An experimental investigation into forced, natural and combined forced and natural convective heat transfer from stationary isothermal circular disks

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Abstract—Experimental heat transfer data are presented and dimensionless correlations developed for forced, natural and combined forced and natural convection for biased stationary isothermal circular disks over wide ranges of the Reynolds, Rayleigh and modified Reynolds numbers, respectively. Experiments with air were performed for a variety of disks ranging in diameter and thickness-to-diameter aspect ratios. The correlation for combined forced and natural convection was developed utilizing the concept of a modified Reynolds number which accounts for a buoyancy-induced velocity. Utilizing this concept, the experimental data and respective empirical correlations for all three convection modes can be collapsed and plotted on the same continuous curve.

INTRODUCTION

Considerable empirical data exist in the literature for forced convection heat transfer involving external flow over a variety of geometries, and for various ranges of Reynolds number. Many current heat transfer textbooks [1–5] present empirical correlations for forced external flow over a flat plate, a sphere, cylinders, and flat tubes. However, for rotating or translating flat plates and circular cylinders, only empirical correlations exist for natural convection heat transfer from geometries such as horizontal and vertical cylinders. Spheres, spheres, flat plates, horizontal and vertical cylinders, and inclined channels, rotating geometries as well as geometries within enclosures, and over a wide range of the Rayleigh number. In the area of combined forced and natural convection, it appears that most of the attention has been focused on vertical and horizontal flat plates and cylinders. A geometry that seems to be missing from all of these lists is that of a thin circular disk. The disk-type geometry is relevant in the cooling of electronic components, such as disk-shaped resistors and power transistors, and the use of disk-type thermostats for temperature and air flow measurements.

Some experimental and theoretical studies have been carried out for natural convection from horizontal and inclined-disk surfaces, however only limited experimental data exist in the available literature for forced [7] or natural [8] convective heat transfer for circular disks, and there appears to be no existing data for this geometry under conditions of combined forced and natural convection.

There have been some experimental research [9–11] and theoretical studies [12, 13] devoted to natural convection heat transfer from stationary and rotating horizontal circular disk surfaces. Husain and Hollenback [3] performed experiments measuring the natural convection heat transfer from a circular disk in both vertical and horizontal configurations, and proposed a characteristic length such that the experimental data obtained could be collapsed with certain other shapes for a limited range of the Rayleigh number, the goal being a type of 'universal correlation'. With the exception of this 'universal correlation', we have to acknowledge the importance of the conditions. From the work of Sparrrow and Asrar [14], Sparrow and Stentox [15] and Lienhard [16] as well as Husain and Hollenback [3].

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1 See, for example, [6] for a discussion of the implications of buoyancy-induced flows as they pertain to applications in technology.

2 The term circular disk in these references is for a single circular disk, not a disk consisting of a number of circular disks.

3 The term circular disk in these references is for a single circular disk, not a disk consisting of a number of circular disks.

4 The term vertical will refer to the configuration such that the direction of the buoyancy force is perpendicular to the axis of the disk, and thus parallel to the two flat sides.

5 Other investigators have arrived, with limited success, to obtain a characteristic length such that experimental data for a variety of shapes could be collapsed to a single curve. The interested reader should refer to the works of Sparrow and Asrar [14], Sparrow and Stentox [15] and Lienhard [16] as well as Husain and Hollenback [3].
NOMENCLATURE

\[ N_a = \frac{C_d \mu \epsilon}{Pr \alpha_\epsilon} \]

This is one of the objectives of the current research. The mode of convection which is most dominated by pure forced nor pure natural convection, but is rather a compromise of these, is appropriately referred to as combined, or mixed, forced and natural convection. In such a situation, the relative direction of the buoyancy force and the externally forced flow is important. In the case where the external forced flow is in the same direction as that of the buoyancy force, the mode of normal energy transport is termed assisting (or aiding) combined convection. Similarly, in the case where the forced flow is in a direction directly opposite that of the buoyancy force, the mode of energy transport is termed opposing combined convection. A third, less common situation occurs when the directions of the forced fluid flow and the buoyancy force are perpendicular to one another, in which case the mode of thermal energy transport is termed traverse combined convection. It is generally accepted that the primary dimensionless parameter influencing combined forced and natural convection phenomena is the Richardson number, \( R_i \), which comes about directly from the dimensionless form of the Navier–Stokes equation [17] and is defined as

\[ R_i = \frac{Gr_i}{Re} \]

