/min]
- 9.55 = a constant [exact value = 60/2$\pi$]

The maximum torque the motor can develop is called the pull-out torque, mentioned previously. It occurs when $\delta = 90^\circ$ (Fig. 17.9).* 

* The remarks in this section apply to motors having smooth rotors. Most synchronous motors have salient poles; in this case the pull-out torque occurs at an angle of about 70°.
Example 17-3

A 150 kW, 1200 r/min, 460 V, 3-phase synchronous motor has a synchronous reactance of 0.8 Ω, per phase. If the excitation voltage $E_n$ is fixed at 300 V, per phase, determine the following:

a. The power versus $\delta$ curve
b. The torque versus $\delta$ curve
c. The pull out torque of the motor

Solution

a. The line-to-neutral voltage is

$$E = E_n \sqrt{3} = 460 \sqrt{3}$$
$$= 266 \text{ V}$$

The mechanical power per phase is

$$P = (E_p/E_n) \sin \delta$$
$$= (266 \times 300/0.8) \sin \delta$$
$$= 99750 \sin \delta \text{ [W]}$$
$$= 100 \sin \delta \text{ [kW]}$$

By selecting different values for $\delta$, we can calculate the corresponding values of $P$ and $T$, per phase.

<table>
<thead>
<tr>
<th>$\delta$ [°]</th>
<th>$P$ [kW]</th>
<th>$T$ [N·m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>30</td>
<td>50</td>
<td>400</td>
</tr>
<tr>
<td>60</td>
<td>86.6</td>
<td>693</td>
</tr>
</tbody>
</table>

These values are plotted in Fig. 17.9.

b. The torque curve can be found by applying Eq. 17.4:

$$T = 9.55 P/n_s$$
$$= 9.55 P/1200$$
$$= P/125$$

c. The pull-out torque $T_{\text{max}}$ coincides with the maximum power output:

$$T_{\text{max}} = 800 \text{ N·m}$$

The actual pull-out torque is 3 times as great (2400 N·m) because this is a 3-phase machine. Similarly, the power and torque values given in Fig. 17.9 must also be multiplied by 3. Consequently, this 150 kW motor can develop a maximum output of 300 kW, or about 400 hp.

17.7 Mechanical and electrical angles

As in the case of synchronous generators, there is a precise relationship between the mechanical angle $\alpha$, the torque angle $\delta$ and the number of poles $p$. It is given by

$$\delta = p\alpha/2$$

Example 17-4

A 3-phase, 6000 kW, 4 kV, 180 r/min, 60 Hz motor has a synchronous reactance of 1.2 Ω. At full-load the rotor poles are displaced by a mechanical angle of 1° from their no-load position. If the line-to-neutral excitation $E_n = 2.4$ kV, calculate the mechanical power developed.

Solution

The number of poles is

$$p = 120 \times 60/180 = 40$$
The electrical torque angle is
\[ \delta = \frac{pa}{2} = (40 \times 1)/2 = 20^\circ \]
Assuming a wye connection, the voltage \( E \) applied to the motor is
\[
E = E_r / \sqrt{3} = 4 \text{kV} / \sqrt{3} \\
= 2.3 \text{kV} \\
= 2309 \text{ V}
\]
and the excitation voltage is
\[ E_o = 2400 \text{ V} \]
The mechanical power developed per phase is
\[
P = (E_o E_i) \sin \delta 
= (2400 \times 2309/1.2) \sin 20^\circ 
= 1573 \text{ kW}
\]
Total power = \( 3 \times 1573 \)
\[ = 4719 \text{ kW} \approx 6300 \text{ hp} \]

17.8 Reluctance torque

If we gradually reduce the excitation of a synchronous motor when it is running at no-load, we find that the motor continues to run at synchronous speed even when the exciting current is zero. The reason is that the flux produced by the stator prefers to cross the short gap between the salient poles and the stator rather than the much longer air gap between the poles. In other words, because the reluctance of the magnetic circuit is less in the axis of the salient poles, the flux is concentrated as shown in Fig. 17.10a. On account of this phenomenon, the motor develops a reluctance torque.

