True fluid temperature reconstruction compensating for conduction error in the temperature measurement of steady fluid flows

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There are three major types of errors that can cause a temperature probe to read a value different from the true fluid temperature into which it is immersed. One type is the transient temperature lag error in general purpose temperature sensors. Another is an error due to radiation heat transfer from walls at high temperatures. The other type is the error due to thermal energy conduction in lead wires and/or thermowell or other casing from a base on which the temperature probe is mounted. Although much study has been devoted to the former, very little exists on the latter. In fact, any prior research on this subject assumed that the entire sensor was completely immersed in the fluid, which in many cases is not true. For all of these types of errors, the cause is the misunderstanding that all physical sensor types only measure their own temperature, which may or may not be equal to the fluid temperature into which they are immersed. This article focuses on conduction error in temperature measurements and quantifies in simple terms a temperature error and lays out a general method for backing out (reconstructing) true fluid temperature from the sensor temperature measurement. Although only thermocouples are experimentally tested here, the theoretical work is valid for other types of physical sensors as well. The experimental verification for the model shows very good agreement. © 2006 American Institute of Physics. [DOI: 10.1063/1.2186211]

I. INTRODUCTION

In many applications, temperature sensors are designed for specific fluid conditions such as those associated with corrosive environments as well as relatively high velocity liquids.1–3 In these cases, typical temperature sensors are encapsulated in a protective “thermowell.” The temperature sensor (thermocouple, thermistor, resistance-temperature device, etc.) is usually bonded at the tip of this thermowell. However, because of the possibility of conduction along the length of the thermowell from the base where it is mounted, which may be at a significantly different temperature than the fluid that the sensor is immersed in, the possibility of the thermowell tip being somewhere between the fluid temperature and the base temperature must be explored.

A fairly large amount of research is available in the archival literature on temperature measurement errors due to the thermal lag of sensors under transient conditions,4–10 which is important in a broad spectrum of scientific investigation.4–10 In contrast, almost no quantitative research is found in the archival literature addressing the possibility of conduction error, except the work of Scadron and Warshawsky11,12 which, as will be shown later in this article, is limited in scope. Also, Fralick and Forney13 addressed some aspects of conduction error on frequency-response characteristics. The purpose of the current article is to outline a discrete-domain type of modeling technique that accurately predicts the steady-state temperature error especially in the case when part of the physical sensor is immersed in the fluid, and part of the sensor is not, which, to the best knowledge of this author, has not been studied before. This type of error may appear in any conceivable application where a thermowell or other physical temperature sensor is used, and may, in fact, be coupled with the transient lag error for time-varying conditions, and radiation in cases where large temperature differences exist between the sensor at nearby boundaries.

II. THEORETICAL ANALYSIS

A temperature probe, either thermowell or bare wire with a probe at the tip, is analogous to the classical fin problem as illustrated in Fig. 1. Superimposed below the sketch is a possible temperature distribution between the base wall (x=0) and the tip (x=L). The only temperature of importance for the current study is the tip temperature (x=L) where the actual temperature measurement is taken. If there is any difference between the tip temperature and the true fluid temperature, then that difference is defined as the conduction error \( E \); thus,

\[
E = [T(x = L) - T_f],
\]

where \( T_f \) is the free-stream fluid temperature, \( T \) the thermocouple junction temperature, and \( T_w \) the base wall temperature. The objective here will be to develop theoretical models for predicting this steady-state conduction error for several cases. The approach taken will then be, with insight from the model, to design and construct the experimental test modules to intentionally incorporate the errors and perform experiments to verify the predictive capability of the model to reconstruct the actual fluid temperature. The forthcoming section deals with the unique effects of thermal energy propagating from a supporting base along thermocouple wires and/or thermowell or other casing from a base on which the temperature probe is mounted. Although much study has been devoted to the former, very little exists on the latter. In fact, any prior research on this subject assumed that the entire sensor was completely immersed in the fluid, which in many cases is not true. For all of these types of errors, the cause is the misunderstanding that all physical sensor types only measure their own temperature, which may or may not be equal to the fluid temperature into which they are immersed. This article focuses on conduction error in temperature measurements and quantifies in simple terms a temperature error and lays out a general method for backing out (reconstructing) true fluid temperature from the sensor temperature measurement. Although only thermocouples are experimentally tested here, the theoretical work is valid for other types of physical sensors as well. The experimental verification for the model shows very good agreement. © 2006 American Institute of Physics. [DOI: 10.1063/1.2186211]
wires (or sensor casing) immersed in a constant temperature fluid on the temperature measurement error characteristics of the probe. Conceptually, a physical temperature sensor may not read the accurate fluid temperature under steady-state conditions because a physical sensor, in actuality, only reads its own temperature. Therefore, if the presence of conduction effects from the supporting base influences the temperature at the junction, the thermocouple junction may be at a different temperature than that of the fluid. It could be argued that the thermal conductivity and the diameter of the thermocouple wires as well as the temperature difference between the fluid and the supporting base may be parameters possibly influencing any present steady-state conduction errors.