Most current heat transfer texts [1–5, 18] note the Richardson number as the primary parameter. However, although this parameter is widely accepted, there is still some question in the literature as to whether it represents the best parameter. For example, Churchill [19] proposed the use of \( R_{ij}/Pr^{1/3} \) as to not confine the work to a single fluid. Aizens [20] noted that the Richardson number is the primary parameter for fluids whose Prandtl number, \( Pr \), is less than unity, and \( R_{ij}/Pr^{1/3} \) is the secondary parameter for fluids whose Prandtl number is much greater than unity. Wilks [21] suggested that the parameter \( R_{ij}/Pr^{1/3} \) may be used with some degree of confidence for fluids with Prandtl numbers as low as 0.4. This same parameter appears in research by Tsuchido and Igauchi [22].

There appears to have been relatively little research done on combined, forced and natural convection as compared with either pure forced or pure natural convection, a good summary of the existing literature being given by Churchill [19] and more recently by Gehart et al. [4]. A common coupling rule [4, 5] for combined forced and natural convection is an addition of correlations for forced and natural convection, each correlation raised to the same power, i.e.
\[ \text{Nu} = \text{Nu}_f \pm \text{Nu}_g \]  

(3)

where the subscripts \( f \) and \( g \) refer to pure forced and pure natural convection respectively, and the magnitude, \( \alpha \), is determined from experimental data and varies from one geometry to the next. The addition of the forced and natural terms in equation (3) is used in the case of assisting flow, while a difference of the terms is used in opposing flow. As will be discussed later, other terms for combined forced and natural convection include replacing the buoyancy force by a pseudo-velocity \( \alpha_u \) and, more notably, by utilizing a modified Reynolds number, \( Re_\alpha \), which incorporates a characteristic buoyancy-induced velocity along with the free-stream velocity. The resulting modified Reynolds number is of the form

\[ Re_\alpha = Re + \alpha u \sqrt{Gr/(4)} \]  

(4)

However, there is a lack of consensus in the available literature on the value of the weighting factor, \( \alpha \).

Although the concept of a modified Reynolds number is less common than the combination rule of equation (3), it will be shown later that the modified Reynolds number is very effective in the current research for developing an empirical correlation for combined forced and natural convection heat transfer.

Although there have been a limited number of experimental and theoretical studies of \( \alpha \) of simple external geometries for combined forced and natural convection, there have not been to the best knowledge of the authors, any published studies for circular disks. Most of the experimental studies for combined forced and natural convection for external flows have been performed on long cylinders and flat plates. However, even for such a familiar geometry as a vertical flat plate, only limited experimental data exist for assisting flow, and even fewer for opposing flow. The experimental data available, such as that presented by Kienle [25], Gryzavitsid introduced the following equation for local heat transfer, for the data being obtained by measuring, usually with an interferometer, local temperature gradients. In addition, Oosthuize and Bassey [30] presented experimental data for the average heat transfer for a vertical flat plate under conditions of both assisting and opposing flow.

**EXPERIMENTAL APPARATUS AND MEASUREMENT TECHNIQUES**

The circular disks that were used as heat transfer models for the experimental data presented in this paper were commercially available disk-type thermometers. Six different circular disk models were tested, ranging in diameter, \( d \), from 2.1 to 19.9 mm and in thickness-to-diameter aspect ratios, \( t/d \), between 0.05 and 0.2. Thermometers were chosen as the heat transfer models because they provided a unique combination for indirectly measuring the surface temperature and the convective heat transfer rate [7, 11].

The thermocouple was shielded by means of Joule heating. Conduction flows through the thermocouple lead wires (1.07 \( \times \) 10 mm diameter) were minimized (less than 1, 0 and 1% for forced, combined forced and natural, and natural convection, respectively) by using constantan wire, which has a low enough thermal conductivity to minimize the 'finite effect' and an electrical resistivity low enough to minimize Joule heating. Also, a one-dimensional analysis was done on the lead wires modeling the 'finite effect' and taking into account the possibility of Joule heating; the result of this analysis indicated that the existence of any Joule heating acted to decrease the conduction losses through the lead wires.