If a mechanical load is applied to the shaft, the rotor poles will fall behind the stator poles, and the stator flux will have the shape shown in Fig. 17.10b. Thus, a considerable reluctance torque can be developed without any dc excitation at all.

The reluctance torque becomes zero when the rotor poles are midway between the stator poles. The reason is that the N and S poles on the stator attract the salient poles in opposite directions (Fig. 17.10c). Consequently, the reluctance torque is zero precisely at that angle where the regular torque \( T \) attains its maximum value, namely at \( \delta = 90^\circ \).

Fig. 17.11 shows the reluctance torque as a function of the angle \( \delta \). The torque reaches a maximum positive value at \( \delta = 45^\circ \). For larger angles it attains a maximum negative value at \( \delta = 135^\circ \). Obviously, to run as a reluctance-torque motor, the angle must lie between zero and 45°. Although a positive torque is still developed between 45° and 90°, this is an unstable region of operation. The reason is that as the angle increases the power decreases.
Figure 17.10c
The reluctance torque is zero when the salient poles are midway between the stator poles.

Figure 17.11
Reluctance torque versus the torque angle.

Figure 17.12
In a synchronous motor, the reluctance torque (1) plus the smooth-rotor torque (2) produce the resultant torque (3). Torque (2) is due to the dc excitation of the rotor.

As in the case of a conventional synchronous motor, the mechanical power curve has exactly the same shape as the torque curve. Thus, in the absence of dc excitation, the mechanical power reaches a peak at $\delta = 45^\circ$.

Does the saliency of the poles modify the power and torque curves shown in Fig. 17.9? The answer is yes. In effect, the curves shown in Fig. 17.9 are those of a smooth-rotor synchronous motor. The torque of a salient-pole motor is equal to the sum of the smooth-rotor component and the reluctance-torque component of Fig. 17.11. Thus, the true torque curve of a synchronous motor has the shape (3) given in Fig. 17.12.

The peak reluctance torque is about 25 percent of the peak smooth-rotor torque. As a result, the peak torque of a salient-pole motor is about 8 percent greater than that of a smooth-rotor motor, as can be seen in Fig. 17.12. However, the difference is not very great, and for this reason we shall continue to use Eqs. 17.2 and 17.5 to describe synchronous motor behavior.

17.9 Losses and efficiency of a synchronous motor

In order to give the reader a sense of the order of magnitude of the pull-out torque, resistance, reactance, and losses of a synchronous motor, we have drawn up Table 17A. It shows the characteristics of a 2000 hp and a 200 hp synchronous motor, respectively labeled Motor A and Motor B.

The following points should be noted:

1. The torque angle at full-load ranges between $27^\circ$ and $37^\circ$. It corresponds to the electrical angle $\delta$ mentioned previously.
2. The power needed to excite the 2000 hp motor (4.2 kW) is only about twice that needed for the 200 hp motor (2.1 kW). In general, the larger the synchronous motor the smaller is the per-unit power needed to excite it.
3. The total losses of Motor A (38 kW) are only four times those of Motor B (9.5 kW) despite the
TABLE 17A  CHARACTERISTICS OF TWO SYNCHRONOUS MOTORS

<table>
<thead>
<tr>
<th>NAMEPLATE RATING</th>
<th>MOTOR A</th>
<th>MOTOR B</th>
</tr>
</thead>
<tbody>
<tr>
<td>power [hp]</td>
<td>2000 hp</td>
<td>200 hp</td>
</tr>
<tr>
<td>power [kW]</td>
<td>1492 kW</td>
<td>149 kW</td>
</tr>
<tr>
<td>line voltage</td>
<td>4000 V</td>
<td>440 V</td>
</tr>
<tr>
<td>line current</td>
<td>220 A</td>
<td>208 A</td>
</tr>
<tr>
<td>speed</td>
<td>1800 r/min</td>
<td>900 r/min</td>
</tr>
<tr>
<td>frequency</td>
<td>60 Hz</td>
<td>60 Hz</td>
</tr>
<tr>
<td>phases</td>
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</table>

