Predicting the Influence of Compressibility and Thermal and Flow Distribution Asymmetry on the Frequency-Response Characteristics of Multitube Two-Phase Condensing Flow Systems

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An equivalent single-tube model concept was extended to predict the frequency-response characteristics of multitube two-phase condensing flow systems, complete with the ability to predict the influence of compressibility and thermal and flow distribution asymmetry. The predictive capability of the equivalent single-tube model was verified experimentally with extensive data that encompassed a three-order-of-magnitude range of frequencies, and a wide range of operating parameters. [S0022-1481(00)00601-0]

Keywords: Condensation, Heat Transfer, Heat Exchangers, Two-Phase, Unsteady

Introduction

The research presented in this paper is associated with the influence of compressibility on the frequency-response characteristics of multitube condensing flow systems. To the best knowledge of the authors, the archival literature does not contain any theoretical models for predicting the frequency-response characteristics of such systems. There is also very little experimental data. Filling these knowledge gaps is the focus of this research.

Kobus et al. [1] extended the predictive capability of the equivalent single-tube model to predict the frequency-response characteristics of multitube condensing flow systems when compressibility effects are negligible. The influence of compressibility on transient flow surges in multitube condensing flow systems was investigated by Wedekind et al. [2]. The effects of compressibility on a single-tube condensing flow system had also been studied earlier ([3]). Given the complexity of the physical mechanisms involved, and the fact that most of them are coupled in some way, a significant step is involved between successfully

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modeling the frequency-response characteristics in a single-tube condenser, and having the same level of success when the condenser is multtube. The major purpose of this research, then, is to investigate the requirements for extending the equivalent single-tube model so that it can successfully predict the influence of compressibility on the frequency-response characteristics for a multtube system, and to verify the model by comparing its predictions with experimental data.

Formulation of Governing Differential Equations

The formulation of the governing differential equations, including the effects of compressibility, was presented in previous research ([2]), but solved for the case of transient flow surges. Therefore, details of the development of these governing equations will not be repeated here. The solution of the equations, however, will be presented for the special case of a sinusoidal inlet vapor flowrate \( m_{v,i}(t) \).

Equivalent Single-Tube Model. The equivalent single-tube model is based on the system mean void fraction model, which is a one-dimensional, two-fluid, distributed parameter integral model developed in previous research ([2]). It represents a way of modeling the transient characteristics of the effective point of complete condensation for a representative /th tube, \( \eta_j(t) \), obtained from the conservation of mass and energy principles ([2]), is expressed as follows:

\[
\tau_{c,j} \frac{d \eta_j(t)}{dt} + \eta_j(t) = x_i \left( h' - h \right) \beta_j m_{v,i}(t)
\]

where

\[
\tau_{c,j} = \frac{\rho' (h' - h) \bar{a} A_{i,j}}{\tilde{f}_{q,j} P_j}
\]

In the above equations, \( \tilde{f}_{q,j} \) represents the spatially averaged heat flux for a representative /th tube, \( x_i \), the inlet flow quality, \( A_{i,j} \) and \( P_j \) the tube cross-sectional area and periphery, respectively, and \( \rho' \) and \( (h' - h) \) the saturated vapor density and heat of vaporization.

The system mean void fraction \( \bar{a} \) is defined in terms of the local area void fraction \( a(z,t) \), and represents the integral form of the mean value theorem. The particular void fraction model used is that of Zivi [4], chosen for its simplicity, yet is sufficiently accurate for these types of condensing flow problems. However, any void fraction-flow quality relationship that is valid over the full range of flow qualities would yield similar results. It was established in previous early research that the system mean void fraction is essentially time invariant. This has the effect of uncoupling the conservation of mass and energy principles in the two-phase region from the transient form of the momentum principle; thus, only the steady-state form of the momentum principle is required. The system mean void fraction \( \bar{a} \) can therefore be expressed as:

\[
\bar{a} = \frac{1}{\eta(t)} \int_{z=0}^{\eta(t)} a(z,t)dz = \frac{1}{1-a} + \frac{a}{x_i(1-a)^2} \ln \left[ \frac{a}{a+(1-a)x_i} \right] ; \quad a = (\rho/\rho')^{2/3}
\]