**A. One-wire model**

Probably the simplest way to analyze and model conduction error is to treat the probe as a single rod protruding from the wall. Although this may be a good approximation for bare thermocouple wires because the thermocouple wires consist of two wires of dissimilar metals. For instance, type T thermocouples consist of one wire of copper and the other of Constantan. The copper and Constantan wires have very different thermal conductivities and therefore will display different thermal characteristics. In this case, just the copper wire is analyzed neglecting any contribution of the Constantan wire because of the large difference in thermal conductivities; that is, the copper wire, because of its relatively higher thermal conductivity, is suspected of being the major cause of the temperature measurement error. Therefore, the Constantan wire is thought to be of secondary influence to the phenomenon.

The analysis begins identically to that of the classical fin problem; that is, referring to Fig. 1, a finite thermodynamic system consisting of a small length of the wire or thermowell, \( \Delta x \), is analyzed utilizing the integral form of the conservation of energy principle. Then, after inclusion of Fourier’s model for conduction and the definition of the convective heat transfer coefficient, the thermodynamic system was reduced to a plane slicing through any cross section of the wire by dividing by \( \Delta x \) and taking the limit as \( \Delta x \) approached zero. The resulting differential equation for the copper wire or thermowell is expressed as

\[
\frac{d^2T}{dx^2} - \left( \frac{hP}{kA} \right) (T - T_f) = 0, \tag{2}
\]

where \( P \) is cross-sectional periphery, \( k \) the thermal conductivity of the material, \( A \) the cross-sectional area, and \( h \) the convective heat transfer coefficient. The equation is general and can be applied to just about any cross-sectional shape as long as the cross-sectional area is axially uniform. For a thermowell, which is basically a hollow cylinder, the periphery and cross-sectional area must be calculated as such. For a bare wire, \( P = \pi d \) and \( A = \pi d^2/4 \), where \( d \) is the wire diameter. It should be noted that implicit in the above analysis is the assumption that the heat transfer was one dimensional and thus that there was not an appreciable cross-sectional temperature variation versus the axial temperature variation which can be large.\(^{15,16}\) Generally, this holds true for the Biot number \( Bi<0.3 \). The convective heat transfer coefficient \( h \) can be obtained from dimensionless heat transfer correlations for the particular shape. Most often, thermowells and bare wires are cylindrical in shape. Therefore, dimensionless correlations for flow over a cylinder are utilized and found in most current heat transfer textbooks.\(^{17,18}\) The simplest dimensionless correlation for forced convection over cylinders is

\[
Nu_d = C \Pr^{1/3} \Re_d^{n} \tag{3}
\]

The values of \( C \) and \( n \) depend on the particular domain of the Reynolds number \( \Re_d \). The Prandtl number \( \Pr \) involves the viscosity \( \mu \), thermal conductivity of the fluid \( k_f \), and the specific heat of the fluid \( c_p \). Thus,

\[
h = \frac{k_f}{d} C \Pr^{1/3} \Re_d^n = k_f C \left( \frac{\mu c_p}{k_f} \right)^{1/3} \left( \frac{\rho v d}{\mu} \right)^n \tag{4}
\]

Table I list the coefficients \( C \) and \( n \) for different ranges of the Reynolds number. Other dimensionless correlations that are more comprehensive but also substantially more complex than Eq. (3) above can be found in most current heat transfer textbooks.\(^{18}\)

The three unique one-dimensional solutions to the fin problem, Eq. (2), differ by the one boundary condition at the fin tip. The “infinite fin” solution assumes the tip temperature to reach the free-stream fluid temperature. For this to happen, the fin needs to be sufficiently long, but certainly not infinite in the mathematical sense. Thus, this solution based on the