LOAD CHARACTERISTICS

<table>
<thead>
<tr>
<th>factor</th>
<th>pull-out torque (pu)</th>
<th>torque angle at full-load connection</th>
<th>dc exciter power</th>
<th>dc exciter voltage</th>
<th>air gap</th>
<th>losses</th>
</tr>
</thead>
<tbody>
<tr>
<td>power</td>
<td>1.0</td>
<td>1.4</td>
<td>36.7° wye</td>
<td>4.2 kW</td>
<td>125 V</td>
<td>windage and friction</td>
</tr>
<tr>
<td>pull-out</td>
<td>2.2</td>
<td>wye</td>
<td>2.1 kW wye</td>
<td>2.1 kW</td>
<td>6 mm</td>
<td>stator core loss</td>
</tr>
<tr>
<td>torque</td>
<td>27°</td>
<td>connection</td>
<td></td>
<td></td>
<td></td>
<td>stray losses</td>
</tr>
<tr>
<td>angle</td>
<td>36.7°</td>
<td>connection</td>
<td></td>
<td></td>
<td></td>
<td>stator IR</td>
</tr>
<tr>
<td>connection</td>
<td>27°</td>
<td>connection</td>
<td></td>
<td></td>
<td></td>
<td>rotor IR</td>
</tr>
<tr>
<td>efficiency</td>
<td>97.5%</td>
<td>efficiency</td>
<td></td>
<td></td>
<td></td>
<td>total losses</td>
</tr>
<tr>
<td></td>
<td></td>
<td>efficiency</td>
<td></td>
<td></td>
<td></td>
<td>efficiency</td>
</tr>
</tbody>
</table>

IMPEDEANCES AND VOLTAGES (line-to-neutral values)

| stator Xs | 7.77 Ω | 0.62 Ω |
| stator resistance Rs | 0.0638 Ω | 0.0262 Ω |
| ratio Xs/Rs | 122 | 23 |
| phase voltage Es | 2309 V | 254 V |
| phase voltage Er | 2873 V | 285 V |

fact that Motor A is ten times as powerful. This is another property of large motors: the more horsepower they develop, the smaller the relative losses are. As a result, the efficiencies improve with increase in power. Compare the efficiencies of the two motors: 97.5% versus 94.0%.

4. The synchronous reactance $X_s$ per phase is much larger than the resistance of the stator winding. Note that for the 2000 hp motor $X_s$ is 122 times larger than $R_s$. As a result, we can al-

ways neglect the effect of $R_s$ as far as motor performance is concerned.

17.10 Excitation and reactive power

Consider a wye-connected synchronous motor connected to a 3-phase source whose line voltage $E_L$ is fixed (Fig. 17.13). It follows that the line-to-neutral voltage $E$ is also fixed. The line currents $I$ produce a magnetomotive force $U_s$ in the stator. On the other hand, the rotor produces a dc magnetomotive force $U_r$. The total flux $\Phi$ is therefore created by the combined action of $U_s$ and $U_r$.

Pursuing our reasoning, flux $\Phi$ induces a line-to-neutral voltage $E_u$ in the stator. If we neglect the very small $IR$ drop in the stator, it follows that $E_u = E$. However, because $E$ is fixed, $\Phi$ is also fixed, as in the case of a transformer (see Section 9.2).

The mmf needed to create the constant flux $\Phi$ may be produced either by the stator or the rotor or by both. If the rotor exciting current $I_r$ is zero, all the

![Figure 17.13](image-url)

The total flux $\Phi$ is due to the mmf produced by the rotor ($U_r$) plus the mmf produced by the stator ($U_s$). For a given $E_u$, the flux $\Phi$ is essentially fixed.
flux has to be produced by the stator. The stator must then absorb considerable reactive power from the 3-phase line (see Section 7.9). But if we excite the rotor with a dc current $I_r$, the rotor mmf helps produce part of the flux $\Phi$. Consequently, less reactive power is drawn from the ac line. If we gradually raise the excitation, the rotor will eventually produce all the required flux by itself. The stator then draws no more reactive power, with the result that the power factor of the motor becomes unity (1.0).

What happens if we excite the motor above this critical level? The stator, instead of absorbing reactive power, actually delivers reactive power to the 3-phase line. The motor then behaves like a source of reactive power, just as if it were a capacitor. Thus, by varying the dc excitation we can cause the motor to either absorb or deliver reactive power. Because of this important property, synchronous motors are sometimes used to correct the power factor of a plant at the same time as they furnish mechanical power to the load they are driving.