Using an electrical circuit suggested by Wedekind [7], as shown in Fig. 1, the thermometer resistance and resistances also heated simultaneously measured during self-heating. This makes it possible to measure indirectly not only the convective heat transfer rate, but also the average temperature of the thermometer, the latter by having pre-calibrated the resistance-temperature characteristics of each thermometer heat transfer model. Thermometers have a high resistance coefficient, therefore the heat transfer surface temperature, \( T \), could be indirectly measured quite accurately without the many difficulties encountered in attempting to measure the surface temperature especially on small heat transfer models by conventional means.\(^*\)
A schematic of the experimental apparatus, which amounts to a miniature wind tunnel made possible by the small size of the thermistor heat transfer models, is shown in Fig. 2, and is the same as that described by Wedekind [7], except that it is in the vertical position. Since the inclination angle of the apparatus is not relevant when the pure forced convection experiment was performed, the forced convection experiment was done with the apparatus in the horizontal position [7]; however, the apparatus was in the vertical position for both natural convection and combined forced and natural convection. Referring to Fig. 2, the diffuser section was replaced by a plug when natural convection experiments were performed to avoid the 'chimney effect'.

The inside diameter of the test section where the thermistor disks were mounted was 3.25 cm and the length of the velocity development section was 26.5 cm. Uniformity of velocity upstream of the heat transfer model was within 8% measured by a slot-wire traverse. The air velocity was varied by controlling the inlet air flow rate. The free-stream air temperature was measured with a thermocouple probe and a variable d.c. power supply was used as the current source to heat the thermistor. Digital multimeters were used to measure simultaneously the voltage drop across the thermistor and standard resistor of known value, which, as shown in Fig. 1, was connected in series with the thermistor.

Experimental uncertainty in the Prandtl number is assumed to be negligible since it is primarily a function of air temperature, which was accurate to ±0.5°C. Maximum experimental uncertainty in the Reynolds number was 5.7%, due primarily to uncertainty in the thermistor temperature, T. The thermistor temperature, T, can be readily determined by rearranging equation (7) and making appropriate substitutions for the thermistor resistance, R; thus

\[ T = \frac{\ln \left( V_0/V \right)}{(R T_c)} \] (8)

A constant temperature may be maintained during a test by adjusting the power supply voltage such that the thermistor-standard resistor voltage ratio, \( V/V_0 \), remains constant. The thermocouple used to measure the temperature was a type E, Wire gauge 28, in series with the thermistor and standard resistor of known value. A lead wire was then used to connect the wires to the bridge circuit. The bridge circuit was used to measure the change in voltage drop across the thermistor and standard resistor.

EXPERIMENTAL DATA AND RESULTS

As was mentioned earlier, six different disk heat transfer models were tested, the diameters ranging from 3.21 to 19.99 mm, and a thickness to diameter aspect ratio, \( \alpha \), of 0.002 to 0.02. The edge of the disk were relatively sharp (edge radius 0.04 mm). For the full range of measurements taken for forced natural, and combined forced and natural convection, air velocities, \( V \), were determined from the mass flow rate and thus (indirectly) the average velocity. Also, a热线 anemometer was used to measure the velocity distribution in the test section, the non-uniformity of which was less than 10% at all times.
**Forced convection**

The experimental results for forced convection are depicted in dimensionless form in Fig. 3, where $N_{\text{Nu}}/Pr^{1/3}$ is plotted as a function of Reynolds number, $Re_c$. Experiments yielding these data have been repeated many times, with excellent repeatability. The
experimental apparatus and heat transfer models, not necessarily the limit of the existing correlation.

**Natural convection**

The experimental results for natural convection are depicted in dimensionless form in Fig. 4 where $N_a$ is plotted as a function of Rayleigh number, $R_a = \frac{Pr Gr}{Pr + Gr}$. An empirical correlation which fits all of the data is given by

$$N_a = C(Pr Gr)^{\frac{1}{3}}, \quad \text{C}_1 = 1.759, \quad a = 0.150$$

$$10^3 < Pr Gr < 10^7$$

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It should be pointed out that these data are only valid at the initial stages of the Rayleigh instability, and the limit of the existing correlation is not necessarily the limit of the existing correlation. For the existing range of Rayleigh number, $R_a = \frac{Pr Gr}{Pr + Gr}$. An empirical correlation which fits all of the data is given by

$$N_a = C(Pr Gr)^{\frac{1}{3}}, \quad \text{C}_1 = 1.759, \quad a = 0.150$$

$$10^3 < Pr Gr < 10^7$$

1. It should be pointed out that there is a small difference between the current correlation and that reported by Wede- kind [7]. Wedekind reported $C = 0.931$ and $a = 0.294$. Although the exponents are virtually identical, the constant in front is not. This difference was traced to a system temperature measurement error in calibrating the thermocouple leads.