The equivalent single-tube model is an approximation technique that has been shown to be successful in predicting various transient characteristics associated with multtube condensing flow systems ([2,1]). This approximation technique has the effect of reducing the equations governing the multtube system, which contain complex summations ([2]), to an approximation where the summations are eliminated. The resulting approximation has the appearance of being an equivalent single-tube condensing flow system, but where the associated parameters are weighted with determinable multtube parameters. The equivalent single-tube model contains an equivalent single-tube condensing flow system time constant \( \tau_{c,j} \), which is a weighted average of the condensing flow system time constants of each of the individual tubes \( \tau_j \), thus

\[
\tau_{c,s} = \sum_{j=1}^{n} \gamma_j \tau_{c,j} = \tau_{c,1} + \sum_{j=2}^{n} \gamma_j \beta_j ; \quad \beta_j = \tilde{f}_{q,j} \tilde{f}_{q,j}^{-1}
\]

where the flow distribution parameter \( \gamma_j \) is defined as the fraction of the total mass flowrate entering tube \( j \). In general, \( 0 \leq \gamma_j \leq 1 \). A flow distribution parameter \( \gamma_j = 1/n \) signifies flow distribution symmetry in an \( n \)-tube system. The parameter \( \beta_j \) is the heat flux ratio between a reference tube (usually designated as tube 1) and the \( j \)th tube in the system. In general, \( \beta_1 \geq 0, \beta_1 = 1 \) signifies thermal symmetry of the multtube system. In this phase of the model development, both the thermal and flow distribution parameters are treated as parameters in the classical sense ([2]).

Outlet Liquid Flowrate. The differential equation governing the transient outlet liquid flowrate \( m_{l,s}(t) \) is the same as that presented in the aforementioned research; thus,

\[
\tau_{f,s} \frac{dm_{l,s}(t)}{dt} + m_{l,s}(t) = [(\rho/\rho') - 1]x_i + 1 \cdot m_{v,i}(t) - [(\rho/\rho') - 1] \sum_{j=1}^{n} \tilde{f}_{q,j} P_j \eta_j(t) / (h' - h)
\]

where

\[
\tau_{f,s} = \frac{\rho}{\rho'} \left\{ \int_{q} \frac{\rho'}{\rho'[\rho'(\rho' - 1)]} \left( V_{u,s} + V_{s,s} + V_{p,s,s} \right) \gamma^2 k_0^n \right\}
\]

The compressible flow system time constant \( \tau_{f,s} \) incorporates fluid properties, system vapor volumes, and flow resistances ([2]). For the case where the effects of compressibility are negligible, \( \tau_{f,s} = 0 \), Eq. (5) reduces to an algebraic equation identical to that obtained in the work of Kobus et al. [1]. Also, for the case of a single-tube condenser, \( n = 1 \), Eq. (5) reduces to the governing differential equation that appears in the work of Bhattacharya and Wedekind [3].

A solution of Eq. (5) is obtained by first solving the set of equations represented by Eq. (1), then substituting these solutions into the summation in Eq. (5), and then solving. As mentioned earlier, this was carried out in previous work where the inlet flowrate to the condensing flow system \( m_{v,i}(t) \) produces transient flow surges ([2]). In this current work, however, the inlet flowrate is a sinusoidal function of the form

\[
m_{v,i}(t) = \bar{m} + \bar{a} \cos(\omega t)
\]

where \( \bar{a} \) and \( \omega \) are the amplitude and angular frequency of the inlet flowrate oscillations, respectively, and \( \bar{m} \) is the mean flowrate about which the oscillations occur. Carrying out the solution, the frequency-response characteristics of the transient outlet liquid flowrate, \( m_{l,s}(t) \), for an \( n \)-tube condensing flow system, can be expressed by the following generalized, yet comparable simple expressions:

\[
G_m = \left[ \frac{1 + [(\rho/\rho') - 1]x_i + 1^2(\omega \tau_{c,s})^2}{1 + (\omega \tau_{f,s})^2} \right]^{1/2}
\]
For the case where the effects of compressibility are negligible, \( t_f, s = 0 \), Eqs. (8) and (9) reduce to Eqs. (1) and (2) in the research of Kobus et al. Note that the above solution is greatly simplified by the equivalent single-tube model, where the summation in Eq. (5) is eventually assimilated by the definition of the condensing flow system time constant \( t_c, s \), Eq. (4).