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**TABLE I. Parameters related to Reynolds number (Ref. 18).**

<table>
<thead>
<tr>
<th>( \Re_d )</th>
<th>( C )</th>
<th>( n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4–4.0</td>
<td>0.989</td>
<td>0.330</td>
</tr>
<tr>
<td>4–40</td>
<td>0.911</td>
<td>0.385</td>
</tr>
<tr>
<td>40–4 000</td>
<td>0.683</td>
<td>0.466</td>
</tr>
<tr>
<td>4 000–40 000</td>
<td>0.193</td>
<td>0.618</td>
</tr>
<tr>
<td>40 000–400 000</td>
<td>0.0266</td>
<td>0.804</td>
</tr>
</tbody>
</table>

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FIG. 1. Schematic illustration of a single thermocouple wire.
free-stream tip temperature would be more appropriately named the “very long” or “sufficiently long” fin solution. Obviously, utilizing this solution would lead to the prediction of no error at the tip, which would be detrimental to the premise of this article. However, it should be noted that practicing engineers who assume the thermowell measures only the true fluid temperature implicitly choose this solution which, as will be shown later, can be detrimental to knowledge of the true fluid temperature.

Probably the most utilized solution is usually termed (inappropriately) the “insulated tip” solution. The tip boundary condition here is that the slope of the temperature profile is approximately zero, as shown approximately in Fig. 1. If the tip was truly insulated, the mathematics would work out the same. In reality, this type of fin is long enough such that the vast majority of the heat dissipation occurs at the periphery, and very little from the tip. This solution should perhaps be more appropriately named the “long fin” solution, with the appropriate physical argument that because of the law of diminishing returns, and the continual “flattening out” of the axial temperature profile as a result (Fig. 1), the slope at the tip is negligible (but not exactly zero in the true mathematical sense).

The third classical solution is appropriately termed the “short fin” solution. In this case, the fin is not sufficiently long to neglect the tip heat transfer. On the contrary, the short fin has a tip area that is comparable to that of the periphery. Thus this heat transfer from the tip must be accounted for in the analysis. The appropriate tip boundary condition in this case is that whatever is axially conducted into the fin tip must be convected off of the tip. A simplification typically showing up here is that the convective heat transfer coefficient is the same as that from the periphery. Again, this third possible tip boundary condition is likely unrealistic in temperature sensors because of the almost nonexistent length that the sensor would need to be such that the peripheral area is of the same order as the tip area.

Based on the above, the long fin solution (also known as the insulated tip condition) will be utilized because it is by far the most realistic. Thus, the differential equation is subject to the following boundary conditions:

\[ T(x)_{x=0} = T_w, \]

\[ \frac{dT}{dx} \bigg|_{x=L} = 0. \]

The corresponding solution for the temperature distribution, subject to the above boundary conditions, is expressed as follows:

\[ T(x) = T_f + (T_w - T_f) \frac{\cosh[m(L - x)]}{\cosh(mL)}, \]

where

\[ m = \left( \frac{hP}{ka} \right)^{1/2}. \]

The thermocouple junction temperature will be the temperature at the full length of the wire, \( T(x)_{x=L} \); therefore, the error, utilizing Eqs. (1) and (7), may be expressed as

\[ E = \frac{(T_w - T_f)}{\cosh(mL)}. \]

It should be pointed out that asymptotic cases of Eq. (9) coincide with intuitive reasoning. For instance, in the numerator, as the temperature difference approaches zero, so does the error (no temperature difference, no heat transfer). The denominator shows the same type of characteristics albeit with more involved physical implications. As the length of the wire increases, the error decreases, as is foreseen since conduction effects will decrease with increasing length. On the other hand, as the length of the wire approaches zero, the hyperbolic cosine of the denominator approaches unity and the error is maximized at \((T_w - T_f)\). There are some additional physics contained in the denominator in the form of the fin parameter \( m \), defined by Eq. (8). As this variable increases or decreases, it will decrease or increase the conduction error, respectively. Therefore, for a constant length, conduction effects will decrease as the wire thermal conductivity decreases, as the heat transfer coefficient increases, such as when the velocity increases, or as the diameter decreases. The inverse of the above argument will increase the error for a constant length.