2. The experimental data of Hassani and Hollands [8] were represented by the Nusselt number based on the inner root of the area and the Rayleigh number based on characteristic length, $L$, which is a function of the height and the average depth of the disk. In this way, the research of Hassani and Hollands attempted to collapse the disk data onto a single curve with a variety of other geometries. From knowledge of the disk geometry, the experimental data were transformed to where the disk diameter, $D$, was the characteristic length.

3. The under-prediction of the correlation could be attributed to the influence of the developing boundary layer; however, the Grahame numbers obtained by Hassani and Hollands [9] are below that necessary for a transition to turbulence, as pointed out by the investigation of Vukovic and Lauterborn [23] for vertical surfaces. It is suggested that experiments with 2.5% turbulence present, typical natural convection data for other geometry, such as flat plates and disks (1) are not linear on a logarithmic plot over the entire range of the Rayleigh number.
convection becomes more dominant, asymptotically approaching pure forced convection.

Aside from the combining rule of equation (3), only a few other methods of dealing with combined forced and natural convection have received significant attention. One such method was the proposal of using a pseudo-velocity function to replace the buoyancy term in the governing differential equation [23, 24]. Although not specifically mentioned in the above referenced research, a pseudo-velocity gives rise to a modified Reynolds number which accounts for a buoyancy-induced velocity as well as the imposed free-stream velocity. Churchill [19] reviewed prior research, some of which postulated that an empirical correlation may be obtained using a modified Reynolds number, $Re_p$, which incorporates some characteristic buoyancy-induced velocity added vectorially to the imposed free-stream velocity. The concept of a modified Reynolds number also appears earlier in research presented by Lemlich and Rose [24] and Hasting et al. [35], who investigated combined forced and natural convective heat transfer for horizontal cylinders.

For the present research, which is for assisting flow,
The form of the modified Reynolds number, \( R_{\text{ef}} \) utilizing a buoyancy-induced velocity averaged over the vertical centerline of the circular disk, \( a \) is
\[
R_{\text{ef}} = Re_{\alpha} - iG_R \alpha^2 \quad (11)
\]
This form of the modified Reynolds number appears in several earlier investigations \([19]\). However, the particular choice of the characteristic buoyancy-induced velocity, which affects the weighting factor, \( \alpha \), is somewhat unclear. It has been proposed that the local maximum velocity in the pure natural convection boundary layer be chosen as the characteristic velocity, as pointed out by a number of investigators \([36]\). Churchill \([18]\) pointed to a number of researchers who took the weighting factor, \( \alpha \), to be unity for geometries such as flat plates, cylinders and spheres. In an effort to obtain the appropriate weighting factor, the authors chose to use the maximum local velocity in the natural convection boundary layer averaged over the vertical centerline of the flat side of the disk \([5]\). Utilizing the integral solution to laminar natural convection, boundary layer theory for a vertical flat plate \([1, 4]\), the averaged maximum velocity in the natural convection boundary layer is
\[
\bar{u}_{\text{max}} = \frac{1}{L} \int_{y=0}^{L} u_{\text{max}}(y) \, dy
\]
\[
= 0.51 \left( \frac{20}{21} \right) + Pr_a^{\frac{1}{3}} \left( \frac{Gr_a}{\rho_d} \right)^{\frac{2}{3}} \quad (12)
\]
where
\[
\bar{u}_{\text{max}}(y) = 5.17 \left( \frac{20}{21} \right) + Pr_a^{\frac{1}{3}} \left( \frac{Gr_a}{\rho_d} \right)^{\frac{2}{3}}
\]
Thus, assuming uniform properties in the non-stratified air, the modified Reynolds number is expressed as
\[
R_{\text{ef}} = \frac{\rho_{\alpha} u_{\text{max}} L \bar{u}}{\mu}
\]
\[
= Re_{\alpha} + 0.723 \left( \frac{20}{21} \right) + Pr_a^{\frac{1}{3}} \left( \frac{Gr_a}{2} \right)^{\frac{2}{3}} \quad (13)
\]
The weighting factor, \( \alpha \), is 0.558 for air at 21°C. The experimental data for combined forced and natural convection were plotted in a form similar to this for forced convection data, except that the data are plotted against the modified Reynolds number.
rather than the standard Reynolds number. Referring to Fig. 7, the result is seen to be very linear with excellent data collapse. The empirical correlation obtained for combined forced and natural convection was

\[ N_u = C_1 \frac{Re^{1.0} Pr^{0.84}}{Pr} \]

\[ C_1 = 1.570 \text{ for } 0.008 \leq Pr \leq 2100. \] (14)

The correlation coefficient of the correlation is 0.983.