**Experimental Verification**

As was pointed out in earlier research, a two-tube condensing flow system with significant thermal asymmetry may very well represent a worst-case situation for the equivalent single-tube model. For this reason, and to keep the experimental apparatus tractable, experimental verification was carried out using a parallel two-tube configuration.

**Experimental Apparatus and Measurement Uncertainties.**

The experimental apparatus used in the present research is similar to that used by Kobus et al. Therefore, the details will not be repeated here. Uncertainties in the experimental measurements were also discussed in detail in the aforementioned research, and will not be repeated. The experimental data pertaining to the gain characteristics had an average maximum uncertainty of \( \pm 10 \) per-

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**Table 1 Physical properties and parameters**

<table>
<thead>
<tr>
<th>Data set</th>
<th>( \alpha )</th>
<th>( \tau_c )</th>
<th>( V_{2,0} )</th>
<th>( V_{2,0} )</th>
<th>( \tau_{c,1} )</th>
<th>( \tau_{c,2} )</th>
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</thead>
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<tr>
<td>1fr-618b</td>
<td>0.831</td>
<td>0.79</td>
<td>245</td>
<td>—</td>
<td>67.7</td>
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</tr>
<tr>
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<td>0.79</td>
<td>245</td>
<td>—</td>
<td>364.4</td>
<td>0.79</td>
</tr>
<tr>
<td>1fr-620b</td>
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<td>0.81</td>
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<td>—</td>
<td>19.3</td>
<td>0.81</td>
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<tr>
<td>2fr-623</td>
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<td>0.79</td>
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<td>120</td>
<td>16.3</td>
<td>0.79</td>
</tr>
<tr>
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<td>0.81</td>
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<td>123</td>
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</tr>
<tr>
<td>2fr-707b</td>
<td>0.830</td>
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<td>123</td>
<td>312.4</td>
<td>0.81</td>
</tr>
<tr>
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<td>0.84</td>
<td>98</td>
<td>343</td>
<td>265.5</td>
<td>2.09</td>
</tr>
</tbody>
</table>

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\[
\Phi_m = \tan^{-1} \left( \frac{\left( \frac{\rho}{\rho^0} - 1 \right) x_2 + 1}{\left( \frac{\rho}{\rho^0} - 1 \right) x_2 + 1} \right) \frac{\left( \omega \tau_{c,1} \right) + \left( \omega \tau_{f,1} \right)}{\left[ 1 - \left( \omega \tau_{c,1} \right) \left( \omega \tau_{f,1} \right) \right]} \frac{\left( \omega \tau_{c,2} \right) + \left( \omega \tau_{f,2} \right)}{\left[ 1 - \left( \omega \tau_{c,2} \right) \left( \omega \tau_{f,2} \right) \right]}.
\]
cent, whereas the uncertainty associated with the phase shift characteristics was slightly higher at ±15 percent. A sample strip-chart trace of the inlet and outlet flowrate variations, \( m_{i}(t) \) and \( m_{o}(t) \), respectively, for the two-tube system is shown in Fig. 1. It demonstrates the clarity and repeatability of the experimental measurements.

In calculating the compressible flow system time constant \( t_{fs} \), the total upstream vapor volume in the two-tube system was 464 cm\(^3\), while that in the single-tube system was 486 cm\(^3\). The test sections were copper tubes with an inside diameter of 0.80 cm. All of the forthcoming experimental data were run at constant conditions of \( \bar{m} = 4.31 \text{ g/s}, \quad x_{i} = 1.0, \quad f_{q,1} = 11.1 \text{ kW/m}^{2}, \) and at a condensing pressure of approximately 690 kPa, which in turn yields a liquid-to-vapor density ratio \( \gamma = 33.7 \). Table 1 lists other relevant parameters associated with the calculations necessary to predict the frequency-response characteristics in the current research.