During the course of experimentation, it was observed that as the velocity increased, the difference between the wall temperature and the free-stream fluid temperature decreases. This was the result of back conduction through the thermocouple wires. To normalize this secondary effect from the models, nondimensionalization of terms was performed for both the temperature distribution and the error. The results of the nondimensionalization yield a proportionate error as follows:

\[ E' = \frac{E}{(T_w - T_f)} = \frac{[T(x)_{x=L} - T_f]}{(T_w - T_f)} = \frac{1}{\cosh(mL)}. \]

It should be pointed out that Eq. (10) represents exactly the model of the only prior research on this topic\(^{13}\) available in the archival literature. The corresponding dimensionless temperature distribution is expressed as

\[ T' = \frac{[T(x) - T_f]}{(T_w - T_f)} = \frac{\cosh[m(L - x)]}{\cosh(mL)}. \]

The above model predicts the worst-case maximum error due to conduction from a base to the tip (or visa versa if the base is at a lower temperature than the free-stream fluid). Although this model may be utilized for purposes of designing a sensor probe to minimize this maximum error, most practical cases involve an existing sensor. In this case, the above model will likely overpredict the conduction error and thus may not give the most accurate reconstruction of the actual fluid temperature for several reasons. One being that if the sensor is bare thermocouple wires, then there will likely be thermal communication between the wires which will have the effect of decreasing the conduction error. Also, in many instances, the sensor extends past the wall of the pipe or vessel containing the fluid. Both of these cases will be investigated separately, and then collectively.
B. One-wire model with adiabatic length

In the case where the thermocouple base temperature is a standard thermocouple connector, a more realistic model is one which is divided into at least two domains, one exposed to the free-stream fluid temperature and the other encapsulated in the thermocouple connector, L and L'', respectively, as illustrated in Fig. 2. The schematic at the top of this figure is an illustration of the real test section that is pictured at the bottom of the figure. Referring especially to the photograph in Fig. 2, you can see that the thermocouple wires extend from the polycarbonate test section and into the blue thermocouple connectors where they are based. The encapsulated section will be assumed adiabatic because, since the wire is totally enclosed by the connector, there will be very little space for any convection to take place. Analyzing each domain separately, the solution for the exposed wire is the same form as that of Eq. (7), except that the base temperature $T_w$ is substituted with the unknown interface temperature $T''_w$ between the two domains. The solution is expressed as

$$T(x) = T_f + (T''_w - T_f) \frac{\cosh[m(L - x)]}{\cosh(mL)}. \quad (12)$$

The solution for the encapsulated length of wire $L''$ may be expressed as a linear function, because of the adiabatic assumption, as

$$T''(x'') = T_w + (T_w - T''_w) \left( \frac{x''}{L''} \right). \quad (13)$$

The two solutions are subject to the following boundary conditions:

$$T''(x'')_{x''=0} = T_w,$$  

$$\left. \frac{dT''}{dx''} \right|_{x''=L''} = 0 \text{ negligible convection off the tip}, \quad (15)$$

$$T''(x'')_{x''=L''} = T(x)_{x=0} = T''_w \text{ interface temperature}, \quad (16)$$

$$k'' \frac{dT''}{dx''} \bigg|_{x''=0} = k \frac{dT}{dx} \bigg|_{x=0} \text{ interface heat flux}. \quad (17)$$

To eliminate the unknown interface temperature $T''_w$ from both equations expressing the temperature distribution in the two domains, the heat fluxes are matched at the contact; that is, both Eqs. (12) and (13) are substituted into Eq. (17) resulting in the following expression for this unknown interface temperature:

$$T''_w = \frac{(T_w + T_mL'')}{(1 + (mL''))}. \quad (18)$$

where

$$m_m = m \tanh(mL). \quad (19)$$

Substituting Eq. (18) into (12), the resulting temperature distribution in the exposed portion of the wire is

$$T(x) = T_f + (T_w - T_f) \frac{\cosh[m(L - x)]}{\cosh(mL) + (mL'') \sinh(mL)}. \quad (20)$$

The thermocouple junction temperature is at the length of the exposed wire, $T(x)_{x=L'}$; therefore, the error in the temperature measurement may be expressed as

$$E = \frac{(T_w - T_f)}{[\cosh(mL) + (mL'') \sinh(mL)]}. \quad (21)$$

Examining the limiting case for the error, as the length of the encapsulated domain $L''$ approaches zero, Eq. (21) reduces to the one-wire model, expressed by Eq. (9).