**SUMMARY AND CONCLUSIONS**

Experimental heat transfer data have been presented and dimensionless correlations proposed for forced, natural, and (assisted) combined forced and natural convection from heated stationary isothermal circular disks over wide ranges of the Reynolds number, Rayleigh number and a modified Reynolds number, respectively. In the case of combined forced and natural convection, the modified Reynolds number, Ref, was utilized which incorporates buoyancy-induced characteristic velocity obtained from natural convection boundary layer theory which includes a weighting factor, a, for the natural convection contribution. The modified Reynolds number is set to reduce to the appropriate asymptotes of pure natural and pure forced convection.

An advantage to using the modified Reynolds number is that experimental data for forced, natural and combined forced and natural convection may be represented on the same graph. Referring to Fig. 8, the same dimensionless transfer data are presented as in Figs. 3, 4 and 7 for forced, natural and combined forced and natural convective heat transfer, respectively, except that the data are plotted against the modified Reynolds number, Ref, defined by equation (11), with a = 0.238. Along with the experimental data, the previously developed empirical correlations for the three domains forced, natural, and combined forced and natural convection are superimposed. It should be noted that the forced convection data and the corresponding empirical correlation, are virtually identical to those presented in Fig. 3 with the standard Reynolds number, because the natural convection contribution to the modified Reynolds number is negligible (less than 3% even at the low end of the forced convection data, where Ref = 2100). Similarly, the standard Reynolds number in equation (11) is zero for pure natural convection and thus the modified Reynolds number is a function only of the Grashof number, as expressed by equation (10), where the characteristic velocity is buoyancy-induced.

The pure natural convection correlation, equation (10), tend to be slightly rearranged to be plotted against the modified Reynolds number. Thus:

\[ N_u = \frac{C_1^{Pr^{-1.0}} (Pr Gr)^{0.84}}{Pr} \]

\[ C_1 = 1.570 \text{ for } 0.5 \leq Pr \leq 2100. \] (15)

The differences in the trends of the data for the three different domains forced, natural, and combined forced and natural convection seem clear especially when the corresponding empirical correlations are superimposed. As can be seen from Fig. 8, a criterion may
be established between the different domains by equating the correlation for combined forced and natural convection with the correlation for pure forced and pure natural convection, respectively. Using this approach, it is seen that forced convection is dominant when \( \text{Re}_f > 2100 \), natural convection is dominant when \( \text{Re}_f < 70 \) and combined forced natural convection must be considered in between the two limits when \( 70 < \text{Re}_f < 2100 \).

It should be pointed out that the established criterion was developed solely from equating the correlations as described above, and thus it was claimed to be a precise distinction of the three convection modes in any absolute sense. Referring again to Fig. 8, there is clearly some overlap in the current data between the natural and combined and between the combined and forced convection data. Also, when the pure natural convection data of Hassan and Hollands [8] were superimposed, the data were plotted right on top of the current data and overlapped most of what has been labeled as combined convection. The point being made here is that the boundaries between pure natural and combined convection, and between combined and pure forced convection, are somewhat 'blurred', the former apparently more than the latter.

\[ \text{Nu} = C \text{Pr}^{1/3} \text{Re}_f^{5/3} \]

(14)

<table>
<thead>
<tr>
<th>Modified Reynolds number, ( \text{Re}_f )</th>
<th>C</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 - 70</td>
<td>3.469</td>
<td>0.300</td>
</tr>
<tr>
<td>70 - 2100</td>
<td>1.570</td>
<td>0.408</td>
</tr>
<tr>
<td>2100 - 20000</td>
<td>0.356</td>
<td>0.600</td>
</tr>
</tbody>
</table>

However, the value of representing the data in terms of the modified Reynolds number is that, as is apparent in Fig. 8, the correct convective heat transfer can be predicted regardless of the distinct convection mode by utilizing Table 1 with the general correlation

Since only air was tested, the convective heat transfer coefficients are recommended for Prandtl numbers near unity, which includes most common gases. The correlations may be valid for Prandtl numbers outside this range, however this is not known at this time since no experimental data are available. This is the subject of ongoing research.

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