Experimental Verification of Equivalent Single-Tube Model. The theoretical predictions of the equivalent single-tube model, Eqs. (6) and (7), for the frequency-response characteristics of a two-tube condensing flow system, are compared with experimental data for several different condensing flow conditions.

Influence of Compressibility. Similar to what was done by Kobus et al. [1], frequency-response tests were carried out initially with a two-tube condenser. But this time the effects of compressibility were made to be more significant. The results, along with the theoretical predictions of the present compressible equivalent single-tube model, are depicted in Fig. 2. Note the very dramatic attenuating effect that compressibility can have on the frequency-response characteristics; the experimental data corresponding to the most significant compressibility, \( t_{fs} = 5.0 \text{ s} \), having roughly a quarter the maximum amplitude in the gain (one-half the gain in decibels, dB) to that of the experimental data corresponding to the lowest magnitude of compressibility, \( t_{fs} = 0.26 \text{ s} \). The agreement between the experimental data and the compressible equivalent single-tube model predictions are seen to be exceptionally good over the entire three-order-of-magnitude range of frequencies.

As a further means of model verification, the compressible equivalent single-tube model was used to design a test for a single-tube condenser, which theoretically would yield the exact same frequency-response characteristics as that of the two-tube condenser, even though the refrigerant flowrate in each of the two tubes was different from what it was for the single-tube condenser. The results of the designed experiments are superimposed in Fig. 2. The equivalent single-tube model predicts that the frequency-response characteristics of symmetric or asymmetric multitube systems are identical to that of a single-tube system as long as the equivalent condensing system time constant \( t_{cs} \) and the compressible flow system time constant \( t_{fs} \) for the multitube system are the same as \( t_{cs} \) and \( t_{fs} \) for a single-tube system. As can be seen from the figure, the measured frequency-response characteristics were virtually identical for both the single- and the two-tube condensers, as the equivalent single-tube model predicted.

Influence of Thermal and Flow Distribution Asymmetry Thermally and hydrodynamically asymmetric experimental data are depicted in Fig. 3 for a two-tube condenser for the case where there is considerable compressibility. One set of data depicts the condition where thermal and flow distribution symmetry was present (\( \beta = 1.0, \quad \gamma = 0.5 \)), the other set of experimental data possessed significant thermal and flow distribution asymmetry be-
An Experimental Study of Electrohydrodynamic Induction Pumping of a Stratified Liquid/Vapor Medium

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Electrohydrodynamic induction pumping of a stratified liquid/vapor medium is quantitatively assessed utilizing Laser Doppler Anemometry. Data are presented suggesting that pumping is due to both interfacial and bulk effects. Values of turbulence intensity associated with this type of flow are briefly discussed for the various cases studied. [S0022-1481(00)00401-1]

Keywords: Electric Fields, Heat Transfer, Pumps, Stratified, Two-Phase

Introduction

Electrohydrodynamic induction pumping is based on charges induced in the fluid and delayed at a gradient or discontinuity of the electric conductivity. A traveling electric wave then attracts or repels these induced charges, leading to fluid motion. Electrohydrodynamic pumps are generally lightweight, produce no vibrations, require little to no maintenance, are easily controllable by adjusting the applied voltage, and have low power consumption. They are also useful for the enhancement of heat transfer, as an increase in mass transport often translates to an augmentation of the heat transfer.

Melcher [1] provided the first theoretical model of electrohydrodynamic induction pumping due to charges at a liquid/air interface. He then presented an improved version of his theoretical model which also described the pumping of a liquid/liquid interface ([2]). This theory was recently examined in more detail by Wawzyniak and Seyed-Yagoobi [3,4]. The above theoretical models are built around a number of simplifying assumptions leading to a linear velocity profile (Couette flow), the most significant of which are that: (1) flow is laminar, isothermal, and one-dimensional, (2) charges are induced and consequently an electric force is present only at the interface, and (3) the pressure is constant in the direction of motion. It will later be shown that these assumptions are not met and that improvements have to be made to the existing theoretical model of Wawzyniak and Seyed-Yagoobi [3,4] to accurately describe and predict the phenomena encountered in two-phase flow electrohydrodynamic induction pump.