17.11 Power factor rating

Most synchronous motors are designed to operate at unity power factor. However, if they also have to deliver reactive power, they are usually designed to operate at a full-load power factor of 0.8 (leading). A motor designed for a power factor of 0.8 can deliver reactive power equal to 75% of its rated mechanical load. Thus, the 3000 kW motor shown in Fig. 17.4 can supply $75\% \times 3000 = 2250$ Mvar to the line at the same time as it develops its rated mechanical output of 3000 kW. Motors designed to operate at leading power factors are bigger and more costly than unity power factor motors are. The reason is that for a given horsepower rating, both the dc exciting current and the stator current are higher. This can be explained as follows.

Fig. 17.4 is the schematic diagram of a unity power factor motor operating at full-load. The line-to-neutral voltage is $E_{ab}$ and the line current is $I_p$. The active power absorbed per phase is, therefore,

$$P = E_{ab}I_p$$  \hspace{1cm} (17.6)

The active power absorbed is equal to the mechanical power of the motor.

![Figure 17.4](image)

Unity power factor synchronous motor and phasor diagram at full-load.

Fig. 17.15 shows an 80% power factor motor also operating at full-load. It develops the same mechanical power as the motor in Fig. 17.14. The line current $I_s$ leads $E_{ab}$ by arccos $0.8 = 36.87^\circ$. This current can be broken up into two components $I_p$ and $I_q$, and it is clear that

$$I_p = 0.8 I_s \hspace{1cm} (17.7)$$

$$I_q = 0.6 I_s \hspace{1cm} (17.8)$$

The active power $P$ is given by

$$P = E_{ab}I_p = 0.8 E_{ab}I_s \hspace{1cm} (17.9)$$

The reactive power delivered by the motor is

$$Q = E_{ab}I_q = 0.6 E_{ab}I_s \hspace{1cm} (17.10)$$

![Figure 17.5](image)

80 percent power factor synchronous motor and phasor diagram at full-load.

It follows from Eqs. 17.9 and 17.10 that

$$Q = 0.75 P$$

as was stated previously.

If we compare $I_p$ with $I_s$, we find that $I_s = 1.25 I_p$. Thus, for the same mechanical power output, a motor designed for a leading power factor of 80% has
to carry a line current that is 25% greater than one that operates at unity power factor.

### 17.12 V-curves

Suppose a synchronous motor is operating at its rated mechanical load. We wish to examine its behavior as the excitation is varied. Because a change in excitation does not affect the speed, the mechanical power remains fixed. Let us begin by adjusting the excitation $I_x$ so that the power factor is unity, thus yielding the phasor diagram shown in Fig. 17.16. We assume $I_x = 100$ A and $P = 800$ kW.

If we reduce the excitation to 70 A, the motor will draw reactive power from the line in addition to the active power. We assume that $S$ increases to $S = 1000$ kVA. As a result, the line current will increase from $I_p$ to $I_{11}$ (Fig. 17.17). Note that the component of $I_{11}$ in phase with $E_{ab}$ is the same as before because the motor is still developing the same mechanical power.

Current $I_{11}$ lags behind $E_{ab}$, and so the power factor of the motor is lagging. The field current $I_x$ in the rotor is smaller than before, but the apparent power $S$ absorbed by the stator is greater.

If we increase the excitation to $I_x = 200$ A, the motor delivers reactive power to the line which it is connected (Fig. 17.18). The apparent power is again greater than in the unity power factor case. We assume $S = 1000$ kVA. The line current becomes $I_{12}$ and it leads $E_{ab}$. However, the in-phase component of $I_{12}$ is still equal to $I_p$ because the mechanical power is the same.

By varying the excitation this way, we can plot the apparent power of the synchronous motor as a function of the dc exciting current. This yields a V-shaped curve (Fig. 17.19). The V-curve is always displayed for a fixed mechanical load. In our case, the V-curve corresponds to full-load. The no-load V-curve is also shown, to illustrate the large reactive power that can be absorbed or delivered by simply changing the excitation.