The normalized form of the error is expressed as

$$E^* = \frac{E}{(T_w - T_f)} = \frac{1}{[\cosh(mL) + (mL'') \sinh(mL)]}. \quad (22)$$

The corresponding dimensionless temperature distribution is

$$T'' = \frac{[T(x) - T_f]}{(T_w - T_f)} = \frac{\cosh[m(L - x)]}{[\cosh(mL) + (mL'') \sinh(mL)]}. \quad (23)$$

It should be noted here that if the adiabatic assumption had not been made, then it would be assumed that the encapsulated length of wire had an effective encapsulated heat transfer coefficient, $h''$, lower than that of the exposed length, which can be found utilizing the appropriate dimensionless correlation for heat transfer in an enclosure in series with conduction through the encapsulation. In this case, the normalized error would be
FIG. 3. Comparison of single wire model with single wire model with adiabatic length.

\[
E^* = \frac{(T_w - T_f)}{\cosh(mL)[\cosh(m''L'' + (m''/m'') \sinh(m''L'')]} \quad (24)
\]

Taking the limit as \(h''\) and thus \(m''\) approach zero, the above expression, utilizing L'Hopital's rule, reduces to Eq. (22).

A comparison of the temperature distribution prediction of the one-wire model with the one-wire model with an adiabatic length illustrates that, as depicted in Fig. 3, the adiabatic wire length \(L''\) can have a significant influence on the temperature measurement error \(E\), which is the temperature difference \((T_f - T_w)\) at \(x/L = 1.0\). The magnitude of the effect of the adiabatic length on the temperature measurement error depends on the relative length of the adiabatic wire \(L''\) to the exposed wire \(L\). As the adiabatic length of wire \(L''\) becomes relatively small compared to the exposed length of wire \(L\) its influence will decrease. Figure 4 illustrates that, for a constant exposed wire length \(L\), increasing the adiabatic length will decrease the thermocouple measurement error \(E\); however, in most practical situations, the adiabatic length will remain constant while the exposed wire length can be varied by design. The adiabatic wire length \(L''\) depends primarily on the connector casing geometry.

FIG. 4. Influence of adiabatic length on single wire model with adiabatic length.

C. Two-wire model

It was hypothesized earlier that the second wire, necessary to complete a thermocouple junction, may have an influence on the error characteristics of the thermocouple due to conduction to or from the base. It is the intent of this section to model and compare the sole influence of the additional exposed thermocouple wire on the steady-state conduction error characteristics to the one-wire model, with implications to actual fluid temperature reconstruction. Therefore, an adiabatic wire length will not be considered here at this time. The derivation will ensue for the equivalent two-base schematic shown in Fig. 5, which simplified the geometry and effectively neglected any convection from the tip of the bonded thermocouple junction extending into the fluid. The bases on the right and left hand sides of the figure are actually the same base, as the thermocouple wires were “unwrapped” to form a simpler geometry for mathematical purposes. The solution for the temperature distribution in the copper wire (type \(T\) thermocouples were utilized in this study and combination can be predicted with this analysis) will have the general form expressed as

\[
T(x) = T_f + c_1 \sinh(mx) + c_2 \cosh(mx). \quad (25)
\]

The solution for the temperature distribution of the Constantan wire will be of the exact same form as Eq. (25) except that every argument \((mx)\) is substituted by \((m'x')\). Both solutions are subject to the following boundary conditions:

\[
T(x)|_{x=0} = T'(x')|_{x'=0} = T_w, \quad (26)
\]

\[
T(x)|_{x=L} = T'(x)|_{x=L'} = T_L \quad \text{junction temperature}, \quad (27)
\]

\[
\frac{dT}{dx}|_{x=0} = -\left(\frac{k'}{k} \frac{dT'}{dx'}\right)_{x'=L'} \quad \text{junction heat flux}. \quad (28)
\]

Substituting Eq. (25) into the above boundary conditions, the temperature distribution for the copper wire is expressed as
\[ T(x) = T_j + \left( \frac{(T_L - T_j) - (T_w - T_j) \cosh(mL)}{\sinh(mL)} \right) \sinh(mx) + (T_w - T_j) \cosh(mx). \]  