For this experimental study, induction pumping of a stratified liquid/vapor medium is carried out. Specifically, velocity and turbulence intensity measurements inside a liquid film of HCFC-123 are conducted by means of a one-dimensional LDA system as functions of location, frequency, and voltage.

Conclusions

It seems appropriate to emphasize the significance of the degree of agreement between all of the single- and the two-tube experimental data presented, and the predictive capability of the compressible equivalent single-tube model. The experimental data represent both single- and two-tube condensers, with different flowrates, heat fluxes, having a wide range of compressibility effects, as well as significant thermal and flow distribution asymmetries. The equivalent single-tube model is seen to predict the effects of all of these different system characteristics very well, over a three order of magnitude range of frequencies.

The experimental data not only confirm the predictive capability of the equivalent single-tube model, they demonstrate its accuracy and its wide range of application. This confirming experimental data also establishes the applicable frequency range of the dynamic viability of the system mean void fraction model, which is an integral part of the equivalent single-tube model. The true value of the equivalent single-tube model can only be appreciated when consideration is given to the complexity of the numerous physical mechanisms involved, and the remarkable accuracy of such a relatively simple model; a model which can be solved, and graphically demonstrated, on typical ‘‘spread-sheet’’ software.

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References


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Experimental Setup and Procedure

The details of the experimental setup and procedure are given in Wawzycki [5]. The AC power supply utilized in this study is capable of generating sine, square, and triangle waves at voltages of 0–12 kV (zero to peak) and frequencies of 0–13 Hz. The pumping channel with a rectangular cross-section width of 50 mm and a height of 36 mm is milled directly into a single piece of PVC (see Fig. 1). Two straight sections measuring 390 mm in length are connected by half-round sections with a centerline radius of 150 mm. The pump is equipped with three view ports along one of the straight pumping sections. These view ports are located at 37.5, 195.0, and 352.5 mm from the start of the straight section, respectively. Each view port measures 45 mm in width by 25 mm in height and extends from the bottom of the pumping channel up. The pumping channel is covered with a plate made from high-density polyethylene, which is equipped with ports for a pressure transducer, a vacuum gauge, and two thermocouple probes. The pumping channel and support lines are initially evacuated to a vacuum better than 250 \( \mu \text{m} \) of mercury. The apparatus is then charged with refrigerant HCFC-123.

Two integrated electrode boards are mounted into the bottoms of the two straight sections of the induction pump. Each board, a schematic of which is shown in Fig. 1, measures 50 mm in width, 390 mm in length, and 1.5 mm in thickness. The board laminate is made from the epoxy resin FR-4, a material commonly used for printed circuit boards, while the electric lines are tin covered copper. The top of each board features 39 electrodes and the bottom holds the three bus lines. The electrodes are 1 mm wide and extend across the entire width of the board, and subsequent electrodes are spaced 10 mm apart (center to center). To completely eliminate the possibility of direct charge injection ([5]), a coating of epoxy 0.8 mm in thickness was applied to the top surface of both electrode plates. This material was compatible with the working fluid. The three bus lines running along the length of the board are also 1 mm in width, but they are located 17 mm from each other (center to center) with the second bus line placed in the middle of the board. The first, second, and third bus lines are connected to electrodes 1, 4, 7, ... 2, 5, 8, ... and 3, 6, 9, ..., respectively, by means of through-plated holes. The bus lines in turn are linked to the high-voltage power supply.

The one-dimensional LDA system measures the main velocity component and turbulence intensity in the \( x \)-direction (see Fig. 1 for the definition of \( x \), \( y \), and \( z \)-directions). Light from an argon-ion laser is passed through a prism, with the green color component (\( \lambda = 514 \text{ nm} \)), being used for the measurement. The optical train consists of a polarization unit, a Bragg cell, a \( 3.75 \times 3 \) beam expander, and a lens with a focal length of 450 mm. Light reflected from particles in the flow is collected in an on-axis electrode plate (electrodes on top side, bus lines on underside)

![Fig. 1 Schematic of electrohydrodynamic induction pump and electrode plate](image-url)