**Example 17-5**

A 4000 hp (3000 kW), 6600 V, 60 Hz, 200 r/min synchronous motor operates at full-load at a leading power factor of 0.8. If the synchronous reactance is 11 $\Omega$, calculate the following:

---

**Figure 17.18**

a. Field excitation raised to 200 A but with same mechanical load. Motor delivers reactive power to the line.

b. Phasor diagram shows current leading the voltage.

---

**Figure 17.16**

a. Synchronous motor operating at unity power factor with a mechanical load of 800 kW. Field excitation is 100 A.

b. Phasor diagram shows current in phase with the voltage.

---

**Figure 17.17**

a. Field excitation reduced to 70 A but with same mechanical load. Motor absorbs reactive power from the line.

b. Phasor diagram shows current lagging behind the voltage.

---

**Figure 17.19**

V-shaped curve showing the apparent power of a synchronous motor as a function of the exciting current.
a. The apparent power of the motor, per phase

\[ P = 3000/3 = 1000 \text{ kW} \]

The apparent power per phase is

\[ S = P/\cos \theta = 1000/0.8 \quad (8.11) \]

\[ S = 1250 \text{ kVA} \]

b. The line-to-neutral voltage is

\[ E = E_L/\sqrt{3} = 6600/\sqrt{3} = 3811 \text{ V} \]

The line current is

\[ I = S/E = 1250 \times 1000/3811 \]

\[ I = 328 \text{ A} \]

\[ I \text{ leads } E \text{ by an angle of } \arccos 0.8 = 36.9^\circ. \]

c. To determine the value and phase of the excitation voltage \( E_o \), we draw the equivalent circuit of one phase (Fig. 17.20). This will enable us to write the circuit equations. Furthermore, we select \( E \) as the reference phasor and so

\[ E = 3815\angle0^\circ \]

It follows that \( I \) is given by

\[ I = 328\angle36.9^\circ \]

Writing the equation for the circuit we find

\[ -E + jIX_1 + E_o = 0 \]

thus

\[ E_o = E - jIX_1 \]

\[ = 3811\angle0^\circ - j(328\angle36.9^\circ)11 \]

\[ = 3811\angle0^\circ - 3608\angle(36.9^\circ + 90^\circ) \]

\[ = 3811(\cos 0^\circ + j \sin 0^\circ) - \]

\[ 3608(\cos 126.9^\circ + j \sin 126.9^\circ) \]

\[ = 3811 + 2166 - j2885 \]

\[ = 5977 - j2885 \]

\[ = 6637\angle-26^\circ \]

d. Consequently, \( E_o \) lags 26° behind \( E \), and the complete phasor diagram is shown in Fig. 17.21. e. The torque angle \( \delta \) is 26°.

17.13 Stopping synchronous motors

Owing to the inertia of the rotor and its load, a large synchronous motor may take several hours to stop after being disconnected from the line. To reduce the time, we use the following braking methods:

1. Maintain full dc excitation with the armature in short-circuit.
2. Maintain full dc excitation with the armature connected to three external resistors.
3. Apply mechanical braking.

In methods 1 and 2, the motor slows down because it functions as a generator, dissipating its energy in the resistive elements of the circuit. Mechanical braking is usually applied only after the motor has reached half-speed or less. A lower speed prevents undue wear of the brake shoes.

**Example 17-6**
A 1500 kW, 4600 V, 600 r/min, 60 Hz synchronous motor possesses a synchronous reactance of 16 Ω and a stator resistance of 0.2 Ω, per phase. The excitation voltage $E_x$ is 2400 V, and the moment of inertia of the motor and its load is 275 kg·m². We wish to stop the motor by short-circuiting the armature while keeping the dc rotor current fixed.