The temperature distribution for the Constantan wire will have exactly the same form except that every parameter \( m \) is replaced with \( m' \). Since the lengths of the two wires are equal, it is not necessary to differentiate the length of the Constantan wire from the copper wire; thus, \( L' = L \). Substituting both forms of Eq. (29), one for each individual wire, into Eq. (28) the following relationship is attained:

\[ (T_L - T_j) = C_{tw}(T_w - T_j), \]  

where

\[ C_{tw} = \frac{\left\{ \left[ 1/\sinh(mL) \right] + \gamma \left[ 1/\sinh(m'L) \right] \right\}}{\left\{ \left[ \cosh(mL)/\sinh(mL) \right] + \gamma \left[ \cosh(m'L)/\sinh(m'L) \right] \right\}}, \]  

and

\[ \gamma = \left( \frac{k'}{k} \right)^{1/2}. \]  

Note that the two-wire influencing coefficient \( C_{tw} \) is primarily a function of the wire diameter and the respective material composing the two wires. Substituting this result into the corresponding temperature distributions for the two wires, and introducing the definition of steady-state temperature measurement error, the equation reduces, after appropriate algebraic rearrangement, to the ensuing simple expression,

\[ E = C_{tw}(T_w - T_j), \]  

or in normalized form,

\[ E^* = \frac{E}{(T_w - T_j)} = C_{tw}. \]  

With the corresponding dimensionless temperature distribution expressed as

\[ T^* = \left( \frac{T(x) - T_j}{T_w - T_j} \right) = \left( C_{tw} - \cosh(mL) \right) \frac{\sinh(mx)}{\cosh(mx)}. \]  

Figure 6 illustrates the two-wire model theoretical prediction in the case where the free-stream air temperature is less than the base temperature; thus any propagating conduction effects will tend to increase the thermocouple junction temperature, causing a temperature measurement error. Figure 6 displays the same physics as was noted by prior researchers, although the prior researchers used an effective one-wire model, similar to Eq. (10) but where the thermal conductivity was taken as a harmonic mean between that of the two wires. This former research did indicate the possibility of modeling the above case of two wires that are very different in conductivities, but did not to so, nor did the article experimentally verify their model as it was not the major focus of the work. Note from Fig. 6 that the Constantan wire almost reaches the free-stream air temperature at some part of its length, due to its relatively low thermal conductivity \( k'/k = 0.0794 \) for copper-Constantan. It is clear from current available thermocouple data that copper-Constantan exhibits the greatest difference in thermal conductivities. However, closer to the thermocouple junction, the temperature increases and eventually equals the copper wire temperature at the junction; thus, the Constantan wire is essentially heated from both the base and the copper wire. In fact, the presence of the Constantan wire tends to effectively cool the thermocouple junction and decrease the error as demonstrated in Fig. 7. It is clear that, in the absolute sense, the shorter copper wire in Fig. 7 is more affected by the Constantan wire than the longer copper wire because, in both cases, the Constantan wire essentially reaches free-stream temperature (refer to Fig. 6). However, for the shorter copper wire, there is a more dramatic difference, close to the junction, at \( (x/L) = 1 \), between the junction temperature and that of the air; therefore, the Constantan wire will have a greater influence on the shorter copper wire.

### D. Two-wire model with adiabatic length

The query remains as to the magnitude of the influence of an adiabatic length \( L^\alpha \) present in both copper and Constantan wires, on the absolute error characteristics of the thermocouple junction.
couple, the possible significance of which was demonstrated in Fig. 3 for the one-wire model. In the case of the two-wire model, an adiabatic length on the copper wire should still have a similar influence on the error characteristics. However, the adiabatic length of the Constantan wire should have little or no relative effect since the Constantan wire essentially reaches free-stream fluid temperature anyway; therefore, an adiabatic length will be considered only on the copper wire. The influence, if any, of the Constantan adiabatic length will be explored later in this section.

Referring to Fig. 8, the analysis will be similar to parts of the prior analyses. The two-wire model will be considered except that an adiabatic length will be added onto the copper wire. Matching several of the boundary conditions from the previous analyses, the following temperature distributions hold for their respective domains:

Adiabatic length

\[ T'_{w}(x') = T_{w} + (T_{w} - T_{w}) \left( \frac{x'}{L'_{w}} \right). \]  \hfill (36)

Two wire

\[ T(x) = T_f + \left( \frac{(T_{L} - T_{f}) - (T_{w} - T_{r})\cosh(mL)}{\sinh(mL)} \right) \sinh(mx) \]
\[ + (T_{w} - T_{r})\cosh(mx). \]  \hfill (37)