**Calculate**
\[ E_x = 2400 \times (150/600) = 600 \text{ V} \]

The frequency is also proportional to the speed, and so
\[ f = 60 \times (15/60) = 15 \text{ Hz} \]

The synchronous reactance is proportional to the frequency; consequently,

**Solution**

a. In Fig. 17.22a the motor has just been disconnected from the line and is now operating as a generator in short-circuit. The speed is still 600 r/min, and the frequency is 60 Hz. Consequently, the impedance per phase is

\[
Z = \sqrt{R^2 + X_L^2} = \sqrt{0.2^2 + 16^2} = 16 \Omega
\]

The current per phase is

\[
I = \frac{E}{Z} = \frac{2400}{16} = 150 \text{ A}
\]

The power dissipated in the 3 phases at 600 r/min is

\[
P = 3I^2R = 3 \times 150^2 \times 0.2 = 13.5 \text{ kW}
\]

b. Because the exciting current is fixed, the induced voltage $E_x$ is proportional to the speed. Consequently, when the speed has dropped to 150 r/min,

\[
E_x = 2400 \times (150/600) = 600 \text{ V}
\]

The time required for the speed to fall from 600 r/min to 150 r/min

**Figure 17.21**
See Example 17-5.

**Figure 17.22a**
Motor turning at 600 r/min (Example 17-6).
\[ P = \frac{Wt}{13.5} = 508.6t \]

whence \( t = 37.7 \text{ s} \)

Note that the motor would stop much sooner if external resistors were connected across the stator terminals.

### 17.14 The synchronous motor versus the induction motor

We have already seen that induction motors have excellent properties for speeds above 600 r/min. But at lower speeds they become heavy, costly, and have relatively low power factors and efficiencies.

Synchronous motors are particularly attractive for low-speed drives because the power factor can always be adjusted to 1.0 and the efficiency is high. Although more complex to build, their weight and cost are often less than those of induction motors of equal power and speed. This is particularly true for speeds below 300 r/min.

A synchronous motor can improve the power factor of a plant while carrying its rated load. Furthermore, its starting torque can be made considerably greater than that of an induction motor. The reason is that the resistance of the squirrel-cage winding can be high without affecting the speed or efficiency at synchronous speed. Figure 17.23 compares the properties of a squirrel-cage induction motor and a synchronous motor having the same nominal rating. The biggest difference is in the starting torque.

High-power electronic converters generating very low frequencies enable us to run synchronous motors at ultra-low-speeds. Thus, huge motors in the 10 MW range drive crushers, rotary kilns, and variable-speed ball mills.

### 17.15 Synchronous capacitor

A synchronous capacitor is essentially a synchronous motor running at no-load. Its only purpose is to absorb or deliver reactive power on a 3-phase system, in order to stabilize the voltage (see Chapter 25). The machine acts as an enormous 3-phase capacitor (or
Figure 17.23
Comparison between the efficiency (a) and starting torque (b) of a squirrel-cage induction motor and a synchronous motor, both rated at 4000 hp, 1800 r/min, 6.9 kV, 60 Hz.

Most synchronous capacitors have ratings that range from 20 Mvar to 200 Mvar and many are hydrogen-cooled (Fig. 17.24). They are started up like synchronous motors. However, if the system cannot furnish the required starting power, a pony motor is used to bring them up to synchronous speed. For example, in one installation, a 160 Mvar inductor whose reactive power can be varied by changing the dc excitation.

Figure 17.24a
Three-phase, 16 kV, 900 r/min synchronous capacitor rated –200 Mvar (supplying reactive power) to +300 Mvar (absorbing reactive power). It is used to regulate the voltage of a 735 kV transmission line. Other characteristics: mass of rotor: 143 t; rotor diameter: 2670 mm; axial length of stator iron: 3200 mm; air gap length: 38.7 mm.

Figure 17.24b
Synchronous capacitor enclosed in its steel housing containing hydrogen under pressure (300 kPa, or about 44 lb/ft²). (Courtesy of Hydro-Québec)
synchronous capacitor is started and brought up to speed by means of a 1270 kW wound-rotor motor.

**Example 17.7**

A synchronous capacitor is rated at 160 Mvar, 16 kV, 1200 r/min, 60 Hz. It has a synchronous reactance of 0.8 pu and is connected to a 16 kV line. Calculate the value of $E_o$ so that the machine

a. Absorbs 160 Mvar
b. Delivers 120 Mvar

**Solution**

a. The nominal impedance of the machine is

$$Z_n = \frac{E_n^2}{S_n}$$

$$= 16\,000^2/(160 \times 10^6)$$

$$= 1.6\,\Omega$$

The synchronous reactance per phase is

$$X_s = X_n(\text{pu}) Z_n = 0.8 \times 1.6$$

$$= 1.28\,\Omega$$

The line current for a reactive load of 160 Mvar is

$$I_n = S_n/\sqrt{3} E_n$$

$$= 160 \times 10^6/(1.73 \times 16\,000)$$

$$= 5780\,A$$

The drop across the synchronous reactance is

$$E_s = I X_s = 5780 \times 1.28$$

$$= 7400\,V$$

The line-to-neutral voltage is

$$E = E_s/\sqrt{3} = 16\,000/1.73$$

$$= 9250\,V$$

Selecting $E$ as the reference phasor, we have

$$E = 9250\angle 0^\circ$$

The current $I$ lags 90° behind $E$ because the machine is absorbing reactive power; consequently,

$$I = 5780\angle -90^\circ$$

b. The load current when the machine is delivering 120 Mvar is

$$I_n = Q/\sqrt{3} E_n$$

$$= 120 \times 10^6/(1.73 \times 16\,000)$$

$$= 4335\,A$$

This time $I$ leads $E$ by 90° and so

$$I = 4335\angle 90^\circ$$

From Fig. 17.25b we can write

$$-E + jIX_s + E_o = 0$$

hence

$$E_o = E - jIX_s$$

$$= 9250\angle 0^\circ - 5780 \times 1.28\angle (90^\circ - 90^\circ)$$

$$= 1850\angle 0^\circ$$

Note that the excitation voltage (1850 V) is much less than the line voltage (9250 V).
The excitation voltage (14,800 V) is now considerably greater than the line voltage (9250 V).

Questions and Problems

Practical level

17-1 Compare the construction of a synchronous generator, a synchronous motor, and a squirrel-cage induction motor.

17-2 Explain how a synchronous motor starts up. When should the dc excitation be applied?

17-3 Why does the speed of a synchronous motor remain constant even under variable load?

17-4 Name some of the advantages of a synchronous motor compared to a squirrel-cage induction motor.

17-5 What is meant by a synchronous capacitor and what is it used for?

17-6 a. What is meant by an under-excited synchronous motor?
   b. If we over-excite a synchronous motor, does its mechanical power output increase?

17-7 A synchronous motor draws 2000 kVA at a power factor of 90% leading. Calculate the approximate power developed by the motor [hp] knowing it has an efficiency of 95%.

17-8 A synchronous motor driving a pump operates at a power factor of 100%. What happens if the dc excitation is increased?

17-9 A 3-phase, 225 r/min synchronous motor connected to a 4 kV, 60 Hz line draws a current of 320 A and absorbs 2000 kW. Calculate
   a. The apparent power supplied to the motor
   b. The power factor
   c. The reactive power absorbed
   d. The number of poles on the rotor

17-10 A synchronous motor draws 150 A from a 3-phase line. If the exciting current is raised, the current drops to 140 A. Was the motor over- or under-excited before the excitation was changed?

Intermediate level

17-11 a. Calculate the approximate full-load current of the 3000 hp motor in Fig. 17.1, if it has an efficiency of 97%.
   b. What is the value of the field resistance?

17-12 Referring to Fig. 17.2, at what speed must the rotor turn to generate the indicated frequencies?

17-13 A 3-phase synchronous motor rated 800 hp, 2.4 kV, 60 Hz operates at unity power factor. The line voltage suddenly drops to 1.8 kV, but the exciting current remains unchanged. Explain how the following quantities are affected:
   a. Motor speed and mechanical power output
   b. Torque angle
   c. Position of the rotor poles
   d. Power factor
   e. Stator current

17-14 A synchronous motor has the following parameters, per phase (Fig. 17.7a):
   \[ E = 2.4 \, \text{kV}, \quad E_o = 3 \, \text{kV} \]
   \[ X_c = 2 \, \Omega \]
   \[ I = 900 \, \text{A} \]
Draw the phasor diagram and determine:

a. Torque angle \( \delta \)
b. Active power, per phase
c. Power factor of the motor
d. Reactive power absorbed (or delivered), per phase

17-15

a. In Problem 17-14 calculate the line current and the new torque angle \( \delta \) if the mechanical load is suddenly removed.
b. Calculate the new reactive power absorbed (or delivered) by the motor, per phase.