The temperature distribution for the Constantan wire, assuming the Constantan wall temperature to be equal to the copper wire wall temperature \( T_w \) will be the same as that expressed in Eq. (37) except that every parameter \( m \) is replaced by \( m' \). The temperature distributions are subject to the remaining boundary conditions,

\[ \left. \frac{dT}{dx} \right|_{x=0} = -\left( \frac{k'}{k} \right) \left. \frac{dT}{dx} \right|_{x=L'_{w}} \]  heat flux at copper

-Constantan interface,

\[ \left. \frac{dT}{dx} \right|_{x=0} = \left. \frac{dT}{dx} \right|_{x=L'_{w}} \]  heat flux at exposed -Encapsulated copper interface. \hfill (38)

Substituting the relevant temperature distributions into their respective boundary conditions, the result yields the following relationships:

\begin{align*}
(T_L - T_f) &= \frac{\left[ \left( T''_{w} - T_f \right)/\sinh(mL) \right] + \gamma \left[ \left( T''_{w} - T_r \right)/\sinh(m'L) \right]}{\left[ \cosh(mL)/\sinh(mL) \right] + \gamma \left[ \cosh(m'L)/\sinh(m'L) \right]}, \hfill (40) \\
(T''_{w} - T_f) &= \frac{\left[ \left( T''_{w} - T_f \right) + (T_L - T_f) \left[ (mL')/\sinh(mL) \right] \right]}{1 + (mL') \left[ \cosh(mL)/\sinh(mL) \right]}, \hfill (41)
\end{align*}

Making the appropriate back substitutions, the temperature measurement error in this case is expressed as

\[ E = (T_w - T_f) \left( \frac{\left[ C_{w} /\sinh(mL) \right] + \gamma \left[ 1/\sinh(m'L) \right]}{\left[ \cosh(mL)/\sinh(mL) \right] + \gamma \left[ \cosh(m'L)/\sinh(m'L) \right]} \right). \]  \hfill (42)

where

\[ C_{w} = \gamma \left( \frac{1 + \gamma \left[ \cosh(mL)/\sinh(mL) \right]}{\cosh(mL)/\sinh(mL) + \gamma \left[ \cosh(m'L)/\sinh(m'L) \right]} \right). \]  \hfill (43)

If it should be pointed out that, to simplify the mathematics, it could be argued that the constantan base temperature assumed to be the same as the copper wire base temperature \( T_w \) would actually be lower if an adiabatic length was present on the Constantan wire. Thus, it could be postulated that the effective wall temperature is \( T''_w \) instead of \( T_w \); this, in fact, contains a higher degree of accuracy since the presence of a Constantan adiabatic length would decrease the effective wall temperature below \( T''_w \). Therefore, the effective two-base schematic, Fig. 8, would assume the Constantan base tem-
perature to be $T_w^*$, the mathematical implication being that the two-wire-adiabatic influencing coefficient $C_{twa}$ would appear in both terms in the numerator of Eq. (42), thus being factored out. The remaining complex fraction is the two-wire influencing coefficient $C_{tw}$ expressed by Eq. (31). Therefore, Eq. (42) may be rewritten as

$$E = C_{twa}C_{tw}(T_w - T_f),$$

or in normalized form, along with the dimensionless temperature distribution, as

$$E^* = \frac{E}{(T_w - T_f)} = C_{twa}C_{tw},$$

$$T^* = \left[ \frac{T(x) - T_f}{T_w - T_f} \right] = C_{twa}\left\{ \frac{C_{tw} - \cosh(mL)}{\sinh(mL)} \right\}\sinh(mx) + \cosh(mx).$$

The Constantan wire will show the equivalent distribution as expressed by Eq. (46) except that every term $m$ will be replaced by $m'$. It is clear from Eq. (43) that as the adiabatic length $L'$ approaches zero, the two-wire-adiabatic influencing coefficient $C_{twa}$ approaches unity and thus the error expressed by Eq. (46) reduces to the two-wire model expressed by Eq. (33). It is clear from Fig. 9 that the two-wire-adiabatic influencing coefficient $C_{twa}$ has an effect on the temperature measurement error of a thermocouple. It should be pointed out that, from Eq. (43), the two-wire-adiabatic influencing coefficient $C_{twa}$ for a fixed exposed thermocouple length $L$ is deferential only on the adiabatic length. The case where the influencing coefficient is unity corresponds to zero adiabatic length and thus corresponds to the error predicted by the simple two-wire model. The case of the influencing coefficient approaching zero corresponds to the adiabatic length approaching infinity.