17-16

A 500 hp synchronous motor drives a compressor and its excitation is adjusted so that the power factor is unity. If the excitation is increased without making any other change, what is the effect upon the following:

a. The active power absorbed by the motor
b. The line current
c. The reactive power absorbed (or delivered) by the motor
d. The torque angle

Advanced level

17-17

The 4000 hp, 6.9 kV motor shown in Fig. 17.4 possesses a synchronous reactance of 10 \( \Omega \), per phase. The stator is connected in wye, and the motor operates at full-load (4000 hp) with a leading power factor of 0.89. If the efficiency is 97%, calculate the following:

a. The apparent power
b. The line current
c. The value of \( E_a \), per phase
d. The mechanical displacement of the poles from their no-load position
e. The total reactive power supplied to the electrical system
f. The approximate maximum power the motor can develop, without pulling out of step (hp)

17-18

In Problem 17-17 we wish to adjust the power factor to unity.

Calculate

a. The exciting voltage \( E_a \), required, per phase
b. The new torque angle

17-19

A 3-phase, unity power factor synchronous motor rated 400 hp, 2300 V, 450 r/min, 80 A, 60 Hz, drives a compressor. The stator has a synchronous reactance of 0.88 pu, and the excitation \( E_a \) is adjusted to 1.2 pu.

Calculate

a. The value of \( X_s \) and of \( E_a \) per phase
b. The pull-out torque [lbf-ft]
c. The line current when the motor is about to pull out of synchronism

17-20

The synchronous capacitor in Fig. 17.24 possesses a synchronous reactance of 0.6 \( \Omega \), per phase. The resistance per phase is 0.007 \( \Omega \). If the machine coasts to a stop, it will run for about 3 h. In order to shorten the stopping time, the stator is connected to three large 0.6 \( \Omega \) braking resistors connected in wye. The dc excitation is fixed at 250 A so that the initial line voltage across the resistors is one-tenth of its rated value, or 1600 V, at 900 r/min.

Calculate

a. The total braking power and braking torque at 900 r/min
b. The braking power and braking torque at 450 r/min
c. The average braking torque between 900 r/min and 450 r/min
d. The time for the speed to fall from 900 r/min to 450 r/min, knowing that the moment of inertia of the rotor is \( 1.7 \times 10^6 \) lbf-ft^2.

Industrial application

17-21

A 500 hp, 3-phase, 2200 V, unity power factor synchronous motor has a rated current of 103 A. It can deliver its rated output so long as the air inlet temperature is 40°C or less. The manufacturer states that the output of the motor must be decreased by 1 percent for each degree Celsius above 40°C. If the air inlet temperature is 46°C, calculate the maximum allowable motor current.
17-22 An 8800 kW, 6.0 kV, 1500 r/min, 3-phase, 50 Hz, 0.9 power factor synchronous motor manufactured by Siemens has the following properties:

1. Rated current: 962 A
2. Rated torque: 56.0 kN-m
3. Pull-out torque: 1.45 pu
4. Locked-rotor current: 4.9 pu
5. Excitation voltage: 160 V
6. Excitation current: 387 A
7. Full-load efficiency, excluding excitation system losses: 97.8%
8. Moment of inertia of rotor: 520 kg·m²
9. Temperature rise of cooling water: 25°C to 32°C
10. Flow of cooling water: 465 L/min
11. Maximum permissible external moment of inertia: 1370 kg·m²
12. Mass of rotor: 6.10 t (t = metric ton)
13. Mass of stator: 7.50 t
14. Mass of enclosure: 3.97 t

Using the above information, calculate the following:

a. The total mass of the motor including its enclosure, in metric tons
b. The flow of cooling water in gallons (U.S.) per minute
c. The maximum total moment of inertia (in lb·ft²), which the motor can pull into synchronism
d. The total losses of the motor at full-load
e. The total efficiency of the motor at full-load
f. The reactive power delivered by the motor at full-load
g. If the iron losses are equal to the stator copper losses, calculate the approximate resistance between two terminals of the stator.
h. Calculate the resistance of the field circuit.