The air velocity has an influence on the magnitude of the conduction effects responsible for the error. It is clear from Fig. 10 that the convective heat transfer coefficient is directly proportional to the velocity. A higher velocity would therefore decrease propagating conduction effects from the base that cause the temperature measurement error.

### E. Summary of conduction error models

In comparing the four different conduction error models, it is apparent from Fig. 11 that the two-wire model with adiabatic length will predict the least amount of conduction error $E$, which is at the thermocouple junction $(x/L)=1$. There appear to be sizable discrepancies in the temperature distribution prediction of the four models, including the thermocouple junction temperature, at $(x/L)=1$. The simple one- and two-wire models predict a higher temperature measurement error because both models lack the influential physics associated with the adiabatic length. Note that the influence of both the adiabatic length and the Constantan wire may be readily compared in Fig. 11 as all the models are present. The two-wire model with the adiabatic length is the most physically precise and should predict the error $E$, at $(x/L)=1$, most accurately. Therefore, this is the most suitable model for reconstructing the actual fluid temperature from the actual readout. The other three models will theoretically predict higher errors. It should be pointed out that in even more complex cases, the technique presented in this article...
can be further extended by adding subdomains as needed with the appropriate boundary condition. As was shown, this will have a cumulative effect of the coefficient as the forward modeling of multiple subdomains is continued and is conceptually similar to the tridiagonal matrix algorithm (TDMA) popular in numerical techniques.

III. EXPERIMENTAL VERIFICATION

Experiments were performed by directly measuring the temperature measurement error of two specially designed experimental thermocouple models. Utilizing the theoretical two-wire model with the adiabatic length, the thermocouple exposed wire lengths were designed to have an error in the temperature measurement. A 44-gage thermocouple was mounted at the base connector plate of each thermocouple to monitor the wall temperature $T_w$. (These are the blue connectors in Figs. 2 and 12 that do not seem to have a thermocouple protruding from them into the test section.) The base of the test section was made from brass with a water jacket so that the base temperature can be controlled and maintained approximately constant. The free-stream air temperature, measured by a very long 44-gage thermocouple, designed very long so as to have negligible steady-state conduction errors, was held constant while the air velocity measured indirectly with a venturi flow meter, and a pitot-tube traverse was varied. Figures 13 and 14 compare the experimental results with the predictions of the theoretical models.

Note that, in both cases, the two-wire theoretical model with the adiabatic length performs better than the other models. The plain one-wire and two-wire models, both of which appeared at least implicitly in the archival literature, overpredicted the measurement errors because the adiabatic length $L''$, which was equivalent to the exposed length $L$, was very influential in this case. The one-wire model with the adiabatic length overpredicted the error as the physical influence of the Constantan wire was not present. The conclusion here is that the two-wire model with the adiabatic length on the copper wire contained all of the dominant physical mechanisms responsible for the error, where the previous three models lacked one or more of the mechanisms. Also, note from the experimental comparisons that the adiabatic length had more influence on the model’s predictive performance than did the physical mechanism associated with the Constantan wire alone.

The two-wire model with the adiabatic length seemed to predict a slightly higher error than was measured in the case of the 24-gage wire, as illustrated in Fig. 14, but well within experimental uncertainty. This could be attributed to a secondary fin effect caused by the thermocouple junction itself, which has a short length beyond what is considered the length of the wire $L$. This would tend to decrease the theoretical error prediction. However, the two-wire model with the adiabatic length predicts the error within the uncertainty of the obtained parameters; the largest uncertainty was prob-
ably in the length measurements, with a lesser uncertainty in the convective heat transfer coefficient, which was a function of velocity.

This article presented a discrete-domain method of reconstructing actual article temperature data from a temperature measurement which included a conduction error due to nonthermal equilibrium with the sensor base. Although only a few discrete domains were necessary in the current article, the technique is general enough that any number of domains can be incorporated as needed, as long as temperature and heat flux compatibility conditions are maintained. The experiments were run only with thermocouple bare wires because of the complexity of a two-wire system versus a classic thermowell design, but the analysis could be used for any physical sensor as appropriate.

1 D. S. Bartran, R. Yee, and D. R. Frikken, Oil Gas J. 100, 60 (2002).